



TABLES  
OF  
THE INCOMPLETE  
BETA-FUNCTION

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## PREFACE

Tables of the Incomplete B-Function have seemed to me essential to the modern theory of statistics ever since I personally learnt, about the year 1894, to appreciate two facts, namely how closely the sum of  $n$  terms of a hypergeometrical series could be represented by the partial area of the curve

$$y = y_0 x^{p-1} (1-x)^{q-1},$$

and secondly how imperfect was Laplace's endeavour to represent such areas by a series based on the normal curve and its differential coefficients. Various methods were given in my lectures on statistics for evaluating the integral

$$B_x(p, q) = \int_0^x x^{p-1} (1-x)^{q-1} dx,$$

and were used in the Biometric Laboratory for many years. In 1921, I asked Mr Herbert E. Soper then a research assistant in that Laboratory to put together various possible methods for evaluating the Incomplete B-Function, and the results of his investigations were published in the Cambridge University Press *Tracts for Computers*, as No. VII. That *Tract* is an essential companion to the present volume, and will be of service to any one seeking values of the function outside the range of arguments in these tables. But the labour required to apply some of the methods of that *Tract*, and the relatively small degree of accuracy provided by others, only emphasised in my mind the already appreciated need for computing tables which would cover some of the field. Accordingly, when the *Tables of the Incomplete Γ-Function*\* had been finished and their publication rendered possible by a contribution from the Department of Industrial and Scientific Research, an application was made to the same Department for help in computing tables of the Incomplete B-Function. This was a still more serious undertaking, owing both to the extent of the computing work necessary—it being a table of triple, not double entry—and to the difficulty of eventually finding means for the publication of such a voluminous work as this promised to be. The Department of Scientific and Industrial Research again came to my aid, at first by granting payment for a definite research assistant for this work, and afterwards by a definite grant for the completion of the work of computing, which extended from 1923–1932. In supervision and proof-reading the aid of members of the Department of Applied Statistics at University College, London, has been frequently drawn upon and readily granted.

The present condition of our national finances did not justify the publication of this sister volume to the *Tables of the Incomplete Γ-Function* in the manner previously adopted, and it seemed for a time as if the printing of the manuscript must be indefinitely delayed. Arrangements have finally been made by which these tables appear as one of the *Biometrika* publications. As only a small edition can be issued the price must necessarily be heavy, but purchasers may be assured that the work is sold without profit, merely at cost price.

I have to thank most cordially Dr Ethel Elderton, Dr Brenda Stoessiger and Mr E. C. Fieller, for the heavy labour of proof-reading of the tables themselves; Dr Egon S. Pearson for much aid in the preparation of the Introduction, Mr E. C. Fieller for computing help therein, and Mr Walter Lewis and the Compositors and Readers of the University Press, Cambridge, for the rapidity and accuracy with which the work has been set up. Such errors as may be found must be due to false copying of figures by the computers on the original working sheets, as the latter have been compared throughout with the text of the tables.

I cannot hope that the work is wholly free from computing errors, and shall be very grateful for any such being pointed out to me, so that eventually a list of errata may be issued.

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## TABLE I

Incomplete B-Function Ratio . . . . .	pp. 1-431
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## TABLE II

Constants of the Curve $y = y_0 x^{p-1} (1-x)^{q-1}$ for various values of $p$ and $q$ . . . . .	433-494
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# COMPUTERS UNDER THE GRANT AND COLLABORATORS

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# INTRODUCTION

## (I) ORIGIN OF THE TABLES AND METHODS ADOPTED FOR COMPUTING THEM

The somewhat exaggerated use made by Laplace of the normal curve

$$y = y_0 e^{-\frac{1}{2} \frac{x^2}{\sigma^2}}$$

to represent almost any function

$$y = f(x)$$

for very considerable distances from its mode, in particular the function

$$y = y_0 x^{p-1} (1-x)^{q-1},$$

led me many years ago to seek by Laplace's own methods for expansions of unimodal functions in the form

$$f(x) = y_0 e^{-\frac{1}{2} \frac{x^2}{\sigma^2}} \times \text{a polynomial in } x,$$

where  $x$  is measured from the mode. Thus the partial area of  $f(x)$ , or what we may term the probability integral of the function, was expressed in what we should now call an incomplete normal moment series, or in another form a tetrachoric function series. But actual experience with the probability integrals of the curves

$$y = y_0 e^{-x} x^{p-1} \quad \text{and} \quad y = y_0 x^{p-1} (1-x)^{q-1}$$

obtained by such series was extremely unsatisfactory, and I was compelled to discard them, and to face the problem of the tabulation of the incomplete  $\Gamma$ - and  $B$ -functions. The work of computing the Incomplete  $\Gamma$ -function was first taken in hand, and the difficulties of the problem soon developed themselves; chief among these was the infinite range of  $x$ , which demanded as the power,  $p-1$ , of  $x$  increased in the function either a change in argument intervals, or what amounts to the same thing the expression of  $x$  in terms of the changing standard deviation. The latter course was chosen and after eight years of work the *Tables of the Incomplete  $\Gamma$ -Function* were published by H.M. Stationery Office in 1922.

In the case of the incomplete  $B$ -function the same problem arose, but in a less aggravated form, because the range of  $x$  is finite. It could have been met in the same manner by expressing the variate  $x$  in terms of the changing standard deviation of the curve instead of in terms of the range. But the variety of cases to which the tabled function can be applied—either directly or by transformation—raised new difficulties. In the case of either or both  $p$  and  $q$  being less than unity, the standard deviation,  $\sigma$ , of the curve

$$y = y_0 x^{p-1} (1-x)^{q-1}$$

given by

$$\sigma^2 = \frac{pq}{(p+q)^2(p+q+1)}$$

was not found to be wholly the best unit for the measurement of  $x$ , while in the case of transformed curves, the above expression is of course not their standard deviation. It was settled therefore to use the range, not the standard deviation, and, as increment of the argument  $x$ , to take  $\frac{1}{100}$ th part of the range.

To lessen the labour of computing the trivariate function, I avoided, except for testing purposes, quadrature and decided that a recurrence formula should be made the basis of the work. This required only the computing, which was easy, of the areas of the curve for the initial low values of  $p$  and  $q$ . The function I proposed to have tabled was to be a *probability integral*; that is to say, if we represent by  $B(p, q)$  the complete  $B$ -function,  $= \int_0^1 x^{p-1} (1-x)^{q-1} dx$ , and by  $B_x(p, q)$  the incomplete  $B$ -function, or  $\int_0^x x^{p-1} (1-x)^{q-1} dx$ , we tabled the ratio

$$I_x(p, q) = B_x(p, q) / B(p, q) \quad \dots\dots(i).$$

The recurrence formula for  $I_x(p, q)$  is the following:

$$I_x(p, q) = x I_x(p-1, q) + (1-x) I_x(p, q-1) \quad \dots\dots(ii).$$

By aid of this formula  $I_x(p, q)$  could be ultimately deduced from values of the function easy to integrate

out\*. In order to test the correctness of the results in any column of this  $I_x(p, q)$  function for a given  $p$  and  $q$  an Euler-Maclaurin summation of the column was provided and was found very useful as a check. It runs†

Sum of column contents

$$= \frac{100q}{p+q} - 0.5 + \frac{1}{B(p, q)} \left[ \frac{1}{12} (\cdot 01) (x^{p-1} (1-x)^{q-1}) - \frac{(\cdot 01)^3}{720} \frac{d^2}{dx^2} (x^{p-1} (1-x)^{q-1}) \right. \\ \left. + \frac{(\cdot 01)^5}{30240} \frac{d^4}{dx^4} (x^{p-1} (1-x)^{q-1}) - \frac{(\cdot 01)^7}{120,9600} \frac{d^6}{dx^6} (x^{p-1} (1-x)^{q-1}) + \dots \right]_0^1 \dots\dots (iii).$$

At the head of each column of the table is given the value of the corresponding complete B-function,  $B(p, q)$ , so that it is possible to obtain rapidly, when it is required, the incomplete B-function itself, instead of the ratio.

In my original plan I proposed to take the argument intervals of  $p$  and  $q$  to be 0.5 from 10 to 50, and when either  $p$  or  $q$  were less than 10 to be 0.1, so that from 0 to 10 both  $p$  and  $q$  would proceed by 0.1. Here also  $x$  was to advance by .005 instead of .01 and some portion of this was actually worked out. Further, to save labour in the use of the tables  $p$  and  $q$  were *both* to run from 0 to 50. But on reckoning out the space the printed tables would take, I found that it would extend to considerably over 2000 pages. The publication of such a table was wholly beyond any funds likely to be at my disposal, and accordingly the table had to be ruthlessly cut down. In the first place I discarded the idea of providing a table containing all the values of *both*  $p$  and  $q$  up to the limit of 50. I have had printed only the values of  $p$  which are equal to or greater than the values of  $q$ . If the user of the tables requires  $I_x(p, q)$  in which  $p$  is less than  $q$  then he must remember that

$$I_x(p, q) = 1 - I_{1-x}(q, p) = 1 - I_{1-x}(p', q'),$$

where  $p' = q$  and  $q' = p$ , so that  $p'$  is now greater than  $q'$ . This reduced the amount to be printed by almost a half.

In the next place the idea of publishing any differences whatever was dropped. It would have been needful to print three sets of differences, and any reasonable number of these would have been quite inadequate at certain parts of the table. When either  $q$  or  $p$  are low and fractional the differential coefficients of the curve at one or other terminal become infinite, and the differences may diverge. The only method of overcoming this difficulty is by the aid of auxiliary tables‡, but that is not feasible when it is important to reduce the matter to be printed. Owing to the large number of differences required at some parts of the table, and to their total inadequacy at other parts I was not loath to omit them. As a matter of fact for many purposes we only need  $p$  and  $q$  to whole or half integers, and accordingly the interpolation requisite will often be with regard to  $x$  alone.

In my opinion far more serious retrenchments were the following:

(a) The adoption throughout of .01 for the increment of  $x$ . When  $p$  and  $q$  approach 50, the standard deviation of the curve is about  $\frac{1}{\sqrt{q}}$ th of the range and 99.9 % of the curve's area falls on less than a third of the range. It would accordingly have been more advantageous if this latter part of the table had proceeded by intervals of .005 in  $x$ , but this would have added upwards of 80 pages to the printed table. The adoption of a smaller interval in the case of  $U$ - and  $J$ -curves would also have been very advantageous.

(b) The adoption of 0.5 and, further on in the table, 1.0 for the increments of  $p$  and  $q$ . This was again enforced by the limitation of space. The restriction affects peculiarly the table as applicable to  $U$ - and  $J$ -curves. In the case of  $U$ -curves, i.e. both  $p$  and  $q$  less than unity, interpolation becomes extremely difficult, and it is doubtful whether any table would be of much service which did not proceed by increments of .01 for  $p$  and  $q$ . This would have involved an addition of some 5000 additional curves, or about 1666 pages of printed matter. Even with intervals of .02, we should have required upward of 200 additional pages. Again, an effective tabulation of  $J$ -curves with increment of  $p$  as large even as .02 and 60 values of  $q$  would have demanded space for 3000 additional curves or some 1000 additional pages. I was convinced at a very early stage of the work that the effective tabulation of  $U$ - and  $J$ -curves must be omitted from the present work, and left for others to undertake at a later date.

\* Use was made of formulae of type  $I_x(p+1, 0.5) = I_x(p, 1.5) - \frac{2\Gamma(p+\frac{1}{2}) x^p \sqrt{1-x}}{\Gamma(p+1) \sqrt{\pi}}$ , for the half-unit values of  $p$  and  $q$ .

† It seems unnecessary here to enter into special variations of this formula, such as arise from altering the limits 0 and 1. When  $p$  and  $q$  are integers, the terms in the square brackets rapidly become negligible as  $p$  and  $q$  increase.

‡ One such auxiliary table for cases  $B_x(\frac{1}{2}, p)$  is given in *Biometrika*, Vol. xxii, p. 283, and is reproduced in the *Tables for Statisticians and Biometricians*, Part II, p. 176. The method will be referred to later, when dealing with interpolation.

It may be asked why certain  $J$ - and  $U$ -curves have been included. The answer lies in the fact that  $B_x(\frac{1}{2})$  or  $I_x(\frac{1}{2}, q)$  have special importance in practical statistics. For example, all symmetrical curves of the B-function type, i.e.  $p=q$ , can have their probability integrals determined by transformation to these types of  $U$ - or  $J$ -curves. Thus

$$I_x(p, p) = \frac{1}{2} \{1 + I_{x'}(\frac{1}{2}, p)\} \\ = 1 - \frac{1}{2} I_{1-x'}(p, \frac{1}{2}) \quad \text{.....(iv)}$$

where  $x' = 4(x - \frac{1}{2})^2$ , or  $x = \frac{1}{2}(1 + \sqrt{x'})$ .

This interchange may be of some service, as interpolating for  $p$  in  $I_x(p, p)$  may involve extracting entries from several pages, while the interpolation for  $I_{1-x'}(p, \frac{1}{2})$  will probably need reference to one page only.

The function was computed to nine decimal places, but these were cut down to seven for publication. They might with but little recomputing of isolated values have been tabulated to eight decimals, but this seemed no particular advantage to be gained by incurring the additional cost of printing. The tables are intended in the first place for statisticians, and there are very few cases in statistical practice, wherein it is needful to ascertain a frequency or a probability to more than five figures. The additional two figures are given to provide greater accuracy for the purposes of interpolation. Should the reader feel that the tables fall short of the completeness desirable in dealing with such an important function, I may venture to remind him that the present is probably the first big attempt at tabling a trivariate function, that provide a table which would effectively cover all regions of the B-function would not only have required another eight years of computing, but would have more than quadrupled the volume of the work, thus preventing or indefinitely delaying its publication; and finally that on studying the following account of the uses of the tables, he may convince himself that they are capable of giving at least a great deal of information in a variety of inquiries.

## (II) USES OF THE TABLES

(a) *To find the subrange frequencies of any distribution graduated by*

$$y = y_0(x + a_1)^{p-1}(a_2 - x)^{q-1} \quad \text{.....(v)}$$

The curve may be transposed to  $x = -a_1$  as origin, then if  $b = a_1 + a_2$ , the curve may be written as

$$y = y_0' x^{p-1} (b - x)^{q-1},$$

or if  $x = bx'$  as

$$y = y_0'' x'^{p-1} (1 - x')^{q-1}.$$

Thus the units of the  $x$  in the table will correspond to  $\frac{1}{100}$ th part of the range  $b$ . If the standard deviation  $\sigma$  of the curve has been found, then

$$b = \sigma(p + q) \sqrt{\frac{p + q + 1}{pq}} \quad \text{.....(vi)}$$

The value of  $b$  will therefore be found by dividing the observed  $\sigma$  by the entry under the corresponding  $p, q$  in the fourth column of Table II, where it is headed " $\sigma$ ." That column gives the ratio of the standard deviation to the range. We do not trouble about  $y_0$  or  $y_0''$ , but simply multiply the entries under the given  $p$  and  $q$  by  $N$ , the total frequency. The frequency on the subrange  $sb/100$  to  $tb/100$ ,  $t > s$ , is given by

$$N \{I_{tb/100}(p, q) - I_{sb/100}(p, q)\}.$$

This is simple enough, if  $p$  and  $q$  are numbers < 51 actually occurring in Table I, and we wish to find the frequencies occurring on subranges, which are integer multiples of hundredths of the range. But as a rule  $p$  and  $q$  will have values for which we must interpolate and I will indicate how we may deal with such cases.

(b) *Trivariate Everett Formula to Third Differences* ( $x, p, q$ ).

The formulae for bivariate interpolation on Everett's lines have been provided in *Tracts for Computations* No. III\*, but as far as I am aware similar formulae for trivariate interpolation have not hitherto been published. I do not propose to discuss such formulae here, but to provide the most needful one. If we proceed to the terms in  $\delta^2$ 's, the bivariate mid-panel formula involves four ordinates and eight  $\delta^2$ 's. The corresponding trivariate formula involves eight ordinates and twenty-four  $\delta^2$ 's. In both cases the interpolated value is correct to the third difference if the fourth is neglected or supposed negligible.

\* Cambridge University Press.

With the bivariate formula twelve tabular values must be used, while for the trivariate thirty-two are required. Hence, while it is relatively easy to use univariate interpolation formulae proceeding up to  $\delta^4$ , and to  $\delta^6$ , and possible though laborious to use bivariate formulae up to  $\delta^4$  terms, it is for practical purposes of small use to provide trivariate formulae going as far as  $\delta^4$ . The number of terms to be dealt with becomes unmanageable. The only remedy is to ascertain what will be the extent of error we are introducing by neglecting the  $\delta^4$  terms in the part of the table dealt with. For a very considerable proportion of the present table the fourth differences only affect the seventh decimal in the interpolate, and for most statistical purposes five-decimal accuracy is ample. A point may be borne in mind here, namely, that while in a bivariate formula the  $\delta^2$  and  $\delta^4$  terms are multiplied by the product of three proper fractions and the inverse factorial, in trivariate formulae they are multiplied by the product of four proper fractions as well as the inverse factorial.

The mention of thirty-two entries being required to provide the terms up to  $\delta^2$  (actually to third differences) need not alarm the reader unnecessarily. We may remind him that

$$\delta^2 z_0 = z_{-1} + z_{+1} - 2z_0 \quad \text{.....(vii),}$$

and the  $\delta^2$  difference can be at once obtained by opening the table, adding the two adjacent values and subtracting twice the value of  $z_0$ . This is done by a continuous operation on the machine. In the case of  $\delta_x^2$  and  $\delta_y^2$  we may usually have to open at one page only or at most two. In the case of  $\delta_z^2$  we may need to turn over several pages for the required values. By aid of (vii) it is possible to replace each  $\delta^2$  by three ordinates, and thus up to and including third differences to provide a formula involving only the thirty-two tabular entries. I shall provide such formulae, but after use prefer in some cases the Everett type for our present purposes. In form it is indicative of the contribution of the successive approximations, the coefficients by their symmetry are remarkably simple, they adapt themselves easily to recalculation when we need to vary one position ratio within the same panel or cell of the table, and twenty-four of the thirty-two  $z$ -values being second differences give far less machine labour.

The following diagram indicates the notation required for the case of a trivariate Everett formula.

Diagram of values of  $Z$  to assist  
the geometrical appreciation  
of trivariate interpolation.

$Z_{\theta\phi\chi}$  = interpolate value.

$\theta_1, \theta_0, \phi_1, \phi_0, \chi_1, \chi_0$  are the argument interval ratios of  $Z_{\theta\phi\chi}$

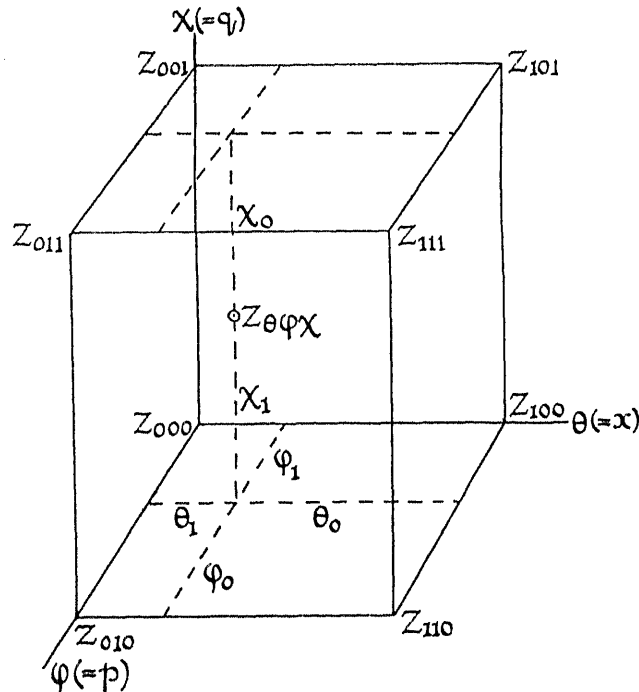


Fig. 1.

Here  $z_{stu}$  marks a tabular entry, and for the geometrical appreciation may be termed an "ordinate".  $\theta_1, \phi_1, \chi_1$  are the position ratios, or the ratios of the three argument intervals in which the ordinate  $z_{011}$  or the interpolate, divides the rectangular six-face. Thus

$$\theta_0 = 1 - \theta_1, \quad \phi_0 = 1 - \phi_1, \quad \chi_0 = 1 - \chi_1.$$

The reader must note that  $\theta_0, \phi_0$  and  $\chi_0$  are measured not from the  $z_{000}$  corner of the cell but from the  $z_{111}$  corner. With this notation the trivariate Everett formula runs thus:

$$\begin{aligned} z_{\theta\phi\chi} = & \theta_0\phi_0\chi_0 z_{000} + \theta_1\phi_0\chi_0 z_{100} + \theta_1\phi_1\chi_0 z_{110} + \theta_0\phi_1\chi_0 z_{010} + \theta_0\phi_0\chi_1 z_{001} + \theta_1\phi_0\chi_1 z_{101} + \theta_1\phi_1\chi_1 z_{111} + \theta_0\phi_1\chi_1 z_{011} \\ & - \frac{1}{6}\theta_1(1+\theta_0)\{\theta_0\phi_0\chi_0\delta_\theta^2 z_{000} + \theta_0\phi_0\chi_1\delta_\theta^2 z_{001} + \theta_0\phi_1\chi_0\delta_\theta^2 z_{010} + \theta_0\phi_1\chi_1\delta_\theta^2 z_{011}\} \\ & - \frac{1}{6}\theta_0(1+\theta_1)\{\theta_1\phi_0\chi_0\delta_\theta^2 z_{100} + \theta_1\phi_0\chi_1\delta_\theta^2 z_{101} + \theta_1\phi_1\chi_0\delta_\theta^2 z_{110} + \theta_1\phi_1\chi_1\delta_\theta^2 z_{111}\} \\ & - \frac{1}{6}\phi_1(1+\phi_0)\{\theta_0\phi_0\chi_0\delta_\phi^2 z_{000} + \theta_0\phi_0\chi_1\delta_\phi^2 z_{001} + \theta_1\phi_0\chi_0\delta_\phi^2 z_{100} + \theta_1\phi_0\chi_1\delta_\phi^2 z_{101}\} \\ & - \frac{1}{6}\phi_0(1+\phi_1)\{\theta_0\phi_1\chi_0\delta_\phi^2 z_{010} + \theta_0\phi_1\chi_1\delta_\phi^2 z_{011} + \theta_1\phi_1\chi_0\delta_\phi^2 z_{110} + \theta_1\phi_1\chi_1\delta_\phi^2 z_{111}\} \\ & - \frac{1}{6}\chi_1(1+\chi_0)\{\theta_0\phi_0\chi_0\delta_\chi^2 z_{000} + \theta_0\phi_1\chi_0\delta_\chi^2 z_{010} + \theta_1\phi_0\chi_0\delta_\chi^2 z_{100} + \theta_1\phi_1\chi_0\delta_\chi^2 z_{110}\} \\ & - \frac{1}{6}\chi_0(1+\chi_1)\{\theta_0\phi_0\chi_1\delta_\chi^2 z_{001} + \theta_0\phi_1\chi_1\delta_\chi^2 z_{011} + \theta_1\phi_0\chi_1\delta_\chi^2 z_{101} + \theta_1\phi_1\chi_1\delta_\chi^2 z_{111}\} \end{aligned} \quad \dots\dots(\text{viii}).$$

When this formula is used in this Introduction,  $\theta$  will stand for  $x$ ,  $\phi$  for  $p$  and  $\chi$  for  $q$  of the table.

It will be noticed at once that with the above notation whether we are dealing with any  $z$ , or any second central difference of any  $z$ , the subscripts of  $z$  define by their order and values the subscripts of the corresponding argument ratio product  $\theta\phi\chi$ . When we have to interpolate inversely for  $x$  to find  $\theta$ , then we only change  $\theta$ .

*Illustration 1.* Given the frequency curve

$$y = y_0 x^{9.2551} (1-x)^{33.2228},$$

find what proportion of the frequency lies beyond  $x = .3914$ , i.e. find the chance of an individual being drawn at random with a greater value of  $x$  than .3914 of the range. We need the area from  $x = .3914$  to  $x = 1$ . Comparing this with the curve

$$y = y_0 x^{p-1} (1-x)^{q-1},$$

we see that  $p = 10.2551$ ,  $q = 34.2228$ , or  $q$  is greater than  $p$ . We must accordingly put  $x = 1 - x'$  and write the curve

$$y = y_0 x'^{33.2228} (1-x')^{9.2551},$$

in which  $q$  will be less than  $p$ , and we must find the relative area from  $x' = 0$  to  $x' = .6086$ .

We have now to find the cell in which our interpolate lies;  $x'$  lies between 60 and 61,  $p$  between 34 and 35,  $q$  between 10 and 10.5, hence pp. 255 and 265 contain the eight  $z$ 's we require. They are:

p. 255	p. 265	
$z_{000} = .006,6131,$	$z_{001} = .009,1302$	} \dots\dots(\text{ix})
$z_{100} = .009,4416,$	$z_{101} = .012,8940$	
$z_{110} = .007,1344,$	$z_{111} = .009,8423$	
$z_{010} = .004,9189,$	$z_{011} = .006,8608$	

Each one of these values gives rise to three  $\delta^2$ 's corresponding to variation of  $x$ ,  $p$  and  $q$ . We may illustrate the finding of these adequately on  $z_{000}$ , reminding the reader that the values would not actually be taken out of the table, but the  $\delta^2$  worked by four turns of the machine handle.

From p. 255:  $\delta_x^2 z_{000} = .004,5686 + .009,4416 - 2 \times .006,6131 = .000,7840$ .

From p. 255:  $\delta_p^2 z_{000} = .008,8437 + .004,9189 - 2 \times .006,6131 = .000,5384$ .

From pp. 245, 255 and 265:  $\delta_q^2 z_{000} = .004,6957 + .009,1302 - 2 \times .006,6131 = .000,5997$ .

In the same manner we find:

$\delta_x^2 z_{100} = .001,0276,$	$\delta_p^2 z_{100} = .000,6834,$	$\delta_q^2 z_{100} = .000,7889$	} \dots\dots(\text{x})
$\delta_x^2 z_{110} = .000,8545,$	$\delta_p^2 z_{110} = .000,5379,$	$\delta_q^2 z_{110} = .000,6428$	
$\delta_x^2 z_{010} = .000,6405,$	$\delta_p^2 z_{010} = .000,4163,$	$\delta_q^2 z_{010} = .000,4797$	
$\delta_x^2 z_{001} = .001,0091,$	$\delta_p^2 z_{001} = .000,6869,$	$\delta_q^2 z_{001} = .000,7260$	
$\delta_x^2 z_{101} = .001,3016,$	$\delta_p^2 z_{101} = .000,8584,$	$\delta_q^2 z_{101} = .000,9395$	
$\delta_x^2 z_{111} = .001,0974,$	$\delta_p^2 z_{111} = .000,6850,$	$\delta_q^2 z_{111} = .000,7766$	
$\delta_x^2 z_{011} = .000,8353,$	$\delta_p^2 z_{011} = .000,5382,$	$\delta_q^2 z_{011} = .000,5888$	

Turning to the argument interval ratios we have:

$$\theta_1 = .86 \text{ (since } x = .6086), \theta_0 = .14, \frac{1}{6}\theta_1(1+\theta_0) = .1634,0000, \frac{1}{6}\theta_0(1+\theta_1) = .0434,0000;$$

$$\phi_1 = .2228, \phi_0 = .7772, \frac{1}{6}\phi_1(1+\phi_0) = .0659,9336, \frac{1}{6}\phi_0(1+\phi_1) = .1583,9336;$$

$$\chi_1 = \frac{.2551}{.5} \text{ (since the interval for } q = .5) = .5102, \chi_0 = .4898, \frac{1}{6}\chi_1(1+\chi_0) = .1266,8266, \frac{1}{6}\chi_0(1+\chi_1) = .1232,8266.$$

We will now write down our argument ratio products in a form useful for a later purpose:

Corresponding to	$\theta$	$\phi\chi$	
$z_{000}$	$\theta_0\phi_0\chi_0 = .14 \times .3806,7256 = .0532,9416$		} .....(xi).
$z_{100}$	$\theta_1\phi_0\chi_0 = .86 \times \quad \quad \quad = .3273,7840$		
$z_{110}$	$\theta_1\phi_1\chi_0 = .86 \times .1091,2744 = .0938,4960$		
$z_{010}$	$\theta_0\phi_1\chi_0 = .14 \times \quad \quad \quad = .0152,7784$		
$z_{001}$	$\theta_0\phi_0\chi_1 = .14 \times .3965,2744 = .0555,1384$		
$z_{101}$	$\theta_1\phi_0\chi_1 = .86 \times \quad \quad \quad = .3410,1360$		
$z_{111}$	$\theta_1\phi_1\chi_1 = .86 \times .1136,7256 = .0977,5840$		
$z_{011}$	$\theta_0\phi_1\chi_1 = .14 \times \quad \quad \quad = .0159,1416$		

Multiplying the  $z$ 's by the corresponding  $\theta\phi\chi$  products as a continuous operation on the machine we find

$$\text{Sum of hyperbolic* terms} = .0101,6336.$$

For the  $\delta^2$  terms we have

$$\begin{aligned} \text{Total } \delta^2 \text{ terms} &= .1634,0000 \times .0001,2088 + .0434,0000 \times .0009,6759 \\ &+ .0659,9336 \times .0005,8329 + .1583,9336 \times .0001,2665 \\ &+ .1266,8266 \times .0003,5788 + .1232,8266 \times .0004,4597 \\ &= .0002,2062, \end{aligned}$$

each individual  $\delta^2$  series and the final sum of products being obtained by continuous operations on the machine.

$$\begin{aligned} \text{Hence the required area} &= .0101,6336 - .0002,2062 \\ &= .0099,4274. \end{aligned}$$

This is the chance that an individual should be drawn with a variate exceeding .3914.

*Illustration 2.* Given the same frequency curve as in *Illustration 1*, find the value of  $x$  for which the relative area is exactly .01.

The former illustration shows that we are not far from the required value of  $x$ . Let us vary  $\theta_1$  from .86 to .88. Thus  $\theta_0 = .12$  and

$$\frac{1}{6}\theta_1(1+\theta_0) = .1642,6667, \quad \frac{1}{6}\theta_0(1+\theta_1) = .0376,0000.$$

The values of the  $z$ 's in (ix) and of the  $\delta^2 z$  in (x) remain unchanged, as well as the  $\phi$  and  $\chi$  coefficients. The form in which we have exhibited the argument ratio products in (xi) enables us to ascertain rapidly the new products. They are:

Corresponding to	$\theta\phi\chi$	
$z_{000} = .006,6131$	$\theta_0\phi_0\chi_0 = .0456,8071$	} .....(xii).
$z_{100} = .009,4416$	$\theta_1\phi_0\chi_0 = .3349,9185$	
$z_{110} = .007,1344$	$\theta_1\phi_1\chi_0 = .0960,3215$	
$z_{010} = .004,9189$	$\theta_0\phi_1\chi_0 = .0130,9529$	
$z_{001} = .009,1302$	$\theta_0\phi_0\chi_1 = .0475,8329$	
$z_{101} = .012,8940$	$\theta_1\phi_0\chi_1 = .3489,4415$	
$z_{111} = .009,8423$	$\theta_1\phi_1\chi_1 = .1000,3185$	
$z_{011} = .006,8608$	$\theta_0\phi_1\chi_1 = .0136,4071$	

$$\text{Hyperbolic terms} = S(z_{stu}\theta_s\phi_t\chi_u) = .0102,2636.$$

$$\text{Sum } \delta^2 \text{ terms} = .0002,1392.$$

$$\text{Total area} = .0100,1244.$$

\* This term is used here, as in *Biometrika*, Vol. xix, p. 356, to denote the part of the interpolation involving only the double or triple products of the interval ratios.



If we suppose linear interpolation adequate between  $\theta_1 = .86$  and  $.88$ , i.e. between  $x' = .6086$  and  $x' = .6088$ , we have

$$\text{Area} = .01 \text{ when } x' = .608,7643.$$

Thus we see that the required value of  $\theta_1$  lies between  $.876$  and  $.877$ . If we require greater accuracy is best to work these out. We find

$$\begin{aligned} \theta_1 = .876: \frac{1}{6}\theta_1(1+\theta_0) &= .1641,0400, \frac{1}{6}\theta_0(1+\theta_1) = .0387,7067, \\ \theta_1 = .877: \frac{1}{6}\theta_1(1+\theta_0) &= .1641,4517, \frac{1}{6}\theta_0(1+\theta_1) = .0384,7850, \end{aligned}$$

and the corresponding triple products are:

$\theta_1 = .876$	$\theta_1 = .877$	
$\theta_0\phi_0\chi_0 = .0472,0340,$	$.0468,2272$	}
$\theta_1\phi_0\chi_0 = .3334,6916,$	$.3338,4984$	
$\theta_1\phi_1\chi_0 = .0955,9564,$	$.0957,0476$	
$\theta_0\phi_1\chi_0 = .0135,3180,$	$.0134,2268$	
$\theta_0\phi_0\chi_1 = .0491,6940,$	$.0487,7287$	
$\theta_1\phi_0\chi_1 = .3473,5804,$	$.3477,5457$	
$\theta_1\phi_1\chi_1 = .0995,7716,$	$.0996,9084$	
$\theta_0\phi_1\chi_1 = .0140,9540,$	$.0139,8172$	

.....(xi)

These give:

Hyperbolic term:	.0102,1376,	.0102,1694,
$\delta^2$ terms:	-2,1615,	-2,1581,
Total relative area:	.0099,9761,	.0100,0113.

$\theta_1$  by linear interpolation\* =  $.876,679$ , or, the relative area =  $.01$  when  $x' = .608,7668$ .

Thus finally the area beyond  $x = .391,2332$  equals  $.01$ , where, without taking into account the differences, we certainly cannot retain more than six figures in  $x$ .

It is of course possible to look upon formula (viii) as a cubic equation to find  $\theta_1$  or  $\theta_0$  whichever is smaller—and solve it by approximation, or even directly. The cubic equation is the following:

$$\begin{aligned} & \frac{1}{6} [\phi_0\chi_0(\delta_\theta^2 z_{000} - \delta_\theta^2 z_{100}) + \phi_0\chi_1(\delta_\theta^2 z_{001} - \delta_\theta^2 z_{101}) + \phi_1\chi_0(\delta_\theta^2 z_{010} - \delta_\theta^2 z_{110}) + \phi_1\chi_1(\delta_\theta^2 z_{011} - \delta_\theta^2 z_{111})] \theta_0^3 \\ & + \frac{1}{2} [\phi_0\chi_0\delta_\theta^2 z_{100} + \phi_0\chi_1\delta_\theta^2 z_{101} + \phi_1\chi_0\delta_\theta^2 z_{110} + \phi_1\chi_1\delta_\theta^2 z_{111}] \theta_0^2 \\ & + [\phi_0\chi_0(z_{000} - z_{100}) + \phi_0\chi_1(z_{001} - z_{101}) + \phi_1\chi_0(z_{010} - z_{110}) + \phi_1\chi_1(z_{011} - z_{111}) \\ & - \frac{1}{6} \{ \phi_0\chi_0(\delta_\theta^2 z_{000} + 2\delta_\theta^2 z_{100}) + \phi_0\chi_1(\delta_\theta^2 z_{001} + 2\delta_\theta^2 z_{101}) + \phi_1\chi_0(\delta_\theta^2 z_{010} + 2\delta_\theta^2 z_{110}) + \phi_1\chi_1(\delta_\theta^2 z_{011} + 2\delta_\theta^2 z_{111}) \\ & - \frac{1}{6} \phi_1(1+\phi_0) \{ \phi_0\chi_0(\delta_\phi^2 z_{000} - \delta_\phi^2 z_{100}) + \phi_0\chi_1(\delta_\phi^2 z_{001} - \delta_\phi^2 z_{101}) \} \\ & - \frac{1}{6} \phi_0(1+\phi_1) \{ \phi_1\chi_0(\delta_\phi^2 z_{010} - \delta_\phi^2 z_{110}) + \phi_1\chi_1(\delta_\phi^2 z_{011} - \delta_\phi^2 z_{111}) \} \\ & - \frac{1}{6} \chi_1(1+\chi_0) \{ \phi_0\chi_0(\delta_\chi^2 z_{000} - \delta_\chi^2 z_{100}) + \phi_1\chi_0(\delta_\chi^2 z_{010} - \delta_\chi^2 z_{110}) \} \\ & - \frac{1}{6} \chi_0(1+\chi_1) \{ \phi_0\chi_1(\delta_\chi^2 z_{001} - \delta_\chi^2 z_{101}) + \phi_1\chi_1(\delta_\chi^2 z_{011} - \delta_\chi^2 z_{111}) \}] \theta_0 \\ & - [z_{\theta\phi\chi} - \phi_0\chi_0 z_{100} - \phi_0\chi_1 z_{101} - \phi_1\chi_0 z_{110} - \phi_1\chi_1 z_{111} \\ & + \frac{1}{6} \phi_1(1+\phi_0) (\phi_0\chi_0\delta_\phi^2 z_{100} + \phi_0\chi_1\delta_\phi^2 z_{101}) + \frac{1}{6} \phi_0(1+\phi_1) (\phi_1\chi_0\delta_\phi^2 z_{110} + \phi_1\chi_1\delta_\phi^2 z_{111}) \\ & + \frac{1}{6} \chi_1(1+\chi_0) (\phi_0\chi_0\delta_\chi^2 z_{100} + \phi_1\chi_0\delta_\chi^2 z_{110}) + \frac{1}{6} \chi_0(1+\chi_1) (\phi_0\chi_1\delta_\chi^2 z_{101} + \phi_1\chi_1\delta_\chi^2 z_{111})] = 0 \end{aligned}$$

.....(xii)

The equation is long and troublesome but it may be worth while seeing to what value of  $\theta_0$  it leads in the inverse interpolation of the previous example.

\* As evidence that we may with our formula linearly interpolate for  $\theta$ , we remark that:

$$\begin{aligned} x = .608700, \quad \text{Area} &= .0099,7667 \\ x = .608720, \quad \text{Area} &= .0099,8377 \end{aligned} \quad \left. \vphantom{\begin{aligned} x = .608700, \quad \text{Area} &= .0099,7667 \\ x = .608720, \quad \text{Area} &= .0099,8377 \end{aligned}} \right\} \text{ or difference in area for } .000010 \text{ in } x = .0000,0355,$$

and

$$\begin{aligned} x = .608760, \quad \text{Area} &= .0099,9761 \\ x = .608770, \quad \text{Area} &= .0100,013 \end{aligned} \quad \left. \vphantom{\begin{aligned} x = .608760, \quad \text{Area} &= .0099,9761 \\ x = .608770, \quad \text{Area} &= .0100,013 \end{aligned}} \right\} \text{ or difference in area for } .000010 \text{ in } x = .0000,0352.$$

Thus there is almost the same difference at  $x = .608710$  as at  $x = .608760$ .

We repeat the values from (xi) for the four argument ratios:

$$\begin{aligned}\phi_0\chi_0 &= \cdot3806,7256, & \text{further: } \frac{1}{6}\phi_1(1+\phi_0) &= \cdot0659,9336, \\ \phi_0\chi_1 &= \cdot3965,2744, & \frac{1}{6}\phi_0(1+\phi_1) &= \cdot1583,9336, \\ \phi_1\chi_0 &= \cdot1091,2744, & \frac{1}{6}\chi_1(1+\chi_0) &= \cdot1266,8266, \\ \phi_1\chi_1 &= \cdot1136,7256, & \frac{1}{6}\chi_0(1+\chi_1) &= \cdot1232,8266.\end{aligned}$$

We will now proceed to the evaluation of the terms of the cubic one by one.

*Coefficient of  $\theta_0^3$*

$$\begin{aligned}&= \frac{1}{6} \left[ \begin{array}{cccc} \cdot3806,7256 & \cdot3965,2744 & \cdot1091,2744 & \cdot1136,7256 \\ \times -\cdot000,2436 & \times -\cdot000,2925 & \times -\cdot000,2140 & \times -\cdot000,2621 \end{array} \right] \\ &= -\cdot0000,4364,4.\end{aligned}$$

*Coefficient of  $\theta_0^2$*

$$\begin{aligned}&= \frac{1}{2} \left[ \begin{array}{cccc} \cdot3806,7256 & \cdot3965,2744 & \cdot1091,2744 & \cdot1136,7256 \\ \times \cdot001,0276 & \times \cdot001,3016 & \times \cdot000,8545 & \times \cdot001,0974 \end{array} \right] \\ &= +\cdot0005,6264,6.\end{aligned}$$

*Coefficient of  $\theta_0$*

$$\begin{aligned}\text{First Line} &= \left[ \begin{array}{cccc} \cdot3806,7256 & \cdot3965,2744 & \cdot1091,2744 & \cdot1136,7256 \\ \times -\cdot002,8285 & \times -\cdot003,7638 & \times -\cdot002,2155 & \times -\cdot002,9815 \end{array} \right] \\ &= -\cdot0031,4986,9.\end{aligned}$$

$$\begin{aligned}\text{Second Line} &= -\frac{1}{6} \left[ \begin{array}{cccc} \cdot3806,7256 & \cdot3965,2744 & \cdot1091,2744 & \cdot1136,7256 \\ \times \cdot002,8392 & \times \cdot003,6123 & \times \cdot002,3495 & \times \cdot003,0301 \end{array} \right] \\ &= -\cdot0005,1900,3.\end{aligned}$$

$$\begin{aligned}\text{Third and Fourth Lines} &= - \left[ \cdot0659,9336 \left( \begin{array}{cc} \cdot3806,7256 & \cdot3965,2744 \\ \times -\cdot000,1450 & \times -\cdot000,1715 \end{array} \right) \right. \\ &\quad \left. + \cdot1583,9336 \left( \begin{array}{cc} \cdot1091,2744 & \cdot1136,7256 \\ \times -\cdot000,1216 & \times -\cdot000,1468 \end{array} \right) \right] \\ &= +\cdot0000,1287,6.\end{aligned}$$

$$\begin{aligned}\text{Fifth and Sixth Lines} &= - \left[ \cdot1266,8266 \left( \begin{array}{cc} \cdot3806,7256 & \cdot1091,2744 \\ \times -\cdot000,1892 & \times -\cdot000,1631 \end{array} \right) \right. \\ &\quad \left. + \cdot1232,8266 \left( \begin{array}{cc} \cdot3965,2744 & \cdot1136,7256 \\ \times -\cdot000,2135 & \times -\cdot000,1878 \end{array} \right) \right] \\ &= +\cdot0000,2444,8.\end{aligned}$$

Hence total coefficient of  $\theta_0 = -\cdot0036,3154,8$ .

*Constant Term*

$$\begin{aligned}&= - \left[ \cdot01 - \left( \begin{array}{cccc} \cdot3806,7256 & \cdot3965,2744 & \cdot1091,2744 & \cdot1136,7256 \\ \times \cdot009,4416 & \times \cdot012,8940 & \times \cdot007,1344 & \times \cdot009,8423 \end{array} \right) \right. \\ &\quad + \cdot0659,9336 \left( \begin{array}{cc} \cdot3806,7256 & \cdot3965,2744 \\ \times \cdot000,6834 & \times \cdot000,8584 \end{array} \right) + \cdot1583,9336 \left( \begin{array}{cc} \cdot1091,2744 & \cdot1136,7256 \\ \times \cdot000,5379 & \times \cdot000,6850 \end{array} \right) \\ &\quad \left. + \cdot1266,8266 \left( \begin{array}{cc} \cdot3806,7256 & \cdot1091,2744 \\ \times \cdot000,7889 & \times \cdot000,6428 \end{array} \right) + \cdot1232,8266 \left( \begin{array}{cc} \cdot3965,2744 & \cdot1136,7256 \\ \times \cdot000,9395 & \times \cdot000,7766 \end{array} \right) \right] \\ &= +\cdot0004,3933,7.\end{aligned}$$

The computing of the terms is not so long as may appear to the reader, and is done by continuous process on the machine. A skilled computer would not write down the individual terms as above. Here they are printed so that the reader can appreciate the amount of labour requisite. The cubic for  $\theta_0$  is

$$F(\theta_0) = 4,3644\theta_0^3 - 56,2646\theta_0^2 + 363,1548\theta_0 - 43,9337 = 0,$$

$$F'(\theta_0) = 13,0932\theta_0^2 - 112,5292\theta_0 + 363,1548.$$

Put  $\theta_0 = .10$ ,

$$F(\theta_0) = -81,765.02.$$

Put  $\theta_0 = .15$ ,

$$F(\theta_0) = +92,882.96.$$

Linear interpolation gives  $\theta_0 = .1234$  approximately for the vanishing of  $F(\theta_0)$ .

Put  $\theta_0 = .1234$ ,

$$F(\theta_0) = +310.31, \quad F'(\theta_0) = 349,4680.74.$$

Hence

$$\epsilon = -F(\theta_0)/F'(\theta_0) = -.0000,8879,5$$

or we have

$$\theta_0 = .123,311.$$

Thus  $x = .391,2331$  for area = .01.

In our previous investigation the value found was  $x = .391,2332$ , a quite sufficient accordance.

It is easy to solve the cubic, but personally I find it less labour to approximate to the proper values  $\theta_1$  and  $\theta_0$  from the general equation (viii).

### (III) SPECIAL CASE OF $I_x(i + 0.5, i' + 0.5)$ OF IMPORTANCE FOR SMALL SAMPLES

In the problem of sampling we frequently have to deal with the  $p$  and  $q$  of  $I_x(p, q)$  in the form  $i + 0.5$  where  $i$  is an integer; accordingly it is desirable to provide special formulae for such cases. If only one other of  $p$  and  $q$  be of this form, while the other is an integer, then, if the values fall within the range of our table, and the values of  $p$  or  $q$  exceed 10 and 10.5 we need a univariate formula to determine  $I_x(i, i' + 0.5)$ .

(a) *Univariate Interpolation Formulae for  $I_x(i, i' + 0.5)$ .*

The formulae available for the special case of  $\theta = \phi = \frac{1}{2}$  are\*:

(a) *Mid-panel Formulae.*

$$z_{\frac{1}{2}} = \frac{1}{2}(z_0 + z_1) - \frac{1}{16}(\delta^2 z_0 + \delta^2 z_1) + \frac{3}{256}(\delta^4 z_0 + \delta^4 z_1) - \frac{5}{2648}(\delta^6 z_0 + \delta^6 z_1) + \frac{35}{65536}(\delta^8 z_0 + \delta^8 z_1) \dots \dots (xv)$$

up to and including the ninth order difference. The  $\delta^8$  terms after  $p, q > 10$  contribute nothing to the interpolation up to seven-figure accuracy. This formula may also be written in the form

$$z_{\frac{1}{2}} = \frac{1}{2}(z_0 + z_1) - \frac{81}{1024}(\delta^2 z_0 + \delta^2 z_1) + \frac{39}{2048}(\delta^2 z_{-1} + \delta^2 z_2) - \frac{5}{2048}(\delta^2 z_{-2} + \delta^2 z_3) \dots \dots (xv) \text{ bis.}$$

This is correct up to and including seventh order differences, which, as I have just indicated, is the order of differences to which it may be profitable to work with our seven-figure table.

Lastly we may replace the  $\delta^2$ , and obtain a formula involving only the tabular entries. It is

$$z_{\frac{1}{2}} = \frac{1}{2048}\{1225(z_0 + z_1) - 245(z_{-1} + z_2) + 49(z_{-2} + z_3) - 5(z_{-3} + z_4)\} \dots \dots (xv) \text{ ter.}$$

This is correct up to and including seventh order differences.

The objection to (xv) bis and (xv) ter is that if we desire to abbreviate our work by omitting some of the  $z$ 's or  $\delta^2 z$ 's, we have no means of doing so unless we have first calculated the differences, or their values in terms of the  $\delta^2 z$ 's or  $z$ 's†.

(b) *Mid-point Formulae.*

The fundamental formula of this type is

$$z_{\frac{1}{2}} = z_0 + \frac{1}{4}(z_1 - z_{-1}) - \frac{1}{32}(\delta^2 z_1 - \delta^2 z_{-1}) + \frac{1}{8}\delta^2 z_0 + \frac{3}{512}(\delta^4 z_1 - \delta^4 z_{-1}) - \frac{1}{128}\delta^4 z_0 - \frac{5}{4096}(\delta^6 z_1 - \delta^6 z_{-1}) + \frac{1}{1624}\delta^6 z_0 \dots (xvi)$$

which includes terms of the seventh order difference.

To the same order we may express the result in terms of second differences only, i.e.

$$z_{\frac{1}{2}} = z_0 + \frac{1}{4}(z_1 - z_{-1}) - \frac{5}{4096}(\delta^2 z_3 - \delta^2 z_{-3}) + \frac{1}{512}(6\delta^2 z_2 - 5\delta^2 z_{-2}) - \frac{1}{4096}(249\delta^2 z_1 - 153\delta^2 z_{-1}) + \frac{75}{512}\delta^2 z_0 \dots (xvi) \text{ bis}$$

where the order of terms indicates nothing as to the order of convergency.

\* See *Tracts for Computers*, No. II, p. 14.

† Of course formula (xv) ter may be written in the form

$z_{\frac{1}{2}} = \frac{1}{2}(z_0 + z_1) - \frac{1}{128}\{z_{-1} + z_2 - z_0 - z_1\} + \frac{3}{2048}\{z_{-2} + z_3 - 3(z_{-1} + z_2) + 2(z_0 + z_1)\} - \frac{5}{2648}\{z_{-3} + z_4 - 5(z_{-2} + z_3) + 9(z_{-1} + z_2) - 5(z_0 + z_1)\}$  where the terms in curled brackets are successively of the order  $\delta^2$ ,  $\delta^4$  and  $\delta^6$ , thus we can follow the order of convergency. But in this form the formula has lost the easy mode of computing peculiar to (xv) ter.

Lastly, expressing the formula in terms of ordinates or table entries only we have

$$z_4 = \frac{1}{4096} (5z_{-4} - 50z_{-3} + 238z_{-2} - 770z_{-1} + 2800z_0 + 2170z_1 - 350z_2 + 58z_3 - 5z_4) \dots\dots(\text{xvi}) \text{ ter.}$$

Undoubtedly (xv) *ter* and (xvi) *ter* are the easiest formulae to apply, for the whole process is one continuous operation on the machine, and we need write down nothing on paper, taking the values direct from table to machine. Going to seventh differences they provide all that our seven-figure table is capable of. At the same time we may be indirectly working differences which are in reality negligible\*.

*Illustration 3.* I will illustrate the applicability of these interpolation formulae to our table by calculating  $I_{.19}(10.5, 10)$  from integer values of  $p$  and  $q$  in the table. The values for which we need to consult the table are  $I_{.19}(6, 10)$  to  $I_{.19}(15, 10)$ , and although it is unnecessary to write them down in the case of formulae (xv) *ter* and (xvi) *ter*, I am doing so here to compare the various methods of ascertaining  $I_{.19}(10.5, 10)$ . We have

		$z$	$\delta^2$	$\delta^4$	$\delta^6$	$\delta^8$
$z_{-4}$	$I_{.19}(6, 10)$	490,286				
$z_{-3}$	$I_{.19}(7, 10)$	204,016	161,947			
$z_{-2}$	$I_{.19}(8, 10)$	79,693	74,128	44,811		
$z_{-1}$	$I_{.19}(9, 10)$	29,498	31,120	24,078	7,901	
$z_0$	$I_{.19}(10, 10)$	10,423	12,190	11,246	6,352	-1,222
$\rightarrow$						
$z_1$	$I_{.19}(11, 10)$	3,538	4,506	4,766	3,581	+910
$z_2$	$I_{.19}(12, 10)$	1,159	1,588	1,867	1,720	
$z_3$	$I_{.19}(13, 10)$	368	537	688		
$z_4$	$I_{.19}(14, 10)$	114	174			
$z_5$	$I_{.19}(15, 10)$	34				

Applying first the mid-panel formula (xv) we have

$$\begin{aligned} z_4 &= \frac{1}{2}(13961) - \frac{1}{16}(16696) + \frac{3}{256}(16012) - \frac{5}{2048}(9933) + \frac{35}{65536}(-312) \\ &= 6980|5 - 1043|5 + 187|64 - 24|25 - 0|17 \\ &= 6100|22, \end{aligned}$$

or introducing the proper number of zeros, omitted for brevity above,

$$z_4 = I_{.19}(10.5, 10) = .000,6100.$$

This differs by a unit in the seventh figure from the value .000,6101 in the table itself. It is as good as we can expect with only seven figures recorded.

Next working with formula (xv) *bis*, which does not regard  $\delta^8$ , we have

$$z_4 = 6980|5 - 1320|67 + 622|86 - 182|29 = 6100|40,$$

or with the zeros reinstated

$$z_4 = .000,6100|40,$$

in complete accord with (xv), if we remember that the  $-0|17$  has not been introduced.

Lastly, the easy formula (xv) *ter* gives us

$$\begin{aligned} z_4 &= \frac{1}{2048} (-5 \times 20430 + 49 \times 80061 - 245 \times 30657 + 1225 \times 13961) \\ &= \frac{1}{2048} (-1,020,650 + 3,922,989 - 7,510,965 + 17,102,225) \\ &= \frac{1}{2048} (1249,3599) = .000,6100|2, \end{aligned}$$

again in complete agreement, as of course it should be. Needless perhaps to repeat that with this last formula nothing but the answer needs to be written down.

\* The Lagrangian which does not regard the values  $z_{-4}$  and  $z_4$  is

$$z_4 = \frac{1}{1024} (-5z_{-3} + 42z_{-2} - 175z_{-1} + 700z_0 + 525z_1 - 70z_2 + 7z_3) \dots\dots(\text{xvi}) \text{ quater.}$$

If (xvi) *ter* and (xvi) *quater* give sensibly the same result, then seventh differences were unnecessary, and we have thus computed terms which were not required.

We now turn to the mid-point formulae also carried to the seventh difference.

First, (xvi) gives us

$$\begin{aligned} z_{\frac{1}{2}} &= 10423 - 6490 + 831|7 + 1523|7 - 113|2 - 87|9 + 5|3 + 6|2 \\ &= 6098|8, \text{ or fully } \cdot 000,6098|8. \end{aligned}$$

Proceeding in the same way with (xvi) *bis*, it gives us

$$\begin{aligned} z_{\frac{1}{2}} &= 10423 - 6490 + 197|0 + 18|6 - 723|9 - 273|9 + 1162|4 + 1785|4 \\ &= \cdot 000,6098|8 \end{aligned}$$

as before, as indeed it should. The advantage of (xvi) *bis* lying in the fact that it does not require the discovery of  $\delta^4$  and  $\delta^6$ .

Next dealing with (xvi) *ter*, the formula of this group most easy to apply, we find

$$\begin{aligned} z_{\frac{1}{2}} &= \frac{1}{4096} \{2,451,430 - 10,200,800 + 18,966,934 - 22,713,460 \\ &\quad + 29,184,400 + 7,677,460 - 405,650 + 21,344 - 570\} \\ &= \cdot 000,6098|9, \text{ as before.} \end{aligned}$$

Comparing this value with that obtainable from (xvi) *quater* in the footnote to p. xv, namely  $\cdot 000,6098$  we see that it is not possible to neglect seventh differences.

Further, comparing the results of the mid-panel formulae with those for the mid-point formulae, we see that the former are one unit in error in the seventh decimal place while the latter are two units off. This is in accordance with the rule that mid-panel formulae give the better result when the interpolant lies in the region from  $\frac{1}{4}$  to  $\frac{3}{4}$  of the argument, and mid-point formulae in the region  $-\frac{1}{4}$  to  $+\frac{1}{4}$  round the point. The formula (xv) *ter* gives a good  $\cdot 5$  interpolate, even at the part of the table where we cease to give arguments ascending by  $0.5$ , and there is little doubt that through the remainder of the table the values will do so likewise.

### ( $\beta$ ) Bivariate Formulae for $I_x(i + 0.5, i' + 0.5)$ .

We now turn to cases in which both  $p$  and  $q$  are of the form  $i + 0.5$ , so that we need bivariate interpolation formulae. The difficulty arising here is that if we go beyond the terms in  $\delta^4, \delta'^4$  and  $\delta^2\delta'^2$ —i.e. beyond the fifth order differences—we have no less than sixteen further terms to take into consideration in order to go to sixth and seventh order differences. Our illustration from the univariate case suggests that it is needful to use these differences, if we require the interpolate to be as accurate as the interpolants. The formula applies of course only to the part of the table where we were applying our formulae. Further, a bivariate formula deals with more “near points” than a univariate formula can do, and accordingly may give a better result with fewer high order differences. It is of interest to see how correctly the bivariate formulae give  $I_x(i + 0.5, i' + 0.5)$ , for not only are such values of themselves often needed, but we shall there test the accuracy with which we can apply bivariate formulae up to  $\delta^4, \delta'^4$  in the part of the table under consideration. As before we have three types of formulae to deal with, each of which may be expressed in a different way.

#### (a) Mid-panel Formulae.

The general mid-panel Everett formula is given on p. 9 of *Tracts for Computers*, No. III\*. In the present case of  $\theta = \phi = \psi = \frac{1}{2}$ , it becomes

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} &= \frac{1}{4}(z_{0,0} + z_{0,1} + z_{1,0} + z_{1,1}) - \frac{1}{32}(\delta^2 z_{0,0} + \delta^2 z_{0,1} + \delta^2 z_{1,0} + \delta^2 z_{1,1}) \\ &\quad - \frac{1}{32}(\delta'^2 z_{0,0} + \delta'^2 z_{0,1} + \delta'^2 z_{1,0} + \delta'^2 z_{1,1}) + \frac{3}{512}(\delta^4 z_{0,0} + \delta^4 z_{0,1} + \delta^4 z_{1,0} + \delta^4 z_{1,1}) \\ &\quad + \frac{3}{512}(\delta'^4 z_{0,0} + \delta'^4 z_{0,1} + \delta'^4 z_{1,0} + \delta'^4 z_{1,1}) + \frac{1}{256}(\delta^2 \delta'^2 z_{0,0} + \delta^2 \delta'^2 z_{0,1} + \delta^2 \delta'^2 z_{1,0} + \delta^2 \delta'^2 z_{1,1}) \\ &\quad - \frac{3}{4096}(\delta^4 \delta'^2 z_{0,0} + \delta^4 \delta'^2 z_{0,1} + \delta^4 \delta'^2 z_{1,0} + \delta^4 \delta'^2 z_{1,1}) - \frac{3}{4096}(\delta'^2 \delta^4 z_{0,0} + \delta'^2 \delta^4 z_{0,1} + \delta'^2 \delta^4 z_{1,0} + \delta'^2 \delta^4 z_{1,1}) \\ &\quad - \frac{5}{4096}(\delta^6 z_{0,0} + \delta^6 z_{0,1} + \delta^6 z_{1,0} + \delta^6 z_{1,1}) - \frac{5}{4096}(\delta'^6 z_{0,0} + \delta'^6 z_{0,1} + \delta'^6 z_{1,0} + \delta'^6 z_{1,1}) \end{aligned}$$

up to and including terms of the seventh order differences.

.....(xvii).

Taking differences only to the fifth order, we have in terms solely of second differences

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & \frac{1}{4} (z_{0,0} + z_{0,1} + z_{1,1} + z_{1,0}) - \frac{5}{128} (\delta^2 z_{0,0} + \delta^2 z_{0,1} + \delta^2 z_{1,0} + \delta^2 z_{1,1}) \\ & - \frac{5}{128} (\delta'^2 z_{0,0} + \delta'^2 z_{1,0} + \delta'^2 z_{0,1} + \delta'^2 z_{1,1}) + \frac{3}{512} (\delta^2 z_{-1,0} + \delta^2 z_{-1,1} + \delta^2 z_{0,0} + \delta^2 z_{0,1}) \\ & + \frac{3}{512} (\delta'^2 z_{0,-1} + \delta'^2 z_{1,-1} + \delta'^2 z_{0,2} + \delta'^2 z_{1,2}) + \frac{1}{512} (\delta^2 z_{0,-1} + \delta^2 z_{1,-1} + \delta^2 z_{0,2} + \delta^2 z_{1,2}) \\ & + \frac{1}{512} (\delta'^2 z_{-1,0} + \delta'^2 z_{-1,1} + \delta'^2 z_{2,0} + \delta'^2 z_{2,1}) \end{aligned} \quad \text{.....(xvii) bis.}$$

If we include differences up to the seventh order we have

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & \frac{1}{4} (z_{0,0} + z_{0,1} + z_{1,1} + z_{1,0}) - \frac{173}{4096} (\delta^2 z_{0,0} + \delta^2 z_{0,1} + \delta^2 z_{1,0} + \delta^2 z_{1,1}) \\ & - \frac{173}{4096} (\delta'^2 z_{0,0} + \delta'^2 z_{1,0} + \delta'^2 z_{0,1} + \delta'^2 z_{1,1}) + \frac{42}{4096} * (\delta^2 z_{-1,0} + \delta^2 z_{-1,1} + \delta^2 z_{0,0} + \delta^2 z_{0,1}) \\ & + \frac{42}{4096} * (\delta'^2 z_{0,-1} + \delta'^2 z_{1,-1} + \delta'^2 z_{0,2} + \delta'^2 z_{1,2}) + \frac{11}{4096} (\delta^2 z_{0,-1} + \delta^2 z_{1,-1} + \delta^2 z_{0,2} + \delta^2 z_{1,2}) \\ & + \frac{11}{4096} (\delta'^2 z_{-1,0} + \delta'^2 z_{-1,1} + \delta'^2 z_{2,0} + \delta'^2 z_{2,1}) - \frac{3}{4096} (\delta^2 z_{-1,-1} + \delta^2 z_{-1,2} + \delta^2 z_{2,-1} + \delta^2 z_{2,2}) \\ & - \frac{3}{4096} (\delta'^2 z_{-1,-1} + \delta'^2 z_{-1,2} + \delta'^2 z_{2,-1} + \delta'^2 z_{2,2}) - \frac{5}{4096} (\delta^2 z_{-2,0} + \delta^2 z_{-2,1} + \delta^2 z_{3,0} + \delta^2 z_{3,1}) \\ & - \frac{5}{4096} (\delta'^2 z_{-2,0} + \delta'^2 z_{-2,1} + \delta'^2 z_{3,0} + \delta'^2 z_{3,1}) \end{aligned} \quad \text{.....(xvii) ter.}$$

While (xvii) *bis* demands only the second differences at the angles of the square and inner octagon (see Fig. 2, p. xix), (xvii) *ter* demands both  $\delta^2$  and  $\delta'^2$  at the mid-points of the sides of the outer octagon, the  $\delta^2$ 's at the top and bottom angles, and the  $\delta'^2$ 's at the lateral angles, or 16 additional second differences beyond the 24 required in going to the fifth order difference in (xvii). The labour is not insuperable, but if (xvii) *bis* is adequate, we certainly do not desire to go further.

#### (b) Mid-point Formulae.

The general mid-point formula is given on p. 29 in *Tracts for Computers*, No. III. In the case of  $\theta = \phi = \chi = \psi = \frac{1}{2}$ , it becomes

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & z_{0,0} + \frac{1}{4} (z_{0,1} - z_{0,-1} + z_{1,0} - z_{-1,0}) + \frac{1}{16} (z_{1,1} - z_{1,-1} + z_{-1,1} - z_{-1,-1}) \\ & + \frac{7}{64} (\delta^2 z_{0,0} + \delta'^2 z_{0,0}) + \frac{1}{128} (\delta^2 z_{0,1} + \delta^2 z_{0,-1} + \delta'^2 z_{1,0} + \delta'^2 z_{-1,0}) \\ & + \frac{1}{32} (\delta^2 z_{0,1} - \delta^2 z_{0,-1} + \delta'^2 z_{1,0} - \delta'^2 z_{-1,0}) - \frac{1}{32} (\delta^2 z_{1,0} - \delta^2 z_{-1,0} + \delta'^2 z_{0,1} - \delta'^2 z_{0,-1}) \\ & + \frac{1}{128} (\delta^2 z_{1,1} - \delta^2 z_{1,-1} - \delta^2 z_{-1,1} + \delta^2 z_{-1,-1}) - \frac{1}{128} (\delta'^2 z_{1,1} - \delta'^2 z_{-1,1} - \delta'^2 z_{1,-1} + \delta'^2 z_{-1,-1}) \\ & - \frac{1}{128} (\delta^4 z_{0,0} + \delta^4 z_{0,0}) - \frac{1}{512} (\delta^4 z_{0,1} - \delta^4 z_{0,-1} - \delta^4 z_{1,0} + \delta^4 z_{-1,0}) \\ & + \frac{3}{512} (\delta^4 z_{1,0} - \delta^4 z_{-1,0} + \delta^4 z_{0,1} - \delta^4 z_{0,-1}) - \frac{1}{256} (\delta^2 \delta'^2 z_{1,0} - \delta^2 \delta'^2 z_{-1,0} + \delta^2 \delta'^2 z_{0,1} - \delta^2 \delta'^2 z_{0,-1}) \\ & + \frac{1}{1024} (\delta^6 z_{0,0} + \delta^6 z_{0,0}) - \frac{1}{1024} (\delta^2 \delta'^2 z_{1,1} - \delta^2 \delta'^2 z_{1,-1} - \delta^2 \delta'^2 z_{-1,1} + \delta^2 \delta'^2 z_{-1,-1}) \\ & + \frac{3}{2048} (\delta^4 z_{1,1} - \delta^4 z_{1,-1} - \delta^4 z_{-1,1} + \delta^4 z_{-1,-1}) + \frac{3}{2048} (\delta^4 z_{1,1} - \delta^4 z_{-1,1} - \delta^4 z_{1,-1} + \delta^4 z_{-1,-1}) \\ & + \frac{1}{4096} (\delta^6 z_{0,1} - \delta^6 z_{0,-1} + \delta^6 z_{1,0} - \delta^6 z_{-1,0}) - \frac{5}{4096} (\delta^6 z_{1,0} - \delta^6 z_{-1,0} + \delta^6 z_{0,1} - \delta^6 z_{0,-1}) \\ & + \frac{1}{4096} (\delta^4 \delta'^2 z_{0,1} - \delta^4 \delta'^2 z_{0,-1} + \delta^2 \delta'^4 z_{1,0} - \delta^2 \delta'^4 z_{-1,0}) + \frac{3}{4096} (\delta^4 \delta'^2 z_{1,0} - \delta^4 \delta'^2 z_{-1,0} + \delta^2 \delta'^4 z_{0,1} - \delta^2 \delta'^4 z_{0,-1}) \end{aligned} \quad \text{.....(xviii).}$$

This includes seventh order difference terms, but is very lengthy and troublesome.

Taken only to fifth order differences and expressed in terms of second differences we have†

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & z_{0,0} + \frac{1}{4} (z_{0,1} - z_{0,-1} + z_{1,0} - z_{-1,0}) + \frac{1}{16} (z_{1,1} - z_{1,-1} + z_{-1,1} + z_{-1,-1}) \\ & + \frac{1}{8} (\delta^2 z_{0,0} + \delta'^2 z_{0,0}) + \frac{3}{64} (\delta^2 z_{0,1} - \delta^2 z_{1,0} + \delta'^2 z_{1,0} - \delta'^2 z_{0,1}) \\ & - \frac{1}{32} (\delta^2 z_{0,-1} - \delta^2 z_{-1,0} + \delta'^2 z_{-1,0} - \delta'^2 z_{0,-1}) - \frac{3}{256} (\delta^2 z_{1,1} + \delta'^2 z_{1,1}) \\ & + \frac{1}{128} (\delta^2 z_{1,-1} + \delta^2 z_{-1,1} + \delta'^2 z_{-1,1} + \delta'^2 z_{1,-1}) - \frac{1}{256} (\delta^2 z_{-1,-1} + \delta'^2 z_{-1,-1}) \\ & + \frac{3}{512} (\delta^2 z_{2,0} - \delta^2 z_{-2,0} + \delta'^2 z_{0,2} - \delta'^2 z_{0,-2}) - \frac{1}{512} (\delta^2 z_{0,2} - \delta^2 z_{0,-2} + \delta'^2 z_{2,0} - \delta'^2 z_{-2,0}) \end{aligned} \quad \text{.....(xviii) bis.}$$

\* The common factor 2 is retained for convenience of continuous machining.

† Erroneously given as  $\delta^2 \delta'^2 z_{1,1}$  in *Tracts for Computers*, No. III, p. 30, third line from top of page.

‡ This must be deduced from Eqn. (xxvii), pp. 29-30 of *Tracts for Computers*, No. III, as in the Eqn. (xxviii), p. 32, the terms  $-\frac{1}{24} \theta^2 (1 - \theta^2) \delta^4 z_{0,0}$  and  $-\frac{1}{24} \chi^2 (1 - \chi^2) \delta^4 z_{0,0}$  have I regret to say been omitted.

The additional terms if we go to sixth, *not to seventh*, order differences are in terms of  $\delta^2$  and  $\delta'^2$

$$\begin{aligned} & \frac{1}{1024} (\delta^2 z_{2,0} - \delta^2 z_{0,2} + \delta^2 z_{-2,0} - \delta^2 z_{0,-2}) + \frac{1}{1024} (\delta'^2 z_{0,2} - \delta'^2 z_{2,0} + \delta'^2 z_{0,-2} - \delta'^2 z_{-2,0}) \\ & - \frac{1}{256} (\delta^2 z_{1,0} - \delta^2 z_{0,1} + \delta^2 z_{-1,0} - \delta^2 z_{0,-1}) - \frac{1}{256} (\delta'^2 z_{0,1} - \delta'^2 z_{1,0} + \delta'^2 z_{0,-1} - \delta'^2 z_{-1,0}) \\ & + \frac{1}{2048} (\delta^2 z_{1,2} - \delta^2 z_{-1,2} + \delta^2 z_{-1,-2} - \delta^2 z_{1,-2}) + \frac{1}{2048} (\delta'^2 z_{2,1} - \delta'^2 z_{-2,1} + \delta'^2 z_{-2,-1} - \delta'^2 z_{1,-1}) \\ & - \frac{1}{256} (\delta^2 z_{1,1} - \delta^2 z_{-1,-1} + \delta^2 z_{-1,1} - \delta^2 z_{1,-1}) - \frac{1}{256} (\delta'^2 z_{1,1} - \delta'^2 z_{-1,-1} + \delta'^2 z_{-1,1} - \delta'^2 z_{1,-1}) \\ & + \frac{3}{2048} (\delta^2 z_{2,1} - \delta^2 z_{-2,1} + \delta^2 z_{-2,-1} - \delta^2 z_{1,-1}) + \frac{3}{2048} (\delta'^2 z_{1,2} - \delta'^2 z_{-1,-2} + \delta'^2 z_{-1,2} - \delta'^2 z_{1,-2}) \dots\dots(x) \end{aligned}$$

If we proceed to seventh order differences, expressing all differences in terms of  $\delta^2$  and  $\delta'^2$  we have

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & z_{0,0} + \frac{1}{4} (z_{0,1} + z_{1,0} - z_{-1,0} - z_{0,-1}) + \frac{1}{16} (z_{1,1} + z_{-1,-1} - z_{-1,1} - z_{1,-1}) \\ & + \frac{1}{8} (\delta^2 z_{0,0} + \delta'^2 z_{0,0}) - \frac{221}{4096} (\delta^2 z_{1,0} + \delta'^2 z_{0,1}) + \frac{218}{4096} (\delta^2 z_{0,1} + \delta'^2 z_{1,0}) \\ & + \frac{125}{4096} (\delta^2 z_{-1,0} + \delta'^2 z_{0,-1}) - \frac{122}{4096} (\delta^2 z_{0,-1} + \delta'^2 z_{-1,0}) - \frac{76}{4096} (\delta^2 z_{1,1} + \delta'^2 z_{1,1}) \\ & + \frac{48}{4096} (\delta^2 z_{-1,1} + \delta'^2 z_{-1,1} + \delta^2 z_{-1,1} + \delta'^2 z_{1,-1}) - \frac{20}{4096} (\delta^2 z_{-1,-1} + \delta'^2 z_{-1,-1}) + \frac{42}{4096} (\delta^2 z_{2,0} + \delta'^2 z_{0,2}) \\ & - \frac{14}{4096} (\delta^2 z_{0,2} + \delta'^2 z_{2,0}) - \frac{34}{4096} (\delta^2 z_{-2,0} + \delta'^2 z_{0,-2}) + \frac{6}{4096} (\delta^2 z_{0,-2} + \delta'^2 z_{-2,0}) \\ & + \frac{10}{4096} (\delta^2 z_{2,1} + \delta'^2 z_{1,2}) + \frac{3}{4096} (\delta^2 z_{1,2} + \delta'^2 z_{2,1}) - \frac{4}{4096} (\delta^2 z_{2,-1} + \delta'^2 z_{-1,2}) \\ & - \frac{1}{4096} (\delta^2 z_{-1,2} + \delta'^2 z_{2,-1}) - \frac{8}{4096} (\delta^2 z_{-2,1} + \delta'^2 z_{1,-2}) - \frac{3}{4096} (\delta^2 z_{1,-2} + \delta'^2 z_{-2,1}) \\ & + \frac{2}{4096} (\delta^2 z_{-2,-1} + \delta'^2 z_{-1,-2}) + \frac{1}{4096} (\delta^2 z_{-1,-2} + \delta'^2 z_{-2,-1}) - \frac{5}{4096} (\delta^2 z_{3,0} + \delta'^2 z_{0,3} - \delta^2 z_{-3,0} - \delta'^2 z_{0,-3}) \dots\dots(xviii) \end{aligned}$$

This formula like (xviii) *ter* is cumbersome and inferior to (xvii) *ter*, but it gives the value correct to seven decimal places. Of course nothing like this number of terms is requisite later in nor indeed for most statistical purposes at this part of the table. It has been used to show that the central difference formula including  $\delta^6$ ,  $\delta'^6$ , will be satisfactory up to the limits of the tabling, and be advantageous when the interpolate is near an interpolant.

(c) *Lagrangian Formulae*: (i) *Mid-point*.

By this term I understand here formulae giving the interpolate in terms of interpolants, and not of their differences.

The formula taken to fifth order differences in terms of tabular entries only runs as follows:

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & \frac{1}{512} \{ 240z_{0,0} + 193(z_{1,0} + z_{0,1}) - 65(z_{-1,0} + z_{0,-1}) + 104z_{1,1} + 8z_{-1,-1} \\ & - 40(z_{-1,1} + z_{1,-1}) - 28(z_{2,0} + z_{0,2}) + 20(z_{-2,0} + z_{0,-2}) - 7(z_{1,2} + z_{2,1}) \\ & - (z_{-2,-1} + z_{-1,-2}) + 3(z_{2,-1} + z_{-1,2}) + 5(z_{-2,1} + z_{1,-2}) \\ & + 3(z_{3,0} + z_{0,3}) - 3(z_{-3,0} + z_{0,-3}) \} \dots \end{aligned}$$

If we proceed to sixth order differences, but do not include the seventh, the extra terms present in (xviii) *ter* in terms of tabular entries are

$$\begin{aligned} & \frac{1}{2048} \{ -32z_{0,0} + 2(z_{1,0} + z_{0,1}) + 2(z_{-1,0} + z_{0,-1}) + 54(z_{1,1} + z_{-1,-1}) \\ & - 22(z_{1,-1} + z_{-1,1}) - 8(z_{2,0} + z_{0,2}) - 8(z_{-2,0} + z_{0,-2}) - 18(z_{1,2} + z_{2,1}) \\ & - 18(z_{-2,-1} + z_{-1,-2}) + 14(z_{2,-1} + z_{-1,2}) + 14(z_{-2,1} + z_{1,-2}) \\ & + 2(z_{3,0} + z_{0,3}) + 2(z_{-3,0} + z_{0,-3}) + 2(z_{2,2} + z_{-2,-2}) - 2(z_{2,-2} + z_{-2,2}) \\ & + 3(z_{3,1} + z_{1,3} + z_{-1,-3} + z_{-3,-1}) - 3(z_{1,-3} + z_{-3,1} + z_{3,-1} + z_{-1,3}) \} \dots\dots(x) \end{aligned}$$

Combining (xviii) *bis* and (xviii) *ter* we have a formula up to sixth but not including seventh order differences as follows:

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} = & \frac{1}{2048} \{ 928z_{0,0} + 774(z_{1,0} + z_{0,1}) - 258(z_{-1,0} + z_{0,-1}) + 470z_{1,1} + 86z_{-1,-1} \\ & - 182(z_{1,-1} + z_{-1,1}) - 120(z_{2,0} + z_{0,2}) + 72(z_{-2,0} + z_{0,-2}) - 46(z_{1,2} + z_{2,1}) \\ & - 22(z_{-2,-1} + z_{-1,-2}) + 26(z_{2,-1} + z_{-1,2}) + 34(z_{-2,1} + z_{1,-2}) \\ & + 14(z_{3,0} + z_{0,3}) - 10(z_{-3,0} + z_{0,-3}) + 2(z_{2,2} + z_{-2,-2}) - 2(z_{2,-2} + z_{-2,2}) \\ & + 3(z_{3,1} + z_{1,3} + z_{-1,-3} + z_{-3,-1}) - 3(z_{1,-3} + z_{-3,1} + z_{3,-1} + z_{-1,3}) \} \dots\dots(x) \end{aligned}$$





Examining the diagram (p. xix) we see that this may be expressed verbally as

$$z_{\frac{1}{2}, \frac{1}{2}} = \frac{1}{512} \{ 174 \times (\text{sum of values at angles of square}) - 27 \times (\text{sum of values at angles of inner octagon}) \\ + 3 \times (\text{sum of values at angles of outer octagon}) + 2 \times (\text{sum of values at mid-points of longer sides of outer octagon}) \} \quad \dots\dots(\text{X})$$

If we now include sixth and seventh order differences we find

$$z_{\frac{1}{2}, \frac{1}{2}} = \frac{1}{4096} \{ 1454(z_{0,0} + z_{0,1} + z_{1,0} + z_{1,1}) - 276(z_{0,-1} + z_{-1,0} + z_{1,-1} + z_{-1,1} + z_{2,0} + z_{0,2} + z_{1,2} + z_{2,1}) \\ + 52(z_{0,-2} + z_{-2,0} + z_{-2,1} + z_{1,-2} + z_{3,0} + z_{0,3} + z_{3,1} + z_{1,3}) + 34(z_{-1,-1} + z_{2,-1} + z_{-1,2} + z_{2,2}) \\ - 5(z_{-3,0} + z_{0,-3} + z_{-3,1} + z_{1,-3} + z_{4,0} + z_{0,4} + z_{4,1} + z_{1,4}) \\ - 3(z_{-2,-1} + z_{-1,-2} + z_{-2,2} + z_{2,-2} + z_{3,-1} + z_{-1,3} + z_{3,2} + z_{2,3}) \} \quad \dots\dots(\text{X})$$

Now examining the diagram we see a square and three octagons, the inner, the mid and the outer and the above result may be read as follows:

$$z_{\frac{1}{2}, \frac{1}{2}} = \frac{1}{4096} \{ 1454 \times (\text{sum of values at angles of square}) \\ - 276 \times (\text{sum of values at angles of inner octagon}) \\ + 52 \times (\text{sum of values at angles of mid-octagon}) \\ + 34 \times (\text{sum of values at points of bisection of longer sides of mid-octagon}) \\ - 5 \times (\text{sum of values at angles of outer octagon}) \\ - 3 \times (\text{sum of values at points of trisection of longer sides of outer octagon}) \} \quad \dots\dots(\text{XX})$$

Undoubtedly formulae (xx) *bis* and (xx) *quater* are the most convenient and rapid to apply of the series, but unless we compute the two we are not in a position to determine (without a previous knowledge of the capacity of the table) whether (xx) *bis* is sufficient for our immediate purpose.

*Illustration 4, and test of what differences are needful in the region, where the table changes the order of argument for p and q.*

We will now find  $I_{.19}(10.5, 10.5)$ , the last value of  $I_x(i+0.5, i'+0.5)$  tabled, from unit values. The value in the table is .000,8006.

The labour of applying (xvii) or (xviii) involving the computing of high order differences including the use of the diagram may be replaced by (xvii) *bis*, (xvii) *ter*, (xviii) *bis* and (xviii) *ter* which involve only a knowledge of the first order differences.

The diagram Fig. 3 shows the octagon system applied to this special example, the  $z$  values and second differences both ways being provided.

Formula (xvii) *bis*.

$$z_{\frac{1}{2}, \frac{1}{2}} = \frac{1}{4} \left| \begin{array}{c} 10,423 \\ 17,309 \\ 6,154 \\ 3,538 \\ 37,424 \end{array} \right| - \frac{5}{128} \left| \begin{array}{c} 12,190 \\ 4,506 \\ 7,109 \\ 18,200 \\ 42,005 \end{array} \right| - \frac{5}{128} \left| \begin{array}{c} 2,412 \\ 3,255 \\ 1,430 \\ 1,000 \\ 8,097 \end{array} \right| + \frac{1}{512} \left| \begin{array}{c} 7,747 \\ 25,982 \\ 2,706 \\ 10,696 \\ 47,131 \end{array} \right| + \frac{1}{512} \left| \begin{array}{c} 5,391 \\ 391 \\ 6,852 \\ 589 \\ 13,223 \end{array} \right| + \frac{3}{512} \left| \begin{array}{c} 31,120 \\ 1,588 \\ 2,637 \\ 43,830 \\ 79,175 \end{array} \right| + \frac{3}{512} \left| \begin{array}{c} 1,712 \\ 4,234 \\ 1,963 \\ 672 \\ 8,581 \end{array} \right|$$

$$= 9356 \quad - 1640.8 \quad - 316.3 \quad + 92.1 \quad + 25.8 \quad + 463.9 \quad + 50.3$$

$$= .000,8031 \text{ (after adding the requisite zeros).}$$

Hence formula (xvii) *bis* is in error about 2 units in the sixth decimal place. This will be accurate for some purposes, but possibly not enough so for all.

We now proceed in the same way with formula (xvii) *ter*. We find

$$z_{4,1} = \frac{1}{4096} [1024 \begin{vmatrix} 10,123 \\ 17,309 \\ 6,154 \\ 3,538 \\ 37,424 \end{vmatrix} 173 \begin{vmatrix} 12,190 \\ 4,506 \\ 7,109 \\ 18,200 \\ 42,005 \end{vmatrix} 173 \begin{vmatrix} 2,412 \\ 3,255 \\ 1,430 \\ 1,000 \\ 8,097 \end{vmatrix} + 42 \begin{vmatrix} 31,120 \\ 1,588 \\ 2,637 \\ 43,830 \\ 79,175 \end{vmatrix} + 42 \begin{vmatrix} 1,712 \\ 4,234 \\ 1,963 \\ 672 \\ 8,581 \end{vmatrix} + 11 \begin{vmatrix} 7,747 \\ 25,982 \\ 2,706 \\ 10,696 \\ 47,131 \end{vmatrix}]$$

$$+ 11 \begin{vmatrix} 5,391 \\ 391 \\ 6,852 \\ 589 \\ 13,223 \end{vmatrix} 3 \begin{vmatrix} 21,034 \\ 59,161 \\ 902 \\ 4,166 \\ 85,263 \end{vmatrix} 3 \begin{vmatrix} 4,080 \\ 248 \\ 8,407 \\ 849 \\ 13,584 \end{vmatrix} 5 \begin{vmatrix} 74,128 \\ 98,008 \\ 537 \\ 935 \\ 173,608 \end{vmatrix} 5 \begin{vmatrix} 1,158 \\ 426 \\ 5,329 \\ 2,602 \\ 9,515 \end{vmatrix}]$$

$$= \frac{1}{4096} (38,322,176 + 7,266,865 + 1,400,781 + 3,325,350 + 360,402 + 518,441 + 145,453 + 225,789 + 10,752 + 868,040 + 47,575)$$

$$= \frac{1}{4096} (-42,671,822 - 9,879,802) = 000,8005[9,$$

INTERPOLATION for  $L_{19}(10.5, 10.5)$  from UNIT VALUES of  $p$  and  $q$ .

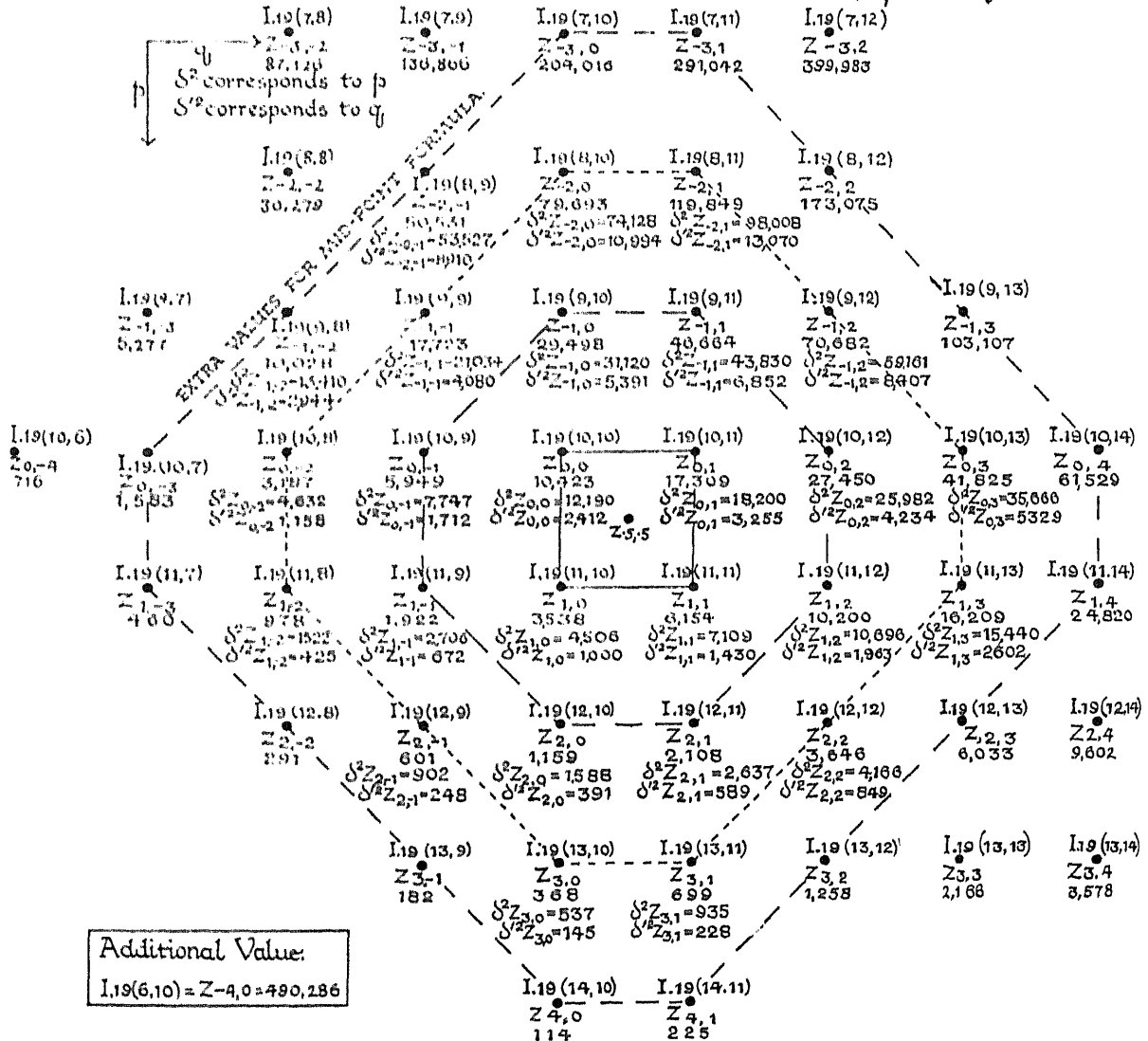


Fig. 3.

introducing the required three zeros and the decimal point, or to seven figures

$$z_{\frac{1}{2}, \frac{1}{2}} = \cdot 000,8006,$$

the exact tabular value.

We now turn to the formula (xviii) *bis* for the mid-point interpolation and find

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} &= 10423 - 3650 - 1544 \cdot 3 + 1825 \cdot 25 + 536 \cdot 2 + 615 \cdot 4 - 100 \cdot 07 + 422 \cdot 34 - 98 \cdot 1 - 407 \cdot 02 - 20 \cdot 99 \\ &= 8001 \cdot 7, \end{aligned}$$

or introducing the three zeros

$$z_{\frac{1}{2}, \frac{1}{2}} = \cdot 000,8002.$$

This result is out 4 units in the seventh decimal place.

We now evaluate (xviii) *ter* to find the addition if we go up to the sixth, but not including seventh differences. We have:

Extra terms =  $38 \cdot 19 - 32 \cdot 28 - 19 \cdot 72 + 79 \cdot 71 - 63 \cdot 52 = 2 \cdot 43$  or  $= \cdot 000,0002 | 4$  with zeros inserted. Adding to the previous value  $\cdot 000,8001 | 7$  we find

$$z_{\frac{1}{2}, \frac{1}{2}} = \cdot 000,8004.$$

This result shows an error of 2 in the last decimal place and is therefore not as good as (xvii) *ter* which includes seventh differences, i.e.  $\delta^6$ . We conclude accordingly that in the worst case, i.e. that of  $z_{\frac{1}{2}, \frac{1}{2}}$ , the answer will, if we do not proceed beyond sixth differences, be given to an error of not more than two units in the seventh decimal place.

If we now apply (xviii) *quater* we have

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} &= 10,423 - 3,650 - 1,544 | 31 + 1,825 | 25 - 418 | 75 + 1,021 | 88 \\ &\quad + 1,001 | 95 - 391 | 32 - 158 | 44 + 59 | 81 + 633 | 52 - 122 | 63 \\ &\quad - 90 | 14 - 624 | 93 + 22 | 89 + 11 | 23 + 8 | 27 - 14 | 50 - 9 | 09 \\ &\quad + 5 | 45 - 10 | 69 - 192 | 25 + 27 | 57 - 192 | 25 + 191 | 43 \\ &= 15232 | 25 - 7227 | 05 = 8005 | 20, \end{aligned}$$

or introducing the three zeros

$$I_{\cdot 19}(10 \cdot 5, 10 \cdot 5) = \cdot 000,8005 | 2,$$

which is as close to the tabled value  $\cdot 000,8006$  as we can hope to get from a mid point formula, and again confirms the view that a mid-panel formula is better than a mid-point when the interpolate is at the middle of the panel, the number of differences used being the same.

### *Lagrangian Type of Formulae.*

These are not so satisfactory for use in the case of the mid-point formulae, as in the case of the more compact and symmetrical mid-panel formulae. But if the interpolant is nearer to a point than to the middle of a panel, the mid-point will probably give the better result.

Starting with (xix),

$$\begin{aligned} I_{\cdot 19}(10 \cdot 5, 10 \cdot 5) &= z_{\frac{1}{2}, \frac{1}{2}} = \frac{1}{512} [240 \times 10,423 + 193 \left| \begin{array}{r} 3,538 \\ 17,309 \\ \hline 20,847 \end{array} \right| - 65 \left| \begin{array}{r} 29,498 \\ 5,949 \\ \hline 35,447 \end{array} \right| + 104 \times 6,154 \\ &\quad + 8 \times 17,723 - 40 \left| \begin{array}{r} 46,664 \\ 1,922 \\ \hline 48,586 \end{array} \right| - 28 \left| \begin{array}{r} 1,159 \\ 27,450 \\ \hline 28,609 \end{array} \right| + 20 \left| \begin{array}{r} 79,693 \\ 3,187 \\ \hline 82,880 \end{array} \right| - 7 \left| \begin{array}{r} 10,200 \\ 2,108 \\ \hline 12,308 \end{array} \right| \\ &\quad - \left| \begin{array}{r} 50,531 \\ 10,028 \\ \hline 60,559 \end{array} \right| + 3 \left| \begin{array}{r} 601 \\ 70,682 \\ \hline 71,283 \end{array} \right| + 5 \left| \begin{array}{r} 119,884 \\ 978 \\ \hline 120,862 \end{array} \right| + 3 \left| \begin{array}{r} 368 \\ 41,825 \\ \hline 42,193 \end{array} \right| - 3 \left| \begin{array}{r} 204,016 \\ 1,583 \\ \hline 205,599 \end{array} \right| \\ &= \frac{1}{512} (990,8954 - 581,2059) = 8001 | 7, \end{aligned}$$

or with the requisite zeros  $= \cdot 000,8002$ , agreeing with the result of (xviii) *bis*, as it must do. In the same manner (xix) *ter* gives us

$$z_{\frac{1}{2}, \frac{1}{2}} = \cdot 000,8004 | 1.$$

y, and here we may put down the values completely, to indicate the extent of the requisite work, *inter* gives us

$$(0.5, 10.5) = \frac{1}{4096} [1856 + 10,423 + 1,556] \begin{array}{|c|} \hline 3,538 \\ \hline 17,309 \\ \hline 20,847 \\ \hline \end{array} \begin{array}{|c|} \hline 524 \\ \hline 5,949 \\ \hline 35,447 \\ \hline \end{array} + 1016 \times 6154 + 96 \times 17,723$$

$$\begin{array}{|c|} \hline 364 \\ \hline 46,664 \\ \hline 48,586 \\ \hline \end{array} \begin{array}{|c|} \hline 1,922 \\ \hline 27,450 \\ \hline 28,609 \\ \hline \end{array} \begin{array}{|c|} \hline 280 \\ \hline 1,159 \\ \hline 3,187 \\ \hline \end{array} \begin{array}{|c|} \hline 184 \\ \hline 79,693 \\ \hline 82,880 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline 52 \\ \hline 41,825 \\ \hline \end{array} \begin{array}{|c|} \hline 368 \\ \hline -44 \\ \hline 42,193 \\ \hline \end{array} \begin{array}{|c|} \hline 204,016 \\ \hline 1,583 \\ \hline 205,599 \\ \hline \end{array}$$

$$116 \begin{array}{|c|} \hline 2,108 \\ \hline 10,200 \\ \hline 12,308 \\ \hline \end{array} \begin{array}{|c|} \hline 20 \\ \hline 50,531 \\ \hline 10,028 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline 44 \\ \hline 60,559 \\ \hline \end{array} \begin{array}{|c|} \hline 601 \\ \hline 70,682 \\ \hline 71,283 \\ \hline \end{array} \begin{array}{|c|} \hline 1 \\ \hline 76 \\ \hline 120,827 \\ \hline \end{array} \begin{array}{|c|} \hline 119,849 \\ \hline 978 \\ \hline \end{array} + 6 \times 3646 + 2 \times 30,279$$

$$4) \begin{array}{|c|} \hline 291 \\ \hline 173,075 \\ \hline 173,366 \\ \hline \end{array} \begin{array}{|c|} \hline 101 \\ \hline 16,209 \\ \hline 16,908 \\ \hline \end{array} \begin{array}{|c|} \hline 699 \\ \hline 2,136,866 \\ \hline 142,143 \\ \hline \end{array} \begin{array}{|c|} \hline 4 \\ \hline 182 \\ \hline 103,107 \\ \hline \end{array} \begin{array}{|c|} \hline -8 \\ \hline 291,042 \\ \hline 460 \\ \hline \end{array} \begin{array}{|c|} \hline 291,502 \\ \hline \end{array}$$

$$5) \begin{array}{|c|} \hline 114 \\ \hline 61,529 \\ \hline 61,643 \\ \hline \end{array} \begin{array}{|c|} \hline 5 \\ \hline 490,286 \\ \hline 716 \\ \hline \end{array} \begin{array}{|c|} \hline 491,002 \\ \hline \end{array}$$

$$\frac{1}{4096} [9249,0962 \quad 5970,1439] \quad 8005[25,$$

the zeros inserted  $I_{19}(10.5, 10.5) = .000,8005[25,$

with the result of (xviii) *quater*. The additions are not necessary and the whole work may be done in a continuous operation on the machine.

Now take the mid panel Lagrangian formulae in terms of the ordinates or tabular entries. These are convenient formulae (xx) *bis* and (xx) *quater*. Up to fifth order differences,

$$z_{4,4} = \frac{1}{512} [174 \begin{array}{|c|} \hline 10,423 \\ \hline 17,309 \\ \hline 3,538 \\ \hline 6,154 \\ \hline 37,424 \\ \hline \end{array} \begin{array}{|c|} \hline 27 \\ \hline 46,664 \\ \hline 27,450 \\ \hline 10,200 \\ \hline 2,108 \\ \hline \end{array} \begin{array}{|c|} \hline 29,498 \\ \hline 119,849 \\ \hline 41,825 \\ \hline 16,209 \\ \hline 699 \\ \hline \end{array} \begin{array}{|c|} \hline 3 \\ \hline 79,693 \\ \hline 82,880 \\ \hline 16,209 \\ \hline 699 \\ \hline \end{array} \begin{array}{|c|} \hline 2 \\ \hline 70,682 \\ \hline 3,646 \\ \hline 601 \\ \hline 17,723 \\ \hline \end{array} \begin{array}{|c|} \hline 124,950 \\ \hline 262,808 \\ \hline \end{array}]$$

$$+ \frac{1}{512} [651,1776 \quad 337,3650 + 78,8424 + 18,5304]$$

$$+ \frac{1}{512} (411,1854) \quad 8031,$$

zeros inserted  $z_{4,4} = .000,8031.$

It is precisely the value which (xvii) *bis* gives, only the work is far less laborious than finding 24 second differences. But it only gives the value correct to five decimal places.

Now apply (xx) *quater* and find

$$z_{4,4} = \frac{1}{4096} [1454 \times 37,424 - 276 \times 124,950 + 52 \times 262,808$$

$$+ 34 \times 92,652 - 5 \times 204,016 - 3 \times 173,075] \\ \begin{array}{|c|} \hline 291,042 \\ \hline 61,529 \\ \hline 24,820 \\ \hline 225 \\ \hline 114 \\ \hline 460 \\ \hline 1,583 \\ \hline 583,789 \\ \hline \end{array} \begin{array}{|c|} \hline 103,107 \\ \hline 6,033 \\ \hline 1,258 \\ \hline 182 \\ \hline 291 \\ \hline 10,028 \\ \hline 50,531 \\ \hline 344,505 \\ \hline \end{array}$$

$$= \frac{1}{4096} (5441,4496 - 3448,6200 + 1366,6016 + 315,0168 - 291,8945 - 103,3515)$$

$$= \frac{1}{4096} (3279,2020) = .000,8005[9,$$

giving the value  $\cdot 000,8006$ , correct to seven decimal places, and agreeing with what one finds from (xvii) *ter* but with far less labour.

The last two results again indicate that interpolation formulae up to  $\delta^4$  will at this part of the table only give accuracy to five decimal places (but of course may be used if five places are adequate), but that formulae up to  $\delta^8$  will give the same accuracy to the interpolate as the interpolants themselves possess. Further, the reader will find with very little experience that (xx) *bis* and (xx) *quater* demand far less labour than (xvii) *bis* and (xvii) *ter*, to say nothing of (xvii) itself.

To work out (xx) *quater* demands, as our example indicates, so little extra work on (xx) *bis*, that even when we want to find the degree of approximation involved in stopping at  $\delta^4$ , it is easier to find (xx) *bis* and (xx) *quater* than to deal with the successive terms in (xvii).

The object of this section of the Introduction has been principally to indicate that when we leave off the 0.5 changes in argument of the table, we require terms up to  $\delta^8$ , to get seven-figure accuracy, but terms to  $\delta^4$  will give five-figure accuracy. The Lagrangians (xx) *bis* and (xx) *quater* are the easier formulae to use, if we want  $I_x(i+0.5, i'+0.5)$ . But in other cases than this particular one we should have to use formula (iv) of *Tracts for Computers*, No. III, and this use is laborious.

(γ) *Univariate Diagonal Formulae to find  $I_x(i+0.5, i'+0.5)$ .*

The reader may ask whether there is no easier method of reaching the value of an interpolate for such a simple case as  $I_x(i+0.5, i'+0.5)$  than these complicated bivariate formulae. We reply: Certainly. They have only been used in the present instances to test how far it is needful to take the differences if we require to go to five, six or seven decimal place accuracy in the general case  $I_x(p, q)$ . If  $p$  and  $q$  are of the form  $i+0.5$  and  $i'+0.5$ , then the interpolate lies on a diagonal of interpolants and we may proceed effectively by univariate formulae. As there will be two diagonals passing through the required interpolate, we have a choice of left-upper to right-lower diagonal and right-upper to left-lower diagonal, and desire to know which it is better to use.

We will start with our example of  $I_{.19}(10.5, 10.5)$ .

*Left-upper to right-lower Diagonal.*

The univariate formula to be used shall be the mid-panel one

$$z_{\frac{1}{2}, \frac{1}{2}} = \frac{1}{2}(z_{0,0} + z_{1,1}) - \frac{1}{16}(\delta^2 z_{0,0} + \delta^2 z_{1,1}) + \frac{3}{256}(\delta^4 z_{0,0} + \delta^4 z_{1,1}) - \frac{5}{2048}(\delta^6 z_{0,0} + \delta^6 z_{1,1}) + \frac{35}{65536}(\delta^8 z_{0,0} + \delta^8 z_{1,1}) - \frac{63}{324288}(\delta^{10} z_{0,0} + \delta^{10} z_{1,1}) \dots \dots \dots \text{(xxi)}$$

Our interpolants and their differences are as follows:

		$z$	$\delta^2$	$\delta^4$	$\delta^6$	$\delta^8$
$I_{.19}(6, 6)$	$z_{-4, -4}$	90,095				
$I_{.19}(7, 7)$	$z_{-3, -3}$	52,035	16,304			
$I_{.19}(8, 8)$	$z_{-2, -2}$	30,279	9,200	3,160		
$I_{.19}(9, 9)$	$z_{-1, -1}$	17,723	5,256	1,719	677	
$I_{.19}(10, 10)$	$z_{0,0}$	10,423	3,031	955	346	174
$I_{.19}(11, 11)$	$z_{1,1}$	6,154	1,761	537	189	68
$I_{.19}(12, 12)$	$z_{2,2}$	3,646	1,028	308	100	
$I_{.19}(13, 13)$	$z_{3,3}$	2,166	603	179		
$I_{.19}(14, 14)$	$z_{4,4}$	1,289	357			
$I_{.19}(15, 15)$	$z_{5,5}$	769				

$$\begin{aligned} z_{\frac{1}{2}, \frac{1}{2}} &= \frac{1}{2}(16577) - \frac{1}{16}(4792) + \frac{3}{256}(1492) - \frac{5}{2048}(535) + \frac{35}{65536}(242) \\ &= 8288|5 - 299|5 + 17|73 - 1|3 + 0|13, \\ &= \cdot 000,8288|5 \text{ by linear interpolation,} \\ &= \cdot 000,7989 \text{ up to third differences,} \\ &= \cdot 000,8006|73 \text{ up to fifth differences,} \\ &= \cdot 000,8005|43 \text{ up to seventh differences,} \\ &= \cdot 000,8005|56, \text{ up to ninth differences, or the correct value, i.e.} \\ &= \cdot 000,8006, \text{ up to seven decimal places using } \delta^8. \end{aligned}$$

taking the value up to  $\delta^4$  would be adequate for most practical purposes.

*upper to left lower Diagonal.*

Following are the interpolant and their central differences:

	$x$	$\delta$	$\delta^2$	$\delta^4$	$\delta^6$	$\delta^8$	$\delta^{10}$
5)	$x_{-1}$	0					
6)	$x_{-1/2}$	1	8				
7)	$x_0$	10	65	321			
8)	$x_{1/2}$	84	443	1,599	3,560		
9)	$x_1$	601	2,420	6,437	9,079	+42	
10)	$x_{3/2}$	3,538	10,834	20,354	14,640	-15,575	-8,909
11)	$x_2$	17,309	39,602	48,911	4,626	-40,101	+39,623
2)	$x_{1/2}$	50,682	117,281	82,094	45,489	-25,004	
3)	$x_1$	241,336	277,054	69,788	120,608		
4)	$x_{3/2}$	689,044	506,615	63,126			
5)	$x_2$	1,643,367	673,050				
6)	$x_{5/2}$	3,270,740					

we see that the differences here are very large and varying rapidly. The success of the application of central difference formula will depend entirely on the rapid convergency of its coefficients. We have

$$z_{4,3} = \frac{1}{2}(208474) - \frac{1}{16}(504306) + \frac{1}{128}(692963) - \frac{1}{2048}(19266) + \frac{35}{65536}(-55676) - \frac{63}{524288}(30714) \\ 10423.5 - 3132.25 + 541.70 - 47.04 - 29.73 + 3.69,$$

$z_{4,3} = .001,0423$  by linear interpolation,

$.000,7271$  up to third differences,

$.000,8083$  up to fifth differences,

$.000,8036$  up to seventh differences,

$.000,8006$  up to ninth differences,

$.000,8007$  up to eleventh differences,

interpolate is worse up to eleventh than it is to ninth differences, which give the correct value. But knowledge of that value we have no reason for stopping at that point and we are ignorant of what further differences would produce.

As that at this part of the table the left upper to right lower diagonal gives a better system of values and a far more rapid approach to the correct value,

we try a similar problem further on in the table and determine  $I_{58}(40.5, 21.5)$ . This is a value midway  $I_{58}(40, 21)$  and  $I_{58}(41, 22)$ .

*upper to right lower Diagonal.*

	$x$	$\delta^2$	$\delta^4$
$I_{58}(36, 17)$		-0650,895	
$I_{58}(37, 18)$		-0751,282	5924
$I_{58}(38, 19)$		-0857,593	5437
$I_{58}(39, 20)$		-0969,341	4943
$I_{58}(40, 21)$		-1086,032	4453
$I_{58}(41, 22)$		-1207,176	3967
$I_{58}(42, 23)$		-1332,287	3498
$I_{58}(43, 24)$		-1460,896	3043
$I_{58}(44, 25)$		-1592,548	2608
$I_{58}(45, 26)$		-1726,808	

$$z_{4,3} = I_{58}(40.5, 21.5) = \frac{1}{2}(-2293,208) - \frac{1}{16}(8420) + \frac{3}{256}(21) \\ = -1146,604 - .0000,526 \cdot 25 + .0000,000 \cdot 25 \\ = -1146,078.$$

This is very satisfactory, the  $\delta^4$  terms have become so small as to be irregular, and they are negligible; accordingly the answer is given by using merely the  $\delta^2$  and this involves taking only four interpolants out of the table. Thus we have good omen of the degree of accuracy that can be obtained by using only  $\delta^2$  at this part of the table. We now turn to the other diagonal.

*Right-upper to left-lower Diagonal.*

	$z$	$\delta^2$	$\delta^4$	$\delta^6$	$\delta^8$	$\delta^{10}$
$I_{.58}(35, 27)$	·5927,339					
$I_{.58}(36, 26)$	·4903,777	2,707				
$I_{.58}(37, 25)$	·3882,922	68,321	-12,184			
$I_{.58}(38, 24)$	·2930,388	121,751	-20,993	5881		
$I_{.58}(39, 23)$	·2099,605	154,188	-23,921	6035	2133	
$I_{.58}(40, 22)$	·1423,010	162,704	-20,814	4056	921	25
$I_{.58}(41, 21)$	·0909,119	150,406	-13,651	1156	316	313
$I_{.58}(42, 20)$	·0545,636	124,457	-5,332	1428	1240	
$I_{.58}(43, 19)$	·0306,610	93,176	+1,559	-2772		
$I_{.58}(44, 18)$	·0160,760	63,454	+5,678			
$I_{.58}(45, 17)$	·0078,364	39,410				
$I_{.58}(46, 16)$	·0035,378					

$$z_{4,4} = I_{.58}(40.5, 21.5) = \frac{1}{2}(2332,129) - \frac{1}{16}(313,110) + \frac{3}{256}(-34465) - \frac{5}{2048}(5212) \\ + \frac{35}{65536}(-605) - \frac{63}{524288}(-288) \\ = .1166,064|5 - 19569|37 - 403|89 - 12|75 - 0|32 + 0|03.$$

We have accordingly  $I_{.58}(40.5, 21.5) = .1166,064|5$  by linear interpolation,  
 $= .1146,495|13$  up to third differences,  
 $= .1146,091|24$  up to fifth differences,  
 $= .1146,078|49$  up to seventh differences,  
 $= .1146,078|17$  up to ninth differences,  
 $= .1146,078|20$  up to eleventh differences.

Thus we do not get the correct answer this way without including  $\delta^6$  or  $\delta^8$ . Accordingly we conclude that to obtain  $I_x(i + \frac{1}{2}, i' + \frac{1}{2})$  we should work with the left-upper to the right-lower diagonal, which will be found far shorter than using the other diagonal. Near the borders of the table we should use the same diagonal, but proceed by forward or backward differences as the case may be.

#### (δ) Diagonal Interpolation.

The effectiveness of the interpolation from the left-upper to the right-lower diagonal leads us to investigate another general method of interpolating into the incomplete B-function. We have seen that the differences converge more rapidly along a vertical than a horizontal line in our diagram (Fig. 3) and much more rapidly than either along the left-upper to the right-lower diagonal. The latter line as it alone concerns us here, we will briefly term the "diagonal."

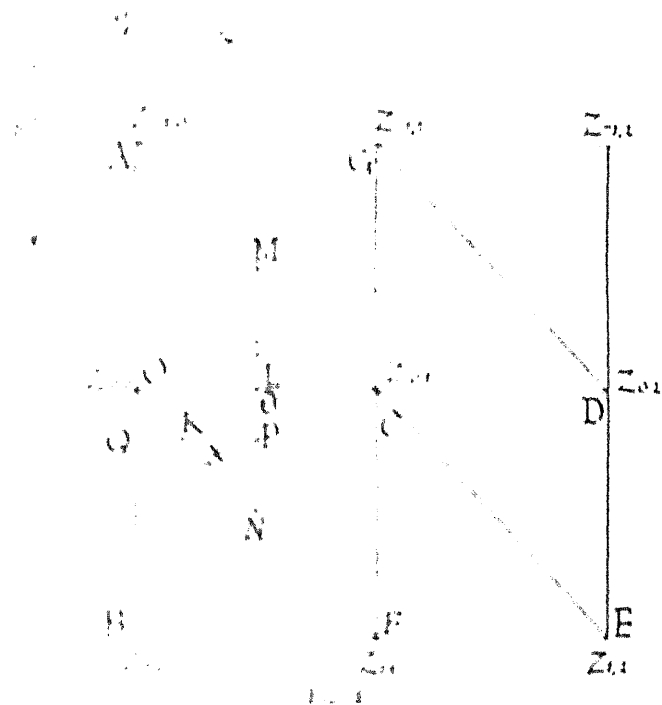
Now if we have rectangular axes and the position of the interpolate be given in the usual way by  $\theta, \chi$ , the value of  $z_{\theta, \chi}$  is given by the expression (xxii) below up to fifth order differences when we replace those differences by the tabulated interpolants:

$$z_{\theta, \chi} = \frac{1}{12} \{ (6 + \theta\phi + \chi\psi) (2 + \theta\phi + \chi\psi) + \theta\phi\chi\psi \} \{ \phi\psi z_{0,0} + \phi\chi z_{0,1} + \theta\chi z_{1,1} + \theta\psi z_{1,0} \} \\ - \frac{1}{4} \theta\phi (1 + \frac{1}{6}\theta\phi + \frac{1}{3}\chi\psi) \{ (1 + \phi) (\psi z_{-1,0} + \chi z_{-1,1}) + (1 + \theta) (\psi z_{2,0} + \chi z_{2,1}) \} \\ - \frac{1}{4} \chi\psi (1 + \frac{1}{6}\chi\psi + \frac{1}{3}\theta\phi) \{ (1 + \chi) (\phi z_{0,2} + \theta z_{1,2}) + (1 + \psi) (\theta z_{1,-1} + \phi z_{0,-1}) \} \\ + \frac{1}{120} \theta\phi (1 + \theta) (1 + \phi) \{ (2 + \phi) (\psi z_{-2,0} + \chi z_{-2,1}) + (2 + \theta) (\chi z_{3,1} + \psi z_{3,0}) \} \\ + \frac{1}{120} \chi\psi (1 + \chi) (1 + \psi) \{ (2 + \psi) (\theta z_{1,-2} + \phi z_{0,-2}) + (2 + \chi) (\phi z_{0,3} + \theta z_{1,3}) \} \\ + \frac{1}{36} \theta\phi\chi\psi (1 + \phi) \{ (1 + \psi) z_{-1,-1} + (1 + \chi) z_{-1,2} \} \\ + \frac{1}{36} \theta\phi\chi\psi (1 + \theta) \{ (1 + \chi) z_{2,2} + (1 + \psi) z_{2,-1} \} \\ \dots\dots (xxii).$$

be found to agree with (xxv) if we put  $\theta = \phi + \frac{1}{2}(\chi - \psi)$ . In other words, we have in (xxvii) will have none of its ordinates (i.e. interpolants) modified if we transform parallel to the "horizontal" or "vertical" of the diagram, Fig. 2 (p. xix). In other words interpolation is tantamount to interpolation if we use oblique instead of rectangular axes. All we need to note the correct values of  $\theta$  and  $\chi$ . If  $\theta, \phi, \chi, \psi$  be the values for rectangular axes and  $\theta', \chi'$  for the oblique axes, all we need for the relation between these values,

which slide along the  $z$ -axis in the Prism  $z_{0,0}, z_{0,1}, z_{1,1}(X + \theta)$ .

important feature of the transformation. Taking the argument changes in the original unit  $\theta$  and  $\chi$  give the plan  $P$  of the interpolate. Instead of the directions  $AOB$  and  $OGD$  of  $\theta$ , we propose to take the directions  $OPD$  and  $ACE$ . We give a uniform slide to the plan of the



that  $P$  slides along  $AB$  and  $Q$  slides along  $AC$ . The line  $OP$  takes the place of the rectangle  $OCFB$ . No change in the direction of the horizontal lines will remain the same. Clearly  $\theta' = \angle OPB = \theta$ , and  $\phi' = \angle OPQ = \phi$ .

$$OP = AB \cdot \cos \theta = AC \cdot \cos \phi = OL \cdot \cos \chi = \theta.$$

$$\cos \chi = \frac{1}{2}(\cos \theta + \cos \phi) = \frac{1}{2}(\chi + \theta).$$

difficultly, we can transform the  $z$ 's of the new diagram into the  $z$ 's of the table. The diagram (xxvii) will show the relation between the formula  $z$ 's and the tabulated values.

It must be supposed that the formula (xxii) to have a dash affixed to them and Fig. 5 will provide the tabular values. To save the reader labour we will repeat formula (xxii) with the dashed  $z$ 's the corresponding dashed or tabulated values. No simplicity is obtained by replacing the  $z$ 's by their values in terms of  $\theta, \phi, \chi, \psi$  at least to substitute their numerical values as deduced from the above, namely,

$$\theta = \theta, \phi = \phi, \chi = \chi, \psi = \psi, \theta' = \theta, \phi' = \phi, \chi' = \chi, \psi' = \psi \quad \dots (xxiii).$$

in the triangle  $OPQ$ , i.e. between  $z_{0,0}, z_{0,1}$  and  $z_{1,1}$ .



We have accordingly for  $\chi > \theta$  the following:

*Formula for Horizontal Slide when the Interpolate lies within the Prism with edges  $z_{0,0}, z_{0,1}, z_{1,1} (\chi' > \theta)$ .*

$$\begin{aligned}
 z_{\theta, \chi} = & \frac{1}{12} \{ (6 + \theta' \phi' + \chi' \psi') (2 + \theta' \phi' + \chi' \psi') + \theta' \phi' \chi' \psi' \} \{ \phi' \psi' z_{0,0} + \phi' \chi' z_{0,1} + \theta' \chi' z_{1,2} + \theta' \psi' z_{1,1} \} \\
 & - \frac{1}{4} \theta' \phi' (1 + \frac{1}{6} \theta' \phi' + \frac{1}{3} \chi' \psi') \{ (1 + \phi') (\psi' z_{-1,-1} + \chi' z_{-1,0}) + (1 + \theta') (\psi' z_{2,2} + \chi' z_{2,3}) \} \\
 & - \frac{1}{4} \chi' \psi' (1 + \frac{1}{6} \chi' \psi' + \frac{1}{3} \theta' \phi') \{ (1 + \chi') (\phi' z_{0,2} + \theta' z_{1,3}) + (1 + \psi') (\theta' z_{1,0} + \phi' z_{0,-1}) \} \\
 & + \frac{1}{120} \theta' \phi' (1 + \theta') (1 + \phi') \{ (2 + \phi') (\psi' z_{-2,-2} + \chi' z_{-2,-1}) + (2 + \theta') (\chi' z_{3,4} + \psi' z_{3,3}) \} \\
 & + \frac{1}{120} \chi' \psi' (1 + \chi') (1 + \psi') \{ (2 + \psi') (\theta' z_{1,-1} + \phi' z_{0,-2}) + (2 + \chi') (\phi' z_{0,3} + \theta' z_{1,4}) \} \\
 & + \frac{1}{36} \theta' \phi' \chi' \psi' (1 + \phi') \{ (1 + \psi') z_{-1,-2} + (1 + \chi') z_{-1,1} \} \\
 & + \frac{1}{36} \theta' \phi' \chi' \psi' (1 + \theta') \{ (1 + \chi') z_{2,4} + (1 + \psi') z_{2,1} \} \dots\dots (xxiv).
 \end{aligned}$$

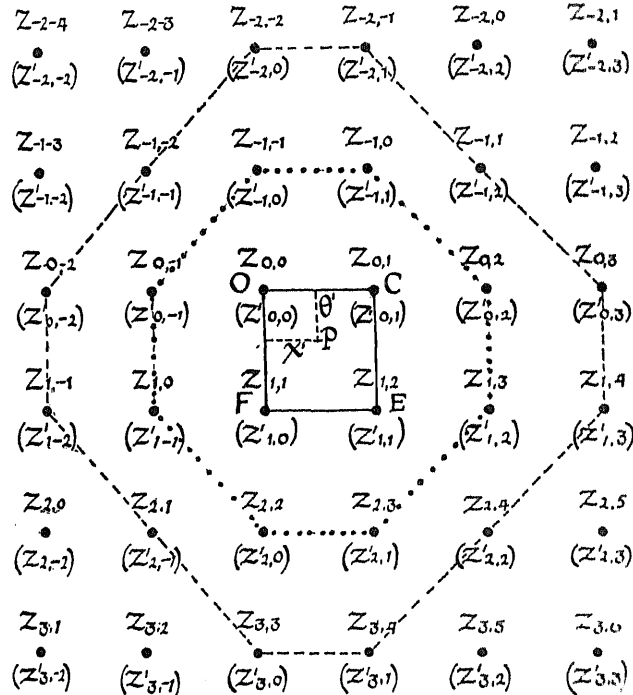


Fig. 5. Horizontal slide for an Interpolate within the prism  $z_{0,0}, z_{0,1}, z_{1,1} (\chi > \theta)$ . Interchange of Interpolants.

To test the accuracy with which this formula provides a result we will take a fairly difficult part of the table where we can use a tabulated value, but we will find it from interpolants differing by 1 and not by 0.5. Let us determine  $I_{19}(10.5, 11)$ .

Here  $\theta = \frac{1}{2}$ ,  $\phi = \frac{1}{2}$ ,  $\chi = 1$ ,  $\psi = 0$ , and accordingly

$$\theta' = \frac{1}{2}, \quad \phi' = \frac{1}{2}, \quad \chi' = \frac{1}{2}, \quad \psi' = \frac{1}{2}.$$

Thus the numerical coefficients are the same as those of formula (xx), and we have

$$\begin{aligned}
 z_{\theta, \chi} = & \frac{1}{512} \{ 174(z_{0,0} + z_{0,1} + z_{1,2} + z_{1,1}) - 27(z_{-1,-1} + z_{-1,0} + z_{2,2} + z_{2,3} + z_{0,2} + z_{1,3} + z_{1,0} + z_{0,-1}) \\
 & + 3(z_{-2,-2} + z_{-2,-1} + z_{3,4} + z_{3,3} + z_{1,-1} + z_{0,-2} + z_{0,3} + z_{1,4}) + 2(z_{-1,-2} + z_{-1,1} + z_{2,4} + z_{2,1}) \} \dots\dots (xxiv) bis.
 \end{aligned}$$

While the coefficients in (xxiv) bis are the same as in (xx), the distribution of interpolants is very different. The actual value of  $I_{19}(10.5, 11)$  as given by the Tables is  $1 - I_{81}(11, 10.5) = 1 - .9989,628 = .0010,372$ .

(xxiv) *bis* we have

$$I_{49}(10.5, 11) = \frac{1}{512} [174 \begin{vmatrix} 10,423 \\ 17,309 \\ 10,200 \\ 6,154 \\ 44,086 \end{vmatrix} - 27 \begin{vmatrix} 17,723 \\ 29,498 \\ 3,646 \\ 6,033 \\ 27,450 \\ 16,209 \\ 3,538 \\ 5,949 \\ 110,046 \end{vmatrix} + 3 \begin{vmatrix} 30,279 \\ 50,531 \\ 3,578 \\ 2,166 \\ 1,922 \\ 3,187 \\ 41,825 \\ 24,820 \\ 158,308 \end{vmatrix} + 2 \begin{vmatrix} 10,028 \\ 46,664 \\ 9,602 \\ 2,108 \\ 68,402 \end{vmatrix}]$$

$$\frac{1}{512} [767,0964 - 297,1242 + 474,924 + 136,804]$$

$$= .0010,373|9,$$

for fifth differences we have an error of two in the seventh decimal place.

To take a univariate interpolation for  $p$  to find  $I_{49}(10.5, 11)$  we have for its values

$$\begin{aligned} &.0010,397|6 \text{ up to fifth differences,} \\ &.0010,372|8 \text{ up to seventh differences,} \\ &.0010,371|3 \text{ up to ninth differences,} \\ &.0010,372|2 \text{ up to eleventh differences,} \end{aligned}$$

is the correct value.

For most practical values the bivariate Lagrangian up to fifth differences is not only easier to use, superior to the univariate Everett formula to the same number of differences; the latter formula gives the correct value with the eleventh differences, but is only a unit out in the seventh decimal place, and needs to either seventh or ninth differences.

There is no doubt of the satisfactory character of (xxiv) for interpolations with six-figure accuracy apart from the table.

*Formula for Horizontal Slide when the Interpolate lies within the Prism with edges  $z_{0,0}$ ,  $z_{1,1}$ ,  $z_{1,0}$  ( $\theta > \chi$ ).*

$$\begin{aligned} &\frac{1}{12} \{ (6 + \theta' \phi' + \chi' \phi') (2 + \theta' \phi' + \chi' \phi') + \theta' \phi' \chi' \phi' + \phi' \psi' z_{0,-1} + \phi' \chi' z_{0,0} + \theta' \chi' z_{1,1} + \theta' \psi' z_{1,0} \} \\ &+ \frac{1}{4} \theta' \phi' (1 + \frac{1}{2} \theta' \phi' + \frac{1}{2} \chi' \phi') (1 + \phi') (\psi' z_{-1,-2} + \chi' z_{-1,-1}) + (1 + \theta') (\psi' z_{2,1} + \chi' z_{2,2}) \\ &+ \frac{1}{4} \chi' \phi' (1 + \frac{1}{2} \chi' \phi' + \frac{1}{2} \theta' \phi') (1 + \chi') (\phi' z_{0,1} + \theta' z_{1,2}) + (1 + \psi') (\theta' z_{1,-1} + \phi' z_{0,-2}) \\ &+ \frac{1}{12} \theta' \phi' (1 + \theta') (1 + \phi') \{ (2 + \phi') (\psi' z_{-2,-3} + \chi' z_{-2,-2}) + (2 + \theta') (\chi' z_{3,3} + \psi' z_{3,2}) \} \\ &+ \frac{1}{12} \chi' \phi' (1 + \chi') (1 + \phi') \{ (2 + \chi') (\phi' z_{0,2} + \theta' z_{1,3}) + (2 + \psi') (\theta' z_{1,-2} + \phi' z_{0,-3}) \} \\ &+ \frac{1}{20} \theta' \phi' \chi' \phi' (1 + \phi') \{ (1 + \psi') z_{-1,-3} + (1 + \chi') z_{-1,0} \} \\ &+ \frac{1}{20} \theta' \phi' \chi' \phi' (1 + \theta') \{ (1 + \phi') z_{2,0} + (1 + \chi') z_{2,3} \} \end{aligned} \quad \dots\dots (xxv).$$

$$\theta' = \theta, \phi' = \phi, \chi' = \chi, \psi' = \theta - \chi (\theta > \chi).$$

*Formula for Vertical Slide when the Interpolate lies in the Prism  $z_{0,0}$ ,  $z_{0,1}$ ,  $z_{1,1}$  ( $\chi > \theta$ ).*

In exactly the same manner we may give a vertical instead of a horizontal slide. Which slide will be needed depends on whether the differences converge more rapidly along the horizontal (or  $q$ ) direction, or the vertical (or  $p$ ) direction. And this again may depend on the value of  $x$ .

The nature of the slide is indicated in the figure (Fig. 6) on p. xxx. If the plan  $P$  of the interpolate is the triangle  $z_{0,0}$ ,  $z_{0,1}$ ,  $z_{1,1}$ , then we must slide the parallelogram  $z_{-1,0}$ ,  $z_{0,1}$ ,  $z_{1,1}$ ,  $z_{0,0}$  vertically upwards. The vertical side  $z_{0,1}$ ,  $z_{1,1}$  takes the place of  $z_{-1,1}$ ,  $z_{0,1}$ . Of course, every vertical line of the figure to the left of the line  $z_{-1,0}$ ,  $z_{0,0}$ ,  $z_{1,0}$ ,  $z_{2,0}$  is slid upwards, and every vertical line to the left downwards, until the left upper to right lower diagonals become horizontal. Clearly this will not change the  $\chi$  and  $\psi$ . Let the new  $\phi$ , or  $\phi' = NM = PM = OM = PM = \chi - \theta$ , with the condition  $\chi > \theta$ . Thus we have

$$\theta' = \psi + \theta, \phi' = \chi - \theta, \chi' = \chi, \psi' = \psi.$$

## VERTICAL SLIDE INTERPOLATION

On the other hand if the interpolate be at  $P'$  within the prism  $z_{0,0}, z_{1,1}, z_{1,0}$ , we have to deal with the parallelogram  $z_{0,0}, z_{1,1}, z_{2,1}, z_{1,0}$  and  $z_{2,1}, z_{1,1}$  is shifted vertically upwards so that  $z_{2,1}, z_{1,1}$  takes the place of  $z_{1,1}, z_{0,1}$ . In this case  $\chi$  and  $\psi$  will not be changed by the slide, but the new  $\theta$  is given by

$$\theta' = P'N' = P'M' - N'M' = \theta - OM' = \theta - \chi$$

and  $\phi' = \phi + \chi$ . Thus we have with the condition  $\theta > \chi$

$$\theta' = \theta - \chi, \quad \phi' = \phi + \chi, \quad \chi' = \chi, \quad \psi' = \psi.$$

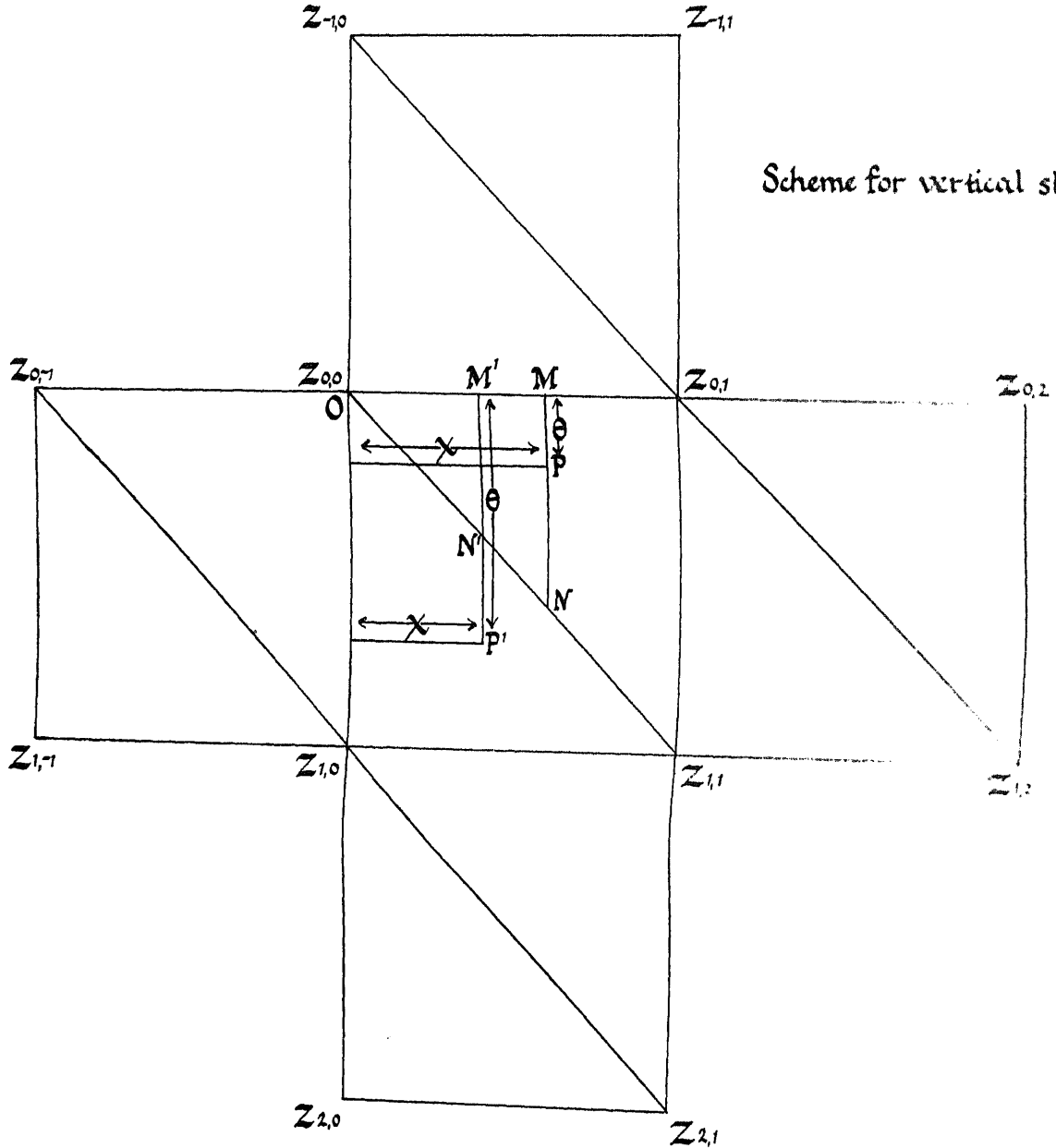


Fig. 6.

If  $\theta < \chi$  we have to slide the parallelogram  $z_{0,0}, z_{-1,0}, z_{0,1}, z_{1,1}$ , so that  $z_{0,0}, z_{-1,0}$  remaining stationary  $z_{1,1}, z_{0,1}$  take the place of  $z_{0,1}, z_{-1,1}$ . If  $\theta > \chi$  it is the parallelogram  $z_{0,0}, z_{1,1}, z_{2,1}, z_{1,0}$  which has to be slid into the rectangle  $z_{0,0}, z_{0,1}, z_{1,1}, z_{1,0}$ , while  $z_{1,0}, z_{0,0}$  remains stationary. Thus no change is made in the subscripts in the vertical through  $z_{0,0}$ , a change of a unit in the first subscript of  $z$ 's on the vertical through  $z_{0,1}$ , a change of two units in the first subscript of  $z$ 's on the vertical through  $z_{0,2}$  and so on. From the  $z$ 's on verticals to the left of the vertical through  $z_{0,0}$ , the corresponding changes in the first subscript are negative.

have accordingly the following formulae:

*Formula for Vertical Slide when the Interpolate lies within the Prism with edges  $z_{0,0}, z_{0,1}, z_{1,1}$  ( $\chi > \theta$ ).*

$$\begin{aligned}
 & \frac{1}{12} \{ (6 + \theta' \phi' + \chi' \psi') (2 + \theta' \phi' + \chi' \psi') + \theta' \phi' \chi' \psi' \} \{ \phi' \psi' z_{-1,0} + \phi' \chi' z_{0,1} + \theta' \chi' z_{1,1} + \theta' \psi' z_{0,0} \} \\
 & - \frac{1}{4} \theta' \phi' (1 + \frac{1}{6} \theta' \phi' + \frac{1}{3} \chi' \psi') \{ (1 + \phi') (\psi' z_{-2,0} + \chi' z_{-1,1}) + (1 + \theta') (\psi' z_{1,0} + \chi' z_{2,1}) \} \\
 & - \frac{1}{4} \chi' \psi' (1 + \frac{1}{6} \chi' \psi' + \frac{1}{3} \theta' \phi') \{ (1 + \chi') (\phi' z_{1,2} + \theta' z_{2,2}) + (1 + \psi') (\theta' z_{-1,-1} + \phi' z_{-2,-1}) \} \\
 & + \frac{1}{120} \theta' \phi' (1 + \theta') (1 + \phi') \{ (2 + \phi') (\psi' z_{-3,0} + \chi' z_{-2,1}) + (2 + \theta') (\chi' z_{3,1} + \psi' z_{2,0}) \} \\
 & + \frac{1}{120} \chi' \psi' (1 + \chi') (1 + \psi') \{ (2 + \psi') (\theta' z_{-2,-2} + \phi' z_{-3,-2}) + (2 + \chi') (\phi' z_{2,3} + \theta' z_{3,3}) \} \\
 & + \frac{1}{36} \theta' \phi' \chi' \psi' (1 + \phi') \{ (1 + \psi') z_{-3,-1} + (1 + \chi') z_{0,2} \} \\
 & + \frac{1}{36} \theta' \phi' \chi' \psi' (1 + \theta') \{ (1 + \chi') z_{3,2} + (1 + \psi') z_{0,-1} \} \\
 & \theta' = \psi + \theta, \phi' = \chi - \theta, \chi' = \chi, \psi' = \psi.
 \end{aligned}
 \tag{xxvi}.$$

*Formula for Vertical Slide when the Interpolate lies within the Prism with edges  $z_{0,0}, z_{1,1}, z_{1,0}$  ( $\theta > \chi$ ).*

$$\begin{aligned}
 & \frac{1}{12} \{ (6 + \theta' \phi' + \chi' \psi') (2 + \theta' \phi' + \chi' \psi') + \theta' \phi' \chi' \psi' \} \{ \phi' \psi' z_{0,0} + \phi' \chi' z_{1,1} + \theta' \chi' z_{2,1} + \theta' \psi' z_{1,0} \} \\
 & - \frac{1}{4} \theta' \phi' (1 + \frac{1}{6} \theta' \phi' + \frac{1}{3} \chi' \psi') \{ (1 + \phi') (\psi' z_{-1,0} + \chi' z_{0,1}) + (1 + \theta') (\psi' z_{2,0} + \chi' z_{3,1}) \} \\
 & - \frac{1}{4} \chi' \psi' (1 + \frac{1}{6} \chi' \psi' + \frac{1}{3} \theta' \phi') \{ (1 + \chi') (\phi' z_{2,2} + \theta' z_{3,2}) + (1 + \psi') (\theta' z_{0,-1} + \phi' z_{-1,-1}) \} \\
 & + \frac{1}{120} \theta' \phi' (1 + \theta') (1 + \phi') \{ (2 + \phi') (\psi' z_{-2,0} + \chi' z_{-1,1}) + (2 + \theta') (\chi' z_{1,1} + \psi' z_{3,0}) \} \\
 & + \frac{1}{120} \chi' \psi' (1 + \chi') (1 + \psi') \{ (2 + \psi') (\theta' z_{-1,-2} + \phi' z_{-2,-2}) + (2 + \chi') (\phi' z_{3,3} + \theta' z_{4,3}) \} \\
 & + \frac{1}{36} \theta' \phi' \chi' \psi' (1 + \phi') \{ (1 + \psi') z_{-2,-1} + (1 + \chi') z_{1,2} \} \\
 & + \frac{1}{36} \theta' \phi' \chi' \psi' (1 + \theta') \{ (1 + \chi') z_{4,2} + (1 + \psi') z_{1,-1} \} \\
 & \theta' = \theta - \chi, \phi' = \phi + \chi, \chi' = \chi, \psi' = \psi.
 \end{aligned}
 \tag{xxvii}.$$

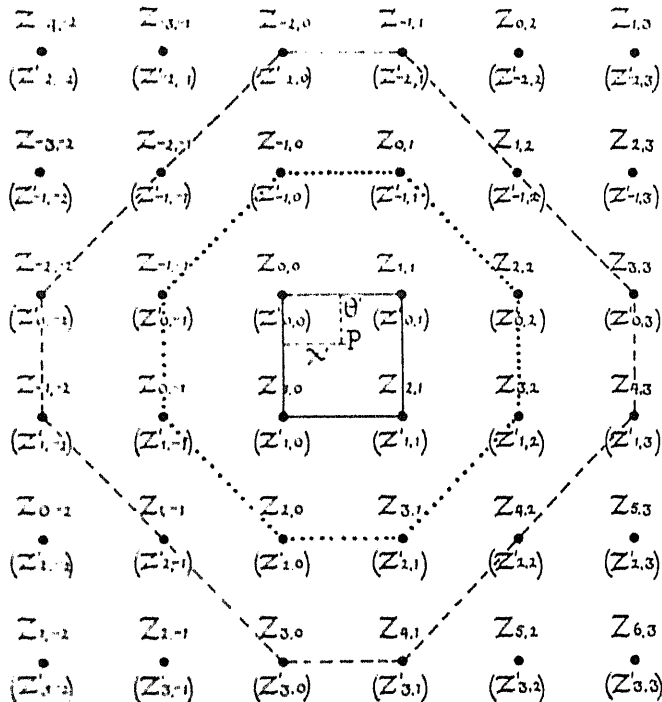


Fig. 7. Vertical slide for an Interpolate within the prism  $z_{0,0}, z_{1,1}, z_{1,0}$  ( $\theta > \chi$ ). Interchange of Interpolants.

The user of this table must be careful to distinguish between these formulae and determine before use in which prism the interpolate lies, and whether it will be better to use a horizontal or vertical slide formula. To the illustration of this point we shall devote the following section.

(e) *Comparison of the usual Everett and the new Slide Formulae on a numerical Example.*

We will take values very early in the table and test in this region the accuracy of the various formulae, which can be applied. We have already stated that values in the case of  $I$ - and  $J$ -curves require a special table for smaller values of the arguments. We choose  $p=4.3$  and  $q=3.1$  and will determine from the table:  $I_{.4}(4.3, 3.1)$  and  $I_{.8}(4.3, 3.1)$ , the exact values of which are  $.1586, 9761$  and  $.8979, 1816$  respectively.

*Table of Values to be extracted for the Determination of  $I_{.4}(4.3, 3.1)$  and  $I_{.8}(4.3, 3.1)$  by three Formulae*

(—, 13, —) $z_{-2, -3}$ *	(—, 14, 18) $z_{-2, -2}$ *	(—, —, 21) $z_{-2, -1}$ *	(13, —, 13) $z_{-2, 0}$ *	(14, —, ) $z_{-2, 1}$ *		
$I(3, 1.5)$ .1169,598 .6958,948	$I(3, 2)$ .1792,000 .8192,000	$I(3, 2.5)$ .2470,920 .8962,464	$I(3, 3)$ .3174,400 .9420,800	$I(3, 3.5)$ .3876,349 .9683,731		
(—, 21, —) $z_{-1, -3}$ *	(—, 5, 17) $z_{-1, -2}$ *	(21, 6, 12) $z_{-1, -1}$ *	(5, 22, 5) $z_{-1, 0}$ *	(6, —, 14) $z_{-1, 1}$ *	(22, —, ) $z_{-1, 2}$ *	
$I(3.5, 1.5)$ .0781,185 .6446,680	$I(3.5, 2)$ .1254,792 .7785,094	$I(3.5, 2.5)$ .1803,149 .8671,827	$I(3.5, 3)$ .2402,319 .9227,026	$I(3.5, 3.5)$ .3029,506 .9561,886	$I(3.5, 4)$ .3664,599 .9756,555	
(—, 20, —) $z_{0, -3}$ *	(18, 12, —) $z_{0, -2}$ *	(12, 1, 11) $z_{0, -1}$ *	(1, 2, 1) $z_{0, 0}$ *	(2, 0, 6) $z_{0, 1}$ *	(9, 17, ) $z_{0, 2}$ *	(19, —, ) $z_{0, 3}$ *
$I(4, 1.5)$ .0518,937 .5957,189	$I(4, 2)$ .0870,400 .7372,800	$I(4, 2.5)$ .1299,720 .8361,409	$I(4, 3)$ .1792,000 .9011,200	$I(4, 3.5)$ .2330,376 .9419,266	$I(4, 4)$ .2897,920 .9666,560	$I(4, 4.5)$ .3478,813 .9812,185
	(17, 19, —) $z_{1, -2}$ *	(11, 11, 24) $z_{1, -1}$ *	(4, 4, 4) $z_{1, 0}$ *	(3, 3, 2) $z_{1, 1}$ *	(10, 10, 22) $z_{1, 2}$ *	(20, 18, ) $z_{1, 3}$ *
	$I(4.5, 2)$ .0599,062 .6960,790	$I(4.5, 2.5)$ .0927,180 .8036,071	$I(4.5, 3)$ .1320,365 .8774,259	$I(4.5, 3.5)$ .1768,111 .9256,723	$I(4.5, 4)$ .2258,659 .9560,696	$I(4.5, 4.5)$ .2777,227 .9744,984
		(24, —, —) $z_{2, -1}$ *	(7, 23, 7) $z_{2, 0}$ *	(8, 7, 3) $z_{2, 1}$ *	(23, 8, 9) $z_{2, 2}$ *	(—, 24, —) $z_{2, 3}$ *
		$I(5, 2.5)$ .0655,576 .7700,249	$I(5, 3)$ .0962,560 .8519,680	$I(5, 3.5)$ .1325,496 .9075,463	$I(5, 4)$ .1736,704 .9437,184	$I(5, 4.5)$ .2186,523 .9664,849
			(16, —, 16) $z_{3, 0}$ *	(15, —, 8) $z_{3, 1}$ *	(—, 16, 16) $z_{3, 2}$ *	(—, 15, 16) $z_{3, 3}$ *
			$I(5.5, 3)$ .0695,236 .8250,368	$I(5.5, 3.5)$ .0983,242 .8876,964	$I(5.5, 4)$ .1320,365 .9298,150	$I(5.5, 4.5)$ .1700,836 .9571,380
				(—, —, 15) $z_{4, 1}$ *	(—, —, 23) $z_{4, 2}$ *	(—, —, 20) $z_{4, 3}$ *
				$I(6, 3.5)$ .0722,568 .8662,899	$I(6, 4)$ .0993,526 .9143,583	$I(6, 4.5)$ .1308,279 .9464,452

The numbers in the round brackets refer to the order of the corresponding  $z$  in the three formulae. The first to the Everett type Formula (xxii), the second to the Horizontal Slide (xxv), and the third to the Vertical Slide (xxvii). A short rule denotes that the  $z$  does not occur in the formula.

*Usual Bivariate Everett Formula.*

We will use first formula (xxi) and the argument difference  $.5$ . We have

$$\theta = 3/.5 = .6, \quad \phi = .4, \quad \chi = 1/.5 = .2, \quad \psi = .8.$$

lower figures in the curled brackets refer to  $I_8(4.3, 3.1)$ . The values of the required interpolants are in the table opposite.

$$\begin{aligned}
 \left. \begin{aligned} I_4(4.3, 3.1) \\ I_8(4.3, 3.1) \end{aligned} \right\} &= 1.2832 \times \left\{ \begin{aligned} &.1605,818|8 \\ &.8959,576|4 \end{aligned} \right\} \\
 &- .0656 \left( 1.4 \times \left\{ \begin{aligned} &.2527,756|4 \\ &.9294,478|0 \end{aligned} \right\} + 1.6 \times \left\{ \begin{aligned} &.1035,147|2 \\ &.8630,836|6 \end{aligned} \right\} \right) \\
 &- .0442,6667 \left( 1.2 \times \left\{ \begin{aligned} &.2514,003|4 \\ &.9602,681|6 \end{aligned} \right\} + 1.8 \times \left\{ \begin{aligned} &.1076,200|0 \\ &.8166,206|2 \end{aligned} \right\} \right) \\
 &+ .00448 \left( 2.4 \times \left\{ \begin{aligned} &.3314,789|8 \\ &.9473,386|2 \end{aligned} \right\} + 2.6 \times \left\{ \begin{aligned} &.0752,837|2 \\ &.8375,687|2 \end{aligned} \right\} \right) \\
 &+ .00288 \left( 2.8 \times \left\{ \begin{aligned} &.0707,597|2 \\ &.7125,594|0 \end{aligned} \right\} + 2.2 \times \left\{ \begin{aligned} &.3057,861|4 \\ &.9771,864|4 \end{aligned} \right\} \right) \\
 &+ .0014,9333 \times \left\{ \begin{aligned} &.7643,187|0 \\ &.27317,154|6 \end{aligned} \right\} \\
 &+ .0017,0667 \times \left\{ \begin{aligned} &.3264,081|6 \\ &.25185,069|0 \end{aligned} \right\}.
 \end{aligned}$$

hence we have

$I_4(4.3, 3.1) =$	$I_8(4.3, 3.1) =$
.2060,586 6	1.1496,928 4
- .0340,798 2	- .1759,497 5
- .0219,295 5	- .1160,777 8
+ .0044,409 7	+ .0199,417 8
+ .0025,080 7	+ .0119,375 3
+ .0011,413 8	+ .0040,793 5
+ .0005,570 7	+ .0042,982 6
.1586,967 8	= .8979,222 3
This is out 8 units in the seventh decimal place.	This is out 4 units in the sixth decimal place.

As these values would be sufficiently accurate for many purposes, it is clear that the usual bivariate formula (xxii), if not taken beyond  $8^4$ , will not give seven-figure accuracy. We will see what improves if we use the diagonal slide formulae.

#### Horizontal Slide Formula.

The plan of the interpolate  $P$  lies in the rectangle  $z_{0,0}, z_{0,1}, z_{1,1}, z_{1,0}$ , but below its left to right diagonal, the triangle  $z_{0,0}, z_{1,1}, z_{1,0}$ . Accordingly the parallelogram for the horizontal slide is  $z_{0,-1}, z_{0,0}, z_{1,1}, z_{1,0}$ . The formula to be used is (xxv).  $\theta$  and  $\phi$  remain unchanged but  $\chi' = \phi + \chi$ ,  $\psi' = \theta - \chi$ , or in our case,

$$\theta' = .6, \quad \phi' = .4, \quad \chi' = .6, \quad \psi' = .4.$$

Substituting we find

$$\begin{aligned}
 z_{\theta, \chi} &= 1.344 \{ .16z_{0,-1} + .24z_{0,0} + .36z_{1,1} + .24z_{1,0} \} \\
 &- .0672 \{ .56z_{-1,-2} + .84z_{-1,-1} + .64z_{2,1} + .96z_{2,2} \} \\
 &- .0672 \{ .64z_{0,1} + .96z_{1,2} + .84z_{1,-1} + .56z_{0,-2} \} \\
 &+ .00448 \{ .96z_{-2,-3} + 1.44z_{-2,-2} + 1.56z_{3,3} + 1.04z_{3,2} \} \\
 &+ .00448 \{ 1.04z_{0,2} + 1.56z_{1,3} + 1.44z_{1,-2} + .96z_{0,-3} \} \\
 &+ .00224 \{ 1.4z_{-1,-3} + 1.6z_{-1,0} \} \\
 &+ .00256 \{ 1.4z_{2,0} + 1.6z_{2,3} \}
 \end{aligned}$$

$$\begin{array}{lcl}
z_{\theta, \chi} = 1.344 \left\{ \begin{array}{l} .1591, 4427 | 6 \\ .8938, 7558 | 8 \end{array} \right\} & & = \left\{ \begin{array}{l} .2138, 8990 | 7 \\ 1.2013, 6879 | 0 \end{array} \right\} \\
- .0672 \left\{ \begin{array}{l} .4732, 8819 | 6 \\ 2.6511, 9802 | 8 \end{array} \right\} & & - \left\{ \begin{array}{l} .0318, 0496 | 7 \\ .1781, 6050 | 7 \end{array} \right\} \\
- .0672 \left\{ \begin{array}{l} .4925, 4337 | 6 \\ 2.6085, 0900 | 4 \end{array} \right\} & & - \left\{ \begin{array}{l} .0330, 9891 | 5 \\ .1752, 9180 | 5 \end{array} \right\} \\
+ .00448 \left\{ \begin{array}{l} .7729, 7778 | 4 \\ 4.3078, 4988 | 8 \end{array} \right\} & & + \left\{ \begin{array}{l} .0034, 6294 | 0 \\ .0192, 9916 | 7 \end{array} \right\} \\
+ .00448 \left\{ \begin{array}{l} .8707, 1397 | 2 \\ 4.0997, 8364 | 8 \end{array} \right\} & & + \left\{ \begin{array}{l} .0039, 0079 | 9 \\ .0183, 6703 | 1 \end{array} \right\} \\
+ .00224 \left\{ \begin{array}{l} .4937, 3694 | 0 \\ 2.3789, 5536 | 0 \end{array} \right\} & & + \left\{ \begin{array}{l} .0011, 0597 | 1 \\ .0053, 2886 | 0 \end{array} \right\} \\
+ .00256 \left\{ \begin{array}{l} .4846, 5008 | 0 \\ 2.7391, 2960 | 0 \end{array} \right\} & & + \left\{ \begin{array}{l} .0012, 4070 | 4 \\ .0070, 1217 | 2 \end{array} \right\}
\end{array}$$

the upper figures in the curled brackets referring to  $I_4(4.3, 3.1)$  and the lower to  $I_8(4.3, 3.1)$ . Thus

$$I_4(4.3, 3.1) = .1586, 9644, \quad I_8(4.3, 3.1) = .8979, 2371.$$

The whole labour is small when once the values of the interpolants have been extracted from the B-function ratio table, as in the Table, p. xxxii.

Compared with the actual value the error in  $I_4(4.3, 3.1) = -.0000, 0117$ , and in  $I_8(4.3, 3.1)$ , is  $+.0000, 0555$ , i.e. errors of 1 and 6 in the sixth decimal place.

#### Vertical Slide Formula.

The interpolate lying in the prism  $z_{0,0}, z_{1,1}, z_{1,0}$ , i.e.  $\theta > \chi$ , we need to use formula (xxvii), and accordingly

$$\theta' = .4, \quad \phi' = .6, \quad \chi' = .2, \quad \psi' = .8,$$

and on calculating the  $\theta', \chi'$  coefficients we have

$$\begin{array}{lcl}
z_{\theta, \chi} = 1.2832 \{ .48z_{0,0} + .12z_{1,1} + .08z_{2,1} + .32z_{1,0} \} \\
\quad - .0656 \{ 1.28z_{-1,0} + .32z_{0,1} + 1.12z_{2,0} + .28z_{3,1} \} \\
\quad - .0442, 6666, 67 \{ .72z_{2,2} + .48z_{3,2} + .72z_{0,-1} + 1.08z_{1,-1} \} \\
\quad + .00448 \{ 2.08z_{-2,0} + .52z_{-1,1} + .48z_{4,1} + 1.92z_{3,0} \} \\
\quad + .00288 \{ 1.12z_{-1,-2} + 1.68z_{-2,-2} + 1.32z_{3,3} + .88z_{4,3} \} \\
\quad + .0017, 0666, 67 \{ 1.8z_{-2,-1} + 1.2z_{1,2} \} \\
\quad + .0014, 9333, 33 \{ 1.2z_{4,2} + 1.8z_{1,-1} \} \\
= 1.2832 \left\{ \begin{array}{l} .1600, 8898 | 0 \\ .8969, 9826 | 8 \end{array} \right\} & & = \left\{ \begin{array}{l} .2054, 2617 | 91 \\ 1.1510, 2817 | 75 \end{array} \right\} \\
- .0656 \left\{ \begin{array}{l} .5174, 0642 | 4 \\ 2.6853, 1179 | 2 \end{array} \right\} & & - \left\{ \begin{array}{l} .0339, 4186 | 14 \\ .1761, 5645 | 34 \end{array} \right\} \\
- \frac{1}{3} (.1328) \left\{ \begin{array}{l} .4767, 4014 | 0 \\ 2.6643, 6721 | 2 \end{array} \right\} & & - \left\{ \begin{array}{l} .0211, 0369 | 69 \\ .1179, 4265 | 53 \end{array} \right\} \\
+ .00448 \left\{ \begin{array}{l} .9859, 7808 | 8 \\ 4.4566, 3428 | 0 \end{array} \right\} & & + \left\{ \begin{array}{l} .0044, 1718 | 18 \\ .0199, 6572 | 16 \end{array} \right\} \\
+ .00288 \left\{ \begin{array}{l} .7812, 3081 | 6 \\ 4.3444, 8046 | 4 \end{array} \right\} & & + \left\{ \begin{array}{l} .0022, 4994 | 48 \\ .0125, 1210 | 37 \end{array} \right\} \\
+ \frac{1}{3} (.00512) \left\{ \begin{array}{l} .7157, 3268 | 0 \\ 2.7604, 5504 | 0 \end{array} \right\} & & + \left\{ \begin{array}{l} .0012, 2151 | 71 \\ .0047, 1117 | 66 \end{array} \right\} \\
+ \frac{1}{3} (.00448) \left\{ \begin{array}{l} .2861, 1552 | 0 \\ 2.5437, 2274 | 0 \end{array} \right\} & & + \left\{ \begin{array}{l} .0004, 2726 | 58 \\ .0037, 9862 | 60 \end{array} \right\}
\end{array}$$

per figures in the curled brackets referring to  $I_4(4\cdot3, 3\cdot1)$  and the lower to  $I_8(4\cdot3, 3\cdot1)$ . Thus

$$I_4(4\cdot3, 3\cdot1) = \cdot1586,9653, \quad I_8(4\cdot3, 3\cdot1) = \cdot8979,1670,$$

errors of  $-.0000,0108$  and  $+.0000,0146$  respectively.

As for  $I_8(4\cdot3, 3\cdot1)$  the Vertical Slide formula gives about one-third of the error of the Everett and one-quarter of the error of the Horizontal Slide formula. In the case of  $I_4(4\cdot3, 3\cdot1)$  there is not much difference between the three formulae, they all give an error of about unity in the sixth decimal, but the Vertical Slide appears to be slightly the best.

### General Remarks as to Interpolation into the Incomplete B-Function Table.

At the limits of the table for  $x$  there may be considerable labour in the work of interpolation, and this is especially so if the values of  $p$  or  $q$  or both are small. If we suppose  $x$  and  $p$  have tabled values, but  $q$  has not, then as at the limits we have to use forward (or backward) difference formulae, we shall, if we use the Vertical Slide, find we have to proceed to eighth or even ninth differences to obtain a result correct to seven decimal places. On the other hand, if we use  $B_x(p, q)$  instead of  $I_x(p, q)$  fourth or fifth differences will suffice for seven-figure accuracy. All we need do is to multiply every  $q$ -value by the complete B-function,  $B(p, q)$ , placed at the head of the column of the table, whence  $I_x(p, q)$  is drawn. When the result has been obtained for  $B_x(p, q)$  we need to find  $B(p, q)$ , as  $q$  will be the interpolate value, and  $B(p, q)$  is not tabled. This must be done from tables of the complete  $\Gamma$ -function\*. But even thus trouble may arise when we need a value as  $I_{\cdot90}(0\cdot5, 3\cdot25)$ , even if we reduce our  $I_x$ -values to  $B_x$ -values. If we use the *twelfth* forward difference for  $I_x(p, q)$ † we shall be in error by more than unity in the fifth decimal place. If we use  $B_x(p, q)$  to *twelfth* difference we shall be in error by less than unity in the sixth decimal place. This may be satisfactory from the mathematician's standpoint, but far fewer differences will satisfy the statistician seeking merely for four or even three decimal place accuracy.

We can, however, proceed in a very simple way to get rapidly converging differences. We have

$$I_{\cdot90}(0\cdot5, 3\cdot25) = 1 - I_{\cdot10}(3\cdot25, 0\cdot5),$$

so we require  $I_{\cdot10}(3\cdot25, 0\cdot5)$ . Now if it is  $p$  we wish to interpolate for, let us interpolate for  $x^{-(p-1)}I_x(p, q)$  in the third column of the table. We have, omitting  $10^{-7}$  in the third and fourth columns:

$p$	$I_{\cdot10}(p, 0\cdot5)$	$I_{\cdot10}(p, 0\cdot5) \times (-10)^{-(p-1)}$	$I_{\cdot10}(p, 0\cdot5) \times (-10)^{-(p-1)}$
3	$\cdot000,3250$	$(\cdot10)^{-2} \times 3250$	$(\cdot10)^{-2} \times 3250$
3.5	$\cdot000,0958$	$(\cdot10)^{-2} \times (\cdot10)^{-\frac{1}{2}} \times 958$	$(\cdot10)^{-2} \times 3029$
4	$\cdot000,0285$	$(\cdot10)^{-2} \times (\cdot10)^{-1} \times 285$	$(\cdot10)^{-2} \times 2850$
4.5	$\cdot000,086$	$(\cdot10)^{-2} \times (\cdot10)^{-1\frac{1}{2}} \times 86$	$(\cdot10)^{-2} \times 2720$

so that  $(\cdot10)^{-\frac{1}{2}} = 3\cdot1622,777$  and  $(\cdot10)^{-1\frac{1}{2}} = 31\cdot6227,77$ , hence the fourth column. We now take the forward differences of the fourth column:

	$\Delta$	$\Delta^2$	$\Delta^3$
3250			
3029	$-\cdot221$		
2850	$-\cdot179$	$+\cdot42$	
2720	$\cdot130$	$+\cdot49$	$\cdot7$

so we have, reinstating the  $10^{-7}$ :

$$\begin{aligned} I_{\cdot10}(3\cdot25, 0\cdot5) \times (-10)^{-2\frac{1}{2}} &= (\cdot10)^{-2} \left\{ \cdot0003,250 + \frac{1}{4} (-\cdot0000,221) - \frac{3}{32} (\cdot0000,042) + \frac{7}{128} (\cdot0000,007) \right\} \\ &= (\cdot10)^{-2} \times \cdot0003,190|43, \\ I_{\cdot10}(3\cdot25, 0\cdot5) &= (\cdot10)^{2\frac{1}{2}} \times \cdot0003,190|43 \\ &= \cdot562341 \times \cdot0003,190|43 \\ &= \cdot0001,794. \end{aligned}$$

Accordingly  $I_{\cdot90}(0\cdot5, 3\cdot25) = \cdot9998,206$ .

For *acts for Computers*, No. III, Legendre's table,  $\text{Log } \Gamma(p)$  to twelve places, for  $p=1$  to 2, argument intervals  $\cdot001$ ; No. VIII, Pearson's table,  $\text{Log } \Gamma(p)$  to ten places, for  $p=2$  to 1200, argument intervals  $0\cdot5, 1$  and  $2$ ; No. IX, Brownlee's table,  $\text{Log } \Gamma(p)$  to 5000 by intervals of  $\cdot01$ .

If we proceed by central differences we have  $I_{\cdot90}(0\cdot5, 3\cdot25) = 1 - I_{\cdot10}(3\cdot25, 0\cdot5)$ , and the  $\delta^2$ 's diverge rapidly. Thus for  $p=3$ ,  $\delta^2 = \cdot0002, \delta^4 = \cdot0010,292, \delta^6 = \cdot0020,423, \delta^8 = \cdot0049,637, \delta^{10} = \cdot0180,511$ , etc., and the reduction of the coefficients hardly keeps up with this divergence. The forward differences here escape this trouble although their convergence is relatively slow.



Thus with the additional trouble of computing two roots of  $\cdot 10$  we have obtained a result with two or three forward differences, where the  $I_x(p, q)$  function required a dozen. This method will often be markedly successful in interpolating for  $x$  when  $p$  and  $q$  are given by table values. Thus suppose we desire

$$I_{.8966}(0.5, 3.5) = 1 - I_{.1034}(3.5, 0.5).$$

Extracting the values from the table:

(a)	(b) $p=3.5$ $q=0.5$	(c) $x^{-2.5}$	(d) $(b) \times (c)$	$\Delta$	$\Delta^2$
$x=.10$	.0000,958	316.2278	.0003,029 46	319 56	0 21
$x=.11$	.0001,344	249.1829	.0003,349 02	319 35	
$x=.12$	.0001,830	164.1125	.0003,668 37		

Linear interpolation is therefore sufficient, or

$$I_{.1034}(3.5, 0.5) \times \frac{(.1034)^{-2.5}}{100} = 2.9087 \{ (.0003,029|46) \times .66 + (.0003,349|02) \times .34 \} = .0003,138|11,$$

or

$$I_{.1034}(3.5, 0.5) \times 2.9087 = .0003,138|11,$$

and accordingly

$$I_{.1034}(3.5, 0.5) = .0001,079$$

and

$$I_{.8966}(0.5, 3.5) = .9998,921.$$

In escaping with a single difference, however, we have had to compute four power terms.

### (7) Applications.

(i) *The Incomplete B-Function Tables may be applied to calculate the Sum of any Number of Terms of a Hypergeometrical Series  $F(\alpha, \beta, \gamma, x)$  of which the fourth element  $x$  is unity.*

*Illustration 5.* Calculate the sum of the last 51 terms of  $F(1, -60, -65, 1)$ .

If  $\epsilon = \gamma - \alpha - \beta - 1$ , and supposing the terms of the series spaced at unity apart, the moments about an origin 0.5 before the first term are\*

$$\mu_1' = \alpha\beta/\epsilon = 9.0714,2857 \text{ in our case,}$$

$$\mu_2' = \frac{\alpha\beta(\alpha+\epsilon)(\beta+\epsilon)}{\epsilon^2(\epsilon-1)} = 143.8214,2857 \text{ in our case,}$$

$$\mu_3' = \frac{\alpha\beta(\alpha+\epsilon)(\beta+\epsilon)(2\alpha+\epsilon)(2\beta+\epsilon)}{\epsilon^3(\epsilon-1)(\epsilon-2)} = 3041.1964,2857 \text{ in our case.}$$

But these are the moments of the discrete ordinates, and our curve by which we replace them must have corrections applied to these moments. In most cases Sheppard's corrections will suffice, but in the present case the first term is the maximum term. There is, however, high contact at the second terminal. The above raw moments were kindly corrected for abruptness† by Dr O. L. Davies, and he found the following values:

$$\mu_1' = 9.063,5495, \quad \mu_2' = 143.738,0331, \quad \mu_3' = 3038.952,2009.$$

These values are slightly below those due to using merely Sheppard's corrections in the first two cases and slightly above in the third case.

To fit a curve starting at 0.5 before the first term of the hypergeometrical series of the form

$$y = y_0 x^{p-1} (b-x)^{q-1}$$

and having the same three moment coefficients, we must calculate the values of  $\lambda_1 = \mu_1'/\mu_2'$  and  $\lambda_2 = \mu_2'\mu_1'/\mu_3'$ ; we find for our series

$$\lambda_1 = .5715,1143|52 \text{ and } \lambda_2 = .4286,9275|04.$$

The values of  $p$  and  $q$  are provided by‡

$$p = \frac{2(\lambda_1 - \lambda_2)}{1 + \lambda_2 - 2\lambda_1} = \frac{.2856,3736|96}{.2856,6988|00} = .9998,8620,$$

$$p+q = \frac{2(\lambda_1 - \lambda_2)}{2\lambda_2 - \lambda_1 - \lambda_1\lambda_2} = \frac{.2856,3736|96}{.0408,7125|69} = 6.9887,1019,$$

$$q = 5.9888,2399.$$

\* Pearson, *Phil. Mag.* Feb. 1899, p. 239.

† *Phil. Trans.* Vol. 186 (1895) A, p. 371, with a different notation and correction of the slip in the value of  $b$ , the range.

‡ *Tables for Statisticians*, Part II, p. cxvii.

range  $b$  is given by

$$b = \frac{\mu_1' (1 + \lambda_2 - 2\lambda_1)}{2\lambda_2 - \lambda_1 - \lambda_1\lambda_2} = \mu_1' \frac{.2856,6988|00}{.0408,7125|69},$$

$$= 63.349,730.$$

the equation to the curve representing our hypergeometrical series  $F(1, -60, -65, 1)$  is

$$y = y_0 x^{-.000,1138} (63.349,730 - x)^{4.988,824}.$$

actual range of the hypergeometrical series is 61, but as the 60th term is .000,001 and the 61st .000,000, the range is very reasonable, the areas of the curve and the last six terms of the series agreeing exactly in decimal figures.

We now have to find from the B-function table the sum of the last 51 terms as a proportion of the whole. This will be given by the incomplete B-function ratio. We have the required sum,  $\bar{S}$ , as approximately\*

$$\bar{S} = y_0 \int_0^{63.349,730} x^{-.000,1138} (63.349,730 - x)^{4.988,824} dx.$$

put  $x = 63.349,730x'$ , and let  $S'$  be the ratio to the total area, then

$$S' = 1 - I_{.142,0685}(.999,8862, 5.988,8240) \\ = I_{.857,9315}(5.988,824, .999,8862),$$

is the value to be found from the table.

To obtain a fairly close approximation to this we must revert to our trivariate interpolation formula

and, remembering that the interval for  $x$  is .01, and for  $p$  and  $q$  at this part of the table .5,

$$\theta_1 = .79315, \quad \phi_1 = .977648, \quad \chi_1 = .999,7724.$$

It will be adequate to take these to five decimals, or

$$\theta_1 = .79315, \quad \phi_1 = .97765, \quad \chi_1 = .99977,$$

accordingly

$$\theta_0 = .20685, \quad \phi_0 = .02235, \quad \chi_0 = .00023.$$

With these values we obtain the triple products

$$\begin{aligned} \theta_0 \phi_0 \chi_0 &= .0000,0107, & \theta_0 \phi_0 \chi_1 &= .0046,2203, \\ \theta_1 \phi_0 \chi_0 &= .0000,0408, & \theta_1 \phi_0 \chi_1 &= .0177,2282, \\ \theta_1 \phi_1 \chi_0 &= .0001,7835, & \theta_1 \phi_1 \chi_1 &= .7752,4475, \\ \theta_0 \phi_1 \chi_0 &= .0000,4651, & \theta_0 \phi_1 \chi_1 &= .2021,8039. \end{aligned}$$

Now turn to the needful  $z$ 's and  $\delta^2 z$ 's,

$$\begin{aligned} z_{000} &= .1910,543, & z_{100} &= .2078,455, & z_{110} &= .1875,144, & z_{010} &= .1712,665, \\ z_{001} &= .4090,761, & z_{101} &= .4362,561, & z_{111} &= .4045,672, & z_{011} &= .3771,495. \end{aligned}$$

Now we find the hyperbolic terms of the interpolation formula (viii) on p. x give us .3995,5574 for the value of  $S'$ . The exact value is .3993,917. It remains to be seen how the terms in  $\delta^2 z$  will modify the deduction from the hyperbolic terms. We have

$$\begin{aligned} \frac{1}{6} \theta_1 (1 + \theta_0) &= .1595,3552, & \frac{1}{6} \theta_0 (1 + \theta_1) &= .0618,1885, \\ \frac{1}{6} \phi_1 (1 + \phi_0) &= .1665,8340, & \frac{1}{6} \phi_0 (1 + \phi_1) &= .0073,6740, \\ \frac{1}{6} \chi_1 (1 + \chi_0) &= .1666,6705, & \frac{1}{6} \chi_0 (1 + \chi_1) &= .0000,7585. \end{aligned}$$

hence

$$\begin{aligned} \delta_p^2 z_{000} &= .0013,884, & \delta_p^2 z_{100} &= .0027,147, & \delta_q^2 z_{000} &= 1.0269,675, \\ \delta_p^2 z_{100} &= .0015,462, & \delta_p^2 z_{110} &= .0026,546, & \delta_q^2 z_{100} &= 1.0205,651, \\ \delta_p^2 z_{010} &= .0014,310, & \delta_p^2 z_{010} &= .0023,161, & \delta_q^2 z_{010} &= 1.0346,165, \\ \delta_p^2 z_{110} &= .0015,996, & \delta_p^2 z_{110} &= .0022,745, & \delta_q^2 z_{110} &= 1.0295,384, \\ \delta_p^2 z_{001} &= .0014,015, & \delta_p^2 z_{001} &= .0027,126, & \delta_q^2 z_{001} &= -.0261,785, \\ \delta_p^2 z_{101} &= .0014,600, & \delta_p^2 z_{101} &= .0024,820, & \delta_q^2 z_{101} &= -.0261,785, \\ \delta_p^2 z_{011} &= .0015,662, & \delta_p^2 z_{011} &= .0024,918, & \delta_q^2 z_{011} &= -.0243,282. \end{aligned}$$

do not need to find  $y_0$ . If we did then we should make the area of the curve equal to the sum of the hypergeometrical series, i.e.

$$\text{Sum} = \frac{\Gamma(\epsilon - \gamma + 1) \Gamma(\beta - \gamma + 1)}{\Gamma(\gamma + \beta - \gamma + 1) \Gamma(1 - \gamma)}, \text{ if } \epsilon < 0, \quad = \frac{\Gamma(\gamma) \Gamma(\gamma - \alpha - \beta)}{\Gamma(\gamma - \alpha) \Gamma(\gamma - \beta)}, \text{ if } \epsilon > 0.$$

In the present case the value of  $\epsilon$  is  $< 0$ , and the Sum =  $(\Gamma(.67) \Gamma(.6)) / (\Gamma(.7) \Gamma(.66)) = 11$ .

The limiting value of  $I_x(p, q)$ , as  $q \rightarrow 0$ , is unity.

Accordingly, if  $S_{\delta^2}$  be the  $\delta^2 z$  terms in (viii), we have

$$S_{\delta^2} = -0.000,5156 - 0.000,8028 - 0.000,0942 - 0.000,1686 - 0.000,3950 + 0.000,1737 \\ = -0.001,8025.$$

Hence, including the hyperbolic term, we have for the total area up to and including third differences 3993.755. Accordingly our results differ from the sum of the 51 terms of the series by less than two units (1.6) in the fifth place of decimals, a result close enough for many statistical purposes. Actually the interpolate differs more from the true value of the curve area ratio\* than it does from the sum of the 51 terms of the series.

We cannot get a better value for the interpolate without going to higher differences than the third, but to do this would involve excessive labour, while the divergence of the partial area of the Pearson curve from the sum of  $n$  terms of the series is of the same order as the divergence of the partial area from a third difference interpolate into the table.

(ii) *On a Method of determining the Probability that the Correlation Coefficient  $r$  in a sample of size  $n$  from a Normal Population with correlation  $\rho$  will not exceed the value  $r$ ; that is, to find the Probability Integral of  $r$ .*

If  $\sigma_1, \sigma_2$  be the standard deviations of the two variates in the sample, the correlation between which is  $r$ , and  $\Sigma_1, \Sigma_2$  the corresponding standard deviations in the parent population, then the correlation surface of  $r, \sigma_1, \sigma_2$ , as was originally shown by Fisher†, is

$$z = z_0 e^{-\frac{1}{2} \frac{n}{1-\rho^2} \left( \frac{\sigma_1^2}{\Sigma_1^2} - \frac{2r\rho\sigma_1\sigma_2}{\Sigma_1\Sigma_2} + \frac{\sigma_2^2}{\Sigma_2^2} \right)} \left( \frac{\sigma_1}{\Sigma_1} \right)^{n-2} \left( \frac{\sigma_2}{\Sigma_2} \right)^{n-2} (1-r^2)^{\frac{n-4}{2}} \quad \dots\dots(\text{xxxviii}),$$

where the element of volume is  $d\sigma_1 d\sigma_2 dr$ , and

$$z_0 = \frac{N n^{n-1}}{\Gamma\left(\frac{n-2}{2}\right) \Gamma\left(\frac{n-1}{2}\right) \sqrt{\pi} (1-\rho^2)^{\frac{1}{2}(n-1)} 2^{n-3} \Sigma_1 \Sigma_2} \quad \dots\dots(\text{xxxix}).$$

Expand  $e^{-\frac{1}{2} \frac{n}{1-\rho^2} \left( \frac{\sigma_1^2}{\Sigma_1^2} - \frac{2r\rho\sigma_1\sigma_2}{\Sigma_1\Sigma_2} + \frac{\sigma_2^2}{\Sigma_2^2} \right)}$  by the Exponential Theorem and we have

$$z = z_0 e^{-\frac{1}{2} \frac{n}{1-\rho^2} \left( \frac{\sigma_1^2}{\Sigma_1^2} - \frac{2r\rho\sigma_1\sigma_2}{\Sigma_1\Sigma_2} + \frac{\sigma_2^2}{\Sigma_2^2} \right)} = \sum_{t=0}^{\infty} \left( \frac{n}{1-\rho^2} \right)^t (r\rho)^t \frac{1}{t!} \left( \frac{\sigma_1}{\Sigma_1} \frac{\sigma_2}{\Sigma_2} \right)^{n+t-2} (1-r^2)^{\frac{n-4}{2}}.$$

Take

$$u = \frac{1}{2} \frac{n}{1-\rho^2} \frac{\sigma_1^2}{\Sigma_1^2}, \quad v = \frac{1}{2} \frac{n}{1-\rho^2} \frac{\sigma_2^2}{\Sigma_2^2}.$$

Substituting, and integrating out for  $u$  and  $v$  by aid of the complete  $\Gamma$ -function we find

$$z = \frac{N (1-\rho^2)^{\frac{n-1}{2}}}{\sqrt{\pi} \Gamma\left(\frac{1}{2}(n-2)\right) \Gamma\left(\frac{1}{2}(n-1)\right)} \sum_{t=0}^{\infty} \frac{(2r\rho)^t}{t!} \Gamma^2\left(\frac{1}{2}(n+t-1)\right) (1-r^2)^{\frac{n-t-4}{2}} \quad \dots\dots(\text{xxx}),$$

for the distribution curve of  $r$ .

To obtain the probability integral, we have  $\frac{1}{2}(1+\alpha_r) = \int_{-1}^r \frac{z dr}{N}$ . Now  $\frac{z}{N}$  may be taken to consist of two series  $\phi_1(r^2)$  corresponding to the even powers of  $r$ , and  $r\phi_2(r^2)$  to the odd powers of  $r$ . It is clear that only the latter will change sign with  $r$ .

Now let us write  $r^2 = u$ , then

$$\int_{-1}^r \frac{z dr}{N} = \frac{1}{2} (1+\alpha_r) = \frac{(1-\rho^2)^{\frac{n-1}{2}}}{\sqrt{\pi} \Gamma\left(\frac{1}{2}(n-2)\right) \Gamma\left(\frac{1}{2}(n-1)\right)} (S_1 + S_2),$$

where

$$S_1 = \int_{-1}^{r^2} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s}}{(2s)!} \Gamma^2\left(\frac{1}{2}(n-1)+s\right) (1-u)^{\frac{n-4}{2}} \frac{1}{2} u^{\frac{2s-1}{2}} du \\ = \int_{-1}^0 + \int_0^{r^2} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s}}{(2s)!} \Gamma^2\left(\frac{1}{2}(n-1)+s\right) u^{s-\frac{1}{2}} \frac{1}{2} (1-u)^{\frac{n-4}{2}} du.$$

3993.494 by quadrature.

† *Biometrika*, Vol. x, pp. 507 et seq.

$\int_0^{r^2} u^{s-\frac{1}{2}} (1-u)^{\frac{n-4}{2}} du$  is the incomplete B-function  $B_{r^2}\left(s+\frac{1}{2}, \frac{n-2}{2}\right)$ , and this equals

$$I_{r^2}\left(s+\frac{1}{2}, \frac{n-2}{2}\right) \times B\left(s+\frac{1}{2}, \frac{n-2}{2}\right),$$

$I_{r^2}\left(s+\frac{1}{2}, \frac{n-2}{2}\right)$  is the incomplete B-function ratio, and  $B\left(s+\frac{1}{2}, \frac{n-2}{2}\right)$  the complete B-function as

$$B\left(s+\frac{1}{2}, \frac{n-2}{2}\right) = \Gamma\left(s+\frac{1}{2}\right) \Gamma\left(\frac{n-2}{2}\right) / \Gamma\left(\frac{1}{2}(n-1)+s\right).$$

or since  $S_1$  depends only on  $r^2$  we have the integral from  $-1$  to  $0$  equals the integral from  $0$  to  $1$ , or

$$I_1\left(s+\frac{1}{2}, \frac{n-2}{2}\right) = 1 \text{ for all values of } S,$$

$$\begin{aligned} I_1 = \frac{1}{2} S \sum_{s=0}^{\infty} \frac{(2\rho)^{2s}}{(2s)!} \Gamma\left(s+\frac{1}{2}\right) \Gamma\left(\frac{n-2}{2}\right) \Gamma\left(\frac{1}{2}(n-1)+s\right) \\ + \frac{1}{2} S \sum_{s=0}^{\infty} \frac{(2\rho)^{2s}}{(2s)!} \Gamma\left(s+\frac{1}{2}\right) \Gamma\left(\frac{n-2}{2}\right) \Gamma\left(\frac{1}{2}(n-1)+s\right) I_{r^2}\left(s+\frac{1}{2}, \frac{n-2}{2}\right) \quad \dots\dots(\text{xxxix}). \end{aligned}$$

the contribution of  $\phi(r^2)$  to  $\int_{-1}^r \frac{z}{N} dr$  is

$$\begin{aligned} (1-\rho^2)^{\frac{n-1}{2}} \frac{1}{\sqrt{\pi} \Gamma\left(\frac{1}{2}(n-1)\right)} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s}}{(2s)!} \Gamma\left(s+\frac{1}{2}\right) \Gamma\left(\frac{1}{2}(n-1)+s\right) \\ + \frac{1}{2} (1-\rho^2)^{\frac{n-1}{2}} \frac{1}{\sqrt{\pi} \Gamma\left(\frac{1}{2}(n-1)\right)} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s}}{(2s)!} \Gamma\left(s+\frac{1}{2}\right) \Gamma\left(\frac{1}{2}(n-1)+s\right) I_{r^2}\left(s+\frac{1}{2}, \frac{n-2}{2}\right). \end{aligned}$$

the first of these series reduces to

$$\begin{aligned} \frac{1}{2} (1-\rho^2)^{\frac{n-1}{2}} \frac{1}{\sqrt{\pi}} \left( \Gamma\left(\frac{1}{2}\right) + \frac{4\rho^2}{2!} \Gamma\left(\frac{3}{2}\right) \frac{1}{2}(n-1) + \frac{16\rho^4}{4!} \Gamma\left(\frac{5}{2}\right) \frac{1}{2}(n+1) \frac{1}{2}(n-1) + \dots \right) \\ = \frac{1}{2} (1-\rho^2)^{\frac{n-1}{2}} \left( 1 + \frac{1}{2}(n-1)\rho^2 + \frac{1}{2!} \frac{1}{2}(n-1) \frac{1}{2}(n+1) \rho^4 + \dots \right) \\ = \frac{1}{2} (1-\rho^2)^{\frac{n-1}{2}} = (1-\rho^2)^{\frac{n-1}{2}} \cdot \frac{1}{2}. \end{aligned}$$

second of the above series may be written

$$\frac{1}{2} (1-\rho^2)^{\frac{n-1}{2}} \left( I_{r^2}\left(\frac{1}{2}, \frac{n-2}{2}\right) + \frac{\rho^2}{1!} \frac{n-1}{2} I_{r^2}\left(\frac{3}{2}, \frac{n-2}{2}\right) + \frac{\rho^4}{2!} \frac{n-1}{2} \frac{n+1}{2} I_{r^2}\left(\frac{5}{2}, \frac{n-2}{2}\right) + \dots \right)$$

$$I_r(p, q) = 1 - I_{1-r}(q, p),$$

$$\frac{1}{2} - \frac{1}{2} (1-\rho^2)^{\frac{n-1}{2}} \sum_{s=0}^{\infty} \frac{\rho^{2s} I_{1-r^2}\left(\frac{n-2}{2}, s+\frac{1}{2}\right)}{s B\left(\frac{n-1}{2}, s\right)}, \quad \dots\dots(\text{xxxii}).$$

advantage of this form is that the values of  $I_{1-r^2}\left(\frac{n-2}{2}, s+\frac{1}{2}\right)$  and  $B\left(\frac{n-1}{2}, s\right)$  are directly given

*Tables of the Incomplete B-Function.*

Thus we have finally

$$\int_{-1}^r \phi(r^2) dr = 1 - \frac{1}{2} (1 - \rho^2)^{\frac{n-1}{2}} \sum_{s=0}^{\infty} \frac{\rho^{2s} I_{1-r^2} \left( \frac{n-2}{2}, s + \frac{1}{2} \right)}{s B \left( \frac{n-1}{2}, s \right)} \quad \dots\dots(\text{xxxiii}).$$

We now turn to the second part of the integral of  $\frac{z}{N}$ , i.e.

$$\int_{-1}^r r \phi(r^2) dr = \int_{-1}^0 + \int_0^r r \phi(r^2) dr.$$

Now

$$\int_0^r r \phi(r^2) dr = \frac{(1 - \rho^2)^{\frac{n-1}{2}}}{\sqrt{\pi} \Gamma \left( \frac{1}{2} (n-2) \right) \Gamma \left( \frac{1}{2} (n-1) \right)} \int_0^r S_2 dr,$$

where

$$S_2 = \sum_{s=0}^{\infty} \frac{(2\rho)^{2s+1}}{(2s+1)!} \Gamma^2 \left( \frac{1}{2} (n+2s) \right) r^{2s+1} (1 - r^2)^{\frac{n-4}{2}}.$$

Put  $u = r^2$  as before and we have

$$\begin{aligned} \int_0^r S_2 du &= \frac{1}{2} \int_0^r \sum_{s=0}^{\infty} \frac{(2\rho)^{2s+1}}{(2s+1)!} \Gamma^2 \left( \frac{1}{2} (n+s) \right) u^s (1-u)^{\frac{n-4}{2}} du \\ &= \frac{1}{2} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s+1}}{(2s+1)!} \Gamma^2 \left( \frac{1}{2} (n+s) \right) B_{r^2} \left( s+1, \frac{n-2}{2} \right) \\ &= \frac{1}{2} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s+1}}{(2s+1)!} \Gamma^2 \left( \frac{1}{2} (n+s) \right) I_{r^2} \left( s+1, \frac{n-2}{2} \right) B \left( s+1, \frac{n-2}{2} \right) \\ &= \frac{1}{2} \sum_{s=0}^{\infty} \frac{(2\rho)^{2s+1}}{(2s+1)!} \Gamma^2 \left( \frac{1}{2} (n+s) \right) I_{r^2} \left( s+1, \frac{n-2}{2} \right) \frac{\Gamma(s+1) \Gamma \left( \frac{1}{2} (n-2) \right)}{\Gamma \left( \frac{1}{2} (n+s) \right)}, \end{aligned}$$

or introducing the factor we have

$$\int_0^r r \phi(r^2) dr = \frac{\frac{1}{2} (1 - \rho^2)^{\frac{n-1}{2}}}{\sqrt{\pi} \Gamma \left( \frac{1}{2} (n-1) \right)} \sum_{s=0}^{\infty} \frac{\Gamma \left( \frac{1}{2} (n+s) \right) \Gamma(s+1)}{(2s+1)!} (2\rho)^{2s+1} I_{r^2} \left( s+1, \frac{n-2}{2} \right).$$

Now change  $I_{r^2} \left( s+1, \frac{n-2}{2} \right)$  as before, and remembering that  $\int_0^1 r \phi(r^2) dr$  will equal  $\int_{-1}^1 r \phi(r^2) dr$ , we see that

$$\begin{aligned} \int_{-1}^r r \phi(r^2) dr &= \frac{\frac{1}{2} (1 - \rho^2)^{\frac{n-1}{2}}}{B \left( \frac{1}{2}, \frac{n-1}{2} \right)} \sum_{s=0}^{\infty} \frac{\Gamma \left( \frac{1}{2} (n+s) \right) \Gamma(s+1)}{\Gamma \left( \frac{1}{2} n \right) \Gamma(2s+2)} (2\rho)^{2s+1} I_{1-r^2} \left( \frac{n-2}{2}, s + \frac{1}{2} \right) \\ &= \frac{\frac{1}{2} (1 - \rho^2)^{\frac{n-1}{2}}}{B \left( \frac{1}{2}, \frac{n-1}{2} \right)} \sum_{s=0}^{\infty} \frac{B \left( \frac{1}{2} n + s, s+1 \right) (2\rho)^{2s+1}}{B \left( \frac{1}{2} n, 2s+2 \right) \left( \frac{1}{2} n + 2s+1 \right)} I_{1-r^2} \left( \frac{n-2}{2}, s + \frac{1}{2} \right) \quad \dots\dots(\text{xxxiv}). \end{aligned}$$

This is a convenient form as the complete B-functions are tabled as well as  $I_{1-r^2} \left( \frac{n-2}{2}, s + \frac{1}{2} \right)$ .

(iii) Thus finally we have the result

$$\begin{aligned} \frac{1}{2} (1 + \alpha_r) &= 1 - \frac{1}{2} (1 - \rho^2)^{\frac{n-1}{2}} \sum_{s=0}^{\infty} \frac{\rho^{2s} I_{1-r^2} \left( \frac{n-2}{2}, s + \frac{1}{2} \right)}{s B \left( \frac{n-1}{2}, s \right)} \\ &\quad - \frac{\frac{1}{2} (1 - \rho^2)^{\frac{n-1}{2}}}{B \left( \frac{n-1}{2}, \frac{1}{2} \right)} \sum_{s=0}^{\infty} \frac{B \left( \frac{1}{2} n + s, s+1 \right) (2\rho)^{2s+1} I_{1-r^2} \left( \frac{n-2}{2}, s + \frac{1}{2} \right)}{B \left( \frac{1}{2} n, 2s+2 \right) \left( \frac{1}{2} n + 2s+1 \right)} \quad \dots\dots(\text{xxxv}). \end{aligned}$$

chief trouble is the slow convergency of the two series. But the values of the factors are easily obtained by machine by continuous multiplication. Representing the first series by

$$\Sigma_1 = \sum_{s=0}^{\infty} f_{2s} \times I_{1-\rho^2} \left( \frac{n-2}{2}, s + \cdot 5 \right),$$

$$f_{2(s+1)} = f_{2s} \times \frac{2s+n-1}{2s+2} \rho^2 \quad \text{.....(xxxvi).}$$

and, if the second series be

$$\Sigma_2 = \sum_{s=0}^{\infty} F_{2s+1} \times I_{1-\rho^2} \left( \frac{n-2}{2}, s + 1 \right)$$

$$F_{2(s+1)+1} = F_{2s+1} \times \frac{2s+n}{2s+3} \rho^2 = F_{2s+1} \times \frac{2s+1+n-1}{2s+1+2} \rho^2 \quad \text{.....(xxxvii).}$$

where  $I_{1-\rho^2} \left( \frac{n-2}{2}, s + \cdot 5 \right)$  and  $I_{1-\rho^2} \left( \frac{n-2}{2}, s + 1 \right)$  are always less than unity and often considerably less than unity, we see that both series ultimately converge more rapidly than a geometrical series of radix  $\rho^2$ . For all values of  $\rho$ , the convergency is rapid. It will be seen that the incomplete B-function ratios required for  $I_x(p, q)$  only for whole numbers, or for whole numbers plus 0.5. Such values are given in our table for  $p$  and  $q$  up to 10.5 each. Further we have directly tabled  $p$  or  $q$  up to 10.5 and  $q$  or  $p$  up to 50. Cases where  $p$  is an integer and  $q > 10.5$  or *vice versa* require an interpolation formula. This is adequately supplied by

$$z_{s+1} = \frac{1}{2}(z_s + z_{s+1}) - \frac{1}{16}(\delta^2 z_s + \delta^2 z_{s+1}) + \frac{3}{256}(\delta^4 z_s + \delta^4 z_{s+1}) - \frac{5}{2048}(\delta^6 z_s + \delta^6 z_{s+1}) \quad \text{.....(xxxviii).}$$

The last term will not influence by a unit the seventh decimal place until the sixth central differences are 100, which will usually not be the case.

When  $p$  and  $q$  both terminate in .5 and are both  $> 10.5$  the matter is more troublesome. We may apply the formula (xx) *quater* or the following formula:

$$\frac{1}{4}(z_{0,0} + z_{0,1} + z_{1,1} + z_{1,0}) - \frac{5}{128}(\delta^2 z_{0,0} + \delta^2 z_{0,1} + \delta^2 z_{1,1} + \delta^2 z_{1,0}) - \frac{5}{128}(\delta'^2 z_{0,0} + \delta'^2 z_{0,1} + \delta'^2 z_{1,1} + \delta'^2 z_{1,0})$$

$$+ \frac{1}{512}(\delta^2 z_{0,-1} + \delta^2 z_{0,2} + \delta^2 z_{1,2} + \delta^2 z_{1,-1}) + \frac{3}{512}(\delta^2 z_{-1,0} + \delta^2 z_{2,0} + \delta^2 z_{-1,1} + \delta^2 z_{2,1})$$

$$+ \frac{1}{512}(\delta'^2 z_{-1,0} + \delta'^2 z_{2,0} + \delta'^2 z_{-1,1} + \delta'^2 z_{2,1}) + \frac{3}{512}(\delta'^2 z_{0,-1} + \delta'^2 z_{0,2} + \delta'^2 z_{1,2} + \delta'^2 z_{1,-1}) \quad \text{.....(xxxix).}$$

where the terms in  $\delta^6$ ,  $\delta'^6$  have been neglected, and the terms in  $\delta^4$ ,  $\delta'^4$  replaced by their equivalents in the formula. Thus this formula will give correct results, if  $\delta^6$ ,  $\delta'^6$  are negligible.

As far we have dealt with the case of  $r$  positive; we will now suppose  $r$  negative:

$$\frac{1}{2}(1 + \alpha_r) = \int_0^r \frac{z}{N} dr + \int_0^r \phi_1(r^2) dr + \int_{-1}^{-r} r \phi_2(r^2) dr.$$

Changing the sign of  $r$  in the two integrals

$$\frac{1}{2}(1 + \alpha_r) = \int_r^1 \phi_1(r^2) dr - \int_r^1 r \phi_2(r^2) dr,$$

where  $r$  is to be considered as positive on the right-hand side. Hence

$$\frac{1}{2}(1 + \alpha_r) = \int_0^1 \phi_1(r^2) dr - \int_0^r \phi_1(r^2) dr - \int_0^r r \phi_2(r^2) dr + \int_0^r r \phi_2(r^2) dr$$

$$= \frac{1}{2} \left[ \frac{1}{2} - \frac{1}{2}(1 - \rho^2)^{\frac{1}{2}(n-1)} \Sigma_1 \right] - \int_0^r r \phi_2(r^2) dr + \left[ \int_0^1 r \phi_2(r^2) dr - \frac{1}{2} \frac{(1 - \rho^2)^{\frac{1}{2}(n-1)}}{B\left(\frac{1}{2}, \frac{n-1}{2}\right)} \Sigma_2 \right]$$

$$= \frac{1}{2}(1 - \rho^2)^{\frac{1}{2}(n-1)} \Sigma_1 - \frac{1}{2} \frac{(1 - \rho^2)^{\frac{1}{2}(n-1)}}{B\left(\frac{1}{2}, \frac{n-1}{2}\right)} \Sigma_2 \quad \text{.....(xl).}$$

Thus in finding  $\Sigma_1$  and  $\Sigma_2$  for the value of  $\frac{1}{2}(1 + \alpha_r)$ , where  $r$  is positive, we have also very readily the value of  $\frac{1}{2}(1 + \alpha_{-r})$ , where  $r$  is negative.

The chance that a sample correlation is greater than  $r$  is

$$\frac{1}{2}(1 - \alpha_r) = \frac{1}{2}(1 - \rho^2)^{\frac{1}{2}(n-1)} \Sigma_1 + \frac{1}{2} \frac{(1 - \rho^2)^{\frac{1}{2}(n-1)}}{B\left(\frac{1}{2}, \frac{n-1}{2}\right)} \Sigma_2 \quad \dots\dots(\text{xli}).$$

Thus the probability that it lies outside the range  $-r$  to  $+r = (1 - \rho^2)^{\frac{1}{2}(n-1)} \Sigma_1$ , or the chance that it lies inside the range is

$$1 - (1 - \rho^2)^{\frac{1}{2}(n-1)} \Sigma_1 \quad \dots\dots(\text{xlii}),$$

or if we merely need this chance we do not require  $\Sigma_2$ .

*Illustration 6.* Samples of 10 are taken from an indefinitely large normal population with a correlation of 0.6 between two variates. What is the probability that the correlation in a sample will exceed 0.9?

We have to determine from the tables the value of  $\Sigma_1$  and  $\Sigma_2$  in a case of very slow convergence. Consider first the external factors of the series  $\Sigma_1$  and  $\Sigma_2$ .

$$\text{For } \Sigma_1 \text{ we need} \quad \frac{1}{2}(1 - \rho^2)^{\frac{1}{2}(n-1)} = \frac{1}{2}(1 - .36)^{\frac{9}{2}} = \frac{1}{2}(.8)^9.$$

This is given at once by tables of the higher powers\*,

$$= .067,108,864.$$

For  $\Sigma_2$  we need to divide this by  $B\left(\frac{1}{2}, \frac{n-1}{2}\right) = B(4.5, 0.5)$ . This is given at the head of the column for  $q = 0.5$ ,  $p = 4.5$  in the *Tables of the Incomplete B-function* = .8590,2924. Thus the required external factor is  $.067,108,864/.8590,2924 = .0781,2174,59$ .

Table II, p. xlv, shows the computing of  $S_1$ . We have  $1 - r^2 = 1 - (.9)^2 = .19$ , but after  $I_{.19}(4, 3.5)$  we are obliged to use the formula

$$I_{.19}(4, s + .5) = 1 - I_{.81}(s + .5, 4)$$

because in the table  $p$  is always equal or greater than  $q$ . Again after  $s = 10$ , we are compelled to use the interpolation formula (xxxviii). The values of  $I_{.19}(4, q)$  were obtained from  $I_{.81}(q, 4)$  for  $q = 8$  to 24 as shown in Table I on the opposite page.

The values of  $\delta^6$  are given merely to show that they are negligible. Subtracting the results in the last column from unity we complete the first column of Table II (p. xlv) beyond 10.5, where it was not feasible to extract the values of  $I_{.19}(4, q + .5)$  without interpolation from the *Tables of the Incomplete B-function*.

Table II shows us by its last column that the contribution of  $\Sigma_1$  to the value required is .012,3844, which is probably hardly a unit wrong in the seventh decimal. We can hardly expect a better result without using the manuscript B-function tables which go to more than seven figures.

For many purposes it would be adequate to get the final result to three-figure accuracy. The last column (f) indicates that ten terms would suffice, in which case we should not need Table I and its interpolations into the B-function table. If we require four-figure accuracy we must go as far as the fifteenth term; five-figure accuracy demands the computation of seventeen terms.

We now turn to the computing of the second series  $\Sigma_2$ , and we have arranged the work in like manner. We are saved here any interpolation corresponding to Table I for all the values can be abstracted from the tables straight off. Table III (p. xlv) corresponds to Table II (p. xlv). The total contribution to  $\frac{1}{2}(1 + \alpha_r) = .0123,8391$ . Thus the chance that a sample taken from a normal population of correlation .6, will not exceed .9 is  $1 - .012,3844 - .012,3839$

$$= .975,2317$$

which will hardly be more than a unit wrong in the last figure, if there be no slip in the somewhat lengthy arithmetic.

\* *Tables for Statisticians and Biometricians*, Part II, p. 259, or Dr Comrie's edition of *Barlow's Tables*, p. 203.

Table I. *Interpolations into B-function Table.*

$q$	$I_{.81}(q, 4)$	$\delta^2$	$\delta^4$	$\delta^6$ $I_{.85}(q + .5, 4)$
8	.8602,980	---	---	---
9	.8204,525	- 31,875	---	---
10	.7774,195	- 22,808	- 266	---
11	.7321,057	- 14,007	- 632	+ 120
12	.6853,912	- 5,838	- 878	+ 122
13	.6380,929	+ 1,455	- 1002	+ 75
14	.5909,401	+ 7,746	- 1051	+ 61
15	.5445,619	+ 12,986	- 1039	+ 44
16	.4994,823	+ 17,187	- 983	+ 30
17	.4561,214	+ 20,405	- 897	+ 15
18	.4148,010	+ 22,726	- 796	+ 6
19	.3757,532	+ 24,251	- 689	0
20	.3391,305	+ 25,087	- 582	+ 1
21	.3050,165	+ 25,341	- 474	- 18
22	.2734,366	+ 25,121	- 384	+ 4
23	.2443,688	+ 24,517	- 290	---
24	.2177,527	+ 23,623		
25	.1934,989			

chance that  $r$  will be negative and exceed  $-.9 = .012,3844 - .012,3839 = .000,0005$ . Finally the chance will lie in the range  $-.9$  to  $+.9 = 1 - 2 \times .012,3844 = .975,2312^*$ .

It is interesting to compare the probability obtained with that deduced by other methods. The probability that a Sample of 10 from a population of correlation  $.6$  will give a correlation:

Relative frequency of $r$	Actual Value	By Quadrature		Garwood's Formula ( <i>Biometrika</i> , Vol. xxv, p. 71)	Fisher's Method	
		From three ordinates	From five ordinates		Simpler Method	More approximate Method
	(a)	(b)	(c)	(d)	(e)	(f)
Area of $+.90$	.024,7683	.02553	.02477	.02477	.02425	.02256
W $-.90$	.000,0005	.000,0006			.000,000,003	.000,000,002

was obtained by the simple quadrature formula for three ordinates: Area  $= h \{ \frac{1}{2} (y_0 + 2y_1 + y_2) - \frac{1}{24} (\delta^2 y_0 + 2\delta^2 y_1 + \delta^2 y_2) \}$ . The ordinates being those at  $+.90$ ,  $+.95$  and  $1.00$ .

was obtained by introducing ordinates at  $+.925$  and  $+.975$ . Only five-figure accuracy could be obtained as only five figures were used in the ordinates (see *Tables for Statisticians*, Part II, p. 192). (d) Garwood's Formula (for  $n=10$ ) is exact, but it depends on ordinates for samples of 10 and earlier samples, and these, as already stated, have only been tabled to five figures. Fisher's method (e) seems at this part of the table to give a better result than his more approximate method (f). Neither is as good as quadrature from five ordinates; both fail in the case of "Below  $-.90$ ."



Table II. *Evaluation of Terms due to  $S_1$ .*

$s$	(a) $I_{1s}(4, s+5)$	(b) $\frac{f_2(s+1)/f_{2s}}{2s+n-1} \rho^2$	(c) Product $f_0 f_2 \dots f_{2s}$	(d) $(a) \times (c)$	(e) $(d) \times .007,108,864$	(f) $S(c)$
0	.0003,872	—	1	.0003,872	.000,0260	.000,0260
1	.0029,529	1.62	1.62	.0047,837	.000,3210	.000,3470
2	.0091,875	.99	1.6038	.0147,349	.000,9888	.001,3358
3	.0202,976	.78	1.250,964	.0253,916	.001,7040	.003,0398
4	.0370,104	.675	.844,4007	.0312,516	.002,0973	.005,1371
5	.0595,727	.612	.516,7732	.0307,856	.002,0660	.007,2031
6	.0878,166	.57	.294,5607	.0258,673	.001,7359	.008,9300
7	.1212,530	.54	.159,0628	.0192,868	.001,2943	.010,2333
8	.1591,699	.5175	.082,3150	.0131,021	.000,8793	.011,1126
9	.2007,223	.50	.041,1575	.0082,612	.000,5544	.011,6670
10	.2450,085	.486	.020,0025	.0049,008	.000,3289	.011,9959
11	.2911,293	.4745,4545	.009,4921	.0027,634	.000,1854	.012,1813
12	.3382,328	.465	.004,4138	.0014,929	.000,1002	.012,2815
13	.3855,434	.4569,2308	.002,0168	.0007,776	.000,0522	.012,3337
14	.4323,810	.45	.000,9075 <sup>5</sup>	.0003,924	.000,0263	.012,3600
15	.4781,688 <sup>5</sup>	.444	.000,4029 <sup>5</sup>	.0001,927	.000,0129	.012,3729
16	.5224,353	.43875	.000,1768	.0000,924	.000,0062	.012,3791
17	.5648,103 <sup>5</sup>	.4341,1765	.000,0767 <sup>5</sup>	.0000,433	.000,0029	.012,3820
18	.6050,182	.43	.000,0330	.0000,200	.000,0013	.012,3833
19	.6428,680	.4263,1579	.000,0141	.0000,091	.000,0006	.012,3839
20	.6782,429	.423	.000,0060	.0000,041	.000,0003	.012,3842
21	.7110,898	.42	.000,0025	.0000,018	.000,0001(2)	.012,3843
22	.7414,083	.4172,7273	.000,0010(4)	.0000,008	.000,0000(5)	.012,3843(5)

Total Contribution of First Series = .012,3844.

The last factor product is obtained for  $s=21$  and should equal  $\frac{\rho^{2s}}{sB\left(\frac{n-1}{2}, s\right)} = \frac{(.36)^{22} \Gamma(26.5)}{22 \times \Gamma(4.5) \Gamma(22)}$ . Evaluating this by logarithms we find  $f_0 f_2 f_4 \dots f_{42} = .000,0010(4)$  confirming the value obtained by continuous product.

(θ) *Further Applications. Uses of the Incomplete B-Function Table in Sampling Tests.*

(i) *Replacement of Type I by a Type III or by a Normal Curve.*

We may make certain remarks which have a bearing on all the tests which lead to a Pearson curve of Type I, or to a Type VI curve which can be transformed to Type I. The equation to Type I being

$$y = y_0 x^{p-1} (b-x)^{q-1} \quad \dots\dots(xliii)$$

has its mean given by  $\bar{x} = \frac{pb}{p+q}$ , its mode by  $\tilde{x} = \frac{p-1}{p+q-2} b$  and its standard deviation by  $\sigma^2 = \frac{pqh^2}{(p+q)^2(p+q+1)}$

Now if  $q$  be large while  $p$  remains moderate (xliii) approaches the form

$$y = y_0' x^{p-1} e^{-\frac{(q-1)x}{b}} \quad \dots\dots(xliv)$$

with mean  $\bar{x} = \frac{pb}{q-1}$ , mode  $\tilde{x} = \frac{p-1}{q-1} b$  and  $\sigma^2 = \frac{pb^2}{(q-1)^2}$ , which are the values we obtain from the original curve if we make  $p(>1)$  negligible as compared with  $q$ . The probability integral will then be given by  $\Gamma_{x'}(p) \Gamma'(p)$ , where  $x' = \frac{q-1}{b} x$  and the unit may be neglected as compared to  $\Gamma_{x'}(p)$ .

This value may be taken from the *Table of the Incomplete  $\Gamma$ -Function*.

If both  $p$  and  $q$  are large (xliii) approaches the form of the Normal Curve

$$y = y_0'' e^{-\frac{1}{2} \frac{(p+q-2)^2}{b^2(p-1)(q-1)} \left(x - \frac{p-1}{p+q-2} b\right)^2} \quad \dots\dots(xlv)$$

where of course we may neglect the units as compared with  $p$  and  $q$ .

Table III. *Evaluation of Terms due to  $\Sigma_2$ .*

(a) $L_{1p}(4, s+1)$	(b) $P_{2s+1} - 1/P_{2s+1}$ $(2s+1)p^2$ $(2s+3)p^2$	(c) Product $f_0 f_1 \dots f_s$	(d) $(a) \times (c)$	(e) $(d) \times .078,121,746$	(f) $S(c)$
-.0013,032	1.2	1.2	-.0015,6384	-.0001,2217	-.0001,2217
-.0055,256	1.2	1.44	-.0079,5686	-.0006,2160	-.0007,4377
-.0140,760	-.864	1.2441,6	-.0175,1280	-.0013,6813	-.0021,1190
-.0279,276	-.72	-.8957,952	-.0250,1741	-.0019,5440	-.0040,6630
-.0475,622	-.64	-.5733,0893	-.0272,6783	-.0021,3021	-.0061,9651
-.0730,086	-.5890,9091	-.3377,3108	-.0246,5757	-.0019,2629	-.0081,2280
-.1039,261	-.5538,4615	-.1870,5106	-.0194,3949	-.0015,1865	-.0096,4145
-.1397,020	-.528	-.0987,6296	-.0137,9738	-.0010,7788	-.0107,1933
-.179,6475	-.5082,3529	-.0501,9482	-.0090,1235	-.0007,0406	-.0114,2339
-.222,5805	-.4926,3158	-.0247,2755	-.0055,0387	-.0004,2997	-.0118,5336
-.267,8943	-.48	-.0118,6923	-.0031,7970	-.0002,4840	-.0121,0176
-.314,6088	-.4695,6522	-.0055,7338	-.0017,5343	-.0001,3698	-.0122,3874
-.361,9071	-.4608	-.0025,6821	-.0009,2945	-.0000,7261	-.0123,1135
-.409,0599	-.4533,3333	-.0011,6426	-.0004,7625	-.0000,3721	-.0123,4856
-.455,4381	-.4468,9655	-.0005,2030	-.0002,3696	-.0000,1851	-.0123,6707
-.500,5177	-.4412,9032	-.0002,2960	-.0001,1492	-.0000,0898	-.0123,7605
-.543,8786	-.4363,6364	-.0001,0019	-.0000,5449	-.0000,0426	-.0123,8031
-.585,1990	-.432	-.0000,4328	-.0000,2533	-.0000,0198	-.0123,8229
-.624,2468	-.4281,0811	-.0000,1853	-.0000,1157	-.0000,0090	-.0123,8319
-.660,8695	-.4246,1538	-.0000,0787	-.0000,0520	-.0000,0041	-.0123,8360
-.694,9835	-.4214,6341	-.0000,0332	-.0000,0231	-.0000,0018	-.0123,8378
-.726,5631	-.4186,0465	-.0000,0139	-.0000,0101	-.0000,0008	-.0123,8386
-.755,6312	-.416	-.0000,057(7)	-.0000,0044	-.0000,0003	-.0123,8389
-.782,2473	-.4136,1702	-.0000,0023(9)	-.0000,0019	-.0000,0001	-.0123,8390
-.806,5011	-.4114,2857	-.0000,0009(8)	-.0000,0008	-.0000,0000(6)	-.0123,8391

Total Contribution of Second Series = .0123,8391.

factor product is obtained for  $s = 24$  and should equal  $B(\frac{1}{2}n + s, s+1) \frac{(2p)^{2s+1}}{B(\frac{1}{2}n, 2s+2) \frac{1}{2}n + 2s+1} = .0000,0009(8)$ , which accords with  $S(c)$ , obtained by continuous product.

$x = \frac{p-1}{p+q-2}b$  and  $\sigma^2 = \frac{(p-1)(q-1)b^2}{(p+q-2)^3}$ , and the probability integral may be found from the of the normal probability integral by entering it with

$$\left(x - \frac{p-1}{p+q-2}b\right) / \frac{b\sqrt{(p-1)(q-1)}}{(p+q-2)^{\frac{3}{2}}}.$$

These approximations will often be adequate when either  $p$  or  $q$  or both lie well outside our table. Other methods of finding more exact values of the incomplete B-function when  $p$  and  $q$  lie outside the of the present table have been discussed by Soper\* and by Wishart†. A consideration and description of these methods of these authors is provided in *Tables for Statisticians and Biometricians*, Part II, pp. cexxv–vi. The methods of Muller‡ and Camp§ (*Tables for Statisticians*, II, pp. xxx–xl) are also dealt with. No method has hitherto been discovered for evaluating numerically the incomplete B-function for *all* of  $p$  and  $q$ .

values of  $\beta_1$  and  $\beta_2$  for the curve

$$y = y_0 x^{p-1} (1-x)^{q-1} \\ \beta_1 = \frac{4(q-p)^2(p+q+1)}{pq(p+q+2)^2}, \quad \beta_2 = \frac{3(p+q+1)\{2(p+q)^2 + pq(p+q-6)\}}{pq(p+q+2)(p+q+3)} \quad \dots\dots(xlvi).$$

\* "Numerical Evaluation of the Incomplete B-Function," *Tracts for Computers*, No. VII. Cambridge University Press.

† *Biometrika*, Vol. XLV, pp. 1–38.

‡ *Ibid.* Vol. XLV, pp. 284–287.

§ *Ibid.* Vol. XLV, pp. 164 *et seq.*; Vol. XLVII, pp. 61 *et seq.*

By aid of these  $\beta$ 's it is possible to test readily whether a given curve of Type I,

$$y = y_0 x^{p-1} (1-x)^{q-1},$$

may be reasonably replaced by either a Type III curve, i.e. (xliv), or a normal curve, i.e. (xlv), so that the *Tables of the Incomplete  $\Gamma$ -Function*, or of the Normal Probability Integral are adequate.

The condition for a Type III curve is that  $2\beta_2 - 3\beta_1 - 6 = 0$ , or substituting from (xlv) that

$$-\frac{12(p+q)^2(pq+p+q+1)}{pq(p+q+2)^2(p+q+3)} = 0 \quad \dots\dots(xlvii).$$

Since  $p$  and  $q$  are always positive, i.e.  $> 0$ , this condition can only be approximately satisfied by either  $p$  or  $q$  or both being large. A brief examination of the  $\beta_1, \beta_2$  columns in Table II, pp. 434-494 of the present volume, will suffice to indicate that up to the value of  $p$ , or  $q=50$ , the expression (xlvii) does not become small enough to allow of Type III replacing Type I for any but the roughest purposes. The conditions for a normal curve are that  $\beta_1$  should be very small, preferably zero, and  $\beta_2$  equal or very nearly equal to 3. An examination shows that for every value of  $p, \beta_2$  falls as  $q$  rises from 0.5 to 50 from a value much above 3 to a value below 3, and then proceeds to rise again and may again pass through the value three.

In all cases in our Table II where  $\beta_2$  first approaches the value 3, the value of  $\beta_1$  is not sufficiently small to justify us in assuming it zero and applying the Normal Curve.

At and after  $p=20$ , we get values such as

$$\begin{aligned} p=20, \quad q=50, \quad \beta_1 &= .0493, \quad \beta_2 = 2.9908, \\ p=21, \quad q=50, \quad \beta_1 &= .0433, \quad \beta_2 = 2.9830, \\ p=31, \quad q=50, \quad \beta_1 &= .0111, \quad \beta_2 = 2.9450, \\ p=40, \quad q=50, \quad \beta_1 &= .0022, \quad \beta_2 = 2.9387. \end{aligned}$$

Thus while  $p$  is rising the second approach of  $\beta_2$  to 3 falls as  $\beta_1$  reaches a value where it might be negligible. When  $p=50$  the first approach of  $\beta_2$  to 3, i.e. when  $q=19, \beta_2=2.9998$ , is accompanied by a  $\beta_1=.0562$ , which is hardly a negligible  $\beta_1$ . By the time  $\beta_1$  has fallen to zero,  $\beta_2$  has passed through its minimum 2.9387 and risen to 2.9417.

It would thus appear that  $p=50, q=50$  would provide as little deviation as will occur anywhere in our table from a normal curve. The true curve is of course  $y=y_0(1-4x^2)^{49}$  with a standard deviation of  $\frac{50}{100\sqrt{101}} = 1/20.099,7512$ . If we take the distance  $x$  from the start of the range as in our table, p. 434, to be .47, the proportionate area = .2745,724. This corresponds to a distance .03 from the centre of the approximate normal curve  $y=y_0'e^{-\frac{1}{2}404x^2}$ .

Accordingly we must find  $\frac{1}{2}(1-\alpha)$  from the probability integral table for

$$x/\sigma = .03 \times 20.099,7512 = .6029,9254,$$

which gives  $\frac{1}{2}(1-\alpha) = .2732,571$ , indicating an error of .0013, or more than unity in the third significant figure. Trying again at  $x=.4$ , where the true value is .0219,304, we have from the normal curve, since its  $x=.1, x/\sigma=2.0099,751$ , the value  $\frac{1}{2}(1-\alpha) = .0222,201$ , making an error of .00029, i.e. an error of 3 in the fourth decimal place, or of 3 in the third significant figure. Thus we cannot expect to be correct to less than a unit in the third significant figure, if we replace the incomplete  $\Gamma$ -function ratio table by the normal probability integral table within the limits of  $p$  and  $q$  in the present table. Outside that table for  $p$  and  $q$  of the order 100, the results are better.

## (ii) A Convenient Univariate Formula.

In our applications to sampling tests we have frequently to interpolate for one variable only. If we have to interpolate into our table for one of the three variates only lying at  $(\theta, \phi)$  between  $z_s$  and  $z_{s+1}$ , then the following formula is convenient:

$$\begin{aligned} z_{\theta, \phi} &= (1 + \frac{1}{2}\theta\phi) \{ \phi z_s + \theta z_{s+1} \} - \frac{1}{6}\theta\phi \{ (1+\phi)z_{s-1} + (1+\theta)z_{s+2} \} \\ &+ \frac{1}{120}\theta\phi(1+\theta)(1+\phi) \{ 10(\phi z_s + \theta z_{s+1}) - 5 \{ (1+\phi)z_{s-1} + (1+\theta)z_{s+2} \} + \{ (2+\phi)z_{s-2} + (2+\theta)z_{s+3} \} \} \dots(xlviii). \end{aligned}$$

The first line is adequate if we only need to go to third differences; the second line carries us to fifth differences. If we wish to use the whole formula, the addition of the terms between curled brackets is easy

of the three  $z$  factors will already have been computed in the first line. To determine whether in the second line, we remark that if

$$2(z_n + z_{n+1}) - 3(z_{n-1} + z_{n+2}) + (z_{n-2} + z_{n+3}) \text{ be } < .0000427 \quad \text{.....(xlix),}$$

correct to six decimals. If it be  $< .000427$ , we shall be correct to five decimals; and if  $< .00427$ , correct to four decimals, by doing so.

of the tables will find it easier to work with this formula than to proceed to find differences.

*of the Difference between the Variances in two Independent Samples.*

variable  $x$  be normally distributed with mean  $m$  and standard deviation  $\sigma$ , and suppose we have independent samples with means  $x_1$  and  $x_2$ , and standard deviations  $s_1$  and  $s_2$ , also drawn from normally distributed populations. Then the distribution curve of the variance  $s^2$  in a sample of size  $n$  is given by

$$df = \frac{1}{\Gamma(\frac{1}{2}(n-1))} \left( \frac{ns^2}{2\sigma^2} \right)^{\frac{1}{2}(n-3)} e^{-\frac{ns^2}{2\sigma^2}} d \left( \frac{ns^2}{2\sigma^2} \right) \quad \text{.....(l).}$$

problem is to test the significance of the difference between  $s_1$  and  $s_2$ , or in other words to test the hypothesis that in the sampled populations  $\sigma_1 = \sigma_2$ .

appropriate criterion\* is the ratio  $\theta = s_1^2/s_2^2$ . If the hypothesis be true, it is assumed that  $\theta$  will be abouthood of unity; while if  $\theta$  be near zero or very large we shall be inclined to reject it in favour of the alternative hypothesis  $\sigma_1 < \sigma_2$  or  $\sigma_1 > \sigma_2$  respectively. If the hypothesis  $\sigma_1 = \sigma_2$  be true the sampling distribution of  $\theta$  is independent of the value of  $\sigma$  (which is frequently unknown) and is given by

$$df = \frac{n_1^{4n_1-1} n_2^{4n_2-1}}{B(\frac{1}{2}(n_1-1), \frac{1}{2}(n_2-1))} \theta^{4n_1-3} (n_2 + n_1 \theta)^{-\frac{1}{2}(n_1+n_2-2)} d\theta \quad \text{.....(li).}$$

Pearson curve Type VI. If we make the appropriate transformation

$$x = \frac{n_1 \theta}{n_2 + n_1 \theta} = \frac{n_1 s_1^2}{n_1 s_1^2 + n_2 s_2^2} \quad \text{.....(lii),}$$

the sampling distribution of  $x$  Type I curve

$$df = \frac{1}{B(\frac{1}{2}(n_1-1), \frac{1}{2}(n_2-1))} x^{4n_1-3} (1-x)^{4n_2-3} dx \quad \text{.....(liii).}$$

accordingly throw light on the hypothesis  $\sigma_1 = \sigma_2$  by entering the B-function table with

$$x = \frac{n_1 s_1^2}{n_1 s_1^2 + n_2 s_2^2}, \quad p = \frac{1}{2}(n_1-1), \quad q = \frac{1}{2}(n_2-1).$$

exceptionally small, i.e. if  $I_c(p, q)$  be small, we shall be inclined to think that  $\sigma_1 < \sigma_2$ ; if  $x$  be exceptionally large, that is if  $1 - I_c(p, q)$  be small, we shall believe that  $\sigma_1 > \sigma_2$ . The confidence in such beliefs is measured by the value of the probability  $I_c(p, q)$ .

Following points may be noted with regard to this test†:

There must be good ground for supposing that the sampling has been made from populations normally distributed.

Tables cover the range up to size 101, and no interpolation will be needful for  $p$  and  $q$  if both  $n_1$  and  $n_2$  are odd. Nor will interpolation be required in other cases unless the larger  $n$  is even and greater than 50.

The mean and standard deviation of  $x$  from (lxiii) are by (xliv) viz

$$\bar{x} = \frac{n_1-3}{n_1+n_2-6}, \quad \sigma_x = \frac{1}{n_1+n_2-2} \sqrt{\frac{2(n_1-1)(n_2-1)}{n_1+n_2}} \quad \text{.....(liv),}$$

by (lv) we have

$$\beta_2 = \frac{3(n_1+n_2)\{4(n_1+n_2-2)^2 + (n_1-1)(n_2-1)(n_1+n_2-14)\}}{(n_1-1)(n_2-1)(n_1+n_2+2)(n_1+n_2+4)} \quad \text{....(lv).}$$

able to select other criteria, and these may give a definite answer when  $\theta$  fails to do so. Thus we might consider  $s_2^2 - s_1^2$ . The Bessel  $K_\nu(x)$  function when  $n_1 = n_2$ , and a double Bessel  $K_{\nu_1, \nu_2}(x)$  function when they are not. Tables for the former are already calculated: see *Biometrika*, Vol. xxiv, pp. 344-346, and for a discussion of the double Bessel  $K$ -function, see pp. 158-178.

in somewhat different form is due to R. A. Fisher, see Section (viii), p. lviii below.

Consequently as  $n_1$  and  $n_2$  grow large, i.e. as the number of observations are increased, the distribution will approximate to the normal\*.

Having regard to the limits suggested in the footnote we may use the normal distribution. We then calculate the ratio

$$R_1 = \frac{x - \text{Mean } x}{\sigma_x},$$

and interpret its value by reference to the normal probability scale. It is easy to show that, if we neglect 3 as compared to  $n_1$  and  $n_2$ ,

$$R_1 \rightarrow \frac{s_1^2 - s_2^2}{n_1 s_1^2 + n_2 s_2^2} \sqrt{\frac{n_1 n_2 (n_1 + n_2)}{2}}.$$

Now we can look at this from another, the older, standpoint. The standard deviations of the distributions of  $s_1^2, s_2^2$  (see p. xliv) are  $\sigma^2 \sqrt{\frac{2(n_1-1)}{n}}$  and  $\sigma^2 \sqrt{\frac{2(n_2-1)}{n_2}}$ . Thus the standard deviation of their difference is  $\sigma^2 \sqrt{2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ , if we may neglect the unit as compared with  $n_1$  or  $n_2$ . Accordingly we should enter the normal probability table with

$$R_2 = \frac{(s_1^2 - s_2^2)}{\sigma^2 \sqrt{\frac{n_1 + n_2}{n_1 n_2}}} \dots\dots (lvi).$$

This will agree with the limiting value of  $R_1$  above if we take as a suitable value for  $\sigma^2$  the weighted mean variance of the two samples, i.e.  $\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$  †.

Thus in the actual formal process of looking up an  $R$  in the normal probability table the two methods agree when  $n_1$  and  $n_2$  are large. But the meaning of the expression  $n_1 s_1^2 + n_2 s_2^2$  is different in the two methods. In (li) and (lii)  $s_1^2$  and  $s_2^2$  are varying from sample pair to sample pair, both in numerator and denominator of  $x$  or  $1-x$ . But in the value

$$R_2 = \frac{s_1^2 - s_2^2}{n_1 s_1^2 + n_2 s_2^2} \sqrt{\frac{n_1 n_2 (n_1 + n_2)}{2}}$$

$s_1^2$  and  $s_2^2$  are supposed to vary in the numerator and  $n_1 s_1^2 + n_2 s_2^2$  stands for a constant  $\sigma^2 (n_1 + n_2)$ . It is only the first method which shows us that the limiting distribution of  $R$  is the same if  $n_1$  and  $n_2$  are large, whether we suppose  $s_1^2$  and  $s_2^2$  to vary or not to vary in the denominator.

(d) If it be not known, whether or not the variables are normally distributed the test must be used with caution for both small‡ and large samples. We know that the means of large samples from any parent population follow closely the normal law; it has not yet been shown that the standard deviations of samples from any non-normal parent population follow a distribution law like (l), but we may argue from the values of  $\beta_1$  and  $\beta_2$  for the standard deviations in the case of large samples from any population that their standard deviations will approach a normal distribution as the size of the sample increases.

*Illustration 7.* Weights were taken of two series of male mice between 160 and 180 days old; the first series was for litters of 5, and the second for litters of 4.

\* But the approximation is not so rapid as some have suggested. Thus if  $n_1 = 80, n_2 = 100$  we have  $\beta_1 = .002,2234$  and  $\beta_2 = 2.938,0815$ . We may perhaps treat  $\beta_1$  as practically zero, but  $\beta_2$  is hardly sufficiently close to 3 to use a normal distribution. This corresponds to values of  $p = 39.5$  and  $q = 49.5$  lying inside our table. If  $n_1 = 101$  and  $n_2 = 201$ , then  $\beta_1 = .001,0929$  and  $\beta_2 = 2.968,8434$  for which a normal distribution might be reasonably adopted for most practical purposes. If we are content with two-figure accuracy we before we can work with that distribution. This means using, say, Muller's process (*Tables for Statisticians*, Part II, pp. cxxxiv-cxxxvi) to determine the incomplete B-function in the field from  $p, q, 50, 50$  to  $70, 70$ . (For  $p = 60, q = 70$ , we have  $\beta_1 = .000,7116, \beta_2 = 2.955,953$ , the latter being not close enough to 3 to provide more than two-figure accuracy.)

† Actually the most probable value of  $\sigma^2$  is  $\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}$ , and is obtained by making the expression

$$\left(\frac{1}{\sigma^2}\right)^{\frac{1}{2}(n_1-1) + \frac{1}{2}(n_2-1)} e^{-\frac{n_1 s_1^2 + n_2 s_2^2}{2\sigma^2}}$$

a maximum for  $\sigma^2$ .

‡ See E. S. Pearson, *Biometrika*, Vol. xxiii, pp. 129-311.

es 1. (Litters of 5.)  $n_1 = 43$ ,  $\bar{x}_1 = 23.849$  gm.,  $s_1^2 = 22.383$  (gm.)<sup>2</sup>.

es 2. (Litters of 4.)  $n_2 = 29$ ,  $\bar{x}_2 = 25.698$  gm.,  $s_2^2 = 19.984$  (gm.)<sup>2</sup>.

assuming that the distribution of weight within a homogeneous group is nearly normally distributed we first apply the test to compare  $s_1^2$  and  $s_2^2$ .

$$x = n_1 s_1^2 / (n_1 s_1^2 + n_2 s_2^2) = .6241, 872, \quad p = \frac{1}{2} (n_1 - 1) = 21, \quad q = \frac{1}{2} (n_2 - 1) = 14.$$

could be adequate to carry out the interpolation to four decimal places, but we will illustrate the formulae (xlvi) and (xlix) on this case. The values needed from our table are:

$x = .60$	$.4907, 854$	$= z_{s-2}$
$.61$	$.5386, 903$	$= z_{s-1}$
$.62$	$.5863, 334$	$= z_s$
$.63$	$.6330, 552$	$= z_{s+1}$
$.64$	$.6782, 206$	$= z_{s+2}$
$.65$	$.7212, 441$	$= z_{s+3}$

so by a continuous operation on the machine we have by (xlix) our criterion = .0000,740. This lies between .000,427 and .0000,427, and accordingly we shall be correct to five decimals, if we use only the formula of (xlvi). We find

$$\theta = .41872, \quad \phi = .58128, \quad \theta\phi = .243, 394, \quad 1 + \frac{1}{2}\theta\phi = 1.121, 697, \quad \frac{1}{6}\theta\phi = .040, 5637,$$

accordingly

$$z_{\theta, \phi} = .679, 6326 - .073, 5872 = .606, 0454.$$

is correct to five decimals, or  $I_x(p, q) = .60605^*$ . It follows that even where there is no difference in the populations sampled we should expect to find  $s_1^2$  still greater than  $s_2^2$  as here occurs in 39% of pairs of random samples of this size. Hence there appears no reason to discard the hypothesis  $\sigma_1 = \sigma_2 = \sigma$  in favour of  $\sigma_1 > \sigma_2$ , i.e. no reason to suppose that the variability in weight of male mice among litters of five mice is greater than among litters of four mice.

We may now compare the means, assuming a common standard deviation,  $\sigma$ . We shall use as an estimate the value of  $\sigma$  that given in the second footnote, p. xlviii, i.e.

$$\sigma_c^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = 22.029,$$

so the ratio of the difference in means to the estimated standard error of that difference is provided by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_c \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -1.639.$$

The sampling distribution of  $t$ , given under Section (vi) below, is:

$$df = \frac{1}{B\left(\frac{1}{2}n, \frac{1}{2}\right)} \left(1 + \frac{t^2}{n}\right)^{-\frac{1}{2}(n+1)} d\left(\frac{t}{\sqrt{n}}\right) \quad \dots\dots (lviii),$$

in our case  $n = n_1 + n_2 - 2$ .

Using the transformation  $x = \left(1 + \frac{t^2}{n}\right)^{-1}$  we find that the chance that  $t \geq t_0$  is equal to  $\frac{1}{2} I_{x_0} \left(\frac{1}{2}n, \frac{1}{2}\right)$ ,

or  $x_0 = 1 / \left(1 + \frac{t_0^2}{n}\right)$ . For our case  $x_0 = .963, 0423$ , and  $n = 70$ .

We may prefer to use the formula  $z_{\theta, \phi} = \phi z_s + \theta z_{s+1} - \frac{1}{4} \theta \phi (\delta^2 z_s + \delta^2 z_{s+1})$  .....(lvii),

correct to four decimal places provided  $\delta^2 z_s$  or  $\delta^2 z_{s+1} < .0060$ , and  $\delta^2 z_s < .0020$ . But formula (xlvi) saves all differencing and the criterion (xlix) is simpler than the double criterion. Besides this it is advisable to use as few formulae as possible and to cover much more ground than (lvii). Applying (lvii) in this case, after cutting down all  $x$  values to four figures, we have  $.6061$  in practical agreement with the above value.

# NUMERICAL ILLUSTRATION OF DIFFERENCE OF MEANS

Thus we have to ascertain the value of  $\frac{1}{2}I_{.963,0423}(35, 0.5)$  from our table. The values we require are:

$$\begin{array}{ll} x_{s-2} = .94 & z_{s-2} = .0380,987 \\ x_{s-1} = .95 & z_{s-1} = .0590,067 \\ x_s = .96 & z_s = .0921,005 \\ x_{s+1} = .97 & z_{s+1} = .1456,708 \\ x_{s+2} = .98 & z_{s+2} = .2360,314 \\ x_{s+3} = .99 & z_{s+3} = .4032,812 \end{array}$$

Applying our criterion (xlix) we have

$$2(z_s + z_{s+1}) - 3(z_{s-1} + z_{s+2}) + (z_{s-2} + z_{s+3}) = .031,8082,$$

and this is less than .0427, i.e. if we retain only the first line of our formula (xlviii), we cannot be sure of being correct in the second significant figure. We therefore retain the second line of our formula (xlviii). We have

$$\begin{aligned} \theta &= .30423, \quad \phi = .69577, \quad \theta\phi = .211,6741, \\ (1 + \frac{1}{2}\theta\phi) &= 1.105,8371, \quad \frac{1}{6}\theta\phi = .035,2790, \quad \frac{1}{120}\theta\phi(1 + \theta)(1 + \phi) = .003,9013, \\ \phi z_s + \theta z_{s+1} &= .108,3982, \quad (1 + \phi)z_{s-1} + (1 + \theta)z_{s+2} = .407,9010, \end{aligned}$$

and

$$(2 + \phi)z_{s-2} + (2 + \theta)z_{s+3} = 1.031,9580.$$

Hence

$$\begin{aligned} z_{\theta, \phi} &= .119,8707 - .014,3903 + .000,2982 \\ &= .052,8893. \end{aligned}$$

This value is probably correct to five decimal places, i.e. the chance that  $\bar{x}_1$  is so much less than  $x_2$  if they were samples of the same population would be .05289.

Had we referred  $t$  to the normal scale as sufficiently representing the distribution curve (lviii) we find  $\frac{1}{2}(1 - \alpha)$ , for  $x = 1.639$ , to be .05061.

Thus the normal probability table does not give a correct answer to three decimals, or is out two units in the second significant figure. This error may be negligible for many statistical purposes, but it confirms the view previously expressed, that for accurate work if only to three decimal places we cannot start with the Normal Curve, where our present table ends.

The value .053 is not clearly significant of a difference in the mean weights of male mice from litters of five and four, but the result suggests that the mice in the larger litters are possibly on the average lighter than those in the smaller litters, and this result is borne out if a study be made for a greater range of litter sizes\*.

## (iv) *Test of Hypotheses regarding the form of Regression Curves.*

Suppose that  $x$  and  $y$  are two variable characters; that the former, which may or may not be continuous, is divided into a number,  $k$ , of categories (or arrays), and that the latter is continuous. In a sample of  $N$  individuals let  $\bar{y}$  and  $s$  represent the mean and standard deviation of the total distribution of  $y$ . Further, let  $n_t$ ,  $y_t$ ,  $\bar{y}_t$  and  $s_t$  represent the number, any individual, the mean and standard deviation respectively of the  $y$ 's falling into the  $t$ th category or array of  $x$ . Let  $\Sigma$  denote the summation for all  $y$ 's in an array and  $S$  the summation of all arrays, then

$$\bar{y}_t = \frac{1}{n_t} \Sigma (y_t), \quad s_t^2 = \frac{1}{n_t} \Sigma (y_t - \bar{y}_t)^2 \quad \dots\dots(\text{lix}),$$

$$\bar{y} = \frac{1}{N} \sum_{t=1}^{t=k} S(n_t \bar{y}_t), \quad s^2 = \frac{1}{N} \sum_{t=1}^{t=k} S \Sigma (y_t - \bar{y})^2 \quad \dots\dots(\text{lx}).$$

The problem is now to test the hypothesis that in the sampled population the regression curve of  $y$  upon  $x$  is of a certain form

$$Y_x = f(x, a_0, a_1, \dots a_{i-1}) \quad \dots\dots(\text{lxi}),$$

where the  $a$ 's represent  $i$  parameters entering into the function  $f$ .

The following test may be used under the conditions indicated.

\* Dr Edgar Schuster made a study many years ago of the inheritance of the size of the long bones in adult mice, with the result that small correlations only were found between the bones of parents and offspring. His data indicated that size of skeleton was influenced to a considerable extent by size of the litter in which the individual was born, and this probably obscured the hereditary influence.

suppose that the parameters  $a_1, a_2, \dots, a_t$  enter into (lxi) in a linear form; for example, the curve may be a  $(t-1)$ th order parabola\*

$$Y_x = a_0 + a_1x + a_2x^2 + \dots + a_{t-1}x^{t-1} \quad \text{.....(lxii).}$$

values of the  $a$ 's can then be determined by minimising

$$\frac{1}{N} \sum_{t=1}^{t-k} n_t (y_t - Y_t)^2,$$

$Y_t$  is the ordinate of (lxii) for  $x = t$ .

The criterion to use in testing the hypothesis—which is a criterion of goodness of fit—may be taken as

$$\psi = \frac{\frac{1}{N} \sum_{t=1}^{t-k} n_t (y_t - Y_t)^2}{\frac{1}{N} \sum_{t=1}^{t-k} n_t s_t^2} \quad \text{.....(lxiii),}$$

ratio of the weighted mean square deviation of the array means from the fitted curve to the weighted array variances.

A. Fisher† has shown that if in the population sampled

- (i) the regression curve be of the supposed form,
- (ii) the standard deviations of the arrays of  $y$  are homoscedastic,
- (iii) the distribution in these arrays is normal,

the distribution of  $\psi$  in repeated samples of  $N$  follows the law

$$df = \frac{1}{B(\frac{1}{2}(N-k), \frac{1}{2}(k-1))} \psi^{k(k-1)/2} (1+\psi)^{-(N-k)/2} d\psi \quad \text{.....(lxiv).}$$

The transformation  $x = 1/(1+\psi)$  applied to (lxiv) leads to the B-function form

$$df = \frac{1}{B(\frac{1}{2}(N-k), \frac{1}{2}(k-1))} x^{k(N-k)/2} (1-x)^{k(k-1)/2} dx \quad \text{.....(lxv).}$$

This result is true whether in repeated sampling the array totals,  $n_t$ 's, are kept the same or vary in a regular manner.

*Special Cases.*

1. To test the hypothesis that  $Y_x = a_0 = \text{a constant}$ , that is to say that the array means are uncorrelated in the population sampled.

In this case  $\psi = \eta^2/(1-\eta^2)$ ,  $x = 1/(1+\eta^2)$  and (lxv) takes the form of the well-known distribution‡ of  $\eta^2$  in repeated samples when the population value is zero, namely,

$$df = (\eta^2)^{k(k-1)/2} (1-\eta^2)^{(N-k)/2-1} d\eta^2 \quad \text{.....(lxvi).}$$

Tables have then to be entered with

$$x = 1/(1+\eta^2) = 1 - \frac{1}{N} \sum_{t=1}^{t-k} n_t (y_t - \bar{y})^2 / s^2,$$

$$p = \frac{1}{2}(N-k), \quad q = \frac{1}{2}(k-1).$$

As  $\eta^2$  increases from 0 towards 1 the hypothesis tested becomes less and less likely. If on the other hand  $\eta^2$  is exceptionally low so that  $1 - I_x(p, q)$  is very small, this shows that the variation in the array means is less than would be expected through chance, and we are naturally led to question whether (lxvi) is the correct distribution, it having been deduced on the basis of the *three* hypotheses (c), (i)–(iii), any one of which—not necessarily (c) (i)—may not hold.

The curve might also be of the form  $Y_x = a_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \dots + \frac{a_{t-1}}{x^{t-1}}$ .

*Journal of the Royal Statistical Society*, Vol. LXXXV, pp. 597–612.

The distribution of  $\eta^2$  is “well known,” but it is not so generally recognised that the proof depends on a series of very restrictive hypotheses; in particular, if the correlation surface be normal and the subranges finite, then the arrays cannot be truly homoscedastic. If the subranges be finite and the arrays truly homoscedastic, then (iii) must be interpreted as applying only to the distribution of their summation will not give a surface which is in itself normal. Hence bivariate normal surfaces are theoretically inadmissible on this test.



## TESTS OF LINEAR REGRESSION

(b)  $i=2$ . To test the hypothesis  $Y_x = a_0 + a_1x$ , that is to say that the regression in the population is linear.

$$\psi = (\eta^2 - r^2)/(1 - \eta^2) \quad \text{and} \quad x = (1 - \eta^2)/(1 - r^2) \quad \dots\dots(\text{lxvii}).$$

Here

The tables have then to be entered with

$$x = (1 - \eta^2)/(1 - r^2), \quad p = \frac{1}{2}(N - k), \quad q = \frac{1}{2}(k - 2),$$

and the hypothesis of linear regression becomes less probable as  $x$  tends from 1 to 0. The following points may be noted with regard to these tests:

(i) There must be good reason for supposing that the standard deviations of the arrays are the same and that the array distributions are normal in the parent population, otherwise the test is not one of linear regression, but of whether one of the three hypotheses (c), (i)–(iii), be incorrect. In anthropological distributions (c) (ii) and (c) (iii) are very often known to be incorrect, and this test of linearity of regression is inapplicable.

(ii) The present B-function table only extending to  $p, q = 50, 50$ , or only admits of  $N$  being  $100 + k$ , or as  $k$  is usually not in excess of 15 to 20 groups, it will not be of service, when  $N$ , as frequently, is over 115 to 120. In such a case  $p$  is large and  $q = \frac{1}{2}(k - 2)$  is small, we are accordingly thrown back on forming

$$I_x(p, q) = 1 - I_{1-x}(q, p).$$

Now  $I_{1-x}(q, p)$  will be a case in which  $p$  is large and  $q$  small, or we may use the curve

$$y = y_0 x^{q-1} e^{-(p-1)x}$$

as indicated on p. xliv. Writing  $(p-1)x = x'$  our curve will be of the form:

$$y = y_0' x'^{q-1} e^{-x'},$$

and accordingly the value  $I_{1-x}(p, q)$  will be given by the incomplete  $\Gamma$ -function ratio

$$I(x', q-1),$$

where

$$x' = (p-1) \left( 1 - \frac{1-\eta^2}{1-r^2} \right) = \frac{1}{2}(N-k-2) \frac{\eta^2-r^2}{1-r^2},$$

and accordingly the required probability is, in the notation of the *Tables of the Incomplete  $\Gamma$ -Function*\*,

$$\left. \begin{aligned} &1 - I(u, \frac{1}{2}(k-4)) \\ &u = \frac{N-k-2}{\sqrt{2}(k-2)} \frac{\eta^2-r^2}{1-r^2} \end{aligned} \right\} \quad \dots\dots(\text{lxviii}).$$

where

(iii) When the samples are *small* the sampling distribution of  $x$  does not appear to be greatly modified when the array distributions differ considerably from the normal†.

*Illustration 8.* (Case  $i=1$ .)

The table below shows the mean and standard deviation of length of life at marked voltage for each of 15 samples of 5 lamps which were withdrawn for testing from time to time during the course of routine production‡. Each sample may be taken as representative of the quality of output at the time it was withdrawn, and the problem is to consider whether there is any evidence for changes in quality with time.

Table IV. *Length of Life of Lamps in Hours.*

Sample No.	Mean	Standard Deviation	Sample No.	Mean	Standard Deviation
1	1295	440	9	1715	385
2	2005	435	10	1650	460
3	2445	580	11	1935	560
4	1900	345	12	1760	280
5	2570	290	13	2175	465
6	1980	510	14	1570	505
7	1990	445	15	1670	380
8	1990	315			

\* Published by H.M. Stationery Office.

† E. S. Pearson, *Biometrika*, Vol. xxiii, pp. 114–33.

‡ These figures represent data of some years past of the General Electric Co. Ltd. of England.

shown below\* that it is justifiable to assume there is no change in the standard deviation within a group of lamps manufactured at the same time, i.e. that the 15 values of  $s_i$  vary only through chance fluctuations from a common  $\sigma$ . Further, in this case there is evidence that the distribution of length of life of a homogeneous group is near enough to normal for the application of the present test. We find

$$\eta^2 = .3449, \quad N = 75, \quad k = 15,$$

consequently have to enter the tables with

$$x = 1 - \eta^2 = .6551, \quad p = \frac{1}{2}(N - k) = 30, \quad q = \frac{1}{2}(k - 1) = 7.$$

As we require only to interpolate for  $x$ . We have:

$x$	$I_x(30,7)$
.63	$z_{s-2} = .00687$
.64	$z_{s-1} = .00951$
.65	$z_s = .01303$
.66	$z_{s+1} = .01766$
.67	$z_{s+2} = .02368$
.68	$z_{s+3} = .03142$

Applying our criterion (xlix), p. xlvii, we find its value to be .00010, this is less than .000427, or working the first line of (xlviii), p. xlvi, we shall have an answer correct to five decimal places,

$$\theta = .51, \quad \phi = .49, \quad \theta\phi = .2499, \quad 1 + \frac{1}{2}\theta\phi = 1.12495, \quad \frac{1}{6}\theta\phi = .04165.$$

$$z_{\theta,\phi} = .01731 - .00209 = .01522,$$

which is correct to five figures.

Accordingly if the difference in sample means was due to chance only, we should not expect an  $\eta^2$  as large as the observed in more than 1% to 2% of trials. It is therefore not improbable that there were variations in the lamp quality from time to time, as measured by mean length of life.

Illustration 9. (Case  $i = 2$ .)

The table below shows the observed relationship between two variables in a sample of thirty. Is there any reason to question the hypothesis that the regression of  $y$  on  $x$  is linear in the population sampled†?

Scale of  $y$  Variable.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	Totals
1	.	.	.	.	.	1	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	1	4
2	.	.	1	.	.	.	.	1	.	1	1	1	.	1	1	.	.	.	.	.	.	.	7
3	1	.	.	.	2	.	3	4	1	.	1	.	.	1	.	.	.	.	.	.	.	.	13
4	.	2	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3
5	.	.	.	.	1	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	3
6	1	2	1	.	3	2	4	5	1	3	3	1	.	2	1	.	.	.	.	.	.	1	30

$$\text{find} \quad r = .439, \quad r^2 = .1925,$$

$$\eta = .572, \quad \eta^2 = .3270.$$

consequently the tables should be entered with

$$x = \frac{1}{1} \frac{\eta^2}{r^2} = .8334, \quad p = \frac{1}{2}(N - k) = 12.5, \quad q = \frac{1}{2}(k - 2) = 1.5.$$

As a bivariate interpolation.

The occurrence of  $p$  at a half unit and  $x$  to be interpolated suggests the modification of formula (viii) will give us a result correct to the fourth, if not to the fifth decimal place. We retain  $\theta_0$  and  $\theta_1$ , put  $\lambda = .5$  and  $\chi_0 = 1$ ,  $\chi_1 = 0$ . Hence we have the convenient formula

$$\begin{aligned} z_{\theta_0, .5, 1} &= \frac{1}{2} \{ \theta_0 (z_{000} + z_{010}) + \theta_1 (z_{100} + z_{110}) \} \\ &\quad - \frac{1}{12} \theta_0 \theta_1 \{ (1 + \theta_0) (\delta_x^2 z_{000} + \delta_x^2 z_{010}) + (1 + \theta_1) (\delta_x^2 z_{100} + \delta_x^2 z_{110}) \} \\ &\quad - \frac{1}{16} \{ \theta_0 (\delta_p^2 z_{000} + \delta_p^2 z_{010}) + \theta_1 (\delta_p^2 z_{100} + \delta_p^2 z_{110}) \} \end{aligned} \quad \text{.....(lix).}$$

p. lvii below.

† This table is taken from the experimental material referred to in *Biometrika*, Vol. xxi, pp. 346-9.

The final subscript 0 to the  $z$ 's merely signifies that the values of the  $z$ 's are to be sought under a constant  $q$ .  
In the present illustration

$$\frac{1}{2}\theta_0 = .33, \quad \frac{1}{2}\theta_1 = .17, \quad \frac{1}{12}\theta_0\theta_1 = .0815, \quad \frac{1}{16}\theta_0 = .04125, \quad \frac{1}{16}\theta_1 = .02125.$$

The values required from the table are:

	$z_{000}$	$z_{010}$	$z_{100}$	$z_{110}$
Tabular values	·2068,222	·1766,323	·2338,525	·2020,264
$\delta_x^2 z$	26474	27613	28325	29970
$\delta_p^2 z$	47286	41685	45466	40640

and we have accordingly

$$z_{\theta_0, .5, 1} = .2006,394 - .0013,684 - .0005,500 \\ = .1987,210,$$

which is probably correct to the fifth decimal place\*.

For many purposes the hyperbolic interpolation, i.e. the first line value in (lxix), which gives ·2006, would be accurate enough.

Since we should expect in 2 samples in 10, or 1 in 5,  $x$  would have a smaller value, or the criterion  $(\eta^2 - r^2)/(1 - \eta^2)$  a larger value, were the hypothesis of linear regression true, there seems no reason to reject it on the slender evidence available.

(v) *Test of the Significance of a Multiple Correlation Coefficient.*

Let  $R$  be the multiple correlation coefficient in a sample of  $N$  individuals, between a dependent variate  $x_0$  and  $n$  independent variates  $x_1, x_2, \dots, x_n$ . If the independent variates in the population sampled are normally correlated and  $x_0$  is normally distributed, but not correlated with them, then the distribution of  $R^2$  in repeated samples of  $N$  takes the form

$$df = \frac{1}{B(\frac{1}{2}n, \frac{1}{2}(N-n-1))} (R^2)^{\frac{1}{2}(n-2)} (1-R^2)^{\frac{1}{2}(N-n-3)} d(R^2) \quad \dots\dots(lxx).$$

The table may therefore be entered with

$$x = 1 - R^2, \quad p = \frac{1}{2}(N-n-1), \quad q = \frac{1}{2}n,$$

and the chance of finding  $R^2 \geq R_0^2$  becomes  $I_{1-R_0^2}(\frac{1}{2}(N-n-1), \frac{1}{2}n)$ ,  $R$  being treated as always positive.

In this case as in (ii), when  $N$  is large, the required incomplete B-function ratio may lie outside the table, but  $n$  being small we can apply a Type III curve as in Section (i) above, and thus get a good approximation by taking out of the *Tables of the Incomplete  $\Gamma$ -Function* the values

$$I(u, \frac{1}{2}n),$$

where

$$u = \frac{N-n-3}{\sqrt{2n}} (1-R_0^2) \quad \dots\dots(lxxi).$$

*Corollary.* In the special case of  $n=1$ ,  $R$  becomes the ordinary bivariate product moment coefficient of correlation and we find for the sampling distribution of  $r$

$$df = \frac{1}{B(\frac{1}{2}, \frac{1}{2}(N-2))} (r^2)^{-\frac{1}{2}} (1-r^2)^{\frac{1}{2}(N-4)} d(r^2) \quad \dots\dots(lxxii).$$

The chance of  $r \geq r_0$  then becomes

$$\frac{1}{2} I_{1-r_0^2}(\frac{1}{2}(N-2), \frac{1}{2}).$$

Our table suffices for values of  $N$  up to 102. Beyond this value we have for our  $\beta_1, \beta_2$ :

$N$	$\beta_1$	$\beta_2$
100	0	2·9406
200	0	2·9701
400	0	2·9850

which indicate the slow degree of approach to normal distributions.

Another way of entering the table is to take the chance of  $r \geq r_0$  to be

$$1 - I_{\frac{1}{2}(1+r_0)}(\frac{1}{2}(N-2), \frac{1}{2}(N-2)).$$

\* Using a  $\delta^2$  formula first to interpolate for  $x$  for  $p=11, 12, 13$  and  $14$ , and, then interpolating for  $p$  from these four values, gave  $q=.1997$ , using  $z$ 's to four figures only.

stration 10. Suppose  $N=102$ ,  $r_0=.20$ , then the chance of  $r$  exceeding .20 in a sample of 102 from a bivariate population of zero correlation is

$$1 - I_{.60}(50, 50) = 1 - .9780,696 = .0219,304.$$

Now suppose we endeavour to replace (lxxii) by a normal curve; we shall have

$$\begin{aligned} df &= \text{const.} \times (1-r^2)^{\frac{1}{2}(N-4)} dr \\ &= \text{const.} \times e^{-\frac{1}{2}r^2(N-4)} \end{aligned} \quad \text{.....(lxxiii),}$$

normal curve of standard deviation  $\sigma = \frac{1}{\sqrt{N-4}} = \frac{1}{9.8994,949}$ . The corresponding area for the normal

will be found by determining  $\frac{1}{2}(1-\alpha)$  for  $x=1.9798,9898$  and the probability integral will be .761,424 = .0238,578. Comparing the two values .02193 and .02386, we see that the normal curve will not be true value correct to the third decimal place; in this case the error is 2 in the second significant figure. While such agreement may for certain purposes be good enough for statisticians, it will not appeal to the mathematician. We must get nearer to the exact value 3 than  $\beta_3=2.94$  before we can use a normal curve in these cases. The mathematician is therefore advised for the range  $N=100$  to 200 to use Wishart's process of determining  $\int_0^\theta \cos^p \theta d\theta$ †. That process is not very lengthy as we need only  $\phi_0(x)$ ,  $\phi_1(x)$  and  $\phi_2(x)$  and the ratio is

$$I_0(p) = \sqrt{\frac{1}{2}p} \frac{\Gamma(\frac{1}{2}p)}{\Gamma(\frac{1}{2}(p+1))} \left\{ \phi_0(x) - \frac{1}{p} \phi_1(x) + \frac{1}{p^2} \phi_2(x) \dots \right\} \quad \text{.....(lxxiv),}$$

$x = 2\sqrt{p} (1 - \cos \theta) / \sin \theta$ .

Factor may be found from tables of the complete  $\Gamma$ -function (*Tracts for Computers*, No. VIII). Tables of  $\phi(x)$ 's are given in the work cited below. In the case given above  $r = \sin \theta = .20$  and  $p = 99$ . We have

$$x = 2.0102,826, \quad \text{External factor} = 1.0025,284,$$

$$\phi_0(x) = .4777,9936, \quad \frac{\phi_1(x)}{p} = .0009,3784, \quad \frac{\phi_2(x)}{p^2} = .0000,0259,$$

$\phi_3(x)/p^3$  contributes a unit in the eighth place. Thus the series in brackets is .4768,6410, or multiplying by external factor

$$I_0(p) = .4780,698.$$

Subtracting this from .5 we have for the desired answer .0219,302, which agrees to within two units in the fifth place with the value given by the B-function table. As we have only used  $\delta^4$  interpolation for  $\phi_0(x)$ ,  $\delta^2$  interpolation for  $\phi_1(x)$  and linear interpolation for  $\phi_2(x)$ , the seventh figure difference is explicable. Wishart's process is not too laborious when  $p$  is  $\geq 100$ , we strongly recommend it for symmetrical situations outside our table. After  $p=400$ , no doubt the normal probability integral table will suffice for the needs of many mathematicians.

*Generalised "Student's" Test for Samples from an  $n$  Variate Normal Population.*

Student's original  $z$  (or  $t$ ) test‡ was developed to measure in small samples the significance of the difference between a sample mean and a hypothetical parent population mean, when only the sample standard deviation is known. The test was later extended by R. A. Fisher§ to deal with the difference between means of two samples. Recently H. Hotelling|| has shown that the result may be generalised still further to meet the case in which not one, but a number of correlated characters have been measured for each individual in the sample. Here a generalised criterion  $T'$  follows the sampling law

$$df = \frac{2}{n^{1/2} B(\frac{1}{2}(n-h+1), \frac{1}{2}h)} T'^{h-1} \left(1 + \frac{T'^2}{n}\right)^{-\frac{1}{2}(n+1)} dT' \quad \text{.....(lxxv),}$$

we take the value of (xliv)  $\sigma^2 = pqb^2/(p+q)^2(p+q+1)$ , and we have  $\sigma^2 = 1/\sqrt{N-1}$ , but the difference is of the order of  $1/N$  when we use the normal curve.

† *Tables for Statisticians*, Part II, pp. cxxii-cxxiv, Table XLVI. The factor outside the curled brackets is  $\sqrt{\frac{p-1}{p}} c_0$ ,

$c_0$  is given in Table XLV for values of  $p=101$  onwards.

‡ *Biometrika*, Vol. VI, pp. 1-25.

§ *Biometrika*, Vol. V, No. 3, pp. 90-104.

|| *Annals of Mathematical Statistics*, Vol. II, pp. 359-378, 1931. See also S. S. Wilks, *Biometrika*, Vol. XXIV, pp. 487-488.

where there are  $h$  variable characters, and  $n$  represents the number of the degrees of freedom depending on the particular form of application. Here  $T$  may take a variety of forms for which reference must be made to Hotelling's paper.  $T$  itself lies between 0 and  $+\infty$ . One illustration of the use of (lxxv) may be cited here.

Case of a sample of  $N$  individuals from a normal population, each individual being measured for  $h$  correlated characters.

We have  $n = \text{degrees of freedom} = N - 1$ .

Let  $s_t$  be the standard deviation in the sample of the  $t$ th character,  $r_{tt'}$  the correlation coefficient in the sample of the  $t$ th and  $t'$ th characters. Let  $R$  be the determinant

$$\begin{vmatrix} 1, & r_{12}, r_{13}, \dots, r_{1h} \\ r_{21}, & 1, & r_{23}, \dots, r_{2h} \\ r_{31}, & r_{32}, & 1, & \dots, r_{3h} \\ \dots & \dots & \dots & \dots \\ r_{h1}, & r_{h2}, & r_{h3}, & \dots, 1 \end{vmatrix}$$

where of course  $r_{tt} = r_{tt}$ .

Let  $R_{tt'}$  be the minor corresponding to the constituent  $r_{tt'}$ , and  $m_t$  the population mean of the  $t$ th character.

$$\text{Then } \frac{T^2}{n} = S \frac{R_{tt}}{R} \frac{(\bar{x}_t - m_t)^2}{s_t^2} + 2S \frac{R_{tt'}}{R} \frac{(\bar{x}_t - m_t)(\bar{x}_{t'} - m_{t'})}{s_t s_{t'}} \dots\dots (lxxv) \text{ bis,}$$

or  $\frac{T^2}{n}$  takes the form familiar in the surface of multiple variation as the power of  $c^{-1}$ , when  $s_t$  the individual value,  $\sigma_t$  the population standard deviation and the  $\rho_{tt'}$ 's of the population are replaced by the sample mean  $\bar{x}_t$ , the sample standard deviation  $s_t$  and the sample correlations  $r_{tt'}$ .

In the case of two variates only

$$\frac{T^2}{n} = \frac{1}{1 - r_{12}^2} \left\{ \frac{(\bar{x}_1 - m_1)^2}{s_1^2} + \frac{(\bar{x}_2 - m_2)^2}{s_2^2} + 2 \frac{r_{12}(\bar{x}_1 - m_1)(\bar{x}_2 - m_2)}{s_1 s_2} \right\} \dots\dots (lxxv) \text{ ter.}$$

For the case of one variate, we have

$$\frac{T^2}{n} = \frac{(\bar{x} - m)^2}{s^2}, \text{ that is "Student's" } z\text{-test,}$$

and (lxxv) may be turned into the B-function type by the transformation

$$u = \frac{1}{1 + \frac{T^2}{n}} \dots\dots (lxxvi),$$

when we have

$$df = \frac{1}{B(\frac{1}{2}(n-h+1), \frac{1}{2}h)} u^{h(n-h-1)} (1-u)^{\frac{1}{2}(h-2)} du \dots\dots (lxxvii).$$

Thus the chance of  $T^2 \geq T_0^2$  is the incomplete  $\beta$ -function ratio

$$I_{\frac{1}{1+\frac{T_0^2}{n}}}(\frac{1}{2}(n-h+1), \frac{1}{2}h).$$

It will be noted that if  $h = 1$ , or if we are dealing with the case of a single variable only we have the simple "Student's" distribution

$$df = \frac{1}{\sqrt{n} B(\frac{1}{2}n, \frac{1}{2})} \left(1 + \frac{t^2}{n}\right)^{-\frac{1}{2}(n+1)} dt \dots\dots (lxxviii),$$

or the chance of finding  $t \geq t_0$  is  $\frac{1}{2} I_u(\frac{1}{2}n, \frac{1}{2})$ .

While the generalised  $T$  as defined by Hotelling is a positive quantity, the  $t$  of the special case  $h = 1$  may be either positive or negative. Illustrations of applications in the case of  $h = 1$  were considered in the Introduction to *Tables for Statisticians and Biometricians*, Part II, pp. cxxi-cxliii, with special tables for symmetrical distributions. It should be noted that in the work just referred to "Student's" original notation, i.e.  $z$  for  $\frac{t}{\sqrt{n}}$ , and  $n$  for the present  $n + 1$ , were adopted. "Student" in his original paper took

$z = \frac{\bar{x}_1 - \bar{x}}{s_1}$  in our present notation, and  $n$  for the size of the sample.

(c) *Tests relating to the Variance and Covariance when more than two Independent Samples are involved.*

test was applied to the lamp data in the *Illustration 1* of Section (ii) to discover whether the *mean* of life remained stable from one sampled batch of lamps to another; we may also ask whether the *variance* within a batch appears to remain stable; and the hypothesis that it does was a necessary assumption in the method adopted in dealing with our illustration as to the means.

Now form of test is involved; in Section (i) a comparison of *two* samples was made, but we now require to test the hypothesis that a number, say  $k$ , of samples have been drawn from populations with a common variance,  $\sigma^2$ , it being assumed that the populations sampled are normal, or approximately so.

Suppose that the  $t$ th sample ( $t=1, 2, 3, \dots k$ ) contains  $n_t$  observations and has a standard deviation  $s_t$ . J. Neyman and E. S. Pearson\* have given a test based on the principle of likelihood. The criterion of the test may be defined as

$$L = \frac{N \sqrt{\prod_{t=1}^{t=k} (s_t^2)^{n_t}}}{\frac{1}{N} \sum_{t=1}^{t=k} (n_t s_t^2)} \quad \dots\dots(\text{lxxix}),$$

$N = \sum_{t=1}^{t=k} n_t$  denoting a summation as to arrays, and  $\Pi$  denoting a continuous product. It is clear

that  $L$  denotes the ratio of the weighted geometric mean to the weighted arithmetic mean of the  $s_t^2$ 's.

As  $L$  decreases from 1 towards 0, the hypothesis of a common  $\sigma^2$  becomes less and less likely. When the population is normal the moment coefficients of the sampling distribution of  $L$  (if the hypothesis be true) have been found.

In the simple case in which the groups contain the same number of individuals, i.e. when

$$n_t = \text{constant} = n = N/k,$$

the moment coefficient of  $L$  about  $L=0$  is

$$\mu'_n = k^n \left\{ \frac{\Gamma(\frac{1}{2}(n-1) + n/k)}{\Gamma(\frac{1}{2}(n-1))} \right\}^k \frac{\Gamma(\frac{1}{2}(N-k))}{\Gamma(\frac{1}{2}(N-k) + n)} \quad \dots\dots(\text{lxxx}).$$

Reasons are given in the paper just referred to for believing that the distribution of  $L$  may in many cases be adequately represented by a Type I distribution of the form

$$df = \frac{1}{B(p, q)} L^{p-1} (1-L)^{q-1} dL \quad \dots\dots(\text{lxxxix})$$

giving the correct mean and standard deviation. In this case  $p$  and  $q$  may be determined from the first two moment coefficients  $\mu'_1$  and  $\mu'_2$ , thus

$$p = \frac{\mu'_1 (\mu'_1 - \mu'_2)}{\mu'_2 - \mu_1'^2}, \quad q = \frac{(1 - \mu'_1) (\mu'_1 - \mu'_2)}{\mu'_2 - \mu_1'^2} \quad \dots\dots(\text{lxxxii}).$$

In a recent paper S. S. Wilks has generalised still further this result, applying it to cases where several uncorrelated characters have been measured for each individual in a number of samples. The sampling distributions of the test criteria were again expressed in terms of products of  $\Gamma$ -functions, and it seems not unlikely that the same method of approximation, using (lxxxix) and (lxxxii), will be again adequate.

*Illustration 11.* Let us take the lamp data already considered in Section (ii). The 15 values of  $s_t$  are given in Table IV on p. lii. In this case

$$N = 75, \quad k = 15, \quad n = 5,$$

using the formula (lxxxii) we find

$$\mu'_1 = .77946, \quad \mu'_2 = .61273,$$

$$p = 25.12, \quad q = 7.11,$$

inserting the observed values of  $s_t$  into (lxxix) gives

$$L = .9138.$$

# lviii RELATION OF FISHER'S TEST TO THE INCOMPLETE B-FUNCTION TEST

We have accordingly to determine

$$I_{.9138}(25.12, 7.11).$$

$$\theta_0 = .62, \theta_1 = .38; \quad \phi_0 = .88, \phi_1 = .12; \quad \chi_0 = .78, \chi_1 = .22.$$

$$\begin{array}{ll} z_{000} = .9817,981, & z_{100} = .9900,699, \\ z_{010} = .9784,655, & z_{110} = .9881,496, \\ z_{001} = .9891,785, & z_{101} = .9944,177, \\ z_{011} = .9870,109, & z_{111} = .9932,401, \\ \theta_0 \phi_0 \chi_0 = .425,568, & \theta_1 \phi_0 \chi_0 = .260,832, \\ \theta_0 \phi_1 \chi_0 = .058,032, & \theta_1 \phi_1 \chi_0 = .035,568, \\ \theta_0 \phi_0 \chi_1 = .120,032, & \theta_1 \phi_0 \chi_1 = .073,568, \\ \theta_0 \phi_1 \chi_1 = .016,368, & \theta_1 \phi_1 \chi_1 = .010,032. \end{array}$$

Whence by continuous operation on the machine the hyperbolic terms of (viii) give

$$I_{.9138}(25.12, 7.11) = .986,0026.$$

The terms in  $\delta^2$  would somewhat reduce this value below .986, but it would be safe to say that if the variance were constant among the sampled batches of lamps, we should expect to find greater diversity than that observed among the 15 values of  $s_i^2$  (as measured by  $L$ ) in 984 to 986 times in 1000 repetitions of the trials.

The variance of these lamps as tested by this  $L$  criterion appears therefore rather unusually stable.

(viii) *Relation of the Incomplete B-Function Ratio Method to R. A. Fisher's Method and Table.*

It may be helpful to some users of the present tables to indicate here the relationship between the Type I distribution leading to the Incomplete B-Function Integral and R. A. Fisher's frequency distribution for which he has provided tables of the 5 % and 1 % probability limits\*. The close relation of the tests described above can be shown to arise from the fact that in each case a comparison is made of two independent estimates of an unknown variance,  $\sigma^2$ , in a population about the nature of which certain restrictions are made. If the hypothesis to be tested be true, then these two estimates will differ only through chance fluctuations: if the test shows that the estimates differ significantly, then we shall conclude that the hypothesis is not true.

If the following notation be adopted:

First estimate of  $\sigma_1 : v_1$ , based on  $u_1$  degrees of freedom.

Second estimate of  $\sigma_2 : v_2$ , based on  $u_2$  degrees of freedom.

Then if  $w = \frac{1}{2} \log_e (v_1/v_2)$  and if the hypothesis to be tested be true, the sampling distribution of  $w$  takes the form

$$df = \frac{2u_1^{\frac{1}{2}}u_2^{\frac{1}{2}}u_2}{B(\frac{1}{2}u_1, \frac{1}{2}u_2)} \frac{e^{u_1w}}{(u_2 + u_1 e^{2w})^{\frac{1}{2}(u_1+u_2)}} dw \quad \text{.....(lxxxiii).}$$

The transformation

$$w = \frac{1}{2} \log_e (v_1/v_2) = \frac{1}{2} \log_e \frac{u_2 x}{u_1 (1-x)} \quad \text{.....(lxxxiv),}$$

or

$$x = \frac{u_1 e^{2w}}{u_2 + u_1 e^{2w}} = \frac{u_1 v_1}{u_2 v_2 + u_1 v_1} \quad \text{.....(lxxxv),}$$

gives us the probability law for  $x$ †,

$$df = \frac{1}{B(\frac{1}{2}u_1, \frac{1}{2}u_2)} x^{\frac{1}{2}u_1-1} (1-x)^{\frac{1}{2}u_2-1} dx \quad \text{.....(lxxxvi).}$$

As  $w$  varies from  $-\infty$  to  $+\infty$ ,  $x$  varies from 0 to 1 and the chance of  $w \geq w_0$  will be identical with

$I_{x_0}(\frac{1}{2}u_1, \frac{1}{2}u_2)$ , where  $x_0$  corresponds to  $w_0$ .

We may illustrate the relationship in the following cases.

\* *Metron*, Vol. v, pp. 90-104.

† The 1 % and 2 % levels of the incomplete B-function are given by Woo's Tables, *Biometrika*, Vol. xxi, pp. 1-66, or *Tables for Statisticians*, Part II, pp. 16-72.

the test in Section (i):

$$\left. \begin{aligned} v_1 &= \sum_{t_1=1}^{t_1=n_1} (x_{t_1} - \bar{x}_1)^2 / (n_1 - 1) = n_1 s_1^2 / (n_1 - 1), & u_1 &= n_1 - 1 \\ v_2 &= \sum_{t_2=1}^{t_2=n_2} (x_{t_2} - \bar{x}_2)^2 / (n_2 - 1) = n_2 s_2^2 / (n_2 - 1), & u_2 &= n_2 - 1 \end{aligned} \right\} \dots\dots(\text{lxxxvii}).$$

the tests in Section (ii):

$$\left. \begin{aligned} v_1 &= \sum_{t=1}^{t=k} S n_t (\bar{y}_t - \bar{Y})^2 / (k - 1) \dots & u_1 &= k - 1 \\ v_2 &= \sum_{t=1}^{t=k} S \sum (y - \bar{y}_t)^2 / (N - k) = \sum_{t=1}^{t=k} S (n_t s_t^2) / (N - k) & u_2 &= N - k \end{aligned} \right\} \dots\dots(\text{lxxxviii}),$$

giving a summation of all  $y$ 's in an array and  $S$  the summation for all arrays.

Our tables\* give only the values of  $w$  which will be surpassed in 5 % and in 1 % of samples, if the hypothesis tested be true. These limits are tabled for

$$u_1 = 1, 2, \dots 6; 8, 12, 24, \infty,$$

$$u_2 = 1, 2, \dots 30; 60, \infty,$$

other values of the  $u$ 's being chosen to form a framework at equidistant values of  $1/u$  from which to interpolate.

In sampling tests a knowledge of the 5 % and 1 % limits may in some cases suffice, but, especially in the case of the relative probability of different hypotheses, an exact value of the probability is often desirable. The Incomplete B-Function Table provides within its range, and as the degrees of freedom are integers, the interpolation will be for  $p$  and  $q$  at most to half intervals, and may be achieved by diagonal interpolation, if not provided by the table itself.

*Concluding Remarks.* There are of course many other purposes to which the *Tables of the Incomplete B-Function* may be applied by either statistician or mathematician. The Editor has found the tables of value in such problems as the following:

The summing of the first  $p$  terms of any binomial, and therefore of any consecutive series of terms.

The sum of first  $p$  terms of  $(1+x)^n = (1+x)^n I_{1+x}^{(n+1-p, p)}$ . See *Biometrika*, Vol. xvi, pp. 202-203.

The sum of first  $p$  terms of  $(1-x)^{-n} = (1-x)^{-n} I_{1-x}^{(n, p)}$ . See *Biometrika*, Vol. xxv, pp. 160-161.

The discussion of whether two  $\chi^2$ 's, namely  $\chi_1^2$  and  $\chi_2^2$ , may be considered as significantly different in terms of the ratio  $\chi_1^2/\chi_2^2$ . A special table drawn from the Incomplete B-Function Table has been provided for this. See *Biometrika*, Vol. xxiv, pp. 305-307, 347-350.

The evaluating of the probability integrals of symmetrical frequency distributions such as occur in the case of the regression coefficient, or approximately in the case of the mean of an array in a sample, when that mean is found from the regression line of the sample. A special table drawn from the Incomplete B-Function Table has been provided for such cases. See *Biometrika*, Vol. xxii, pp. 253-283, *Tables for Statisticians*, Part II, pp. 169-178.

The determination of the probability integrals of a great variety of statistical constants which are required for the application of the  $P_{\lambda_n}$  test for randomness. See *Biometrika*, Vol. xxv, pp. 379-410.

The above mentioned special tables while to some extent shortening the work are far from absolute necessities for those possessing the more comprehensive *Tables of the Incomplete  $\Gamma$ - and B-Functions*.

*Statistical Methods for Research Workers.* Fisher's notation is  $z$  for our  $w$  and  $n_1$  and  $n_2$  for the number of degrees of freedom; we have here used to avoid confusion with the  $n_1$  and  $n_2$  used by us for the size of samples.  $z$  in this Introduction has also a different meaning.





# TABLES OF THE INCOMPLETE BETA-FUNCTION

## TABLE I THE $I_x(p, q)$ FUNCTION

The corresponding value of the Complete Beta-Function  
is given at the top of each column

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .01$  to  $.60$  $q = 0.5$  $p = 0.5$  to  $3$ 

	$p = 0.5$	$p = 1$	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$
$B(p, q) = 3.14159245^+$	$2.00000000$	$1.57079633$	$1.33333333$	$1.17809725^-$	$1.06666667$	
$x$						
.01	.0637 686	.0050 126	.0004 257	.0000 376	.0000 034	.0000 003
.02	.0903 345	.0100 505 <sup>+</sup>	.0012 077	.0001 510	.0000 193	.0000 025
.03	.1108 247	.0151 142	.0022 255	.0003 409	.0000 535 <sup>+</sup>	.0000 085
.04	.1281 884	.0202 041	.0034 369	.0006 082	.0001 102	.0000 203
.05	.1435 663	.0253 206	.0048 182	.0009 536	.0001 933	.0000 398
.06	.1575 424	.0304 640	.0063 536	.0013 779	.0003 061	.0000 601
.07	.1704 634	.0356 349	.0080 318	.0018 821	.0004 517	.0001 101
.08	.1825 549	.0408 337	.0098 443	.0024 670	.0006 330	.0001 650
.09	.1939 734	.0460 608	.0117 844	.0031 335 <sup>+</sup>	.0008 531	.0002 350
.10	.2048 328	.0513 167	.0138 468	.0038 825 <sup>+</sup>	.0011 144	.0003 250
.11	.2152 190	.0566 019	.0160 272	.0047 150	.0014 198	.0004 343
.12	.2251 989	.0619 168	.0183 220	.0056 319	.0017 718	.0005 062
.13	.2348 255	.0672 621	.0207 281	.0066 341	.0021 729	.0007 220
.14	.2441 418	.0726 382	.0232 430	.0077 228	.0026 257	.0009 067
.15	.2531 833	.0780 456	.0258 646	.0088 990	.0031 327	.0011 200
.16	.2619 798	.0834 849	.0285 911	.0101 636	.0036 963	.0013 651
.17	.2705 563	.0889 566	.0314 210	.0115 180	.0043 190	.0016 445
.18	.2789 343	.0944 615	.0343 530	.0129 630	.0050 032	.0019 607
.19	.2871 326	.1000 000 <sup>0</sup>	.0373 861	.0145 000 <sup>0</sup>	.0057 515 <sup>+</sup>	.0023 162
.20	.2951 672	.1055 728	.0405 193	.0161 301	.0065 663	.0027 137
.21	.3030 525 <sup>+</sup>	.1111 806	.0437 521	.0178 545 <sup>+</sup>	.0074 500	.0031 557
.22	.3108 011	.1168 239	.0470 837	.0196 745 <sup>+</sup>	.0084 052	.0036 449
.23	.3184 242	.1225 036	.0505 139	.0215 915	.0094 344	.0041 841
.24	.3259 319	.1282 202	.0540 424	.0236 066	.0105 400	.0047 762
.25	.3333 333	.1339 746	.0576 689	.0257 214	.0117 248	.0054 240
.26	.3406 367	.1397 675	.0613 934	.0279 372	.0129 913	.0061 304
.27	.3478 494	.1455 996	.0652 160	.0302 556	.0143 420	.0068 684
.28	.3549 784	.1514 719	.0691 369	.0326 779	.0157 798	.0077 312
.29	.3620 301	.1573 850 <sup>+</sup>	.0731 562	.0352 059	.0173 072	.0086 319
.30	.3690 101	.1633 400	.0772 743	.0378 410	.0189 271	.0096 037
.31	.3759 240	.1693 376	.0814 916	.0405 849	.0206 423	.0106 499
.32	.3827 767	.1753 789	.0858 087	.0434 395	.0224 556	.0117 740
.33	.3895 729	.1814 647	.0902 262	.0464 064	.0243 699	.0129 795
.34	.3963 171	.1875 962	.0947 447	.0494 875 <sup>+</sup>	.0263 883	.0142 698
.35	.4030 133	.1937 742	.0993 650 <sup>+</sup>	.0526 847	.0285 138	.0156 487
.36	.4096 655 <sup>+</sup>	.2000 000 <sup>0</sup>	.1040 880	.0560 000 <sup>0</sup>	.0307 494	.0171 200
.37	.4162 774	.2062 746	.1089 147	.0594 354	.0330 985 <sup>+</sup>	.0186 875
.38	.4228 526	.2125 992	.1138 459	.0629 931	.0355 643	.0203 553
.39	.4293 943	.2189 750 <sup>+</sup>	.1188 830	.0666 752	.0381 501	.0221 275
.40	.4359 058	.2254 033	.1240 271	.0704 840	.0408 594	.0240 082
.41	.4423 902	.2318 854	.1292 794	.0744 219	.0436 958	.0260 019
.42	.4488 506	.2384 227	.1346 415	.0784 915	.0466 629	.0281 131
.43	.4552 897	.2450 166	.1401 147	.0826 951	.0497 646	.0303 465
.44	.4617 105 <sup>+</sup>	.2516 685 <sup>+</sup>	.1457 008	.0870 356	.0530 046	.0327 067
.45	.4681 157	.2583 802	.1514 014	.0915 157	.0563 871	.0351 980
.46	.4745 080	.2651 531	.1572 183	.0961 383	.0599 161	.0378 282
.47	.4808 899	.2719 890	.1631 535 <sup>+</sup>	.1009 064	.0635 961	.0405 998
.48	.4872 642	.2788 897	.1692 091	.1058 233	.0674 314	.0435 194
.49	.4936 334	.2858 572	.1753 872	.1108 922	.0714 267	.0465 925
.50	.5000 000 <sup>0</sup>	.2928 932	.1816 901	.1161 165 <sup>+</sup>	.0755 868	.0498 253
.51	.5063 666	.3000 000 <sup>0</sup>	.1881 204	.1215 000 <sup>0</sup>	.0799 167	.0532 237
.52	.5127 358	.3071 797	.1946 807	.1270 464	.0844 215 <sup>+</sup>	.0567 944
.53	.5191 101	.3144 345 <sup>+</sup>	.2013 737	.1327 597	.0891 068	.0605 439
.54	.5254 920	.3217 670	.2082 024	.1386 441	.0939 781	.0644 793
.55	.5318 843	.3291 796	.2151 699	.1447 040	.0990 414	.0686 078
.56	.5382 895	.3366 750 <sup>+</sup>	.2222 797	.1509 441	.1043 027	.0729 370
.57	.5447 103	.3442 561	.2295 352	.1573 691	.1097 687	.0774 750
.58	.5511 494	.3519 259	.2369 403	.1639 844	.1154 461	.0822 299
.59	.5576 098	.3596 876	.2444 990	.1707 954	.1213 421	.0872 106
.60	.5640 942	.3675 445	.2522 155	.1778 078	.1274 640	.0924 263

TABLE I. THE  $I_w(p, q)$  FUNCTION

3

 $x = .61$  to  $1.00$  $q = 0.5$  $p = 0.5$  to  $3$ 

	$p = 0.5$	$p = 1$	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$
$I_3(p, q) = 3.14159245^+$	2.0000 0000 <sup>a</sup>	1.5707 9633	1.3333 3333	1.1780 9725 <sup>-</sup>	1.0666 6667	
$x$						
.61	.5706 057	.3755 002	.2600 945 <sup>-</sup>	.1850 278	.1338 109	.0978 866
.62	.5771 474	.3835 586	.2681 408	.1924 618	.1404 181	.1036 017
.63	.5837 226	.3917 237	.2763 598	.2001 167	.1472 674	.1095 824
.64	.5903 345 <sup>-</sup>	.4000 000 <sup>a</sup>	.2847 570	.2080 000 <sup>a</sup>	.1543 773	.1158 400 <sup>a</sup>
.65	.5969 867	.4083 920	.2933 384	.2161 194	.1617 575 <sup>-</sup>	.1223 865 <sup>+</sup>
.66	.6036 820	.4169 048	.3021 105 <sup>+</sup>	.2244 834	.1694 187	.1292 348
.67	.6104 271	.4255 437	.3110 804	.2331 009	.1773 722	.1363 984
.68	.6172 233	.4343 146	.3202 554	.2419 815 <sup>+</sup>	.1856 299	.1438 917
.69	.6240 760	.4432 236	.3296 437	.2511 357	.1942 048	.1517 302
.70	.6309 899	.4522 774	.3392 541	.2605 745 <sup>+</sup>	.2031 107	.1599 305 <sup>+</sup>
.71	.6379 699	.4614 835 <sup>+</sup>	.3490 960	.2703 102	.2123 624	.1685 104
.72	.6450 216	.4708 497	.3591 800	.2803 556	.2219 760	.1774 888
.73	.6521 506	.4803 848	.3695 172	.2907 252	.2319 690	.1868 866
.74	.6593 633	.4900 980	.3801 201	.3014 343	.2423 601	.1967 260
.75	.6666 667	.5000 000 <sup>a</sup>	.3910 022	.3125 000 <sup>a</sup>	.2531 700	.2070 312
.76	.6740 681	.5101 021	.4021 785 <sup>-</sup>	.3239 408	.2644 211	.2178 289
.77	.6815 758	.5204 168	.4136 655 <sup>-</sup>	.3357 773	.2761 382	.2291 480
.78	.6891 989	.5309 584	.4254 815 <sup>-</sup>	.3480 322	.2883 484	.2410 204
.79	.6969 475 <sup>-</sup>	.5417 424	.4376 470	.3607 307	.3010 821	.2534 812
.80	.7048 328	.5527 864	.4501 849	.3739 010	.3143 726	.2665 697
.81	.7128 674	.5641 101	.4631 209	.3875 747	.3282 578	.2803 294
.82	.7210 657	.5757 359	.4764 843	.4017 877	.3427 799	.2948 095 <sup>-</sup>
.83	.7294 437	.5876 804	.4903 085 <sup>-</sup>	.4165 806	.3579 870	.3100 653
.84	.7380 202	.6000 000 <sup>a</sup>	.5046 316	.4320 000 <sup>a</sup>	.3739 339	.3261 600 <sup>a</sup>
.85	.7468 167	.6127 017	.5194 980	.4480 999	.3906 840	.3431 662
.86	.7558 582	.6258 343	.5349 594	.4649 430	.4083 108	.3611 681
.87	.7651 745 <sup>+</sup>	.6394 449	.5510 771	.4826 034	.4269 006	.3802 643
.88	.7748 011	.6535 898	.5679 242	.5011 694	.4465 564	.4005 719
.89	.7847 810	.6683 375 <sup>+</sup>	.5855 892	.5207 477	.4674 020	.4222 315 <sup>+</sup>
.90	.7951 672	.6837 722	.6041 813	.5414 697	.4895 897	.4454 156
.91	.8060 266	.7000 000 <sup>a</sup>	.6238 377	.5635 000 <sup>a</sup>	.5133 097	.4703 387
.92	.8174 451	.7171 573	.6447 345 <sup>-</sup>	.5870 496	.5388 053	.4972 754
.93	.8295 306	.7354 249	.6671 049	.6123 974	.5663 973	.5265 858
.94	.8424 576	.7550 510	.6912 688	.6390 250 <sup>+</sup>	.5965 238	.5587 612
.95	.8564 337	.7763 032	.7176 856	.6701 800	.6298 119	.5945 030
.96	.8718 116	.8000 000 <sup>a</sup>	.7470 601	.7040 000 <sup>a</sup>	.6672 191	.6348 800 <sup>a</sup>
.97	.8891 753	.8267 949	.7805 761	.7427 905 <sup>-</sup>	.7103 486	.6816 772
.98	.9096 655 <sup>+</sup>	.8585 786	.8205 388	.7892 822	.7623 093	.7383 493
.99	.9362 314	.9000 000 <sup>a</sup>	.8728 886	.8505 000 <sup>a</sup>	.8310 823	.8137 462
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .02$  to  $.60$  $q = 0.5$  $p = 3.5$  to  $6$ 

	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$
$B(p, q) = .9817\ 4770$	$.9142\ 8571$	$.8590\ 2924$	$.8126\ 9841$	$.7731\ 2432$	$.7388\ 1674$	
$x$						
.02	.0000 003					
.03	.0000 014	.0000 002				
.04	.0000 038	.0000 007	.0000 001			
.05	.0000 083	.0000 017	.0000 004	.0000 001		
.06	.0000 158	.0000 036	.0000 008	.0000 002		
.07	.0000 272	.0000 068	.0000 017	.0000 004	.0000 001	
.08	.0000 435 <sup>+</sup>	.0000 116	.0000 031	.0000 008	.0000 002	.0000 001
.09	.0000 660	.0000 186	.0000 053	.0000 015 <sup>+</sup>	.0000 004	.0000 001
.10	.0000 958	.0000 285 <sup>+</sup>	.0000 085 <sup>+</sup>	.0000 026	.0000 008	.0000 002
.11	.0001 344	.0000 419	.0000 132	.0000 042	.0000 013	.0000 004
.12	.0001 830	.0000 596	.0000 196	.0000 065 <sup>-</sup>	.0000 021	.0000 007
.13	.0002 432	.0000 825 <sup>+</sup>	.0000 282	.0000 097	.0000 033	.0000 012
.14	.0003 166	.0001 115 <sup>-</sup>	.0000 395 <sup>+</sup>	.0000 141	.0000 050 <sup>+</sup>	.0000 018
.15	.0004 049	.0001 476	.0000 542	.0000 200	.0000 074	.0000 028
.16	.0005 098	.0001 920	.0000 728	.0000 277	.0000 106	.0000 041
.17	.0006 331	.0002 458	.0000 960	.0000 377	.0000 140	.0000 059
.18	.0007 769	.0003 104	.0001 248	.0000 505 <sup>-</sup>	.0000 205 <sup>-</sup>	.0000 083
.19	.0009 431	.0003 872	.0001 600	.0000 664	.0000 277	.0000 116
.20	.0011 338	.0004 776	.0002 025 <sup>-</sup>	.0000 863	.0000 369	.0000 159
.21	.0013 512	.0005 834	.0002 535 <sup>-</sup>	.0001 107	.0000 486	.0000 214
.22	.0015 978	.0007 061	.0003 141	.0001 404	.0000 630	.0000 284
.23	.0018 757	.0008 477	.0003 856	.0001 763	.0000 809	.0000 373
.24	.0021 876	.0010 101	.0004 694	.0002 192	.0001 028	.0000 484
.25	.0025 360	.0011 953	.0005 670	.0002 703	.0001 294	.0000 622
.26	.0029 236	.0014 055 <sup>+</sup>	.0006 800	.0003 306	.0001 614	.0000 791
.27	.0033 532	.0016 430	.0008 101	.0004 015 <sup>-</sup>	.0001 998	.0000 998
.28	.0038 278	.0019 103	.0009 593	.0004 842	.0002 454	.0001 248
.29	.0043 503	.0022 098	.0011 295 <sup>+</sup>	.0005 802	.0002 993	.0001 519
.30	.0049 238	.0025 444	.0013 230	.0006 913	.0003 627	.0001 910
.31	.0055 517	.0029 167	.0015 419	.0008 191	.0004 369	.0002 339
.32	.0062 372	.0033 299	.0017 887	.0009 656	.0005 234	.0002 846
.33	.0069 839	.0037 871	.0020 661	.0011 328	.0006 236	.0003 444
.34	.0077 954	.0042 914	.0023 769	.0013 229	.0007 393	.0004 145 <sup>+</sup>
.35	.0086 754	.0048 466	.0027 239	.0015 384	.0008 723	.0004 963
.36	.0096 279	.0054 560 <sup>e</sup>	.0031 104	.0017 818	.0010 248	.0005 914
.37	.0106 569	.0061 236	.0035 397	.0020 560	.0011 990	.0007 015 <sup>+</sup>
.38	.0117 666	.0068 534	.0040 154	.0023 640	.0013 972	.0008 286
.39	.0129 614	.0076 494	.0045 412	.0027 088	.0016 222	.0009 747
.40	.0142 458	.0085 163	.0051 211	.0030 941	.0018 767	.0011 421
.41	.0156 244	.0094 584	.0057 593	.0035 234	.0021 640	.0013 334
.42	.0171 021	.0104 807	.0064 602	.0040 008	.0024 873	.0015 514
.43	.0186 841	.0115 882	.0072 287	.0045 304	.0028 502	.0017 990
.44	.0203 755 <sup>+</sup>	.0127 861	.0080 697	.0051 167	.0032 568	.0020 796
.45	.0221 819	.0140 801	.0089 885 <sup>+</sup>	.0057 646	.0037 112	.0023 968
.46	.0241 090	.0154 760	.0099 907	.0064 792	.0042 179	.0027 546
.47	.0261 625 <sup>+</sup>	.0169 797	.0110 821	.0072 660	.0047 819	.0031 570
.48	.0283 488	.0185 978	.0122 691	.0081 307	.0054 084	.0036 090
.49	.0306 742	.0203 368	.0135 582	.0090 797	.0061 032	.0041 153
.50	.0331 455 <sup>+</sup>	.0222 039	.0149 564	.0101 196	.0068 723	.0046 816
.51	.0357 696	.0242 063	.0164 710	.0112 573	.0077 223	.0053 137
.52	.0385 538	.0263 519	.0181 098	.0125 005 <sup>+</sup>	.0086 602	.0060 181
.53	.0415 056	.0286 487	.0198 811	.0138 572	.0096 936	.0068 017
.54	.0446 332	.0311 052	.0217 936	.0153 359	.0108 305 <sup>+</sup>	.0076 720
.55	.0479 448	.0337 304	.0238 564	.0169 456	.0120 798	.0086 372
.56	.0514 491	.0365 338	.0260 793	.0186 962	.0134 508	.0097 060
.57	.0551 552	.0395 252	.0284 726	.0205 978	.0149 534	.0108 880
.58	.0590 728	.0427 152	.0310 472	.0226 615 <sup>+</sup>	.0165 985 <sup>-</sup>	.0121 935 <sup>-</sup>
.59	.0632 120	.0461 148	.0338 148	.0248 990	.0183 976	.0136 335 <sup>-</sup>
.60	.0675 833	.0497 356	.0367 875 <sup>-</sup>	.0273 229	.0203 631	.0152 201

TABLE I. THE  $I_x(p, q)$  FUNCTION

5

 $x = .61$  to  $1.00$  $q = 0.5$  $p = 3.5$  to  $6$ 

	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$
$B(p, q)$	.9817 4770	.9142 8571	.8590 2924	.8126 9841	.7731 2432	.7388 1674
$x$						
.61	.0721 979	.0535 890	.0399 784	.0299 465 <sup>-</sup>	.0225 083	.0169 663
.62	.0770 676	.0576 907	.0434 014	.0327 810	.0248 475 <sup>+</sup>	.0188 860
.63	.0822 040	.0620 519	.0470 711	.0358 507	.0273 902	.0209 946
.64	.0876 228	.0666 880 <sup>+</sup>	.0510 033	.0391 629	.0301 708	.0233 084
.65	.0933 354	.0716 146	.0552 145 <sup>+</sup>	.0427 380	.0331 891	.0258 452
.66	.0993 574	.0768 481	.0597 227	.0465 947	.0364 704	.0286 243
.67	.1057 046	.0824 061	.0645 469	.0507 532	.0400 352	.0316 664
.68	.1123 936	.0883 074	.0697 074	.0552 348	.0439 059	.0349 944
.69	.1194 425 <sup>+</sup>	.0945 721	.0752 260	.0600 628	.0481 065 <sup>+</sup>	.0386 326
.70	.1268 704	.1012 215 <sup>+</sup>	.0811 262	.0652 622	.0526 631	.0426 079
.71	.1346 977	.1082 788	.0874 332	.0708 600	.0576 040	.0469 493
.72	.1429 466	.1157 687	.0941 741	.0768 851	.0629 597	.0516 885 <sup>-</sup>
.73	.1516 408	.1237 181	.1013 783	.0833 692	.0687 636	.0568 600
.74	.1608 062	.1321 558	.1090 777	.0903 466	.0750 519	.0625 017
.75	.1704 707	.1411 133	.1173 068	.0978 546	.0818 642	.0686 550 <sup>+</sup>
.76	.1806 646	.1506 247	.1261 033	.1059 339	.0892 440	.0753 654
.77	.1914 214	.1607 275 <sup>+</sup>	.1355 083	.1146 292	.0972 389	.0826 831
.78	.2027 774	.1714 627	.1455 671	.1239 896	.1059 013	.0906 634
.79	.2147 730	.1828 753	.1563 205 <sup>-</sup>	.1340 690	.1152 891	.0993 677
.80	.2274 528	.1950 155 <sup>+</sup>	.1678 507	.1449 276	.1254 669	.1088 643
.81	.2408 665 <sup>-</sup>	.2079 389	.1801 920	.1566 321	.1365 063	.1192 294
.82	.2550 607	.2217 077	.1931 221	.1692 572	.1484 877	.1305 487
.83	.2701 255 <sup>-</sup>	.2363 922	.2076 183	.1828 871	.1615 019	.1429 189
.84	.2861 051	.2520 720 <sup>+</sup>	.2228 683	.1976 173	.1750 516	.1564 496
.85	.3030 905 <sup>+</sup>	.2688 382	.2392 724	.2135 568	.1910 543	.1712 665 <sup>-</sup>
.86	.3211 765 <sup>+</sup>	.2867 961	.2569 461	.2308 312	.2078 455 <sup>+</sup>	.1875 144
.87	.3404 738	.3060 685 <sup>+</sup>	.2760 240	.2495 869	.2261 829	.2053 619
.88	.3611 134	.3268 003	.2966 650 <sup>+</sup>	.2699 963	.2462 521	.2250 075 <sup>-</sup>
.89	.3832 528	.3491 653	.3190 589	.2922 651	.2682 745 <sup>-</sup>	.2466 879
.90	.4070 838	.3733 749	.3434 364	.3166 429	.2925 185 <sup>-</sup>	.2706 900
.91	.4328 453	.3996 915 <sup>-</sup>	.3700 831	.3434 386	.3193 155 <sup>-</sup>	.2973 674
.92	.4608 414	.4284 484	.3993 614	.3730 427	.3490 843	.3271 668
.93	.4914 709	.4600 818	.4317 438	.4059 641	.3823 694	.3606 677
.94	.5252 755 <sup>+</sup>	.4951 828	.4678 698	.4428 896	.4199 041	.3986 496
.95	.5630 278	.5345 921	.5086 465 <sup>-</sup>	.4847 912	.4627 245 <sup>-</sup>	.4422 114
.96	.6059 013	.5795 840 <sup>+</sup>	.5554 454	.5331 354	.5123 898	.4930 037
.97	.6558 521	.6322 773	.6105 421	.5903 492	.5714 749	.5537 459
.98	.7166 574	.6967 541	.6783 097	.6610 862	.6449 047	.6296 271
.99	.7979 717	.7834 244	.7698 750 <sup>-</sup>	.7571 581	.7451 499	.7337 548
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .10$  to  $.70$  $q = 0.5$  $p = 6.5$  to  $9$ 

	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$
$B(p, q) =$	.7086 9912	.6819 8468	.6580 7776	.6365 1904	.6169 4790	.5990 7674
$x$						
.10	.0000 001					
.11	.0000 001					
.12	.0000 002	.0000 001				
.13	.0000 004	.0000 001				
.14	.0000 007	.0000 002	.0000 001			
.15	.0000 010	.0000 004	.0000 001	.0000 001		
.16	.0000 016	.0000 006	.0000 002	.0000 001		
.17	.0000 023	.0000 009	.0000 004	.0000 001	.0000 001	
.18	.0000 034	.0000 014	.0000 006	.0000 002	.0000 001	.0000 001
.19	.0000 049	.0000 021	.0000 009	.0000 004	.0000 002	.0000 001
.20	.0000 068	.0000 030	.0000 013	.0000 006	.0000 002	
.21	.0000 094	.0000 042	.0000 019	.0000 008	.0000 004	.0000 002
.22	.0000 128	.0000 058	.0000 026	.0000 012	.0000 005 <sup>+</sup>	.0000 003
.23	.0000 172	.0000 080	.0000 037	.0000 017	.0000 008	.0000 004
.24	.0000 228	.0000 108	.0000 051	.0000 024	.0000 012	.0000 006
.25	.0000 300	.0000 145 <sup>-</sup>	.0000 070	.0000 034	.0000 017	.0000 008
.26	.0000 389	.0000 192	.0000 095 <sup>-</sup>	.0000 047	.0000 023	.0000 012
.27	.0000 500 <sup>-</sup>	.0000 251	.0000 126	.0000 064	.0000 032	.0000 016
.28	.0000 637	.0000 325 <sup>+</sup>	.0000 167	.0000 086	.0000 044	.0000 023
.29	.0000 804	.0000 419	.0000 218	.0000 114	.0000 060	.0000 031
.30	.0001 008	.0000 534	.0000 283	.0000 151	.0000 080	.0000 043
.31	.0001 255 <sup>+</sup>	.0000 676	.0000 364	.0000 197	.0000 107	.0000 058
.32	.0001 553	.0000 849	.0000 465 <sup>+</sup>	.0000 255 <sup>+</sup>	.0000 140	.0000 077
.33	.0001 908	.0001 060	.0000 590	.0000 329	.0000 184	.0000 103
.34	.0002 331	.0001 314	.0000 742	.0000 420	.0000 238	.0000 135 <sup>+</sup>
.35	.0002 832	.0001 620	.0000 929	.0000 533	.0000 307	.0000 177
.36	.0003 423	.0001 986	.0001 155 <sup>-</sup>	.0000 672	.0000 392	.0000 229
.37	.0004 116	.0002 421	.0001 427	.0000 843	.0000 499	.0000 295 <sup>+</sup>
.38	.0004 928	.0002 938	.0001 755 <sup>-</sup>	.0001 050 <sup>+</sup>	.0000 630	.0000 378
.39	.0005 873	.0003 547	.0002 147	.0001 302	.0000 791	.0000 481
.40	.0006 970	.0004 264	.0002 614	.0001 605 <sup>+</sup>	.0000 988	.0000 608
.41	.0008 239	.0005 103	.0003 168	.0001 970	.0001 227	.0000 765 <sup>+</sup>
.42	.0009 703	.0006 084	.0003 822	.0002 406	.0001 517	.0000 958
.43	.0011 386	.0007 224	.0004 593	.0002 925 <sup>+</sup>	.0001 866	.0001 192
.44	.0013 316	.0008 547	.0005 497	.0003 542	.0002 286	.0001 477
.45	.0015 522	.0010 076	.0006 554	.0004 271	.0002 788	.0001 822
.46	.0018 038	.0011 840	.0007 787	.0005 131	.0003 387	.0002 238
.47	.0020 900	.0013 868	.0009 221	.0006 142	.0004 098	.0002 738
.48	.0024 147	.0016 194	.0010 882	.0007 326	.0004 940	.0003 335 <sup>+</sup>
.49	.0027 823	.0018 855 <sup>-</sup>	.0012 803	.0008 709	.0005 934	.0004 048
.50	.0031 977	.0021 892	.0015 018	.0010 320	.0007 103	.0004 896
.51	.0036 661	.0025 351	.0017 565 <sup>+</sup>	.0012 192	.0008 476	.0005 901
.52	.0041 931	.0029 281	.0020 489	.0014 361	.0010 082	.0007 088
.53	.0047 850 <sup>+</sup>	.0033 739	.0023 836	.0016 869	.0011 957	.0008 487
.54	.0054 487	.0038 784	.0027 661	.0019 762	.0014 140	.0010 132
.55	.0061 916	.0044 483	.0032 021	.0023 090	.0016 676	.0012 059
.56	.0070 217	.0050 911	.0036 984	.0026 913	.0019 614	.0014 314
.57	.0079 480	.0058 147	.0042 621	.0031 294	.0023 012	.0016 945 <sup>-</sup>
.58	.0089 801	.0066 280	.0049 013	.0036 305 <sup>+</sup>	.0026 933	.0020 007
.59	.0101 284	.0075 407	.0056 248	.0042 027	.0031 449	.0023 564
.60	.0114 043	.0085 635 <sup>+</sup>	.0064 425 <sup>+</sup>	.0048 549	.0036 639	.0027 688
.61	.0128 203	.0097 082	.0073 652	.0055 970	.0042 595 <sup>-</sup>	.0032 459
.62	.0143 899	.0109 874	.0084 049	.0064 400	.0049 416	.0037 968
.63	.0161 278	.0124 152	.0095 748	.0073 963	.0057 217	.0044 320
.64	.0180 501	.0140 071	.0108 895 <sup>+</sup>	.0084 795 <sup>-</sup>	.0066 123	.0051 620
.65	.0201 741	.0157 799	.0123 651	.0097 048	.0076 277	.0060 028
.66	.0225 190	.0177 521	.0140 194	.0110 890	.0087 836	.0069 663
.67	.0251 054	.0199 440	.0158 718	.0126 510	.0100 978	.0080 700
.68	.0279 559	.0223 778	.0179 442	.0144 114	.0115 901	.0093 327
.69	.0310 952	.0250 780	.0202 603	.0163 933	.0132 826	.0107 755 <sup>-</sup>
.70	.0345 503	.0280 714	.0228 466	.0186 226	.0152 002	.0124 219

TABLE I. THE  $I_w(p, q)$  FUNCTION

7

 $x = .71$  to  $1.00$  $q = 0.5$  $p = 6.5$  to  $9$ 

	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$
$R(p, q)$	.7086 9912	.6819 8468	.6580 7776	.6365 1904	.6169 4790	.5990 7674
$x$						
.71	.0383 506	.0313 875 <sup>-</sup>	.0257 323	.0211 278	.0173 705 <sup>-</sup>	.0142 986
.72	.0425 285 <sup>-</sup>	.0350 587	.0289 496	.0239 405 <sup>+</sup>	.0198 245 <sup>+</sup>	.0164 358
.73	.0471 192	.0391 209	.0325 343	.0270 964	.0225 970	.0188 671
.74	.0521 618	.0436 136	.0365 260	.0306 347	.0257 270	.0216 307
.75	.0576 988	.0485 803	.0400 600	.0345 997	.0292 580	.0247 696
.76	.0637 776	.0540 694	.0459 120	.0390 404	.0332 393	.0283 323
.77	.0704 503	.0601 345 <sup>-</sup>	.0514 098	.0440 122	.0377 260	.0323 739
.78	.0777 746	.0668 352	.0575 234	.0495 768	.0427 805 <sup>-</sup>	.0369 566
.79	.0858 147	.0742 382	.0643 210	.0558 039	.0484 730	.0421 510
.80	.0946 423	.0824 179	.0718 796	.0627 720	.0548 834	.0480 375 <sup>+</sup>
.81	.1043 378	.0914 580	.0802 856	.0705 608	.0621 022	.0547 079
.82	.1149 913	.1014 529	.0896 370	.0792 984	.0702 325 <sup>+</sup>	.0622 672
.83	.1267 050	.1125 097	.1000 452	.0890 728	.0793 927	.0708 361
.84	.1395 951	.1247 504	.1116 375 <sup>+</sup>	.1000 251	.0897 187	.0805 539
.85	.1537 948	.1383 153	.1245 605 <sup>-</sup>	.1123 074	.1013 679	.0915 823
.86	.1694 578	.1533 663	.1389 839	.1260 965 <sup>+</sup>	.1145 234	.1041 103
.87	.1867 031	.1700 924	.1551 060	.1415 997	.1294 004	.1183 603
.88	.2059 217	.1887 165 <sup>-</sup>	.1731 610	.1590 616	.1462 536	.1345 963
.89	.2271 852	.2095 046	.1934 288	.1787 752	.1653 885 <sup>-</sup>	.1531 354
.90	.2508 583	.2327 788	.2162 484	.2010 959	.1871 760	.1743 635 <sup>-</sup>
.91	.2773 168	.2589 365 <sup>-</sup>	.2420 379	.2264 623	.2120 743	.1987 577
.92	.3070 344	.2884 781	.2713 244	.2554 269	.2406 613	.2269 203
.93	.3406 256	.3220 524	.3047 901	.2887 054	.2736 849	.2596 310
.94	.3789 153	.3605 294	.3433 496	.3272 500	.3121 466	.2979 337
.95	.4230 646	.4051 315 <sup>+</sup>	.3882 859	.3724 217	.3574 488	.3432 896
.96	.4748 140	.4576 879	.4415 160	.4262 064	.4116 810	.3978 730
.97	.5370 247	.5211 995 <sup>+</sup>	.5061 785 <sup>-</sup>	.4918 845 <sup>-</sup>	.4782 524	.4652 262
.98	.6151 439	.6013 664	.5882 217	.5756 491	.5635 970	.5520 214
.99	.7228 973	.7125 164	.7025 619	.6929 921	.6837 719	.6748 712
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



$x = .21$  to  $.80$  $q = 0.5$  $p = 9.5$  to  $13$ 

	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$B(p, q) =$	.5826 7301	.5675 4639	.5535 3936	.5405 2037	.5170 1948	.4963 3870
$x$						
.21	.0000 001					
.22	.0000 001	.0000 001				
.23	.0000 002	.0000 001				
.24	.0000 003	.0000 001	.0000 001			
.25	.0000 004	.0000 002	.0000 001			
.26	.0000 006	.0000 003	.0000 001	.0000 001		
.27	.0000 008	.0000 004	.0000 002	.0000 001		
.28	.0000 012	.0000 006	.0000 003	.0000 002		
.29	.0000 016	.0000 009	.0000 005 <sup>-</sup>	.0000 002	.0000 001	
.30	.0000 023	.0000 012	.0000 007	.0000 004	.0000 001	
.31	.0000 031	.0000 017	.0000 009	.0000 005 <sup>+</sup>	.0000 002	
.32	.0000 043	.0000 024	.0000 013	.0000 007	.0000 002	.0000 001
.33	.0000 058	.0000 032	.0000 018	.0000 010	.0000 003	.0000 001
.34	.0000 077	.0000 044	.0000 025 <sup>-</sup>	.0000 014	.0000 005 <sup>-</sup>	.0000 002
.35	.0000 102	.0000 059	.0000 034	.0000 020	.0000 007	.0000 002
.36	.0000 134	.0000 079	.0000 046	.0000 027	.0000 009	.0000 003
.37	.0000 175 <sup>+</sup>	.0000 104	.0000 062	.0000 037	.0000 013	.0000 005
.38	.0000 227	.0000 137	.0000 082	.0000 050 <sup>-</sup>	.0000 018	.0000 007
.39	.0000 293	.0000 179	.0000 109	.0000 067	.0000 025 <sup>-</sup>	.0000 009
.40	.0000 375 <sup>+</sup>	.0000 232	.0000 143	.0000 089	.0000 034	.0000 013
.41	.0000 478	.0000 299	.0000 187	.0000 117	.0000 046	.0000 018
.42	.0000 605 <sup>+</sup>	.0000 383	.0000 243	.0000 154	.0000 062	.0000 025 <sup>+</sup>
.43	.0000 763	.0000 489	.0000 313	.0000 201	.0000 083	.0000 034
.44	.0000 956	.0000 619	.0000 402	.0000 261	.0000 110	.0000 047
.45	.0001 193	.0000 782	.0000 513	.0000 337	.0000 146	.0000 063
.46	.0001 481	.0000 981	.0000 651	.0000 432	.0000 191	.0000 085 <sup>-</sup>
.47	.0001 831	.0001 227	.0000 822	.0000 552	.0000 249	.0000 113
.48	.0002 255 <sup>+</sup>	.0001 526	.0001 034	.0000 701	.0000 324	.0000 150 <sup>-</sup>
.49	.0002 766	.0001 891	.0001 295 <sup>-</sup>	.0000 887	.0000 418	.0000 197
.50	.0003 379	.0002 334	.0001 615 <sup>-</sup>	.0001 118	.0000 537	.0000 259
.51	.0004 113	.0002 870	.0002 005 <sup>-</sup>	.0001 402	.0000 687	.0000 338
.52	.0004 989	.0003 516	.0002 480	.0001 751	.0000 875 <sup>+</sup>	.0000 439
.53	.0006 032	.0004 291	.0003 056	.0002 179	.0001 110	.0000 567
.54	.0007 268	.0005 220	.0003 753	.0002 701	.0001 402	.0000 730
.55	.0008 732	.0006 330	.0004 593	.0003 336	.0001 764	.0000 936
.56	.0010 459	.0007 651	.0005 602	.0004 106	.0002 211	.0001 194
.57	.0012 492	.0009 220	.0006 812	.0005 037	.0002 761	.0001 518
.58	.0014 880	.0011 079	.0008 257	.0006 160	.0003 436	.0001 922
.59	.0017 678	.0013 276	.0009 980	.0007 509	.0004 262	.0002 426
.60	.0020 948	.0015 866	.0012 029	.0009 128	.0005 269	.0003 050 <sup>+</sup>
.61	.0024 764	.0018 914	.0014 460	.0011 065 <sup>-</sup>	.0006 494	.0003 822
.62	.0029 207	.0022 491	.0017 336	.0013 375 <sup>+</sup>	.0007 980	.0004 774
.63	.0034 370	.0026 682	.0020 734	.0016 126	.0009 778	.0005 945 <sup>+</sup>
.64	.0040 359	.0031 582	.0024 738	.0019 394	.0011 948	.0007 381
.65	.0047 295 <sup>-</sup>	.0037 301	.0029 448	.0023 268	.0014 561	.0009 137
.66	.0055 313	.0043 964	.0034 977	.0027 851	.0017 700	.0011 279
.67	.0064 568	.0051 713	.0041 457	.0033 263	.0021 463	.0013 887
.68	.0075 235 <sup>-</sup>	.0060 711	.0049 037	.0039 642	.0025 965 <sup>+</sup>	.0017 053
.69	.0087 513	.0071 145 <sup>-</sup>	.0057 892	.0047 147	.0031 341	.0020 890
.70	.0101 626	.0083 225 <sup>+</sup>	.0068 218	.0055 964	.0037 749	.0025 530
.71	.0117 829	.0097 193	.0080 244	.0066 306	.0045 373	.0031 129
.72	.0136 410	.0113 325 <sup>+</sup>	.0094 231	.0078 419	.0054 429	.0037 876
.73	.0157 696	.0131 934	.0110 479	.0092 587	.0065 170	.0045 989
.74	.0182 057	.0153 378	.0129 330	.0109 139	.0077 891	.0055 730
.75	.0209 915 <sup>+</sup>	.0178 066	.0151 179	.0128 454	.0092 937	.0067 409
.76	.0241 746	.0206 463	.0176 480	.0150 970	.0110 712	.0081 391
.77	.0278 093	.0239 103	.0205 753	.0177 192	.0131 687	.0098 108
.78	.0319 575 <sup>-</sup>	.0276 597	.0239 598	.0207 707	.0156 415 <sup>+</sup>	.0118 075 <sup>-</sup>
.79	.0366 896	.0319 644	.0278 706	.0243 194	.0185 543	.0141 897
.80	.0420 863	.0369 048	.0323 874	.0284 440	.0219 830	.0170 296

TABLE I. THE  $I_x(p, q)$  FUNCTION

9

 $x = .81$  to  $1.00$  $q = 0.5$  $p = 9.5$  to  $13$ 

	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$B(p, q)$	.5826 7301	.5675 4639	.5535 3936	.5405 2037	.5170 1948	.4963 3870
$x$						
.81	.0482 400	.0425 736	.0376 025 <sup>+</sup>	.0332 362	.0260 167	.0204 125 <sup>+</sup>
.82	.0552 568	.0490 775 <sup>-</sup>	.0436 231	.0388 027	.0307 603	.0244 403
.83	.0632 595 <sup>+</sup>	.0565 405 <sup>-</sup>	.0505 737	.0452 684	.0363 378	.0292 343
.84	.0723 000	.0651 067	.0586 000	.0527 709	.0428 960	.0349 395 <sup>-</sup>
.85	.0828 138	.0749 447	.0678 729	.0615 098	.0506 093	.0417 298
.86	.0947 249	.0862 527	.0785 943	.0716 629	.0596 861	.0498 152
.87	.1083 520	.0992 653	.0910 038	.0834 832	.0703 770	.0594 496
.88	.1230 679	.1142 629	.1053 887	.0972 642	.0829 853	.0709 434
.89	.1410 006	.1315 837	.1220 966	.1133 617	.0978 814	.0846 779
.90	.1625 500 <sup>-</sup>	.1516 409	.1415 531	.1322 131	.1155 229	.1011 275 <sup>+</sup>
.91	.1864 114	.1749 472	.1642 873	.1543 630	.1364 828	.1208 898
.92	.2141 107	.2021 512	.1909 698	.1805 030	.1614 919	.1447 305 <sup>-</sup>
.93	.2464 587	.2340 939	.2224 710	.2115 320	.1915 031	.1736 523
.94	.2845 412	.2719 021	.2599 578	.2486 559	.2277 976	.2090 079
.95	.3298 768	.3171 516	.3050 620	.2935 620	.2721 706	.2526 955 <sup>-</sup>
.96	.3847 242	.3721 840	.3602 077	.3487 557	.3272 869	.3075 355 <sup>+</sup>
.97	.4527 575 <sup>-</sup>	.4408 042	.4293 290	.4182 993	.3974 618	.3780 016
.98	.5408 843	.5301 526	.5197 973	.5097 927	.4907 470	.4728 599
.99	.6662 641	.6579 282	.6498 437	.6419 932	.6269 347	.6126 479
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .34$  to  $1.00$  $q = 0.5$  $p = 14$  to  $19$ 

	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$
$B(p, q) =$	.4779 5579	.4614 7455 <sup>+</sup>	.4465 8828	.4330 5530	.4206 8229	.4093 1250 <sup>-</sup>
$x$						
.34	.0000 001					
.35	.0000 001					
.36	.0000 001					
.37	.0000 002	.0000 001				
.38	.0000 002	.0000 001				
.39	.0000 004	.0000 001				
.40	.0000 005 <sup>+</sup>	.0000 002	.0000 001			
.41	.0000 007	.0000 003	.0000 001			
.42	.0000 010	.0000 004	.0000 002	.0000 001		
.43	.0000 014	.0000 006	.0000 002	.0000 001		
.44	.0000 020	.0000 008	.0000 004	.0000 002	.0000 001	
.45	.0000 027	.0000 012	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
.46	.0000 038	.0000 017	.0000 007	.0000 003	.0000 001	.0000 001
.47	.0000 051	.0000 023	.0000 011	.0000 005 <sup>-</sup>	.0000 002	.0000 001
.48	.0000 069	.0000 032	.0000 015 <sup>+</sup>	.0000 007	.0000 003	.0000 002
.49	.0000 093	.0000 044	.0000 021	.0000 010	.0000 005 <sup>-</sup>	.0000 002
.50	.0000 125 <sup>+</sup>	.0000 061	.0000 029	.0000 014	.0000 007	.0000 003
.51	.0000 166	.0000 082	.0000 041	.0000 020	.0000 010	.0000 005 <sup>-</sup>
.52	.0000 220	.0000 111	.0000 056	.0000 028	.0000 014	.0000 007
.53	.0000 291	.0000 149	.0000 077	.0000 040	.0000 020	.0000 011
.54	.0000 381	.0000 199	.0000 104	.0000 055 <sup>-</sup>	.0000 029	.0000 015 <sup>+</sup>
.55	.0000 497	.0000 265 <sup>+</sup>	.0000 141	.0000 076	.0000 041	.0000 022
.56	.0000 646	.0000 351	.0000 191	.0000 104	.0000 057	.0000 031
.57	.0000 837	.0000 462	.0000 256	.0000 142	.0000 079	.0000 044
.58	.0001 078	.0000 606	.0000 341	.0000 192	.0000 109	.0000 061
.59	.0001 384	.0000 791	.0000 453	.0000 260	.0000 149	.0000 086
.60	.0001 770	.0001 029	.0000 600	.0000 350 <sup>-</sup>	.0000 204	.0000 120
.61	.0002 255 <sup>+</sup>	.0001 333	.0000 790	.0000 469	.0000 278	.0000 166
.62	.0002 863	.0001 721	.0001 036	.0000 625 <sup>-</sup>	.0000 377	.0000 228
.63	.0003 624	.0002 213	.0001 354	.0000 830	.0000 509	.0000 313
.64	.0004 571	.0002 836	.0001 763	.0001 098	.0000 685 <sup>-</sup>	.0000 427
.65	.0005 747	.0003 622	.0002 287	.0001 447	.0000 916	.0000 581
.66	.0007 204	.0004 611	.0002 957	.0001 899	.0001 221	.0000 787
.67	.0009 006	.0005 852	.0003 810	.0002 484	.0001 622	.0001 060
.68	.0011 226	.0007 405 <sup>-</sup>	.0004 893	.0003 238	.0002 146	.0001 424
.69	.0013 956	.0009 342	.0006 264	.0004 207	.0002 830	.0001 906
.70	.0017 305 <sup>+</sup>	.0011 753	.0007 997	.0005 449	.0003 719	.0002 541
.71	.0021 406	.0014 748	.0010 179	.0007 036	.0004 871	.0003 376
.72	.0026 416	.0018 459	.0012 922	.0009 059	.0006 360	.0004 471
.73	.0032 525 <sup>+</sup>	.0023 048	.0016 360	.0011 631	.0008 280	.0005 601
.74	.0039 962	.0028 711	.0020 662	.0014 892	.0010 748	.0007 767
.75	.0048 999	.0035 685 <sup>+</sup>	.0026 033	.0019 019	.0013 914	.0010 101
.76	.0059 964	.0044 261	.0032 724	.0024 231	.0017 905 <sup>+</sup>	.0013 330
.77	.0073 247	.0054 788	.0041 048	.0030 799	.0023 139	.0017 405
.78	.0089 319	.0067 691	.0051 384	.0039 061	.0029 732	.0022 658
.79	.0108 743	.0083 486	.0064 199	.0049 438	.0038 120	.0029 427
.80	.0132 192	.0102 798	.0080 067	.0062 450 <sup>+</sup>	.0048 771	.0038 132
.81	.0160 478	.0126 386	.0099 692	.0078 745 <sup>+</sup>	.0062 278	.0049 310
.82	.0194 573	.0155 171	.0123 939	.0099 129	.0079 383	.0063 641
.83	.0235 652	.0190 278	.0153 874	.0124 602	.0101 021	.0081 993
.84	.0285 131	.0233 077	.0190 809	.0156 413	.0128 371	.0105 470
.85	.0344 726	.0285 243	.0236 368	.0196 122	.0162 919	.0135 480
.86	.0416 527	.0348 837	.0292 563	.0245 681	.0206 548	.0173 828
.87	.0503 085 <sup>+</sup>	.0426 397	.0361 903	.0307 546	.0261 647	.0222 824
.88	.0607 541	.0521 077	.0447 526	.0384 823	.0331 268	.0285 448
.89	.0733 788	.0636 817	.0553 390	.0481 460	.0419 325 <sup>+</sup>	.0365 561
.90	.0886 700	.0778 587	.0684 528	.0602 521	.0530 885 <sup>-</sup>	.0468 204
.91	.1072 459	.0952 734	.0847 416	.0754 572	.0672 568	.0600 018
.92	.1299 031	.1167 490	.1050 507	.0946 246	.0853 147	.0769 875 <sup>-</sup>
.93	.1576 897	.1433 745 <sup>+</sup>	.1305 053	.1189 109	.1084 452	.0989 825 <sup>-</sup>
.94	.1920 248	.1766 309	.1626 429	.1499 051	.1382 837	.1276 631
.95	.2349 057	.2186 090	.2036 432	.1898 700	.1771 703	.1654 407
.96	.2893 035 <sup>+</sup>	.2724 259	.2567 635 <sup>-</sup>	.2421 974	.2286 252	.2159 579
.97	.3600 252	.3431 267	.3272 815 <sup>+</sup>	.3123 920	.2983 739	.2851 541
.98	.4560 048	.4400 767	.4249 874	.4106 621	.3970 362	.3840 537
.99	.5990 480	.5860 650 <sup>-</sup>	.5736 402	.5617 240	.5502 740	.5392 534
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$B(p, q)$	.3988 1731	.3890 9006	.3800 4145 <sup>-</sup>	.3715 9608	.3636 8978	.3562 6754
$x$						
.48	.0000 001					
.49	.0000 001	.0000 001				
.50	.0000 002	.0000 001				
.51	.0000 002	.0000 001	.0000 001			
.52	.0000 004	.0000 002	.0000 001			
.53	.0000 005 <sup>+</sup>	.0000 003	.0000 001	.0000 001		
.54	.0000 008	.0000 004	.0000 002	.0000 001	.0000 001	
.55	.0000 012	.0000 006	.0000 003	.0000 002	.0000 001	.0000 001
.56	.0000 017	.0000 009	.0000 005 <sup>+</sup>	.0000 003	.0000 002	.0000 001
.57	.0000 024	.0000 014	.0000 008	.0000 004	.0000 002	.0000 001
.58	.0000 035 <sup>-</sup>	.0000 020	.0000 011	.0000 006	.0000 004	.0000 002
.59	.0000 050 <sup>-</sup>	.0000 029	.0000 017	.0000 010	.0000 006	.0000 003
.60	.0000 070	.0000 041	.0000 024	.0000 014	.0000 008	.0000 005 <sup>-</sup>
.61	.0000 090	.0000 059	.0000 035 <sup>+</sup>	.0000 021	.0000 013	.0000 008
.62	.0000 138	.0000 084	.0000 051	.0000 031	.0000 019	.0000 011
.63	.0000 193	.0000 119	.0000 073	.0000 045 <sup>+</sup>	.0000 028	.0000 017
.64	.0000 267	.0000 167	.0000 105 <sup>-</sup>	.0000 066	.0000 041	.0000 026
.65	.0000 369	.0000 234	.0000 149	.0000 095 <sup>-</sup>	.0000 061	.0000 039
.66	.0000 507	.0000 327	.0000 211	.0000 137	.0000 088	.0000 057
.67	.0000 694	.0000 455 <sup>-</sup>	.0000 298	.0000 196	.0000 129	.0000 085 <sup>-</sup>
.68	.0000 946	.0000 629	.0000 419	.0000 279	.0000 186	.0000 124
.69	.0001 285 <sup>-</sup>	.0000 867	.0000 586	.0000 396	.0000 268	.0000 182
.70	.0001 738	.0001 190	.0000 816	.0000 560	.0000 384	.0000 264
.71	.0002 342	.0001 627	.0001 131	.0000 787	.0000 548	.0000 382
.72	.0003 146	.0002 216	.0001 562	.0001 103	.0000 779	.0000 550 <sup>+</sup>
.73	.0004 211	.0003 008	.0002 150 <sup>+</sup>	.0001 539	.0001 102	.0000 790
.74	.0005 618	.0004 068	.0002 949	.0002 139	.0001 553	.0001 128
.75	.0007 473	.0005 485 <sup>-</sup>	.0004 030	.0002 963	.0002 180	.0001 605 <sup>+</sup>
.76	.0009 910	.0007 372	.0005 488	.0004 090	.0003 050 <sup>-</sup>	.0002 276
.77	.0013 106	.0009 878	.0007 452	.0005 627	.0004 251	.0003 215 <sup>-</sup>
.78	.0017 285 <sup>+</sup>	.0013 199	.0010 088	.0007 716	.0005 907	.0004 525 <sup>-</sup>
.79	.0022 740	.0017 589	.0013 617	.0010 551	.0008 181	.0006 348
.80	.0029 845	.0023 380	.0018 332	.0014 385 <sup>+</sup>	.0011 297	.0008 877
.81	.0039 082	.0031 004	.0024 617	.0019 561	.0015 555 <sup>+</sup>	.0012 378
.82	.0051 072	.0041 023	.0032 980	.0026 534	.0021 363	.0017 211
.83	.0066 615 <sup>-</sup>	.0054 170	.0044 087	.0035 908	.0029 267	.0023 870
.84	.0086 739	.0071 398	.0058 819	.0048 492	.0040 007	.0033 027
.85	.0112 772	.0093 952	.0078 335 <sup>+</sup>	.0065 363	.0054 577	.0045 599
.86	.0146 429	.0123 456	.0104 169	.0087 959	.0074 322	.0062 838
.87	.0189 937	.0162 040	.0138 348	.0118 204	.0101 060	.0086 455 <sup>-</sup>
.88	.0246 188	.0212 593	.0183 505 <sup>+</sup>	.0158 679	.0137 256	.0118 796
.89	.0318 971	.0278 542	.0243 417	.0212 866	.0186 267	.0163 087
.90	.0413 275 <sup>-</sup>	.0365 075 <sup>-</sup>	.0322 728	.0285 482	.0252 680	.0223 790
.91	.0535 734	.0478 699	.0428 032	.0382 974	.0342 862	.0307 120
.92	.0695 281	.0628 370	.0568 277	.0514 249	.0465 623	.0421 819
.93	.0904 137	.0826 441	.0755 903	.0691 794	.0633 468	.0580 356
.94	.1179 424	.1090 335 <sup>-</sup>	.1008 584	.0933 485 <sup>-</sup>	.0864 426	.0800 864
.95	.1545 908	.1445 411	.1352 212	.1265 685 <sup>+</sup>	.1185 272	.1110 470
.96	.2041 173	.1930 344	.1826 482	.1729 041	.1637 531	.1551 511
.97	.2726 684	.2608 599	.2496 785 <sup>+</sup>	.2390 790	.2290 210	.2194 680
.98	.3716 657	.3598 289	.3485 051	.3376 600	.3272 628	.3172 858
.99	.5286 301	.5183 759	.5084 660	.4988 782	.4895 926	.4805 913
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .68$  to  $1.00$  $q = 0.5$  $p = .38$  to  $.43$ 

	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$B(p, q) = .2884\ 7734$		$.2847\ 3088$	$.2811\ 2669$	$.2776\ 5599$	$.2743\ 1074$	$.2710\ 8355$
$x$						
.68	.0000 001					
.69	.0000 001	.0000 001	.0000 001			
.70	.0000 002	.0000 001	.0000 001	.0000 001		
.71	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.72	.0000 006	.0000 005	.0000 003	.0000 002	.0000 002	.0000 001
.73	.0000 011	.0000 008	.0000 006	.0000 004	.0000 003	.0000 002
.74	.0000 019	.0000 014	.0000 010	.0000 007	.0000 005 <sup>+</sup>	.0000 004
.75	.0000 031	.0000 023	.0000 017	.0000 013	.0000 010	.0000 007
.76	.0000 053	.0000 040	.0000 030	.0000 022	.0000 017	.0000 013
.77	.0000 089	.0000 068	.0000 051	.0000 039	.0000 030	.0000 023
.78	.0000 148	.0000 114	.0000 088	.0000 068	.0000 052	.0000 040
.79	.0000 245 <sup>+</sup>	.0000 192	.0000 150 <sup>-</sup>	.0000 117	.0000 091	.0000 071
.80	.0000 405 <sup>-</sup>	.0000 320	.0000 253	.0000 200	.0000 158	.0000 125 <sup>1</sup>
.81	.0000 664	.0000 531	.0000 425 <sup>+</sup>	.0000 341	.0000 273	.0000 219
.82	.0001 084	.0000 878	.0000 712	.0000 577	.0000 468	.0000 380
.83	.0001 762	.0001 445 <sup>+</sup>	.0001 186	.0000 973	.0000 799	.0000 650
.84	.0002 852	.0002 368	.0001 967	.0001 634	.0001 358	.0001 128
.85	.0004 599	.0003 865 <sup>-</sup>	.0003 248	.0002 731	.0002 296	.0001 931
.86	.0007 391	.0006 284	.0005 344	.0004 546	.0003 868	.0003 292
.87	.0011 839	.0010 184	.0008 762	.0007 541	.0006 402	.0005 590
.88	.0018 911	.0016 456	.0014 324	.0012 471	.0010 860	.0009 459
.89	.0030 139	.0026 528	.0023 355 <sup>+</sup>	.0020 567	.0018 116	.0015 960
.90	.0047 945 <sup>+</sup>	.0042 681	.0038 004	.0033 848	.0030 152	.0026 860
.91	.0076 187	.0068 587	.0061 760	.0055 625 <sup>+</sup>	.0050 110	.0045 152
.92	.0121 029	.0110 175 <sup>-</sup>	.0100 317	.0091 361	.0083 223	.0075 824
.93	.0192 416	.0177 105 <sup>-</sup>	.0163 048	.0150 138	.0138 270	.0127 381
.94	.0306 608	.0285 325 <sup>-</sup>	.0265 575 <sup>-</sup>	.0247 242	.0230 219	.0214 409
.95	.0490 735 <sup>-</sup>	.0461 689	.0434 449	.0408 894	.0384 914	.0362 403
.96	.0791 535 <sup>+</sup>	.0752 859	.0716 206	.0681 459	.0648 509	.0617 253
.97	.1293 941	.1244 282	.1196 731	.1151 184	.1107 541	.1065 712
.98	.2168 109	.2108 239	.2050 319	.1994 266	.1940 004	.1887 461
.99	.3836 977	.3774 712	.3713 860	.3654 370	.3596 193	.3539 283
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$x = .63$  to  $1.00$  $q = .05$  $p = 32$  to  $37$ 

	$p = 32$	$p = 33$	$p = 34$	$p = 35$	$p = 36$	$p = 37$
$B(p, q)$	.3145 5482	.3007 1551	.3050 9289	.3000 7126	.2904 3645 <sup>†</sup>	.2023 7568
$x$						
.63	.0000 001					
.64	.0000 001	.0000 001				
.65	.0000 002	.0000 001	.0000 001			
.66	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001	
.67	.0000 005	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.68	.0000 007	.0000 005	.0000 003	.0000 002	.0000 002	.0000 001
.69	.0000 012	.0000 008	.0000 006	.0000 004	.0000 003	.0000 002
.70	.0000 019	.0000 013	.0000 009	.0000 006	.0000 004	.0000 003
.71	.0000 031	.0000 022	.0000 015 <sup>†</sup>	.0000 011	.0000 007	.0000 005 <sup>†</sup>
.72	.0000 049	.0000 035	.0000 025	.0000 018	.0000 013	.0000 009
.73	.0000 078	.0000 056	.0000 040	.0000 029	.0000 021	.0000 015 <sup>†</sup>
.74	.0000 122	.0000 089	.0000 065 <sup>†</sup>	.0000 048	.0000 035	.0000 025 <sup>†</sup>
.75	.0000 192	.0000 142	.0000 105	.0000 078	.0000 057	.0000 043
.76	.0000 298	.0000 223	.0000 167	.0000 126	.0000 094	.0000 071
.77	.0000 462	.0000 350 <sup>†</sup>	.0000 266	.0000 202	.0000 154	.0000 117
.78	.0000 711	.0000 547	.0000 421	.0000 324	.0000 250	.0000 192
.79	.0001 092	.0000 851	.0000 663	.0000 517	.0000 403	.0000 314
.80	.0001 668	.0001 316	.0001 039	.0000 820	.0000 648	.0000 512
.81	.0002 539	.0002 029	.0001 621	.0001 296	.0001 037	.0000 829
.82	.0003 850	.0003 114	.0002 520	.0002 040	.0001 651	.0001 338
.83	.0005 817	.0004 763	.0003 901	.0003 196	.0002 620	.0002 148
.84	.0008 759	.0007 259	.0006 018	.0004 991	.0004 140	.0003 436
.85	.0013 150	.0011 029	.0009 253	.0007 766	.0006 519	.0005 475
.86	.0019 687	.0016 707	.0014 184	.0012 045 <sup>†</sup>	.0010 232	.0008 605
.87	.0029 402	.0025 245 <sup>†</sup>	.0021 684	.0018 631	.0016 013	.0013 767
.88	.0043 819	.0038 062	.0033 073	.0028 747	.0024 995	.0021 738
.89	.0065 199	.0057 286	.0050 350 <sup>†</sup>	.0044 268	.0038 933	.0034 250 <sup>†</sup>
.90	.0096 897	.0086 110	.0076 549	.0068 070	.0060 548	.0053 872
.91	.0143 930	.0129 356	.0116 294	.0104 582	.0094 077	.0084 650
.92	.0213 844	.0194 350 <sup>†</sup>	.0176 687	.0160 676	.0146 156	.0132 984
.93	.0318 120	.0292 347	.0268 742	.0247 112	.0227 283	.0209 090
.94	.0474 497	.0440 809	.0409 795	.0380 987	.0354 294	.0329 552
.95	.0711 046	.0668 614	.0627 753	.0590 067	.0554 777	.0521 717
.96	.1073 812	.1020 001	.0969 127	.0921 005	.0875 468	.0832 360
.97	.1642 933	.1578 009	.1515 986	.1456 708	.1400 031	.1345 817
.98	.2573 552	.2499 948	.2428 909	.2360 314	.2294 051	.2230 016
.99	.4243 560	.4171 535 <sup>†</sup>	.4101 312	.4032 812	.3965 967	.3900 709
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .68$  to  $1.00$  $q = 0.5$  $p = 38$  to  $43$ 

	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$B(p, q) =$	.2884 7734	.2847 3088	.2811 2669	.2776 5599	.2743 1074	.2710 8355 <sup>-</sup>
$x$						
.68	.0000 001					
.69	.0000 001	.0000 001	.0000 001			
.70	.0000 002	.0000 001	.0000 001	.0000 001		
.71	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.72	.0000 006	.0000 005 <sup>-</sup>	.0000 003	.0000 002	.0000 002	.0000 001
.73	.0000 011	.0000 008	.0000 006	.0000 004	.0000 003	.0000 002
.74	.0000 019	.0000 014	.0000 010	.0000 007	.0000 005 <sup>+</sup>	.0000 004
.75	.0000 031	.0000 023	.0000 017	.0000 013	.0000 010	.0000 007
.76	.0000 053	.0000 040	.0000 030	.0000 022	.0000 017	.0000 013
.77	.0000 089	.0000 068	.0000 051	.0000 039	.0000 030	.0000 023
.78	.0000 148	.0000 114	.0000 088	.0000 068	.0000 052	.0000 040
.79	.0000 245 <sup>+</sup>	.0000 192	.0000 150 <sup>-</sup>	.0000 117	.0000 091	.0000 071
.80	.0000 405 <sup>-</sup>	.0000 320	.0000 253	.0000 200	.0000 158	.0000 125 <sup>+</sup>
.81	.0000 664	.0000 531	.0000 425 <sup>+</sup>	.0000 341	.0000 273	.0000 219
.82	.0001 084	.0000 878	.0000 712	.0000 577	.0000 468	.0000 380
.83	.0001 762	.0001 445 <sup>+</sup>	.0001 186	.0000 973	.0000 799	.0000 656
.84	.0002 852	.0002 368	.0001 967	.0001 634	.0001 358	.0001 128
.85	.0004 599	.0003 865 <sup>-</sup>	.0003 248	.0002 731	.0002 296	.0001 931
.86	.0007 391	.0006 284	.0005 344	.0004 546	.0003 868	.0003 292
.87	.0011 839	.0010 184	.0008 762	.0007 541	.0006 402	.0005 590
.88	.0018 911	.0016 456	.0014 324	.0012 471	.0010 860	.0009 459
.89	.0030 139	.0026 528	.0023 355 <sup>+</sup>	.0020 567	.0018 116	.0015 960
.90	.0047 945 <sup>+</sup>	.0042 681	.0038 004	.0033 848	.0030 152	.0026 866
.91	.0076 187	.0068 587	.0061 760	.0055 625 <sup>+</sup>	.0050 110	.0045 152
.92	.0121 029	.0110 175 <sup>-</sup>	.0100 317	.0091 361	.0083 223	.0075 824
.93	.0192 416	.0177 105 <sup>-</sup>	.0163 048	.0150 138	.0138 279	.0127 381
.94	.0306 608	.0285 325 <sup>-</sup>	.0265 575 <sup>-</sup>	.0247 242	.0230 219	.0214 409
.95	.0490 735 <sup>-</sup>	.0461 689	.0434 449	.0408 804	.0384 914	.0362 403
.96	.0791 535 <sup>+</sup>	.0752 859	.0716 206	.0681 459	.0648 500	.0617 253
.97	.1293 941	.1244 282	.1196 731	.1151 184	.1107 541	.1065 712
.98	.2168 109	.2108 239	.2050 319	.1994 266	.1940 004	.1887 461
.99	.3836 977	.3774 712	.3713 860	.3654 370	.3596 193	.3539 283
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

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$p = 44$  to 50

[illegible]



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .01$  to  $.60$  $q = 1$  $p = 1$  to  $3.5$ 

	$p = 1$	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$
$B(p, q) = 1.0000\ 0000$		.6666 6667	.5000 0000	.4000 0000	.3333 3333	.2857 1429
$x$						
.01	.0100 000 <sup>e</sup>	.0010 000 <sup>e</sup>	.0001 000 <sup>e</sup>	.0000 100 <sup>e</sup>	.0000 010 <sup>e</sup>	.0000 001
.02	.0200 000 <sup>e</sup>	.0028 284	.0004 000 <sup>e</sup>	.0000 566	.0000 080 <sup>e</sup>	.0000 011
.03	.0300 000 <sup>e</sup>	.0051 962	.0009 000 <sup>e</sup>	.0001 559	.0000 270 <sup>e</sup>	.0000 047
.04	.0400 000 <sup>e</sup>	.0080 000 <sup>e</sup>	.0016 000 <sup>e</sup>	.0003 200 <sup>e</sup>	.0000 640 <sup>e</sup>	.0000 128 <sup>e</sup>
.05	.0500 000 <sup>e</sup>	.0111 803	.0025 000 <sup>e</sup>	.0005 590	.0001 250 <sup>e</sup>	.0000 280
.06	.0600 000 <sup>e</sup>	.0146 969	.0036 000 <sup>e</sup>	.0008 818	.0002 160 <sup>e</sup>	.0000 520
.07	.0700 000 <sup>e</sup>	.0185 203	.0049 000 <sup>e</sup>	.0012 964	.0003 430 <sup>e</sup>	.0000 907
.08	.0800 000 <sup>e</sup>	.0226 274	.0064 000 <sup>e</sup>	.0018 102	.0005 120 <sup>e</sup>	.0001 448
.09	.0900 000 <sup>e</sup>	.0270 000 <sup>e</sup>	.0081 000 <sup>e</sup>	.0024 300 <sup>e</sup>	.0007 290 <sup>e</sup>	.0002 187 <sup>e</sup>
.10	.1000 000 <sup>e</sup>	.0316 228	.0100 000 <sup>e</sup>	.0031 623	.0010 000 <sup>e</sup>	.0003 162
.11	.1100 000 <sup>e</sup>	.0364 829	.0121 000 <sup>e</sup>	.0040 131	.0013 310 <sup>e</sup>	.0004 414
.12	.1200 000 <sup>e</sup>	.0415 692	.0144 000 <sup>e</sup>	.0049 883	.0017 280 <sup>e</sup>	.0005 986
.13	.1300 000 <sup>e</sup>	.0468 722	.0169 000 <sup>e</sup>	.0060 934	.0021 970 <sup>e</sup>	.0007 921
.14	.1400 000 <sup>e</sup>	.0523 832	.0196 000 <sup>e</sup>	.0073 336	.0027 440 <sup>e</sup>	.0010 267
.15	.1500 000 <sup>e</sup>	.0580 948	.0225 000 <sup>e</sup>	.0087 142	.0033 750 <sup>e</sup>	.0013 071
.16	.1600 000 <sup>e</sup>	.0640 000 <sup>e</sup>	.0256 000 <sup>e</sup>	.0102 400 <sup>e</sup>	.0040 960 <sup>e</sup>	.0016 381 <sup>e</sup>
.17	.1700 000 <sup>e</sup>	.0700 928	.0289 000 <sup>e</sup>	.0119 158	.0049 130 <sup>e</sup>	.0020 257
.18	.1800 000 <sup>e</sup>	.0763 675 <sup>+</sup>	.0324 000 <sup>e</sup>	.0137 462	.0058 320 <sup>e</sup>	.0024 743
.19	.1900 000 <sup>e</sup>	.0828 191	.0361 000 <sup>e</sup>	.0157 356	.0068 590 <sup>e</sup>	.0029 808
.20	.2000 000 <sup>e</sup>	.0894 427	.0400 000 <sup>e</sup>	.0178 885 <sup>+</sup>	.0080 000 <sup>e</sup>	.0035 777
.21	.2100 000 <sup>e</sup>	.0962 341	.0441 000 <sup>e</sup>	.0202 092	.0092 610 <sup>e</sup>	.0042 430
.22	.2200 000 <sup>e</sup>	.1031 891	.0484 000 <sup>e</sup>	.0227 016	.0106 480 <sup>e</sup>	.0049 944
.23	.2300 000 <sup>e</sup>	.1103 041	.0529 000 <sup>e</sup>	.0253 699	.0121 670 <sup>e</sup>	.0058 351
.24	.2400 000 <sup>e</sup>	.1175 755 <sup>+</sup>	.0576 000 <sup>e</sup>	.0282 181	.0138 240 <sup>e</sup>	.0067 723
.25	.2500 000 <sup>e</sup>	.1250 000 <sup>e</sup>	.0625 000 <sup>e</sup>	.0312 500 <sup>e</sup>	.0156 250 <sup>e</sup>	.0078 125 <sup>e</sup>
.26	.2600 000 <sup>e</sup>	.1325 745 <sup>+</sup>	.0676 000 <sup>e</sup>	.0344 694	.0175 760 <sup>e</sup>	.0089 620
.27	.2700 000 <sup>e</sup>	.1402 961	.0729 000 <sup>e</sup>	.0378 800	.0196 830 <sup>e</sup>	.0102 276
.28	.2800 000 <sup>e</sup>	.1481 621	.0784 000 <sup>e</sup>	.0414 854	.0219 520 <sup>e</sup>	.0116 159
.29	.2900 000 <sup>e</sup>	.1561 698	.0841 000 <sup>e</sup>	.0452 892	.0243 890 <sup>e</sup>	.0131 339
.30	.3000 000 <sup>e</sup>	.1643 168	.0900 000 <sup>e</sup>	.0492 950 <sup>+</sup>	.0270 000 <sup>e</sup>	.0147 885 <sup>+</sup>
.31	.3100 000 <sup>e</sup>	.1726 007	.0961 000 <sup>e</sup>	.0535 062	.0297 910 <sup>e</sup>	.0165 860
.32	.3200 000 <sup>e</sup>	.1810 193	.1024 000 <sup>e</sup>	.0579 262	.0327 680 <sup>e</sup>	.0185 361
.33	.3300 000 <sup>e</sup>	.1895 706	.1089 000 <sup>e</sup>	.0625 583	.0359 370 <sup>e</sup>	.0206 442
.34	.3400 000 <sup>e</sup>	.1982 524	.1156 000 <sup>e</sup>	.0674 058	.0393 040 <sup>e</sup>	.0229 180
.35	.3500 000 <sup>e</sup>	.2070 628	.1225 000 <sup>e</sup>	.0724 720	.0428 750 <sup>e</sup>	.0253 652
.36	.3600 000 <sup>e</sup>	.2160 000 <sup>e</sup>	.1296 000 <sup>e</sup>	.0777 600 <sup>e</sup>	.0466 560 <sup>e</sup>	.0279 036 <sup>e</sup>
.37	.3700 000 <sup>e</sup>	.2250 622	.1369 000 <sup>e</sup>	.0832 730	.0506 530 <sup>e</sup>	.0308 110
.38	.3800 000 <sup>e</sup>	.2342 477	.1444 000 <sup>e</sup>	.0890 141	.0548 720 <sup>e</sup>	.0338 254
.39	.3900 000 <sup>e</sup>	.2435 549	.1521 000 <sup>e</sup>	.0949 864	.0593 190 <sup>e</sup>	.0370 447
.40	.4000 000 <sup>e</sup>	.2529 822	.1600 000 <sup>e</sup>	.1011 929	.0640 000 <sup>e</sup>	.0404 772
.41	.4100 000 <sup>e</sup>	.2625 281	.1681 000 <sup>e</sup>	.1076 365 <sup>+</sup>	.0689 210 <sup>e</sup>	.0441 310
.42	.4200 000 <sup>e</sup>	.2721 911	.1764 000 <sup>e</sup>	.1143 203	.0740 880 <sup>e</sup>	.0480 145 <sup>+</sup>
.43	.4300 000 <sup>e</sup>	.2819 699	.1849 000 <sup>e</sup>	.1212 470	.0795 070 <sup>e</sup>	.0521 362
.44	.4400 000 <sup>e</sup>	.2918 630	.1936 000 <sup>e</sup>	.1284 197	.0851 840 <sup>e</sup>	.0565 047
.45	.4500 000 <sup>e</sup>	.3018 692	.2025 000 <sup>e</sup>	.1358 411	.0911 250 <sup>e</sup>	.0611 285 <sup>+</sup>
.46	.4600 000 <sup>e</sup>	.3119 872	.2116 000 <sup>e</sup>	.1435 141	.0973 360 <sup>e</sup>	.0660 165 <sup>+</sup>
.47	.4700 000 <sup>e</sup>	.3222 158	.2209 000 <sup>e</sup>	.1514 414	.1038 230 <sup>e</sup>	.0711 775 <sup>+</sup>
.48	.4800 000 <sup>e</sup>	.3325 538	.2304 000 <sup>e</sup>	.1596 258	.1105 920 <sup>e</sup>	.0766 204
.49	.4900 000 <sup>e</sup>	.3430 000 <sup>e</sup>	.2401 000 <sup>e</sup>	.1680 700	.1176 490 <sup>e</sup>	.0823 543 <sup>e</sup>
.50	.5000 000 <sup>e</sup>	.3535 534	.2500 000 <sup>e</sup>	.1767 767	.1250 000 <sup>e</sup>	.0883 883
.51	.5100 000 <sup>e</sup>	.3642 128	.2601 000 <sup>e</sup>	.1857 486	.1326 510 <sup>e</sup>	.0947 318
.52	.5200 000 <sup>e</sup>	.3749 773	.2704 000 <sup>e</sup>	.1949 882	.1406 080 <sup>e</sup>	.1013 939
.53	.5300 000 <sup>e</sup>	.3858 458	.2809 000 <sup>e</sup>	.2044 983	.1488 770 <sup>e</sup>	.1083 841
.54	.5400 000 <sup>e</sup>	.3968 173	.2916 000 <sup>e</sup>	.2142 814	.1574 640 <sup>e</sup>	.1157 119
.55	.5500 000 <sup>e</sup>	.4078 909	.3025 000 <sup>e</sup>	.2243 400	.1663 750 <sup>e</sup>	.1233 870
.56	.5600 000 <sup>e</sup>	.4190 656	.3136 000 <sup>e</sup>	.2346 768	.1756 160 <sup>e</sup>	.1314 190
.57	.5700 000 <sup>e</sup>	.4303 406	.3249 000 <sup>e</sup>	.2452 941	.1851 930 <sup>e</sup>	.1398 176
.58	.5800 000 <sup>e</sup>	.4417 148	.3364 000 <sup>e</sup>	.2561 946	.1951 120 <sup>e</sup>	.1485 929
.59	.5900 000 <sup>e</sup>	.4531 876	.3481 000 <sup>e</sup>	.2673 807	.2053 790 <sup>e</sup>	.1577 546
.60	.6000 000 <sup>e</sup>	.4647 580	.3600 000 <sup>e</sup>	.2788 548	.2160 000 <sup>e</sup>	.1673 129

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION $q = .61$  to  $1.00$  $q = 1$  $p = 1$  to  $3.5$ 

	$p = 1$	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$
$B(p, q) = 1.0000\ 0000$		.6666 6667	.5000 0000	.4000 0000	.3333 3333	.2857 1429
$x$						
.61	.6100 000 <sup>a</sup>	.4764 252	.3721 000 <sup>a</sup>	.2906 194	.2269 810 <sup>a</sup>	.1772 778
.62	.6200 000 <sup>a</sup>	.4881 885 <sup>-</sup>	.3844 000 <sup>a</sup>	.3026 769	.2383 280 <sup>a</sup>	.1876 597
.63	.6300 000 <sup>a</sup>	.5000 470	.3969 000 <sup>a</sup>	.3150 296	.2500 470 <sup>a</sup>	.1984 687
.64	.6400 000 <sup>a</sup>	.5120 000 <sup>a</sup>	.4096 000 <sup>a</sup>	.3276 800 <sup>a</sup>	.2621 440 <sup>a</sup>	.2097 152 <sup>a</sup>
.65	.6500 000 <sup>a</sup>	.5240 468	.4225 000 <sup>a</sup>	.3406 304	.2746 250 <sup>a</sup>	.2214 098
.66	.6600 000 <sup>a</sup>	.5361 865 <sup>+</sup>	.4356 000 <sup>a</sup>	.3538 831	.2874 960 <sup>a</sup>	.2335 629
.67	.6700 000 <sup>a</sup>	.5484 186	.4489 000 <sup>a</sup>	.3674 405 <sup>-</sup>	.3007 630 <sup>a</sup>	.2461 852
.68	.6800 000 <sup>a</sup>	.5607 424	.4624 000 <sup>a</sup>	.3813 048	.3144 320 <sup>a</sup>	.2592 873
.69	.6900 000 <sup>a</sup>	.5731 570	.4761 000 <sup>a</sup>	.3954 784	.3285 090 <sup>a</sup>	.2728 801
.70	.7000 000 <sup>a</sup>	.5856 620	.4900 000 <sup>a</sup>	.4099 634	.3430 000 <sup>a</sup>	.2869 744
.71	.7100 000 <sup>a</sup>	.5982 566	.5041 000 <sup>a</sup>	.4247 622	.3579 110 <sup>a</sup>	.3015 812
.72	.7200 000 <sup>a</sup>	.6109 403	.5184 000 <sup>a</sup>	.4398 770	.3732 480 <sup>a</sup>	.3167 114
.73	.7300 000 <sup>a</sup>	.6237 123	.5329 000 <sup>a</sup>	.4553 100	.3890 170 <sup>a</sup>	.3323 763
.74	.7400 000 <sup>a</sup>	.6365 721	.5476 000 <sup>a</sup>	.4710 633	.4052 240 <sup>a</sup>	.3485 869
.75	.7500 000 <sup>a</sup>	.6495 191	.5625 000 <sup>a</sup>	.4871 393	.4218 750 <sup>a</sup>	.3653 545 <sup>-</sup>
.76	.7600 000 <sup>a</sup>	.6625 526	.5776 000 <sup>a</sup>	.5035 400	.4389 700 <sup>a</sup>	.3826 904
.77	.7700 000 <sup>a</sup>	.6756 723	.5929 000 <sup>a</sup>	.5202 676	.4565 330 <sup>a</sup>	.4006 061
.78	.7800 000 <sup>a</sup>	.6888 773	.6084 000 <sup>a</sup>	.5373 243	.4745 520 <sup>a</sup>	.4191 130
.79	.7900 000 <sup>a</sup>	.7021 674	.6241 000 <sup>a</sup>	.5547 122	.4930 390 <sup>a</sup>	.4382 226
.80	.8000 000 <sup>a</sup>	.7155 418	.6400 000 <sup>a</sup>	.5724 334	.5120 000 <sup>a</sup>	.4579 467
.81	.8100 000 <sup>a</sup>	.7290 000 <sup>a</sup>	.6561 000 <sup>a</sup>	.5904 900 <sup>a</sup>	.5314 410 <sup>a</sup>	.4782 969 <sup>a</sup>
.82	.8200 000 <sup>a</sup>	.7425 416	.6724 000 <sup>a</sup>	.6088 841	.5513 680 <sup>a</sup>	.4992 850 <sup>-</sup>
.83	.8300 000 <sup>a</sup>	.7561 660	.6889 000 <sup>a</sup>	.6276 178	.5717 870 <sup>a</sup>	.5209 227
.84	.8400 000 <sup>a</sup>	.7698 727	.7056 000 <sup>a</sup>	.6466 931	.5927 040 <sup>a</sup>	.5432 222
.85	.8500 000 <sup>a</sup>	.7836 613	.7225 000 <sup>a</sup>	.6661 121	.6141 250 <sup>a</sup>	.5661 953
.86	.8600 000 <sup>a</sup>	.7975 312	.7396 000 <sup>a</sup>	.6858 768	.6360 560 <sup>a</sup>	.5898 541
.87	.8700 000 <sup>a</sup>	.8114 820	.7569 000 <sup>a</sup>	.7059 893	.6585 030 <sup>a</sup>	.6142 107
.88	.8800 000 <sup>a</sup>	.8255 132	.7744 000 <sup>a</sup>	.7264 516	.6814 720 <sup>a</sup>	.6392 774
.89	.8900 000 <sup>a</sup>	.8396 243	.7921 000 <sup>a</sup>	.7472 656	.7049 690 <sup>a</sup>	.6650 664
.90	.9000 000 <sup>a</sup>	.8538 150 <sup>-</sup>	.8100 000 <sup>a</sup>	.7684 335 <sup>-</sup>	.7290 000 <sup>a</sup>	.6915 901
.91	.9100 000 <sup>a</sup>	.8680 847	.8281 000 <sup>a</sup>	.7899 571	.7535 710 <sup>a</sup>	.7188 609
.92	.9200 000 <sup>a</sup>	.8824 330	.8464 000 <sup>a</sup>	.8118 384	.7786 880 <sup>a</sup>	.7468 913
.93	.9300 000 <sup>a</sup>	.8968 595 <sup>+</sup>	.8649 000 <sup>a</sup>	.8340 794	.8043 570 <sup>a</sup>	.7756 938
.94	.9400 000 <sup>a</sup>	.9113 638	.8836 000 <sup>a</sup>	.8566 820	.8305 840 <sup>a</sup>	.8052 811
.95	.9500 000 <sup>a</sup>	.9259 455 <sup>-</sup>	.9025 000 <sup>a</sup>	.8796 482	.8573 750 <sup>a</sup>	.8356 658
.96	.9600 000 <sup>a</sup>	.9406 041	.9216 000 <sup>a</sup>	.9029 799	.8847 360 <sup>a</sup>	.8668 667
.97	.9700 000 <sup>a</sup>	.9553 392	.9409 000 <sup>a</sup>	.9266 790	.9126 730 <sup>a</sup>	.8988 787
.98	.9800 000 <sup>a</sup>	.9701 505 <sup>+</sup>	.9604 000 <sup>a</sup>	.9507 475 <sup>-</sup>	.9411 920 <sup>a</sup>	.9317 325 <sup>+</sup>
.99	.9900 000 <sup>a</sup>	.9850 376	.9801 000 <sup>a</sup>	.9751 872	.9702 990 <sup>a</sup>	.9654 353
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .02$  to  $.60$  $q = 1$  $p = 4$  to  $6.5$ 

	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$
$B(p, q) =$	.2500 0000	.2222 2222	.2000 0000	.1818 1818	.1666 6667	.1538 4615 <sup>+</sup>
$x$						
.02	.0000 002					
.03	.0000 008	.0000 001				
.04	.0000 026	.0000 005 <sup>+</sup>	.0000 001			
.05	.0000 062	.0000 014	.0000 003	.0000 001		
.06	.0000 130	.0000 032	.0000 008	.0000 002		
.07	.0000 240	.0000 064	.0000 017	.0000 004	.0000 001	
.08	.0000 410	.0000 116	.0000 033	.0000 009	.0000 003	.0000 001
.09	.0000 656	.0000 197	.0000 059	.0000 018	.0000 005 <sup>+</sup>	.0000 002
.10	.0001 000 <sup>e</sup>	.0000 316	.0000 100 <sup>a</sup>	.0000 032	.0000 010 <sup>a</sup>	.0000 003
.11	.0001 464	.0000 486	.0000 161	.0000 053	.0000 018	.0000 006
.12	.0002 074	.0000 718	.0000 249	.0000 086	.0000 030	.0000 010
.13	.0002 856	.0001 030	.0000 371	.0000 134	.0000 048	.0000 017
.14	.0003 842	.0001 437	.0000 538	.0000 201	.0000 075 <sup>+</sup>	.0000 028
.15	.0005 062	.0001 961	.0000 759	.0000 294	.0000 114	.0000 041
.16	.0006 554	.0002 621	.0001 049	.0000 419	.0000 168	.0000 067
.17	.0008 352	.0003 444	.0001 420	.0000 585 <sup>+</sup>	.0000 241	.0000 100
.18	.0010 498	.0004 454	.0001 890	.0000 802	.0000 340	.0000 141
.19	.0013 032	.0005 681	.0002 476	.0001 079	.0000 470	.0000 205 <sup>+</sup>
.20	.0016 000 <sup>e</sup>	.0007 155 <sup>+</sup>	.0003 200 <sup>a</sup>	.0001 431	.0000 640 <sup>a</sup>	.0000 286
.21	.0019 448	.0008 912	.0004 084	.0001 872	.0000 858	.0000 393
.22	.0023 426	.0010 988	.0005 154	.0002 417	.0001 134	.0000 532
.23	.0027 984	.0013 421	.0006 436	.0003 087	.0001 480	.0000 710
.24	.0033 178	.0016 254	.0007 963	.0003 901	.0001 911	.0000 936
.25	.0039 062	.0019 531	.0009 766	.0004 883	.0002 441	.0001 221
.26	.0045 698	.0023 301	.0011 881	.0006 058	.0003 080	.0001 575 <sup>+</sup>
.27	.0053 144	.0027 614	.0014 349	.0007 456	.0003 874	.0002 013
.28	.0061 466	.0032 525 <sup>-</sup>	.0017 210	.0009 107	.0004 819	.0002 550
.29	.0070 728	.0038 088	.0020 511	.0011 046	.0005 948	.0003 203
.30	.0081 000 <sup>e</sup>	.0044 366	.0024 300 <sup>a</sup>	.0013 310	.0007 290 <sup>a</sup>	.0003 903
.31	.0092 352	.0051 419	.0028 629	.0015 940	.0008 875 <sup>+</sup>	.0004 941
.32	.0104 858	.0059 316	.0033 554	.0018 981	.0010 737	.0006 074
.33	.0118 592	.0068 126	.0039 135 <sup>+</sup>	.0022 482	.0012 915 <sup>-</sup>	.0007 410
.34	.0133 634	.0077 921	.0045 435 <sup>+</sup>	.0026 493	.0015 418	.0009 008
.35	.0150 003	.0088 778	.0052 522	.0031 072	.0018 383	.0010 875 <sup>+</sup>
.36	.0167 962	.0100 777	.0060 466	.0036 280	.0021 768	.0013 061
.37	.0187 416	.0114 001	.0069 344	.0042 180	.0025 657	.0015 607
.38	.0208 514	.0128 536	.0079 235 <sup>+</sup>	.0048 844	.0030 100	.0018 561
.39	.0231 344	.0144 474	.0090 224	.0056 345 <sup>-</sup>	.0035 187	.0021 975
.40	.0256 000 <sup>e</sup>	.0161 909	.0102 400 <sup>a</sup>	.0064 763	.0040 060 <sup>a</sup>	.0025 905 <sup>+</sup>
.41	.0282 576	.0180 937	.0115 856	.0074 184	.0047 501	.0030 416
.42	.0311 170	.0201 661	.0130 691	.0084 698	.0054 890	.0035 573
.43	.0341 880	.0224 186	.0147 008	.0096 400	.0063 214	.0041 452
.44	.0374 810	.0248 621	.0164 916	.0109 393	.0072 563	.0048 133
.45	.0410 062	.0275 078	.0184 528	.0123 785 <sup>+</sup>	.0083 038	.0055 703
.46	.0447 746	.0303 676	.0205 963	.0139 691	.0094 743	.0064 258
.47	.0487 968	.0334 534	.0229 345 <sup>+</sup>	.0157 231	.0107 792	.0073 890
.48	.0530 842	.0367 778	.0254 804	.0176 533	.0122 306	.0084 736
.49	.0576 480	.0403 536	.0282 475 <sup>+</sup>	.0197 733	.0138 413	.0096 880
.50	.0625 000 <sup>e</sup>	.0441 942	.0312 500 <sup>a</sup>	.0220 971	.0156 250 <sup>a</sup>	.0110 485 <sup>+</sup>
.51	.0676 520	.0483 132	.0345 025 <sup>+</sup>	.0246 397	.0175 963	.0125 663
.52	.0731 162	.0527 248	.0380 204	.0274 169	.0197 706	.0142 568
.53	.0789 048	.0574 436	.0418 195 <sup>+</sup>	.0304 451	.0221 644	.0161 350
.54	.0850 306	.0624 844	.0459 165 <sup>+</sup>	.0337 416	.0247 949	.0182 205 <sup>-</sup>
.55	.0915 063	.0678 629	.0503 284	.0373 246	.0276 806	.0205 285 <sup>+</sup>
.56	.0983 450 <sup>-</sup>	.0735 946	.0550 732	.0412 130	.0308 410	.0230 793
.57	.1055 000	.0796 961	.0601 692	.0454 268	.0342 964	.0258 932
.58	.1131 650 <sup>-</sup>	.0861 839	.0656 357	.0499 866	.0380 687	.0289 923
.59	.1211 736	.0930 752	.0714 924	.0549 144	.0421 805 <sup>+</sup>	.0323 995
.60	.1296 000 <sup>e</sup>	.1003 877	.0777 600 <sup>a</sup>	.0602 326	.0466 560 <sup>a</sup>	.0361 396

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$p = 4$  to  $6.5$

	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$
$B(p, q)$	$\cdot 2500\ 0000$	$\cdot 2222\ 2222$	$\cdot 2000\ 0000$	$\cdot 1818\ 1818$	$\cdot 1666\ 6667$	$\cdot 1538\ 4615^+$
$\cdot 01$	$\cdot 1384\ 584$	$\cdot 1081\ 305^-$	$\cdot 0844\ 596$	$\cdot 0659\ 651$	$\cdot 0515\ 204$	$\cdot 0402\ 387$
$\cdot 02$	$\cdot 1477\ 034$	$\cdot 1163\ 490$	$\cdot 0910\ 133$	$\cdot 0721\ 304$	$\cdot 0568\ 002$	$\cdot 0447\ 246$
$\cdot 03$	$\cdot 1575\ 296$	$\cdot 1250\ 353$	$\cdot 0992\ 437$	$\cdot 0787\ 722$	$\cdot 0625\ 235^+$	$\cdot 0496\ 265^-$
$\cdot 04$	$\cdot 1677\ 722$	$\cdot 1342\ 177$	$\cdot 1073\ 742$	$\cdot 0858\ 993$	$\cdot 0687\ 195$	$\cdot 0549\ 756$
$\cdot 05$	$\cdot 1785\ 062$	$\cdot 1430\ 103$	$\cdot 1160\ 291$	$\cdot 0935\ 456$	$\cdot 0754\ 189$	$\cdot 0608\ 047$
$\cdot 06$	$\cdot 1897\ 474$	$\cdot 1541\ 515^-$	$\cdot 1252\ 333$	$\cdot 1017\ 400$	$\cdot 0826\ 540$	$\cdot 0671\ 484$
$\cdot 07$	$\cdot 2015\ 112$	$\cdot 1649\ 440$	$\cdot 1350\ 125^+$	$\cdot 1105\ 125^+$	$\cdot 0904\ 584$	$\cdot 0740\ 434$
$\cdot 08$	$\cdot 2138\ 138$	$\cdot 1763\ 153$	$\cdot 1453\ 934$	$\cdot 1198\ 944$	$\cdot 0988\ 675^-$	$\cdot 0815\ 282$
$\cdot 09$	$\cdot 2266\ 712$	$\cdot 1882\ 872$	$\cdot 1564\ 031$	$\cdot 1299\ 182$	$\cdot 1079\ 182$	$\cdot 0896\ 436$
$\cdot 70$	$\cdot 2401\ 000^0$	$\cdot 2008\ 821$	$\cdot 1680\ 700^0$	$\cdot 1400\ 175^-$	$\cdot 1176\ 490^0$	$\cdot 0984\ 322$
$\cdot 71$	$\cdot 2541\ 168$	$\cdot 2141\ 226$	$\cdot 1804\ 229$	$\cdot 1520\ 271$	$\cdot 1281\ 003$	$\cdot 1079\ 392$
$\cdot 72$	$\cdot 2687\ 386$	$\cdot 2280\ 322$	$\cdot 1934\ 918$	$\cdot 1641\ 832$	$\cdot 1393\ 141$	$\cdot 1182\ 119$
$\cdot 73$	$\cdot 2839\ 824$	$\cdot 2426\ 347$	$\cdot 2073\ 072$	$\cdot 1771\ 233$	$\cdot 1513\ 342$	$\cdot 1293\ 000$
$\cdot 74$	$\cdot 2998\ 058$	$\cdot 2579\ 543$	$\cdot 2219\ 097$	$\cdot 1908\ 802$	$\cdot 1642\ 065^-$	$\cdot 1412\ 558$
$\cdot 75$	$\cdot 3164\ 062$	$\cdot 2740\ 159$	$\cdot 2373\ 047$	$\cdot 2055\ 119$	$\cdot 1779\ 785^+$	$\cdot 1541\ 339$
$\cdot 76$	$\cdot 3330\ 218$	$\cdot 2908\ 447$	$\cdot 2535\ 525^+$	$\cdot 2210\ 420$	$\cdot 1926\ 999$	$\cdot 1679\ 919$
$\cdot 77$	$\cdot 3515\ 394$	$\cdot 3084\ 607$	$\cdot 2700\ 784$	$\cdot 2375\ 193$	$\cdot 2084\ 224$	$\cdot 1828\ 899$
$\cdot 78$	$\cdot 3701\ 506$	$\cdot 3269\ 081$	$\cdot 2887\ 174$	$\cdot 2549\ 883$	$\cdot 2251\ 996$	$\cdot 1988\ 909$
$\cdot 79$	$\cdot 3895\ 008$	$\cdot 3461\ 959$	$\cdot 3077\ 056$	$\cdot 2734\ 948$	$\cdot 2430\ 875^-$	$\cdot 2160\ 609$
$\cdot 80$	$\cdot 4096\ 000^0$	$\cdot 3663\ 574$	$\cdot 3276\ 800^0$	$\cdot 2930\ 859$	$\cdot 2621\ 440^0$	$\cdot 2344\ 687$
$\cdot 81$	$\cdot 4304\ 672$	$\cdot 3874\ 205^-$	$\cdot 3486\ 784$	$\cdot 3138\ 106$	$\cdot 2824\ 295^+$	$\cdot 2541\ 866$
$\cdot 82$	$\cdot 4521\ 218$	$\cdot 4094\ 737$	$\cdot 3707\ 398$	$\cdot 3357\ 192$	$\cdot 3040\ 067$	$\cdot 2752\ 897$
$\cdot 83$	$\cdot 4745\ 832$	$\cdot 4323\ 659$	$\cdot 3939\ 041$	$\cdot 3588\ 637$	$\cdot 3269\ 404$	$\cdot 2978\ 569$
$\cdot 84$	$\cdot 4978\ 714$	$\cdot 4563\ 066$	$\cdot 4182\ 119$	$\cdot 3832\ 976$	$\cdot 3512\ 980$	$\cdot 3219\ 700$
$\cdot 85$	$\cdot 5220\ 062$	$\cdot 4812\ 600$	$\cdot 4437\ 053$	$\cdot 4090\ 761$	$\cdot 3771\ 495^+$	$\cdot 3477\ 147$
$\cdot 86$	$\cdot 5470\ 082$	$\cdot 5072\ 745^-$	$\cdot 4704\ 270$	$\cdot 4362\ 561$	$\cdot 4045\ 672$	$\cdot 3751\ 802$
$\cdot 87$	$\cdot 5728\ 976$	$\cdot 5343\ 933$	$\cdot 4984\ 209$	$\cdot 4648\ 961$	$\cdot 4330\ 262$	$\cdot 4044\ 596$
$\cdot 88$	$\cdot 5996\ 954$	$\cdot 5625\ 641$	$\cdot 5277\ 319$	$\cdot 4950\ 564$	$\cdot 4644\ 041$	$\cdot 4356\ 496$
$\cdot 89$	$\cdot 6274\ 224$	$\cdot 5919\ 091$	$\cdot 5584\ 059$	$\cdot 5267\ 991$	$\cdot 4969\ 813$	$\cdot 4688\ 512$
$\cdot 90$	$\cdot 6561\ 000^0$	$\cdot 6224\ 311$	$\cdot 5904\ 900^0$	$\cdot 5601\ 880$	$\cdot 5314\ 410^0$	$\cdot 5041\ 692$
$\cdot 91$	$\cdot 6857\ 496$	$\cdot 6541\ 634$	$\cdot 6240\ 321$	$\cdot 5952\ 887$	$\cdot 5678\ 693$	$\cdot 5417\ 127$
$\cdot 92$	$\cdot 7163\ 930$	$\cdot 6871\ 400$	$\cdot 6590\ 815^+$	$\cdot 6321\ 688$	$\cdot 6063\ 550^+$	$\cdot 5815\ 953$
$\cdot 93$	$\cdot 7480\ 520$	$\cdot 7213\ 952$	$\cdot 6956\ 884$	$\cdot 6708\ 970$	$\cdot 6460\ 902$	$\cdot 6230\ 347$
$\cdot 94$	$\cdot 7807\ 490$	$\cdot 7560\ 642$	$\cdot 7339\ 040$	$\cdot 7115\ 463$	$\cdot 6898\ 698$	$\cdot 6688\ 536$
$\cdot 95$	$\cdot 8145\ 062$	$\cdot 7938\ 825^-$	$\cdot 7737\ 809$	$\cdot 7541\ 884$	$\cdot 7350\ 919$	$\cdot 7164\ 789$
$\cdot 96$	$\cdot 8493\ 466$	$\cdot 8321\ 803$	$\cdot 8153\ 727$	$\cdot 7988\ 988$	$\cdot 7827\ 578$	$\cdot 7669\ 429$
$\cdot 97$	$\cdot 8852\ 928$	$\cdot 8719\ 123$	$\cdot 8587\ 340$	$\cdot 8457\ 549$	$\cdot 8329\ 720$	$\cdot 8203\ 823$
$\cdot 98$	$\cdot 9223\ 682$	$\cdot 9130\ 979$	$\cdot 9039\ 208$	$\cdot 8948\ 359$	$\cdot 8858\ 424$	$\cdot 8760\ 392$
$\cdot 99$	$\cdot 9605\ 960$	$\cdot 9557\ 810$	$\cdot 9509\ 900$	$\cdot 9462\ 232$	$\cdot 9414\ 801$	$\cdot 9367\ 609$
$\cdot 100$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .10$  to  $.70$  $q = 1$  $p = 7$  to  $9.5$ 

	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$
$B(p, q) = .1428\ 5714$		.1333 3333	.1250 0000	.1176 4706	.1111 1111	.1052 6316
$\cdot 10$	.0000 001 <sup>e</sup>					
$\cdot 11$	.0000 002	.0000 001				
$\cdot 12$	.0000 004	.0000 001				
$\cdot 13$	.0000 006	.0000 002	.0000 001			
$\cdot 14$	.0000 011	.0000 004	.0000 001	.0000 001		
$\cdot 15$	.0000 017	.0000 007	.0000 003	.0000 001		
$\cdot 16$	.0000 027	.0000 011	.0000 004	.0000 002	.0000 001	
$\cdot 17$	.0000 041	.0000 017	.0000 007	.0000 003	.0000 001	
$\cdot 18$	.0000 061	.0000 026	.0000 011	.0000 005 <sup>-</sup>	.0000 002	.0000 001
$\cdot 19$	.0000 089	.0000 039	.0000 017	.0000 007	.0000 003	.0000 001
$\cdot 20$	.0000 128 <sup>e</sup>	.0000 057	.0000 026	.0000 011	.0000 005 <sup>+</sup>	.0000 002
$\cdot 21$	.0000 180	.0000 083	.0000 038	.0000 017	.0000 008	.0000 004
$\cdot 22$	.0000 249	.0000 117	.0000 055 <sup>-</sup>	.0000 026	.0000 012	.0000 006
$\cdot 23$	.0000 340	.0000 163	.0000 078	.0000 038	.0000 018	.0000 009
$\cdot 24$	.0000 459	.0000 225 <sup>-</sup>	.0000 110	.0000 054	.0000 026	.0000 013
$\cdot 25$	.0000 610	.0000 305 <sup>+</sup>	.0000 153	.0000 076	.0000 038	.0000 019
$\cdot 26$	.0000 803	.0000 410	.0000 209	.0000 106	.0000 054	.0000 028
$\cdot 27$	.0001 046	.0000 544	.0000 282	.0000 147	.0000 076	.0000 040
$\cdot 28$	.0001 349	.0000 714	.0000 378	.0000 200	.0000 106	.0000 056
$\cdot 29$	.0001 725 <sup>-</sup>	.0000 929	.0000 500 <sup>+</sup>	.0000 269	.0000 145 <sup>+</sup>	.0000 078
$\cdot 30$	.0002 187 <sup>e</sup>	.0001 198	.0000 656	.0000 359	.0000 197	.0000 108
$\cdot 31$	.0002 751	.0001 532	.0000 853	.0000 475 <sup>-</sup>	.0000 264	.0000 147
$\cdot 32$	.0003 436	.0001 944	.0001 100	.0000 622	.0000 352	.0000 199
$\cdot 33$	.0004 262	.0002 448	.0001 406	.0000 808	.0000 464	.0000 267
$\cdot 34$	.0005 252	.0003 063	.0001 786	.0001 041	.0000 607	.0000 354
$\cdot 35$	.0006 434	.0003 806	.0002 252	.0001 332	.0000 788	.0000 466
$\cdot 36$	.0007 836	.0004 702	.0002 821	.0001 693	.0001 016	.0000 609
$\cdot 37$	.0009 493	.0005 774	.0003 512	.0002 137	.0001 300	.0000 791
$\cdot 38$	.0011 442	.0007 053	.0004 348	.0002 680	.0001 652	.0001 018
$\cdot 39$	.0013 723	.0008 570	.0005 352	.0003 342	.0002 087	.0001 364
$\cdot 40$	.0016 384 <sup>e</sup>	.0010 362	.0006 554	.0004 145 <sup>-</sup>	.0002 621	.0001 658
$\cdot 41$	.0019 475 <sup>+</sup>	.0012 470	.0007 985 <sup>-</sup>	.0005 113	.0003 274	.0002 096
$\cdot 42$	.0023 054	.0014 941	.0009 683	.0006 275 <sup>+</sup>	.0004 067	.0002 636
$\cdot 43$	.0027 182	.0017 824	.0011 688	.0007 664	.0005 026	.0003 296
$\cdot 44$	.0031 928	.0021 178	.0014 048	.0009 319	.0006 181	.0004 100
$\cdot 45$	.0037 367	.0025 067	.0016 815 <sup>+</sup>	.0011 280	.0007 567	.0005 076
$\cdot 46$	.0043 582	.0029 559	.0020 048	.0013 597	.0009 222	.0006 255
$\cdot 47$	.0050 662	.0034 732	.0023 811	.0016 324	.0011 191	.0007 672
$\cdot 48$	.0058 707	.0040 673	.0028 179	.0019 523	.0013 526	.0009 371
$\cdot 49$	.0067 822	.0047 476	.0033 233	.0023 263	.0016 284	.0011 399
$\cdot 50$	.0078 125 <sup>e</sup>	.0055 243	.0039 062	.0027 621	.0019 531	.0013 811
$\cdot 51$	.0089 741	.0064 088	.0045 768	.0032 685 <sup>-</sup>	.0023 342	.0016 606
$\cdot 52$	.0102 807	.0074 135 <sup>+</sup>	.0053 460	.0038 550 <sup>+</sup>	.0027 799	.0020 646
$\cdot 53$	.0117 471	.0085 520	.0062 260	.0045 326	.0032 998	.0024 023
$\cdot 54$	.0133 893	.0098 391	.0072 302	.0053 131	.0039 043	.0028 601
$\cdot 55$	.0152 244	.0112 907	.0083 734	.0062 099	.0046 054	.0034 151
$\cdot 56$	.0172 709	.0129 244	.0096 717	.0072 377	.0054 162	.0040 531
$\cdot 57$	.0195 490	.0147 592	.0111 429	.0084 127	.0063 515 <sup>-</sup>	.0047 052
$\cdot 58$	.0220 798	.0168 155 <sup>+</sup>	.0128 063	.0097 530	.0074 277	.0056 567
$\cdot 59$	.0248 865 <sup>+</sup>	.0191 157	.0146 830	.0112 783	.0086 630	.0066 542
$\cdot 60$	.0279 936 <sup>e</sup>	.0216 837	.0167 962	.0130 102	.0100 777	.0078 061
$\cdot 61$	.0314 274	.0245 456	.0191 707	.0149 728	.0116 941	.0091 334
$\cdot 62$	.0352 161	.0277 292	.0218 340	.0171 921	.0135 371	.0106 591
$\cdot 63$	.0393 898	.0312 647	.0248 156	.0196 968	.0156 338	.0124 090
$\cdot 64$	.0439 805 <sup>-</sup>	.0351 844	.0281 475 <sup>-</sup>	.0225 180	.0180 144	.0144 115 <sup>+</sup>
$\cdot 65$	.0490 223	.0395 230	.0318 645 <sup>-</sup>	.0256 900	.0207 119	.0166 985 <sup>-</sup>
$\cdot 66$	.0545 516	.0443 179	.0360 041	.0292 498	.0237 627	.0193 049
$\cdot 67$	.0606 071	.0496 091	.0406 068	.0332 381	.0272 065 <sup>+</sup>	.0222 695 <sup>+</sup>
$\cdot 68$	.0672 299	.0554 392	.0457 163	.0376 986	.0310 871	.0256 351
$\cdot 69$	.0744 635 <sup>+</sup>	.0618 541	.0513 798	.0426 793	.0354 521	.0294 487
$\cdot 70$	.0823 543 <sup>e</sup>	.0689 026	.0576 480	.0482 318	.0403 536	.0337 622

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .71$  to  $1.00$  $q = 1$  $p = 7$  to  $9.5$ 

	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$
$I(p, q)$	.1428 5714	.1333 3333	.1250 0000	.1176 4706	.1111 1111	.1052 6316
$x$						
.71	.0000 512	.0766 368	.0645 754	.0544 122	.0458 485 <sup>+</sup>	.0386 326
.72	.1003 061	.0851 126	.0722 204	.0612 811	.0510 987	.0441 224
.73	.1104 740	.0943 800	.0806 460	.0680 040	.0588 716	.0502 999
.74	.1215 128	.1045 293	.0899 195 <sup>-</sup>	.0773 517	.0665 404	.0572 402
.75	.1334 830	.1156 004	.1001 120	.0867 003	.0750 847	.0650 252
.76	.1464 519	.1276 738	.1113 035 <sup>-</sup>	.0970 321	.0845 906	.0737 444
.77	.1604 852	.1408 252	.1235 736	.1084 354	.0951 517	.0834 953
.78	.1756 557	.1551 349	.1370 114	.1210 052	.1068 680	.0943 841
.79	.1920 301	.1706 881	.1517 109	.1348 436	.1198 516	.1065 264
.80	.2097 152 <sup>+</sup>	.1875 750 <sup>-</sup>	.1677 722	.1500 600	.1342 177	.1200 480
.81	.2287 679	.2058 911	.1853 020	.1667 718	.1500 946	.1350 852
.82	.2492 855 <sup>-</sup>	.2257 376	.2044 141	.1851 048	.1676 196	.1517 860
.83	.2713 605 <sup>+</sup>	.2472 212	.2252 202	.2051 936	.1860 403	.1703 107
.84	.2950 903	.2704 548	.2478 759	.2271 820	.2082 157	.1908 329
.85	.3205 771	.2955 575 <sup>-</sup>	.2724 905 <sup>+</sup>	.2512 239	.2316 169	.2135 403
.86	.3479 278	.3226 550 <sup>-</sup>	.2992 179	.2774 833	.2573 274	.2380 350
.87	.3772 548	.3518 798	.3282 117	.3061 355 <sup>-</sup>	.2855 442	.2663 379
.88	.4086 756	.3833 717	.3596 345 <sup>+</sup>	.3373 671	.3164 784	.2968 830
.89	.4423 133	.4172 776	.3936 580	.3713 770	.3503 564	.3305 256
.90	.4782 969 <sup>+</sup>	.4537 523	.4304 672	.4083 771	.3874 205 <sup>-</sup>	.3675 393
.91	.5167 610	.4920 586	.4702 525 <sup>+</sup>	.4485 923	.4279 298	.4082 190
.92	.5578 466	.5350 677	.5132 180	.4922 623	.4721 614	.4528 813
.93	.6017 000	.5802 593	.5595 818	.5396 412	.5204 111	.5018 663
.94	.6484 770	.6287 224	.6095 680	.5909 990	.5729 948	.5555 391
.95	.6983 373	.6806 550 <sup>+</sup>	.6634 204	.6466 223	.6302 494	.6142 911
.96	.7514 475 <sup>-</sup>	.7362 652	.7213 896	.7068 145 <sup>+</sup>	.6925 340	.6785 420
.97	.8070 828	.7957 708	.7837 434	.7718 977	.7602 311	.7487 408
.98	.8661 255 <sup>+</sup>	.8594 004	.8507 630	.8422 124	.8337 478	.8253 682
.99	.9320 653	.9273 933	.9227 447	.9181 104	.9135 172	.9089 382
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .19$  to  $.80$  $q = 1$  $p = 10$  to  $14$ 

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$B(p, q) = .1000\ 0000$		$.9523\ 8095 \times \frac{1}{10}$	$.9090\ 9091 \times \frac{1}{10}$	$.8363\ 6364 \times \frac{1}{10}$	$.7692\ 3077 \times \frac{1}{10}$	$.7142\ 8571 \times \frac{1}{10}$
$x$						
.19	.0000 001					
.20	.0000 001					
.21	.0000 002	.0000 001				
.22	.0000 003	.0000 001	.0000 001			
.23	.0000 004	.0000 002	.0000 001			
.24	.0000 006	.0000 003	.0000 002			
.25	.0000 010	.0000 005	.0000 002	.0000 001		
.26	.0000 014	.0000 007	.0000 004	.0000 001		
.27	.0000 021	.0000 011	.0000 006	.0000 002		
.28	.0000 030	.0000 016	.0000 008	.0000 002	.0000 001	
.29	.0000 042	.0000 023	.0000 012	.0000 004	.0000 001	
.30	.0000 059	.0000 032	.0000 018	.0000 005 <sup>+</sup>	.0000 002	
.31	.0000 082	.0000 046	.0000 025 <sup>+</sup>	.0000 008	.0000 002	.0000 001
.32	.0000 113	.0000 064	.0000 036	.0000 012	.0000 004	.0000 001
.33	.0000 153	.0000 088	.0000 051	.0000 017	.0000 006	.0000 002
.34	.0000 206	.0000 120	.0000 070	.0000 024	.0000 008	.0000 003
.35	.0000 276	.0000 163	.0000 097	.0000 034	.0000 012	.0000 004
.36	.0000 366	.0000 219	.0000 132	.0000 047	.0000 017	.0000 006
.37	.0000 481	.0000 292	.0000 178	.0000 066	.0000 024	.0000 009
.38	.0000 628	.0000 387	.0000 239	.0000 091	.0000 034	.0000 013
.39	.0000 814	.0000 508	.0000 317	.0000 124	.0000 048	.0000 019
.40	.0001 049	.0000 663	.0000 419	.0000 168	.0000 067	.0000 027
.41	.0001 342	.0000 859	.0000 550 <sup>+</sup>	.0000 226	.0000 093	.0000 038
.42	.0001 708	.0001 107	.0000 717	.0000 301	.0000 127	.0000 053
.43	.0002 161	.0001 417	.0000 929	.0000 400	.0000 172	.0000 074
.44	.0002 720	.0001 804	.0001 197	.0000 527	.0000 232	.0000 102
.45	.0003 405 <sup>+</sup>	.0002 284	.0001 532	.0000 690	.0000 310	.0000 140
.46	.0004 242	.0002 877	.0001 951	.0000 898	.0000 413	.0000 190
.47	.0005 260	.0003 606	.0002 472	.0001 162	.0000 546	.0000 257
.48	.0006 493	.0004 498	.0003 116	.0001 496	.0000 718	.0000 345
.49	.0007 979	.0005 585 <sup>+</sup>	.0003 910	.0001 916	.0000 939	.0000 460
.50	.0009 766	.0006 905 <sup>+</sup>	.0004 883	.0002 441	.0001 221	.0000 610
.51	.0011 904	.0008 501	.0006 071	.0003 096	.0001 579	.0000 805 <sup>+</sup>
.52	.0014 456	.0010 424	.0007 517	.0003 909	.0002 033	.0001 057
.53	.0017 489	.0012 732	.0009 269	.0004 913	.0002 604	.0001 380
.54	.0021 083	.0015 493	.0011 385 <sup>-</sup>	.0006 148	.0003 320	.0001 703
.55	.0025 330	.0018 785 <sup>-</sup>	.0013 931	.0007 662	.0004 214	.0002 318
.56	.0030 331	.0022 697	.0016 985 <sup>+</sup>	.0009 512	.0005 327	.0002 983
.57	.0036 203	.0027 333	.0020 636	.0011 762	.0006 705 <sup>-</sup>	.0003 822
.58	.0043 080	.0032 809	.0024 987	.0014 492	.0008 406	.0004 875 <sup>+</sup>
.59	.0051 112	.0039 260	.0030 156	.0017 792	.0010 497	.0006 193
.60	.0060 466	.0046 837	.0036 280	.0021 768	.0013 061	.0007 836
.61	.0071 334	.0055 714	.0043 514	.0026 543	.0016 192	.0009 877
.62	.0083 930	.0066 086	.0052 037	.0032 203	.0020 003	.0012 402
.63	.0098 493	.0078 176	.0062 051	.0039 092	.0024 628	.0015 510
.64	.0115 292	.0092 234	.0073 787	.0047 224	.0030 223	.0019 343
.65	.0134 627	.0108 540	.0087 508	.0056 880	.0036 972	.0024 032
.66	.0156 834	.0127 412	.0103 510	.0068 317	.0045 089	.0029 759
.67	.0182 284	.0149 206	.0122 130	.0081 827	.0054 824	.0036 732
.68	.0211 392	.0174 319	.0143 747	.0097 748	.0066 468	.0045 199
.69	.0244 619	.0203 196	.0168 787	.0116 403	.0080 360	.0055 448
.70	.0282 475 <sup>+</sup>	.0236 336	.0197 733	.0138 413	.0096 889	.0067 822
.71	.0325 524	.0274 292	.0231 122	.0164 097	.0116 509	.0082 721
.72	.0374 391	.0317 681	.0269 561	.0194 084	.0139 741	.0100 613
.73	.0429 763	.0367 189	.0313 727	.0229 020	.0167 185 <sup>-</sup>	.0122 045 <sup>+</sup>
.74	.0492 399	.0423 578	.0364 375 <sup>+</sup>	.0269 638	.0199 532	.0147 654
.75	.0563 135 <sup>+</sup>	.0487 680	.0422 351	.0316 764	.0237 573	.0178 179
.76	.0642 889	.0560 458	.0488 596	.0371 333	.0282 213	.0214 482
.77	.0732 668	.0642 914	.0564 154	.0434 399	.0334 487	.0257 555 <sup>+</sup>
.78	.0833 578	.0736 196	.0650 191	.0507 149	.0395 576	.0308 549
.79	.0946 828	.0841 559	.0747 994	.0590 915 <sup>+</sup>	.0466 823	.0368 790
.80	.1073 742	.0960 384	.0858 993	.0687 195 <sup>-</sup>	.0549 756	.0439 805 <sup>-</sup>

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION

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 $x = .81$  to  $1.00$  $q = 1$  $p = 10$  to  $14$ 

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$B(p, q)$	1.0000 0000	.9523 8005 $\times \frac{1}{10}$	.9090 9091 $\times \frac{1}{10}$	.8363 6364 $\times \frac{1}{10}$	.7692 3077 $\times \frac{1}{10}$	.7142 8571 $\times \frac{1}{10}$
$x$						
.81	.1215 767	.1094 190	.0984 771	.0797 664	.0646 108	.0523 348
.82	.1374 480	.1244 645 <sup>-</sup>	.1127 074	.0924 201	.0757 844	.0621 432
.83	.1551 604	.1413 579	.1287 831	.1068 900	.0887 187	.0736 365 <sup>+</sup>
.84	.1749 012	.1602 906	.1460 170	.1234 103	.1036 647	.0870 783
.85	.1968 744	.1815 002	.1673 432	.1422 418	.1209 055 <sup>-</sup>	.1027 697
.86	.2213 016	.2052 266	.1903 194	.1636 746	.1407 602	.1210 538
.87	.2484 234	.2317 139	.2161 284	.1880 317	.1635 876	.1423 212
.88	.2785 010	.2612 571	.2450 809	.2156 712	.1897 906	.1670 157
.89	.3118 172	.2941 078	.2775 173	.2460 904	.2198 215 <sup>-</sup>	.1956 411
.90	.3486 784	.3307 854	.3138 106	.2824 295 <sup>+</sup>	.2541 866	.2287 679
.91	.3894 161	.3714 793	.3543 687	.3224 755 <sup>-</sup>	.2934 527	.2670 420
.92	.4343 885 <sup>-</sup>	.4166 508	.3996 374	.3676 664	.3382 531	.3111 928
.93	.4839 823	.4667 356	.4501 035 <sup>+</sup>	.4185 963	.3892 946	.3620 439
.94	.5386 151	.5222 067	.5062 982	.4750 203	.4473 651	.4205 232
.95	.5987 369	.5835 766	.5688 001	.5403 601	.5133 421	.4876 750 <sup>-</sup>
.96	.6648 326	.6514 003	.6382 393	.6127 098	.5882 014	.5646 733
.97	.7374 241	.7262 785 <sup>+</sup>	.7153 014	.6938 424	.6730 271	.6528 363
.98	.8170 728	.8088 608	.8007 314	.7847 167	.7690 224	.7536 419
.99	.9043 821	.8998 488	.8953 383	.8863 849	.8775 210	.8687 458
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .33$  to  $1.00$  $q = 1$  $p = 15$  to  $20$ 

	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$
$B(p, q) = .6666\ 6667 \times \frac{x}{10}$	$.6250\ 0000 \times \frac{x}{10}$	$.5882\ 3529 \times \frac{x}{10}$	$.5555\ 5556 \times \frac{x}{10}$	$.5263\ 1579 \times \frac{x}{10}$	$.5000\ 0000 \times \frac{x}{10}$	
$x$						
.33	.0000 001					
.34	.0000 001					
.35	.0000 001	.0000 001				
.36	.0000 002	.0000 001				
.37	.0000 003	.0000 001				
.38	.0000 005	.0000 002	.0000 001			
.39	.0000 007	.0000 003	.0000 001			
.40	.0000 011	.0000 004	.0000 002	.0000 001		
.41	.0000 016	.0000 006	.0000 003	.0000 001		
.42	.0000 022	.0000 009	.0000 004	.0000 002	.0000 001	
.43	.0000 032	.0000 014	.0000 006	.0000 003	.0000 001	
.44	.0000 045	.0000 020	.0000 009	.0000 004	.0000 002	.0000 001
.45	.0000 063	.0000 028	.0000 013	.0000 006	.0000 003	.0000 001
.46	.0000 087	.0000 040	.0000 018	.0000 009	.0000 004	.0000 002
.47	.0000 121	.0000 057	.0000 027	.0000 013	.0000 006	.0000 003
.48	.0000 165 <sup>+</sup>	.0000 079	.0000 038	.0000 018	.0000 009	.0000 004
.49	.0000 225 <sup>+</sup>	.0000 110	.0000 054	.0000 027	.0000 013	.0000 006
.50	.0000 305 <sup>+</sup>	.0000 153	.0000 076	.0000 038	.0000 019	.0000 010
.51	.0000 411	.0000 209	.0000 107	.0000 054	.0000 028	.0000 014
.52	.0000 550	.0000 286	.0000 149	.0000 077	.0000 040	.0000 021
.53	.0000 731	.0000 388	.0000 205 <sup>+</sup>	.0000 109	.0000 058	.0000 031
.54	.0000 968	.0000 523	.0000 282	.0000 152	.0000 082	.0000 044
.55	.0001 275	.0000 701	.0000 386	.0000 212	.0000 117	.0000 064
.56	.0001 670	.0000 935 <sup>+</sup>	.0000 524	.0000 293	.0000 164	.0000 092
.57	.0002 178	.0001 242	.0000 708	.0000 403	.0000 230	.0000 131
.58	.0002 828	.0001 640	.0000 951	.0000 552	.0000 320	.0000 186
.59	.0003 654	.0002 156	.0001 272	.0000 750 <sup>+</sup>	.0000 443	.0000 261
.60	.0004 702	.0002 821	.0001 693	.0001 016	.0000 609	.0000 366
.61	.0006 025	.0003 675 <sup>+</sup>	.0002 242	.0001 368	.0000 834	.0000 509
.62	.0007 689	.0004 767	.0002 956	.0001 833	.0001 136	.0000 704
.63	.0009 775	.0006 158	.0003 880	.0002 444	.0001 540	.0000 970
.64	.0012 379	.0007 923	.0005 071	.0003 245 <sup>+</sup>	.0002 077	.0001 329
.65	.0015 621	.0010 153	.0006 600	.0004 290	.0002 788	.0001 812
.66	.0019 641	.0012 963	.0008 556	.0005 647	.0003 727	.0002 400
.67	.0024 611	.0016 489	.0011 048	.0007 402	.0004 959	.0003 323
.68	.0030 735 <sup>+</sup>	.0020 900	.0014 212	.0009 664	.0006 572	.0004 409
.69	.0038 259	.0026 399	.0018 215 <sup>+</sup>	.0012 569	.0008 672	.0005 984
.70	.0047 476	.0033 233	.0023 263	.0016 284	.0011 399	.0007 979
.71	.0058 732	.0041 700	.0029 607	.0021 021	.0014 925	.0010 597
.72	.0072 442	.0052 158	.0037 554	.0027 039	.0019 468	.0014 017
.73	.0089 093	.0065 038	.0047 478	.0034 659	.0025 301	.0018 470
.74	.0109 264	.0080 855 <sup>+</sup>	.0059 833	.0044 276	.0032 764	.0024 246
.75	.0133 635	.0100 226	.0075 169	.0056 377	.0042 283	.0031 712
.76	.0163 006	.0123 885	.0094 152	.0071 556	.0054 382	.0041 331
.77	.0198 317	.0152 704	.0117 582	.0090 538	.0069 715	.0053 680
.78	.0240 668	.0187 721	.0146 423	.0114 210	.0089 084	.0069 485 <sup>+</sup>
.79	.0291 344	.0230 162	.0181 828	.0143 644	.0113 479	.0089 648
.80	.0351 844	.0281 475	.0225 180	.0180 144	.0144 115 <sup>+</sup>	.0115 292
.81	.0423 912	.0343 368	.0278 128	.0225 284	.0182 480	.0147 809
.82	.0509 575	.0417 851	.0342 638	.0280 963	.0230 390	.0188 920
.83	.0611 183	.0507 282	.0421 044	.0349 467	.0290 057	.0240 748
.84	.0731 458	.0614 425	.0516 117	.0433 538	.0364 172	.0305 904
.85	.0873 542	.0742 511	.0631 134	.0536 464	.0455 994	.0387 595 <sup>+</sup>
.86	.1041 062	.0895 314	.0769 970	.0662 174	.0569 470	.0489 744
.87	.1238 194	.1077 229	.0937 189	.0815 355	.0709 359	.0617 142
.88	.1469 739	.1293 370	.1138 166	.1001 586	.0881 395 <sup>+</sup>	.0775 628
.89	.1741 206	.1549 673	.1379 209	.1227 496	.1092 472	.0972 300
.90	.2058 911	.1853 020	.1667 718	.1500 946	.1350 852	.1215 767
.91	.2430 082	.2211 374	.2012 351	.1831 239	.1666 428	.1516 449
.92	.2862 974	.2633 936	.2423 221	.2229 304	.2051 014	.1886 933
.93	.3367 009	.3131 318	.2912 126	.2708 277	.2518 608	.2342 389
.94	.3952 918	.3715 743	.3492 798	.3283 230	.3086 237	.2901 062
.95	.4632 912	.4401 267	.4181 203	.3972 143	.3773 536	.3584 859
.96	.5420 864	.5204 029	.4995 868	.4796 033	.4604 192	.4420 024
.97	.6332 512	.6142 537	.5958 260	.5779 513	.5606 127	.5437 943
.98	.7385 691	.7237 977	.7093 218	.6951 353	.6812 326	.6676 080
.99	.8600 584	.8514 578	.8429 432	.8345 138	.8261 686	.8179 069
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .45$  to  $1.00$  $q = 1$  $p = 21$  to  $26$ 

	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$I(p, q) = .4761\ 9048 \times \frac{x}{10}$	$.4545\ 4545 \times \frac{x}{10}$	$.4347\ 8261 \times \frac{x}{10}$	$.4166\ 6667 \times \frac{x}{10}$	$.4000\ 0000 \times \frac{x}{10}$	$.3846\ 1538 \times \frac{x}{10}$	
.45	.0000 001					
.46	.0000 001					
.47	.0000 001	.0000 001				
.48	.0000 002	.0000 001				
.49	.0000 003	.0000 002	.0000 001			
.50	.0000 005 <sup>-</sup>	.0000 002	.0000 001	.0000 001		
.51	.0000 007	.0000 004	.0000 002	.0000 001		
.52	.0000 011	.0000 006	.0000 003	.0000 002	.0000 001	
.53	.0000 016	.0000 009	.0000 005 <sup>-</sup>	.0000 002	.0000 001	.0000 001
.54	.0000 024	.0000 013	.0000 007	.0000 004	.0000 002	.0000 001
.55	.0000 035 <sup>+</sup>	.0000 019	.0000 011	.0000 006	.0000 003	.0000 002
.56	.0000 052	.0000 029	.0000 016	.0000 009	.0000 005 <sup>+</sup>	.0000 003
.57	.0000 075 <sup>-</sup>	.0000 043	.0000 024	.0000 014	.0000 008	.0000 004
.58	.0000 108	.0000 062	.0000 036	.0000 021	.0000 012	.0000 007
.59	.0000 154	.0000 091	.0000 054	.0000 032	.0000 019	.0000 011
.60	.0000 219	.0000 132	.0000 079	.0000 047	.0000 028	.0000 017
.61	.0000 310	.0000 189	.0000 116	.0000 070	.0000 043	.0000 026
.62	.0000 437	.0000 271	.0000 168	.0000 104	.0000 065 <sup>-</sup>	.0000 040
.63	.0000 611	.0000 385 <sup>+</sup>	.0000 243	.0000 153	.0000 096	.0000 061
.64	.0000 851	.0000 544	.0000 348	.0000 223	.0000 143	.0000 091
.65	.0001 178	.0000 766	.0000 498	.0000 324	.0000 210	.0000 137
.66	.0001 623	.0001 071	.0000 707	.0000 467	.0000 308	.0000 203
.67	.0002 226	.0001 492	.0000 999	.0000 670	.0000 449	.0000 301
.68	.0003 039	.0002 066	.0001 405 <sup>+</sup>	.0000 955 <sup>+</sup>	.0000 650 <sup>-</sup>	.0000 442
.69	.0004 129	.0002 849	.0001 966	.0001 356	.0000 936	.0000 646
.70	.0005 585 <sup>+</sup>	.0003 910	.0002 737	.0001 916	.0001 341	.0000 939
.71	.0007 524	.0005 342	.0003 793	.0002 693	.0001 912	.0001 357
.72	.0010 092	.0007 266	.0005 232	.0003 767	.0002 712	.0001 953
.73	.0013 483	.0009 842	.0007 185 <sup>-</sup>	.0005 245 <sup>+</sup>	.0003 829	.0002 795 <sup>+</sup>
.74	.0017 942	.0013 277	.0009 825 <sup>-</sup>	.0007 270	.0005 380	.0003 981
.75	.0023 784	.0017 838	.0013 379	.0010 034	.0007 525 <sup>+</sup>	.0005 644
.76	.0031 411	.0023 873	.0018 143	.0013 789	.0010 479	.0007 964
.77	.0041 334	.0031 827	.0024 507	.0018 870	.0014 530	.0011 188
.78	.0054 198	.0042 275 <sup>-</sup>	.0032 974	.0025 720	.0020 062	.0015 648
.79	.0070 822	.0055 949	.0044 200	.0034 918	.0027 585 <sup>+</sup>	.0021 792
.80	.0092 234	.0073 787	.0059 030	.0047 224	.0037 779	.0030 223
.81	.0119 725 <sup>+</sup>	.0096 977	.0078 552	.0063 627	.0051 538	.0041 746
.82	.0154 914	.0127 030	.0104 164	.0085 415 <sup>-</sup>	.0070 040	.0057 433
.83	.0199 820	.0165 851	.0137 656	.0114 255 <sup>-</sup>	.0094 831	.0078 710
.84	.0256 960	.0215 846	.0181 311	.0152 301	.0127 933	.0107 464
.85	.0329 456	.0280 038	.0238 032	.0202 327	.0171 978	.0146 181
.86	.0421 180	.0362 215 <sup>-</sup>	.0311 505 <sup>-</sup>	.0267 894	.0230 389	.0198 134
.87	.0536 913	.0467 115 <sup>-</sup>	.0406 390	.0353 559	.0307 596	.0267 609
.88	.0682 553	.0600 646	.0528 569	.0465 140	.0409 324	.0360 205 <sup>-</sup>
.89	.0865 347	.0770 150	.0685 441	.0610 043	.0542 938	.0483 215 <sup>-</sup>
.90	.1094 190	.0984 771	.0886 294	.0797 664	.0717 898	.0646 108
.91	.1379 969	.1255 772	.1142 752	.1039 904	.0946 313	.0861 145 <sup>-</sup>
.92	.1735 979	.1597 100	.1469 332	.1351 786	.1243 643	.1144 151
.93	.2178 422	.2025 932	.1884 117	.1752 229	.1629 573	.1515 503
.94	.2726 999	.2563 379	.2409 576	.2265 001	.2129 101	.2001 355 <sup>+</sup>
.95	.3405 616	.3235 335 <sup>+</sup>	.3073 569	.2919 890	.2773 896	.2635 201
.96	.4243 223	.4073 494	.3910 555 <sup>-</sup>	.3754 132	.3603 967	.3459 808
.97	.5274 805 <sup>+</sup>	.5116 561	.4963 064	.4814 172	.4669 747	.4529 655 <sup>-</sup>
.98	.6542 558	.6411 707	.6283 473	.6157 803	.6034 647	.5913 954
.99	.8097 279	.8016 306	.7936 143	.7856 781	.7778 214	.7700 431
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .54$  to  $1.00$  $q = 1$  $p = 27$  to  $32$ 

	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$
$B(p, q) = .3703\ 7037 \times \frac{x}{10}$	$.3703\ 7037 \times \frac{x}{10}$	$.3571\ 4286 \times \frac{x}{10}$	$.3448\ 2759 \times \frac{x}{10}$	$.3333\ 3333 \times \frac{x}{10}$	$.3225\ 8065 \times \frac{x}{10}$	$.3125\ 0000 \times \frac{x}{10}$
$x$						
.54	.0000 001					
.55	.0000 001	.0000 001				
.56	.0000 002	.0000 001				
.57	.0000 003	.0000 001	.0000 001	.0000 001		
.58	.0000 004	.0000 002	.0000 001	.0000 001	.0000 001	
.59	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001	.0000 001
.60	.0000 010	.0000 006	.0000 004	.0000 002		
.61	.0000 016	.0000 010	.0000 006	.0000 004	.0000 002	.0000 001
.62	.0000 025 <sup>-</sup>	.0000 015 <sup>+</sup>	.0000 010	.0000 006	.0000 004	.0000 002
.63	.0000 038	.0000 024	.0000 015 <sup>+</sup>	.0000 010	.0000 006	.0000 004
.64	.0000 058	.0000 037	.0000 024	.0000 015 <sup>+</sup>	.0000 010	.0000 006
.65	.0000 089	.0000 058	.0000 038	.0000 024	.0000 016	.0000 010
.66	.0000 134	.0000 089	.0000 058	.0000 039	.0000 025 <sup>+</sup>	.0000 017
.67	.0000 201	.0000 135 <sup>-</sup>	.0000 090	.0000 061	.0000 041	.0000 027
.68	.0000 300	.0000 204	.0000 139	.0000 094	.0000 064	.0000 044
.69	.0000 446	.0000 307	.0000 212	.0000 146	.0000 101	.0000 070
.70	.0000 657	.0000 460	.0000 322	.0000 225 <sup>+</sup>	.0000 158	.0000 110
.71	.0000 964	.0000 684	.0000 486	.0000 345 <sup>-</sup>	.0000 245 <sup>-</sup>	.0000 174
.72	.0001 406	.0001 012	.0000 729	.0000 525 <sup>-</sup>	.0000 378	.0000 272
.73	.0002 040	.0001 489	.0001 087	.0000 794	.0000 579	.0000 424
.74	.0002 946	.0002 180	.0001 613	.0001 194	.0000 883	.0000 644
.75	.0004 233	.0003 175 <sup>-</sup>	.0002 381	.0001 786	.0001 339	.0001 095
.76	.0006 053	.0004 600	.0003 496	.0002 657	.0002 019	.0001 535
.77	.0008 615 <sup>-</sup>	.0006 633	.0005 108	.0003 933	.0003 028	.0002 442
.78	.0012 205 <sup>+</sup>	.0009 520	.0007 426	.0005 792	.0004 518	.0003 524
.79	.0017 216	.0013 601	.0010 744	.0008 488	.0006 706	.0005 297
.80	.0024 179	.0019 343	.0015 474	.0012 379	.0009 904	.0007 024
.81	.0033 814	.0027 389	.0022 185 <sup>+</sup>	.0017 970	.0014 556	.0011 790
.82	.0047 095 <sup>-</sup>	.0038 618	.0031 667	.0025 967	.0021 293	.0017 460
.83	.0065 329	.0054 223	.0045 005 <sup>+</sup>	.0037 354	.0031 004	.0025 744
.84	.0090 269	.0075 826	.0063 694	.0053 503	.0044 943	.0037 552
.85	.0124 254	.0105 616	.0089 774	.0076 308	.0064 861	.0054 142
.86	.0170 396	.0146 540	.0126 025 <sup>-</sup>	.0108 381	.0093 208	.0080 159
.87	.0232 820	.0202 553	.0176 221	.0153 313	.0133 382	.0116 042
.88	.0316 980	.0278 943	.0245 469	.0216 013	.0190 092	.0167 281
.89	.0430 061	.0382 754	.0340 651	.0303 180	.0269 830	.0240 149
.90	.0581 497	.0523 348	.0471 013	.0423 912	.0381 520	.0343 368
.91	.0783 642	.0713 114	.0648 934	.0590 530	.0537 382	.0489 018
.92	.1052 619	.0968 410	.0890 937	.0819 662	.0754 080	.0693 762
.93	.1409 417	.1310 758	.1219 005 <sup>+</sup>	.1133 675 <sup>-</sup>	.1054 317	.0980 515 <sup>+</sup>
.94	.1881 274	.1768 398	.1662 294	.1562 556	.1468 803	.1380 675
.95	.2503 441	.2378 269	.2259 355 <sup>+</sup>	.2146 388	.2039 068	.1937 115
.96	.3321 416	.3188 559	.3061 017	.2938 576	.2821 033	.2708 162
.97	.4393 765 <sup>+</sup>	.4261 952	.4134 093	.4010 071	.3889 769	.3773 076
.98	.5795 675 <sup>+</sup>	.5679 762	.5566 167	.5454 843	.5345 746	.5238 831
.99	.7623 427	.7547 193	.7471 721	.7397 004	.7323 034	.7249 804
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION

27

 $x = .61$  to  $1.00$  $q = 1$  $p = 33$  to  $38$ 

	$p = 33$	$p = 34$	$p = 35$	$p = 36$	$p = 37$	$p = 38$
$I(p, q) = .3030 \ 3030 \times \frac{x}{10}$	$.2941 \ 1765 \times \frac{x}{10}$	$.2857 \ 1429 \times \frac{x}{10}$	$.2777 \ 7778 \times \frac{x}{10}$	$.2702 \ 7027 \times \frac{x}{10}$	$.2631 \ 5789 \times \frac{x}{10}$	
.61	.0000 001	.0000 001				
.62	.0000 001	.0000 001	.0000 001			
.63	.0000 002	.0000 002	.0000 001	.0000 001		
.64	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	
.65	.0000 007	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001
.66	.0000 011	.0000 007	.0000 005 <sup>-</sup>	.0000 003	.0000 002	.0000 001
.67	.0000 018	.0000 012	.0000 008	.0000 005 <sup>+</sup>	.0000 004	.0000 002
.68	.0000 030	.0000 020	.0000 014	.0000 009	.0000 006	.0000 004
.69	.0000 048	.0000 033	.0000 023	.0000 016	.0000 011	.0000 008
.70	.0000 077	.0000 054	.0000 038	.0000 027	.0000 019	.0000 013
.71	.0000 123	.0000 088	.0000 062	.0000 044	.0000 031	.0000 022
.72	.0000 196	.0000 141	.0000 102	.0000 073	.0000 053	.0000 038
.73	.0000 309	.0000 225 <sup>+</sup>	.0000 165 <sup>-</sup>	.0000 120	.0000 088	.0000 064
.74	.0000 484	.0000 358	.0000 265 <sup>-</sup>	.0000 196	.0000 145 <sup>+</sup>	.0000 107
.75	.0000 753	.0000 565 <sup>+</sup>	.0000 424	.0000 318	.0000 238	.0000 179
.76	.0001 166	.0000 886	.0000 674	.0000 512	.0000 389	.0000 296
.77	.0001 796	.0001 383	.0001 065 <sup>-</sup>	.0000 820	.0000 631	.0000 486
.78	.0002 749	.0002 144	.0001 672	.0001 304	.0001 017	.0000 794
.79	.0004 185 <sup>-</sup>	.0003 306	.0002 612	.0002 063	.0001 630	.0001 288
.80	.0006 338	.0005 071	.0004 056	.0003 245 <sup>+</sup>	.0002 596	.0002 077
.81	.0009 550 <sup>+</sup>	.0007 736	.0006 266	.0005 075 <sup>+</sup>	.0004 111	.0003 330
.82	.0014 317	.0011 740	.0009 627	.0007 894	.0006 473	.0005 308
.83	.0021 359	.0017 728	.0014 714	.0012 213	.0010 137	.0008 413
.84	.0031 711	.0026 638	.0022 376	.0018 796	.0015 788	.0013 262
.85	.0046 862	.0039 833	.0033 858	.0028 779	.0024 462	.0020 793
.86	.0068 936	.0059 285 <sup>+</sup>	.0050 985 <sup>+</sup>	.0043 847	.0037 709	.0032 430
.87	.0100 957	.0087 832	.0076 414	.0066 480	.0057 838	.0050 319
.88	.0147 207	.0129 512	.0113 997	.0100 317	.0088 279	.0077 686
.89	.0213 732	.0190 222	.0169 207	.0150 675 <sup>-</sup>	.0134 100	.0119 349
.90	.0309 032	.0278 128	.0250 316	.0225 284	.0202 756	.0182 480
.91	.0445 006	.0404 956	.0368 510	.0335 344	.0305 163	.0277 608
.92	.0638 261	.0587 200	.0540 224	.0497 006	.0457 246	.0420 666
.93	.0911 879	.0848 048	.0788 684	.0733 476	.0682 133	.0634 384
.94	.1297 834	.1219 964	.1146 766	.1077 960	.1013 283	.0952 486
.95	.1840 259	.1748 246	.1660 834	.1577 792	.1498 903	.1423 957
.96	.2599 864	.2495 870	.2396 035 <sup>-</sup>	.2300 194	.2208 186	.2119 858
.97	.3659 883	.3550 087	.3443 584	.3340 277	.3240 068	.3142 866
.98	.5134 055 <sup>-</sup>	.5031 374	.4930 746	.4832 131	.4735 489	.4640 779
.99	.7177 305 <sup>+</sup>	.7105 532	.7034 477	.6964 132	.6894 491	.6825 546
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .65$  to  $1.00$  $q = 1$  $p = 39$  to  $44$ 

	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$
$B(p, q) = .2564\ 1026 \times \frac{x}{10}$	$.2500\ 0000 \times \frac{x}{10}$	$.2439\ 0244 \times \frac{x}{10}$	$.2380\ 9524 \times \frac{x}{10}$	$.2325\ 5814 \times \frac{x}{10}$	$.2272\ 7273 \times \frac{x}{10}$	
$x$						
.65	.0000 001					
.66	.0000 001	.0000 001				
.67	.0000 002	.0000 001	.0000 001			
.68	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001	
.69	.0000 005 <sup>+</sup>	.0000 004	.0000 002	.0000 002	.0000 001	.0000 001
.70	.0000 009	.0000 006	.0000 004	.0000 003	.0000 002	.0000 002
.71	.0000 016	.0000 011	.0000 008	.0000 006	.0000 004	.0000 003
.72	.0000 027	.0000 020	.0000 014	.0000 010	.0000 007	.0000 005 <sup>+</sup>
.73	.0000 047	.0000 034	.0000 025 <sup>-</sup>	.0000 018	.0000 013	.0000 010
.74	.0000 079	.0000 059	.0000 044	.0000 032	.0000 024	.0000 018
.75	.0000 134	.0000 101	.0000 075 <sup>+</sup>	.0000 057	.0000 042	.0000 032
.76	.0000 225 <sup>-</sup>	.0000 171	.0000 130	.0000 099	.0000 075 <sup>-</sup>	.0000 057
.77	.0000 374	.0000 288	.0000 222	.0000 171	.0000 132	.0000 101
.78	.0000 619	.0000 483	.0000 377	.0000 294	.0000 220	.0000 170
.79	.0001 017	.0000 804	.0000 635 <sup>-</sup>	.0000 502	.0000 396	.0000 313
.80	.0001 662	.0001 329	.0001 063	.0000 851	.0000 681	.0000 544
.81	.0002 697	.0002 185 <sup>-</sup>	.0001 770	.0001 433	.0001 161	.0000 940
.82	.0004 353	.0003 569	.0002 927	.0002 400	.0001 968	.0001 614
.83	.0006 983	.0005 796	.0004 811	.0003 993	.0003 314	.0002 751
.84	.0011 140	.0009 358	.0007 861	.0006 603	.0005 546	.0004 659
.85	.0017 674	.0015 023	.0012 770	.0010 854	.0009 226	.0007 842
.86	.0027 889	.0023 985 <sup>-</sup>	.0020 627	.0017 730	.0015 256	.0013 120
.87	.0043 777	.0038 086	.0033 135 <sup>+</sup>	.0028 828	.0025 080	.0021 820
.88	.0068 363	.0060 160	.0052 941	.0046 588	.0040 907	.0036 078
.89	.0106 221	.0094 537	.0084 138	.0074 882	.0066 645 <sup>+</sup>	.0059 314
.90	.0164 232	.0147 809	.0133 028	.0119 725 <sup>+</sup>	.0107 753	.0096 077
.91	.0252 705 <sup>+</sup>	.0229 962	.0209 265 <sup>+</sup>	.0190 431	.0173 203	.0157 606
.92	.0387 013	.0356 052	.0327 568	.0301 362	.0277 253	.0255 073
.93	.0589 977	.0548 679	.0510 271	.0474 552	.0441 333	.0410 440
.94	.0895 337	.0841 616	.0791 119	.0743 052	.0699 033	.0657 091
.95	.1352 760	.1285 122	.1220 865 <sup>+</sup>	.1159 822	.1101 831	.1046 740
.96	.2035 064	.1953 662	.1875 515 <sup>+</sup>	.1800 494	.1728 475 <sup>-</sup>	.1659 336
.97	.3048 580	.2957 123	.2868 409	.2782 357	.2698 886	.2617 920
.98	.4547 963	.4457 004	.4367 864	.4280 507	.4194 807	.4110 999
.99	.6757 290	.6689 718	.6622 820	.6556 592	.6491 026	.6426 116
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION $x = .69$  to  $1.00$  $q = 1$  $p = 45$  to  $50$ 

	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$B(p, q) = .2222\ 2222 \times \frac{x}{10}$		$.2173\ 9130 \times \frac{x}{10}$	$.2127\ 6596 \times \frac{x}{10}$	$.2083\ 3333 \times \frac{x}{10}$	$.2040\ 8163 \times \frac{x}{10}$	$.2000\ 0000 \times \frac{x}{10}$
$x$						
.69	.0000 001					
.70	.0000 001	.0000 001	.0000 001			
.71	.0000 002	.0000 001	.0000 001	.0000 001	.0000 001	
.72	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.73	.0000 007	.0000 005 <sup>+</sup>	.0000 004	.0000 003	.0000 002	.0000 001
.74	.0000 013	.0000 010	.0000 007	.0000 005 <sup>+</sup>	.0000 004	.0000 003
.75	.0000 024	.0000 018	.0000 013	.0000 010	.0000 008	.0000 006
.76	.0000 043	.0000 033	.0000 025 <sup>+</sup>	.0000 019	.0000 014	.0000 011
.77	.0000 078	.0000 060	.0000 046	.0000 036	.0000 027	.0000 021
.78	.0000 139	.0000 109	.0000 085 <sup>-</sup>	.0000 066	.0000 052	.0000 040
.79	.0000 247	.0000 195 <sup>+</sup>	.0000 154	.0000 122	.0000 096	.0000 076
.80	.0000 436	.0000 348	.0000 279	.0000 223	.0000 178	.0000 143
.81	.0000 762	.0000 617	.0000 500 <sup>-</sup>	.0000 405 <sup>-</sup>	.0000 328	.0000 266
.82	.0001 323	.0001 085 <sup>+</sup>	.0000 890	.0000 730	.0000 598	.0000 491
.83	.0002 283	.0001 895 <sup>-</sup>	.0001 573	.0001 305 <sup>+</sup>	.0001 083	.0000 899
.84	.0003 914	.0003 287	.0002 761	.0002 320	.0001 948	.0001 637
.85	.0006 666	.0005 666	.0004 816	.0004 094	.0003 480	.0002 958
.86	.0011 283	.0009 704	.0008 345 <sup>+</sup>	.0007 177	.0006 172	.0005 308
.87	.0018 983	.0016 515 <sup>+</sup>	.0014 368	.0012 500 <sup>+</sup>	.0010 875 <sup>+</sup>	.0009 462
.88	.0031 748	.0027 938	.0024 586	.0021 636	.0019 039	.0016 755 <sup>-</sup>
.89	.0052 790	.0046 983	.0041 815 <sup>-</sup>	.0037 215 <sup>+</sup>	.0033 122	.0029 478
.90	.0087 280	.0078 552	.0070 697	.0063 627	.0057 264	.0051 538
.91	.0143 504	.0130 588	.0118 835 <sup>+</sup>	.0108 140	.0098 408	.0089 551
.92	.0234 667	.0215 894	.0198 622	.0182 732	.0168 114	.0154 665 <sup>-</sup>
.93	.0381 709	.0354 900	.0330 140	.0307 031	.0285 538	.0265 551
.94	.0617 666	.0580 606	.0545 769	.0513 023	.0482 242	.0453 307
.95	.0994 403	.0944 682	.0897 448	.0852 576	.0809 947	.0769 450 <sup>-</sup>
.96	.1592 062	.1529 244	.1468 074	.1409 351	.1352 977	.1298 858
.97	.2530 382	.2463 201	.2389 305 <sup>-</sup>	.2317 625 <sup>+</sup>	.2248 097	.2180 654
.98	.4028 779	.3948 203	.3869 239	.3791 854	.3716 017	.3641 697
.99	.6361 855 <sup>-</sup>	.6298 236	.6235 254	.6172 901	.6111 172	.6050 061
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .01$  to  $.60$  $q = 1.5$  $p = 1.5$  to  $4$ 

	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$	$p = 4$
$B(p, q) = .3926\ 9908$		.2666 6667	.1963 4954	.1523 8095 <sup>+</sup>	.1227 1846	.1015 8730
$x$						
.01	.0016 926	.0001 869	.0000 203	.0000 022	.0000 002	
.02	.0047 728	.0007 450 <sup>-</sup>	.0001 144	.0000 174	.0000 026	.0000 004
.03	.0087 414	.0016 705 <sup>+</sup>	.0003 141	.0000 584	.0000 108	.0000 020
.04	.0134 171	.0029 597	.0006 425 <sup>+</sup>	.0001 379	.0000 203	.0000 062
.05	.0186 930	.0046 086	.0011 183	.0002 083	.0000 638	.0000 151
.06	.0244 963	.0066 134	.0017 575 <sup>-</sup>	.0004 617	.0001 203	.0000 311
.07	.0307 722	.0089 702	.0025 741	.0007 303	.0002 051	.0000 571
.08	.0374 780	.0116 750 <sup>+</sup>	.0035 806	.0010 858	.0003 205 <sup>-</sup>	.0000 975 <sup>+</sup>
.09	.0445 784	.0147 239	.0047 883	.0015 399	.0004 910	.0001 555 <sup>+</sup>
.10	.0520 440	.0181 128	.0062 074	.0021 038	.0007 070	.0002 300
.11	.0598 494	.0218 377	.0078 471	.0027 887	.0009 828	.0003 441
.12	.0679 724	.0258 945 <sup>-</sup>	.0097 159	.0036 056	.0013 269	.0004 852
.13	.0763 934	.0302 790	.0118 216	.0045 652	.0017 484	.0006 653
.14	.0850 940	.0349 873	.0141 714	.0056 780	.0022 503	.0008 908
.15	.0940 602	.0400 149	.0167 718	.0069 542	.0028 509	.0011 686
.16	.1032 755 <sup>-</sup>	.0453 578	.0196 290	.0084 039	.0035 688	.0015 059
.17	.1127 270	.0510 117	.0227 484	.0100 369	.0043 927	.0019 103
.18	.1224 023	.0569 722	.0261 351	.0118 028	.0053 414	.0023 808
.19	.1322 897	.0632 350 <sup>o</sup>	.0297 938	.0138 908	.0064 247	.0029 570
.20	.1423 785 <sup>-</sup>	.0697 957	.0337 287	.0161 301	.0076 528	.0036 681
.21	.1526 583	.0766 499	.0379 437	.0185 895 <sup>-</sup>	.0090 357	.0043 646
.22	.1631 194	.0837 931	.0424 423	.0212 775 <sup>+</sup>	.0105 836	.0052 318
.23	.1737 527	.0912 208	.0472 276	.0242 026	.0123 066	.0062 103
.24	.1845 494	.0989 284	.0523 023	.0273 727	.0142 151	.0073 371
.25	.1955 011	.1069 113	.0576 689	.0307 958	.0163 192	.0085 053
.26	.2065 999	.1151 648	.0633 295 <sup>+</sup>	.0344 793	.0186 291	.0100 047
.27	.2178 381	.1236 843	.0692 860	.0384 306	.0211 551	.0115 757
.28	.2292 081	.1324 648	.0755 397	.0426 566	.0230 071	.0133 103
.29	.2407 030	.1415 017	.0820 920	.0471 641	.0268 954	.0152 460
.30	.2523 158	.1507 901	.0889 437	.0519 596	.0301 298	.0173 680
.31	.2640 397	.1603 249	.0960 955 <sup>-</sup>	.0570 402	.0336 202	.0196 078
.32	.2758 682	.1701 013	.1035 476	.0624 388	.0373 765 <sup>+</sup>	.0222 148
.33	.2877 950 <sup>+</sup>	.1801 141	.1113 002	.0681 339	.0414 083	.0250 215 <sup>+</sup>
.34	.2998 139	.1903 583	.1193 530	.0741 399	.0457 250	.0280 300
.35	.3119 188	.2008 287	.1277 955 <sup>+</sup>	.0804 617	.0503 360	.0313 110
.36	.3241 038	.2115 200 <sup>o</sup>	.1363 570	.0871 040 <sup>o</sup>	.0552 504	.0348 493
.37	.3363 631	.2224 269	.1453 064	.0940 711	.0604 772	.0386 642
.38	.3486 910	.2335 441	.1545 524	.1013 670	.0660 252	.0427 686
.39	.3610 818	.2448 060	.1640 934	.1089 955 <sup>-</sup>	.0719 020	.0471 744
.40	.3735 300	.2563 872	.1739 277	.1169 598	.0781 185 <sup>+</sup>	.0518 937
.41	.3860 303	.2681 020	.1840 529	.1252 630	.0846 801	.0569 383
.42	.3985 771	.2800 048	.1944 669	.1339 076	.0915 953	.0623 200
.43	.4111 652	.2920 898	.2051 669	.1428 961	.0988 717	.0680 566
.44	.4237 894	.3043 511	.2161 499	.1522 302	.1065 163	.0741 415 <sup>+</sup>
.45	.4364 443	.3167 827	.2274 128	.1619 116	.1145 358	.0806 043
.46	.4491 248	.3293 787	.2389 521	.1719 414	.1229 368	.0874 501
.47	.4618 257	.3421 329	.2507 640	.1823 204	.1317 252	.0946 808
.48	.4745 420	.3550 390	.2628 445 <sup>+</sup>	.1930 488	.1409 067	.1023 343
.49	.4872 685 <sup>-</sup>	.3680 907	.2751 892	.2041 266	.1504 866	.1103 938
.50	.5000 000 <sup>o</sup>	.3812 816	.2877 934	.2155 534	.1604 605 <sup>-</sup>	.1188 787
.51	.5127 315 <sup>+</sup>	.3946 050 <sup>o</sup>	.3006 523	.2273 282	.1708 597	.1277 085 <sup>-</sup>
.52	.5254 580	.4080 543	.3137 605 <sup>+</sup>	.2394 496	.1816 613	.1371 627
.53	.5381 743	.4216 227	.3271 125 <sup>+</sup>	.2519 157	.1928 773	.1469 802
.54	.5508 752	.4353 032	.3407 025 <sup>+</sup>	.2647 242	.2045 106	.1572 505
.55	.5635 557	.4490 888	.3545 243	.2778 723	.2165 635 <sup>-</sup>	.1680 685
.56	.5762 106	.4629 721	.3685 712	.2913 567	.2290 374	.1792 346
.57	.5888 348	.4769 459	.3828 364	.3051 734	.2419 335 <sup>-</sup>	.1909 447
.58	.6014 229	.4910 026	.3973 126	.3193 181	.2552 519	.2031 449
.59	.6139 697	.5051 345 <sup>+</sup>	.4119 924	.3337 857	.2689 924	.2158 406
.60	.6264 700	.5193 338	.4268 676	.3485 708	.2831 539	.2290 367

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION

31

 $x = .61$  to  $1.00$  $q = 1.5$  $p = 1.5$  to  $4$ 

	$p = 1.5$	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$	$p = 4$
$B(p, q)$	.3926 9908	.2666 6667	.1963 4954	.1523 8095 <sup>+</sup>	.1227 1846	.1015 8730
$x$						
.61	.6389 182	.5335 923	.4419 299	.3636 671	.2977 344	.2427 370
.62	.6513 090	.5479 019	.4571 705 <sup>-</sup>	.3790 678	.3127 314	.2569 445 <sup>+</sup>
.63	.6630 360	.5622 540	.4725 802	.3947 655 <sup>+</sup>	.3281 413	.2716 615 <sup>-</sup>
.64	.6758 962	.5766 400 <sup>e</sup>	.4881 494	.4107 520 <sup>e</sup>	.3439 598	.2868 890
.65	.6880 812	.5910 510	.5038 679	.4270 184	.3601 815 <sup>+</sup>	.3026 271
.66	.7001 801	.6054 778	.5197 252	.4435 552	.3768 001	.3188 748
.67	.7122 050 <sup>-</sup>	.6199 110	.5357 101	.4603 518	.3938 083	.3356 297
.68	.7241 318	.6343 409	.5518 112	.4773 972	.4111 976	.3528 885 <sup>-</sup>
.69	.7359 603	.6487 576	.5680 161	.4946 791	.4289 583	.3706 459
.70	.7476 842	.6631 506	.5843 121	.5121 846	.4470 796	.3888 957
.71	.7592 970	.6775 094	.6006 859	.5298 997	.4655 493	.4076 296
.72	.7707 919	.6918 229	.6171 234	.5478 094	.4843 539	.4268 380
.73	.7821 619	.7060 796	.6330 098	.5658 975 <sup>+</sup>	.5034 782	.4465 091
.74	.7934 001	.7202 678	.6501 297	.5841 460	.5229 056	.4666 292
.75	.8044 989	.7343 750 <sup>e</sup>	.6666 667	.6025 391	.5426 177	.4871 826
.76	.8154 506	.7483 884	.6832 036	.6210 541	.5625 942	.5081 511
.77	.8262 473	.7622 946	.6997 222	.6396 709	.5828 130	.5295 139
.78	.8368 866	.7760 796	.7162 035 <sup>+</sup>	.6583 665 <sup>+</sup>	.6032 498	.5512 477
.79	.8473 417	.7897 285 <sup>+</sup>	.7326 272	.6771 166	.6238 778	.5733 259
.80	.8576 215 <sup>+</sup>	.8032 260	.7489 717	.6958 948	.6446 680	.5957 189
.81	.8677 103	.8165 557	.7652 143	.7146 727	.6655 882	.6183 933
.82	.8775 977	.8297 094	.7813 305 <sup>+</sup>	.7334 200	.6866 036	.6413 118
.83	.8872 730	.8420 417	.7972 914	.7521 037	.7076 757	.6644 327
.84	.8967 245 <sup>+</sup>	.8553 600 <sup>e</sup>	.8130 780	.7706 880 <sup>e</sup>	.7287 624	.6877 094
.85	.9059 398	.8678 344	.8286 514	.7891 342	.7498 173	.7110 808
.86	.9149 054	.8800 425 <sup>-</sup>	.8439 821	.8074 001	.7707 803	.7345 155 <sup>+</sup>
.87	.9236 066	.8919 597	.8590 348	.8254 393	.7916 219	.7579 211
.88	.9320 276	.9035 594	.8737 710	.8432 000	.8122 521	.7812 328
.89	.9401 506	.9148 125 <sup>-</sup>	.8881 482	.8606 286	.8326 097	.8043 676
.90	.9479 560	.9256 865 <sup>-</sup>	.9021 194	.8776 594	.8526 158	.8272 309
.91	.9554 216	.9361 450 <sup>e</sup>	.9156 315 <sup>+</sup>	.8942 224	.8721 808	.8497 147
.92	.9625 220	.9461 467	.9286 247	.9102 370	.8912 020	.8716 939
.93	.9692 278	.9556 440	.9410 296	.9256 099	.9095 605 <sup>+</sup>	.8930 229
.94	.9755 037	.9645 804	.9527 649	.9402 312	.9271 156	.9135 283
.95	.9813 070	.9728 877	.9637 322	.9539 684	.9436 970	.9329 996
.96	.9865 829	.9804 800 <sup>e</sup>	.9738 084	.9666 560 <sup>e</sup>	.9590 921	.9511 731
.97	.9912 586	.9872 434	.9828 313	.9780 765 <sup>-</sup>	.9730 219	.9677 025 <sup>-</sup>
.98	.9952 272	.9930 138	.9905 689	.9879 205 <sup>-</sup>	.9850 906	.9820 972
.99	.9983 074	.9975 150 <sup>e</sup>	.9966 352	.9956 773	.9946 486	.9935 548
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .02$  to  $.60$  $q = 1.5$  $p = 4.5$  to  $7$ 

	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$
$B(p, q) = .8590\ 2924 \times \frac{x}{10}$	$.7388\ 1674 \times \frac{x}{10}$	$.6442\ 7193 \times \frac{x}{10}$	$.5683\ 2057 \times \frac{x}{10}$	$.5062\ 1366 \times \frac{x}{10}$	$.4540\ 5645 \times \frac{x}{10}$	
$x$						
.02	.0000 001					
.03	.0000 004	.0000 001				
.04	.0000 013	.0000 003	.0000 001			
.05	.0000 035 <sup>+</sup>	.0000 008	.0000 002			
.06	.0000 080	.0000 021	.0000 005 <sup>+</sup>	.0000 001		
.07	.0000 160	.0000 044	.0000 012	.0000 003	.0000 001	.0000 001
.08	.0000 290	.0000 086	.0000 025 <sup>+</sup>	.0000 007	.0000 002	.0000 001
.09	.0000 490	.0000 154	.0000 048	.0000 015 <sup>-</sup>	.0000 005 <sup>-</sup>	.0000 001
.10	.0000 784	.0000 259	.0000 085 <sup>+</sup>	.0000 028	.0000 009	.0000 003
.11	.0001 198	.0000 415 <sup>+</sup>	.0000 144	.0000 049	.0000 017	.0000 006
.12	.0001 765 <sup>-</sup>	.0000 639	.0000 231	.0000 083	.0000 030	.0000 011
.13	.0002 518	.0000 949	.0000 356	.0000 133	.0000 050	.0000 019
.14	.0003 499	.0001 368	.0000 533	.0000 207	.0000 080	.0000 031
.15	.0004 750 <sup>+</sup>	.0001 923	.0000 775 <sup>+</sup>	.0000 312	.0000 125 <sup>+</sup>	.0000 050 <sup>+</sup>
.16	.0006 321	.0002 642	.0001 101	.0000 457	.0000 180	.0000 078
.17	.0008 265 <sup>-</sup>	.0003 561	.0001 529	.0000 654	.0000 279	.0000 119
.18	.0010 638	.0004 715 <sup>+</sup>	.0002 083	.0000 917	.0000 403	.0000 177
.19	.0013 503	.0006 149	.0002 790	.0001 262	.0000 579	.0000 250
.20	.0016 926	.0007 907	.0003 681	.0001 708	.0000 791	.0000 365 <sup>+</sup>
.21	.0020 977	.0010 040	.0004 789	.0002 277	.0001 080	.0000 511
.22	.0025 734	.0012 605 <sup>+</sup>	.0006 153	.0002 995 <sup>-</sup>	.0001 454	.0000 764
.23	.0031 274	.0015 662	.0007 816	.0003 889	.0001 930	.0000 950
.24	.0037 684	.0019 275 <sup>+</sup>	.0009 825 <sup>+</sup>	.0004 994	.0002 532	.0001 281
.25	.0045 050 <sup>+</sup>	.0023 516	.0012 233	.0006 345 <sup>+</sup>	.0003 283	.0001 695 <sup>-</sup>
.26	.0053 468	.0028 459	.0015 096	.0007 985 <sup>-</sup>	.0004 213	.0002 218
.27	.0063 033	.0034 185 <sup>+</sup>	.0018 477	.0009 958	.0005 354	.0002 872
.28	.0073 847	.0040 780	.0022 444	.0012 317	.0006 743	.0003 683
.29	.0086 016	.0048 335 <sup>-</sup>	.0027 070	.0015 117	.0008 421	.0004 681
.30	.0099 650 <sup>-</sup>	.0056 946	.0032 434	.0018 421	.0010 436	.0005 900
.31	.0114 862	.0066 715 <sup>+</sup>	.0038 622	.0022 205 <sup>+</sup>	.0012 839	.0007 378
.32	.0131 768	.0077 749	.0045 725 <sup>-</sup>	.0026 815 <sup>+</sup>	.0015 688	.0009 158
.33	.0150 490	.0090 160	.0053 840	.0032 061	.0019 045 <sup>+</sup>	.0011 290
.34	.0171 152	.0104 067	.0063 071	.0038 118	.0022 982	.0013 828
.35	.0193 881	.0119 591	.0073 529	.0045 083	.0027 576	.0016 832
.36	.0218 808	.0136 861	.0085 330	.0053 055 <sup>+</sup>	.0032 909	.0020 371
.37	.0246 066	.0156 010	.0098 598	.0062 144	.0039 075	.0024 518
.38	.0275 791	.0177 177	.0113 464	.0072 464	.0046 171	.0029 358
.39	.0308 123	.0200 504	.0130 063	.0084 142	.0054 307	.0034 979
.40	.0343 200	.0226 139	.0148 541	.0097 308	.0063 598	.0041 482
.41	.0381 168	.0254 235 <sup>+</sup>	.0169 046	.0112 104	.0074 170	.0048 973
.42	.0422 170	.0284 949	.0191 738	.0128 676	.0086 158	.0057 573
.43	.0466 352	.0318 441	.0216 778	.0147 184	.0099 705	.0067 407
.44	.0513 862	.0354 876	.0244 337	.0167 792	.0114 965 <sup>+</sup>	.0078 614
.45	.0564 848	.0394 425 <sup>-</sup>	.0274 593	.0190 674	.0132 104	.0091 345 <sup>+</sup>
.46	.0619 459	.0437 258	.0307 728	.0216 013	.0151 295 <sup>+</sup>	.0105 760
.47	.0677 844	.0483 552	.0343 931	.0244 002	.0172 724	.0122 031
.48	.0740 152	.0533 484	.0383 397	.0274 839	.0196 587	.0140 344
.49	.0806 531	.0587 236	.0426 327	.0308 734	.0223 690	.0160 896
.50	.0877 129	.0644 991	.0472 926	.0345 904	.0252 452	.0183 898
.51	.0952 093	.0706 933	.0523 406	.0386 573	.0284 901	.0209 574
.52	.1031 566	.0773 249	.0577 983	.0430 976	.0320 678	.0238 163
.53	.1115 691	.0844 124	.0636 876	.0479 354	.0360 034	.0269 915
.54	.1204 608	.0919 746	.0700 309	.0531 954	.0403 231	.0305 096
.55	.1298 453	.1000 302	.0768 508	.0589 033	.0450 542	.0343 986
.56	.1397 359	.1085 977	.0841 704	.0650 853	.0502 250 <sup>-</sup>	.0386 879
.57	.1501 453	.1176 955 <sup>+</sup>	.0920 128	.0717 683	.0558 649	.0434 082
.58	.1610 860	.1273 419	.1004 012	.0789 795 <sup>+</sup>	.0620 043	.0485 919
.59	.1725 696	.1375 546	.1093 591	.0867 469	.0686 745 <sup>-</sup>	.0542 724
.60	.1846 073	.1483 512	.1189 096	.0950 988	.0759 075 <sup>-</sup>	.0604 847

TABLE I. THE  $I_w(p, q)$  FUNCTION

33

 $p = .61$  to  $1.00$  $q = 1.5$  $p = 4.5$  to  $7$ 

$p$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$
$I(p, q) \times 10^4$	$8590.2024 \times 10^4$	$7388.1674 \times 10^4$	$6142.7193 \times 10^4$	$5083.2057 \times 10^4$	$5062.1366 \times 10^4$	$4546.5645 \times 10^4$
.61	.1072 006	.1507 487	.1200 761	.1040 635 <sup>+</sup>	.0837 363	.0672 649
.62	.2103 860	.1717 035 <sup>+</sup>	.1308 814	.1136 701	.0921 946	.0746 507
.63	.2241 453	.1844 115 <sup>+</sup>	.1513 482	.1239 472	.1013 166	.0826 804
.64	.2384 955 <sup>+</sup>	.1977 076	.1634 986	.1349 239	.1111 371	.0913 938
.65	.2534 431	.2116 050	.1763 542	.1466 287	.1216 912	.1008 316
.66	.2680 038	.2262 096	.1899 359	.1590 899	.1330 141	.1110 351
.67	.2831 520	.2416 205 <sup>+</sup>	.2042 635 <sup>+</sup>	.1723 356	.1451 413	.1220 464
.68	.3010 207	.2576 393	.2193 560	.1863 929	.1581 070	.1339 081
.69	.3193 013	.2743 652	.2352 300	.2012 881	.1719 488	.1466 629
.70	.3372 936	.2918 056	.2519 045 <sup>+</sup>	.2170 463	.1866 982	.1603 538
.71	.3558 957	.3099 664	.2693 911	.2336 915 <sup>+</sup>	.2023 893	.1750 233
.72	.3751 036	.3288 512	.2877 033	.2512 456	.2190 543	.1907 133
.73	.3949 112	.3484 613	.3068 514	.2697 290	.2367 237	.2074 648
.74	.4153 193	.3687 957	.3268 431	.2891 593	.2554 260	.2253 174
.75	.4362 890	.3898 506	.3476 835 <sup>+</sup>	.3095 517	.2751 874	.2443 089
.76	.4578 364	.4116 190	.3693 742	.3309 181	.2960 310	.2644 744
.77	.4799 329	.4340 904	.3919 133	.3532 668	.3179 768	.2858 463
.78	.5025 596	.4572 509	.4152 948	.3766 017	.3410 403	.3084 530
.79	.5256 927	.4810 820	.4395 079	.4009 220	.3652 324	.3323 184
.80	.5493 045 <sup>+</sup>	.5055 606	.4645 370	.4262 214	.3905 580	.3574 607
.81	.5733 629	.5306 587	.4903 602	.4524 871	.4170 159	.3838 916
.82	.5978 309	.5563 420	.5169 491	.4796 992	.4445 967	.4116 149
.83	.6226 659	.5825 700	.5442 680	.5078 293	.4732 823	.4406 250 <sup>+</sup>
.84	.6478 193	.6092 947	.5722 725 <sup>+</sup>	.5368 395 <sup>+</sup>	.5030 441	.4709 052
.85	.6732 356	.6364 599	.6009 084	.5666 809	.5338 413	.5024 260
.86	.6988 513	.6630 997	.6301 105 <sup>+</sup>	.5972 918	.5656 191	.5351 422
.87	.7245 942	.6918 376	.6598 007	.6285 958	.5983 058	.5689 903
.88	.7503 817	.7198 845 <sup>+</sup>	.6898 861	.6604 992	.6318 104	.6038 853
.89	.7761 192	.7480 363	.7202 562	.6928 880	.6660 184	.6397 158
.90	.8016 970	.7761 721	.7507 799	.7256 239	.7007 878	.6763 394
.91	.8266 920	.8041 498	.7813 011	.7585 394	.7359 425 <sup>+</sup>	.7135 751
.92	.8518 548	.8318 018	.8116 331	.7914 310	.7712 652	.7511 948
.93	.8761 134	.8589 288	.8415 513	.8240 505 <sup>+</sup>	.8064 865 <sup>+</sup>	.7889 107
.94	.8995 668	.8852 900	.8707 814	.8560 916	.8412 695 <sup>+</sup>	.8263 578
.95	.9219 445 <sup>+</sup>	.9105 892	.8989 835 <sup>+</sup>	.8871 703	.8751 875 <sup>+</sup>	.8630 684
.96	.9429 462	.9344 516	.9257 240	.9167 937	.9076 876	.8984 295 <sup>+</sup>
.97	.9621 475 <sup>+</sup>	.9563 819	.9504 273	.9443 028	.9380 252	.9316 097
.98	.9789 550 <sup>+</sup>	.9756 769	.9722 740	.9687 560	.9651 314	.9614 082
.99	.9924 008	.9911 908	.9899 283	.9886 165 <sup>+</sup>	.9872 580	.9858 555 <sup>+</sup>
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .10$  to  $.70$  $q = 1.5$  $p = 7.5$  to  $10$ 

	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$B(p, q) = .4112\ 9860 \times \frac{x}{10}$	$.3744\ 2296 \times \frac{x}{10}$	$.3427\ 4883 \times \frac{x}{10}$	$.3153\ 0355 \times \frac{x}{10}$	$.2913\ 3651 \times \frac{x}{10}$	$.2702\ 0018 \times \frac{x}{10}$	
$x$						
.10	.0000 001					
.11	.0000 002	.0000 001				
.12	.0000 004	.0000 001				
.13	.0000 007	.0000 003	.0000 001			
.14	.0000 012	.0000 005	.0000 002	.0000 001		
.15	.0000 020	.0000 008	.0000 003	.0000 001	.0000 001	
.16	.0000 032	.0000 013	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
.17	.0000 051	.0000 021	.0000 009	.0000 004	.0000 002	.0000 001
.18	.0000 077	.0000 034	.0000 015 <sup>-</sup>	.0000 006	.0000 003	.0000 001
.19	.0000 115 <sup>+</sup>	.0000 052	.0000 023	.0000 010	.0000 005 <sup>-</sup>	.0000 002
.20	.0000 168	.0000 077	.0000 036	.0000 016	.0000 007	.0000 003
.21	.0000 241	.0000 114	.0000 054	.0000 025 <sup>+</sup>	.0000 012	.0000 006
.22	.0000 340	.0000 164	.0000 079	.0000 038	.0000 018	.0000 009
.23	.0000 473	.0000 233	.0000 115 <sup>-</sup>	.0000 057	.0000 028	.0000 014
.24	.0000 647	.0000 326	.0000 164	.0000 082	.0000 041	.0000 021
.25	.0000 873	.0000 449	.0000 231	.0000 118	.0000 061	.0000 031
.26	.0001 165 <sup>+</sup>	.0000 611	.0000 320	.0000 167	.0000 087	.0000 046
.27	.0001 538	.0000 822	.0000 439	.0000 234	.0000 124	.0000 066
.28	.0002 008	.0001 093	.0000 594	.0000 322	.0000 175	.0000 095
.29	.0002 597	.0001 439	.0000 796	.0000 439	.0000 242	.0000 134
.30	.0003 329	.0001 875 <sup>+</sup>	.0001 055 <sup>-</sup>	.0000 593	.0000 332	.0000 186
.31	.0004 232	.0002 423	.0001 385 <sup>+</sup>	.0000 791	.0000 451	.0000 257
.32	.0005 336	.0003 104	.0001 803	.0001 046	.0000 606	.0000 351
.33	.0006 680	.0003 946	.0002 327	.0001 371	.0000 807	.0000 474
.34	.0008 304	.0004 979	.0002 981	.0001 782	.0001 064	.0000 635
.35	.0010 255 <sup>+</sup>	.0006 238	.0003 789	.0002 298	.0001 392	.0000 843
.36	.0012 586	.0007 764	.0004 782	.0002 942	.0001 807	.0001 109
.37	.0015 357	.0009 603	.0005 996	.0003 739	.0002 320	.0001 449
.38	.0018 633	.0011 807	.0007 471	.0004 721	.0002 980	.0001 879
.39	.0022 489	.0014 436	.0009 253	.0005 923	.0003 787	.0002 419
.40	.0027 008	.0017 556	.0011 396	.0007 387	.0004 783	.0003 004
.41	.0032 279	.0021 241	.0013 958	.0009 160	.0006 005 <sup>-</sup>	.0003 932
.42	.0038 403	.0025 576	.0017 009	.0011 207	.0007 495 <sup>-</sup>	.0004 967
.43	.0045 491	.0030 652	.0020 625 <sup>-</sup>	.0013 860	.0009 303	.0006 238
.44	.0053 663	.0036 574	.0024 892	.0016 920	.0011 488	.0007 792
.45	.0063 052	.0043 455 <sup>-</sup>	.0029 907	.0020 557	.0014 114	.0009 680
.46	.0073 801	.0051 420	.0035 777	.0024 802	.0017 257	.0011 966
.47	.0086 067	.0060 610	.0042 623	.0029 038	.0021 004	.0014 721
.48	.0100 020	.0071 174	.0050 578	.0035 898	.0025 450 <sup>+</sup>	.0018 624
.49	.0115 844	.0083 280	.0059 789	.0042 872	.0030 707	.0021 972
.50	.0133 735 <sup>-</sup>	.0097 109	.0070 419	.0051 002	.0036 899	.0026 608
.51	.0153 906	.0112 857	.0082 645 <sup>+</sup>	.0060 448	.0044 165 <sup>-</sup>	.0032 235 <sup>+</sup>
.52	.0176 587	.0130 738	.0096 665 <sup>-</sup>	.0071 386	.0052 660	.0038 808
.53	.0202 021	.0150 983	.0112 691	.0084 010	.0062 561	.0046 542
.54	.0230 468	.0173 842	.0130 957	.0098 535 <sup>+</sup>	.0074 060	.0055 610
.55	.0262 208	.0199 583	.0151 718	.0115 197	.0087 374	.0066 207
.56	.0297 533	.0228 494	.0175 249	.0134 255 <sup>-</sup>	.0102 741	.0078 549
.57	.0336 757	.0260 883	.0201 847	.0155 990	.0120 424	.0092 879
.58	.0380 211	.0297 081	.0231 834	.0180 710	.0140 713	.0109 465
.59	.0428 241	.0337 438	.0265 556	.0208 750 <sup>-</sup>	.0163 926	.0128 606
.60	.0481 215 <sup>-</sup>	.0382 328	.0303 385 <sup>-</sup>	.0240 472	.0190 410	.0150 630
.61	.0539 516	.0432 144	.0345 717	.0276 267	.0220 545 <sup>+</sup>	.0175 899
.62	.0603 545 <sup>+</sup>	.0487 306	.0392 976	.0316 558	.0254 744	.0204 812
.63	.0673 722	.0548 253	.0445 615 <sup>-</sup>	.0361 798	.0293 454	.0237 805 <sup>-</sup>
.64	.0750 480	.0615 447	.0504 112	.0412 472	.0337 161	.0275 352
.65	.0834 271	.0689 372	.0568 973	.0469 102	.0386 385 <sup>+</sup>	.0317 972
.66	.0925 559	.0770 534	.0640 733	.0532 238	.0441 690	.0366 225 <sup>+</sup>
.67	.1024 824	.0859 459	.0719 955 <sup>-</sup>	.0602 469	.0503 677	.0426 719
.68	.1132 555 <sup>+</sup>	.0956 691	.0807 226	.0680 415 <sup>-</sup>	.0572 989	.0482 110
.69	.1249 254	.1062 794	.0903 161	.0766 732	.0650 310	.0551 100
.70	.1375 427	.1178 345 <sup>-</sup>	.1008 400	.0862 107	.0736 368	.0628 442

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $1.00$  $q = 1.5$  $p = 7.5$  to  $10$ 

	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$I_x(p, q) \times 10^4$	$1.112\ 9800 \times 10^4$	$1.3744\ 2290 \times 10^4$	$1.3127\ 4883 \times 10^4$	$1.3153\ 0355 \times 10^4$	$1.2913\ 3651 \times 10^4$	$1.2702\ 6018 \times 10^4$
.71	.1511 588	.1303 936	.1123 602	.0967 261	.0831 928	.0714 941
.72	.1658 250 <sup>+</sup>	.1440 169	.1249 449	.1082 942	.0937 798	.0811 449
.73	.1815 925 <sup>+</sup>	.1587 653	.1386 638	.1209 928	.1054 823	.0918 870
.74	.1985 120	.1746 990	.1535 879	.1349 019	.1183 885 <sup>+</sup>	.1038 152
.75	.2166 328	.1918 816	.1697 891	.1501 036	.1325 897	.1170 293
.76	.2360 625 <sup>+</sup>	.2103 703	.1873 393	.1666 812	.1481 798	.1316 328
.77	.2566 664	.2302 245 <sup>+</sup>	.2063 101	.1847 189	.1652 549	.1477 329
.78	.2786 666	.2515 003	.2267 716	.2043 007	.1839 125 <sup>+</sup>	.1654 397
.79	.3020 410	.2742 593	.2487 917	.2255 005 <sup>+</sup>	.2042 503	.1848 650 <sup>+</sup>
.80	.3268 224	.2985 220	.2724 346	.2484 259	.2263 649	.2061 217
.81	.3530 371	.3243 604	.2977 595 <sup>+</sup>	.2731 265 <sup>+</sup>	.2503 508	.2293 214
.82	.3807 640	.3517 979	.3248 191	.2996 824	.2762 979	.2545 735 <sup>+</sup>
.83	.4098 320	.3808 611	.3536 573	.3281 568	.3042 897	.2819 821
.84	.4404 191	.4115 644	.3843 070	.3586 027	.3344 003	.3116 434
.85	.4724 492	.4439 082	.4167 870	.3910 593	.3666 910	.3436 421
.86	.5058 902	.4778 758	.4510 988	.4255 486	.4012 065 <sup>+</sup>	.3780 472
.87	.5406 898	.5134 295 <sup>+</sup>	.4872 222	.4620 705 <sup>+</sup>	.4379 691	.4149 050
.88	.5767 725 <sup>+</sup>	.5505 061	.5251 102	.5005 972	.4769 731	.4542 371
.89	.6140 336	.5890 124	.5646 826	.5410 659	.5181 766	.4960 229
.90	.6523 338	.6288 151	.6058 180	.5833 699	.5614 912	.5401 970
.91	.6914 911	.6697 350 <sup>+</sup>	.6483 436	.6273 470	.6067 697	.5866 310
.92	.7312 700	.7115 334	.6920 213	.6727 643	.6537 884	.6351 153
.93	.7713 677	.7538 963	.7365 299	.7192 977	.7022 249	.6853 335 <sup>+</sup>
.94	.8113 943	.7964 117	.7814 304	.7665 031	.7516 255 <sup>+</sup>	.7368 270
.95	.8508 424	.8385 360	.8261 728	.8137 737	.8013 580	.7889 420
.96	.8890 497	.8795 405 <sup>+</sup>	.8699 403	.8602 738	.8505 374	.8407 502
.97	.9250 698	.9184 180	.9116 653	.9048 222	.8978 981	.8909 017
.98	.9575 932	.9536 930	.9497 133	.9456 595 <sup>+</sup>	.9415 367	.9373 494
.99	.9844 110	.9829 268	.9814 047	.9798 463	.9782 533	.9766 271
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION $c = .81$  to  $1.00$  $q = 1.5$  $p = 10.5$  to  $15$ 

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .2516\ 0880 \times \frac{1}{10}$	$.2350\ 0886 \times \frac{1}{10}$	$.2068\ 0779 \times \frac{1}{10}$	$.1838\ 2915 \times \frac{1}{10}$	$.1648\ 1234 \times \frac{1}{10}$	$.1488\ 6276 \times \frac{1}{10}$	
$x$						
.81	.2099 286	.1920 652	.1605 160	.1338 963	.1115 051	.0927 205 <sup>-</sup>
.82	.2344 164	.2157 347	.1824 393	.1539 995 <sup>+</sup>	.1297 819	.1092 143
.83	.2611 580	.2417 407	.2068 222	.1766 323	.1506 109	.1282 418
.84	.2902 722	.2702 252	.2338 525 <sup>+</sup>	.2020 264	.1742 643	.1501 112
.85	.3218 683	.3013 223	.2637 153	.2304 175 <sup>+</sup>	.2010 258	.1751 506
.86	.3560 408	.3351 534	.2905 880	.2620 398	.2311 856	.2037 033
.87	.3928 636	.3718 209	.3326 332	.2971 193	.2650 339	.2361 227
.88	.4323 830	.4114 004	.3719 905 <sup>+</sup>	.3358 649	.3028 516	.2727 623
.89	.4746 078	.4539 301	.4147 648	.3784 552	.3448 968	.3139 631
.90	.5194 974	.4993 986	.4610 110	.4250 227	.3913 874	.3600 345 <sup>+</sup>
.91	.5669 463	.5477 269	.5107 149	.4756 307	.4424 761	.4112 278
.92	.6167 629	.5987 463	.5637 659	.5302 431	.4982 159	.4676 986
.93	.6686 422	.6521 674	.6199 209	.5886 821	.5585 126	.5294 529
.94	.7221 254	.7075 367	.6787 524	.6505 677	.6230 551	.5962 697
.95	.7765 432	.7641 736	.7395 734	.7152 295 <sup>+</sup>	.6912 133	.6675 831
.96	.8309 243	.8210 705 <sup>-</sup>	.8013 191	.7815 678	.7618 772	.7422 992
.97	.8838 410	.8767 236	.8623 457	.8478 181	.8331 843	.8184 826
.98	.9331 019	.9287 983	.9200 372	.9110 937	.9019 919	.8927 536
.99	.9749 692	.9732 807	.9698 173	.9662 456	.9625 736	.9588 085 <sup>+</sup>
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $p = 16 \text{ to } 21$  $x = .33 \text{ to } 1.00$  $q = 1.5$ 

	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$B(p, q) = .1353\ 2978 \times \frac{x}{10}$	$.1237\ 3009 \times \frac{x}{10}$	$.1136\ 9792 \times \frac{x}{10}$	$.1049\ 5192 \times \frac{x}{10}$	$.9727\ 2514 \times \frac{x}{100}$	$.9048\ 6059 \times \frac{x}{100}$	
$x$						
.33	.0000 001					
.34	.0000 001					
.35	.0000 002	.0000 001				
.36	.0000 003	.0000 001				
.37	.0000 005	.0000 002	.0000 001			
.38	.0000 007	.0000 003	.0000 001	.0000 001		
.39	.0000 011	.0000 004	.0000 002	.0000 001		
.40	.0000 016	.0000 006	.0000 003			
.41	.0000 023	.0000 010	.0000 004	.0000 002	.0000 001	
.42	.0000 034	.0000 015	.0000 006	.0000 003	.0000 001	.0000 001
.43	.0000 049	.0000 022	.0000 010	.0000 004	.0000 002	.0000 001
.44	.0000 070	.0000 032	.0000 014	.0000 006	.0000 003	.0000 002
.45	.0000 099	.0000 046	.0000 021	.0000 010	.0000 005	.0000 003
.46	.0000 140	.0000 066	.0000 031	.0000 015	.0000 007	.0000 004
.47	.0000 195 <sup>+</sup>	.0000 094	.0000 046	.0000 022	.0000 011	.0000 005
.48	.0000 271	.0000 134	.0000 066	.0000 032	.0000 016	.0000 008
.49	.0000 374	.0000 189	.0000 095	.0000 048	.0000 024	.0000 012
.50	.0000 513	.0000 263	.0000 135 <sup>+</sup>	.0000 069	.0000 035 <sup>+</sup>	.0000 018
.51	.0000 697	.0000 366	.0000 191	.0000 100	.0000 052	.0000 027
.52	.0000 943	.0000 504	.0000 269	.0000 143	.0000 076	.0000 041
.53	.0001 267	.0000 690	.0000 375 <sup>+</sup>	.0000 204	.0000 111	.0000 060
.54	.0001 692	.0000 939	.0000 520	.0000 288	.0000 159	.0000 088
.55	.0002 248	.0001 270	.0000 717	.0000 404	.0000 228	.0000 128
.56	.0002 969	.0001 709	.0000 982	.0000 563	.0000 323	.0000 185 <sup>+</sup>
.57	.0003 902	.0002 285 <sup>+</sup>	.0001 336	.0000 780	.0000 455 <sup>+</sup>	.0000 265 <sup>+</sup>
.58	.0005 101	.0003 039	.0001 809	.0001 075	.0000 638	.0000 378
.59	.0006 636	.0004 022	.0002 434	.0001 471	.0000 888	.0000 536
.60	.0008 590	.0005 294	.0003 258	.0002 003	.0001 230	.0000 754
.61	.0011 068	.0006 934	.0004 339	.0002 711	.0001 602	.0001 055 <sup>+</sup>
.62	.0014 197	.0009 040	.0005 748	.0003 651	.0002 316	.0001 468
.63	.0018 130	.0011 729	.0007 578	.0004 890	.0003 152	.0002 030
.64	.0023 052	.0015 149	.0009 941	.0006 516	.0004 267	.0002 791
.65	.0029 188	.0019 479	.0012 982	.0008 642	.0005 746	.0003 817
.66	.0036 807	.0024 938	.0016 875	.0011 405	.0007 700	.0005 193
.67	.0046 230	.0031 794	.0021 837	.0014 981	.0010 266	.0007 028
.68	.0057 840	.0040 367	.0028 136	.0019 589	.0013 623	.0009 465 <sup>+</sup>
.69	.0072 089	.0051 046	.0036 099	.0025 499	.0017 093	.0012 684
.70	.0089 515	.0064 295 <sup>+</sup>	.0046 122	.0033 048	.0023 655	.0016 914
.71	.0110 746	.0080 670	.0058 688	.0042 648	.0030 959	.0022 453
.72	.0136 523	.0100 833	.0074 380	.0054 806	.0040 341	.0029 666
.73	.0167 706	.0125 566	.0093 899	.0070 139	.0052 339	.0039 019
.74	.0205 298	.0155 793	.0118 081	.0089 399	.0067 616	.0051 094
.75	.0250 458	.0192 599	.0147 927	.0113 493	.0086 988	.0066 612
.76	.0304 522	.0237 252	.0184 623	.0143 514	.0111 449	.0086 471
.77	.0369 021	.0291 230	.0229 569	.0180 771	.0142 208	.0111 772
.78	.0445 703	.0356 241	.0284 409	.0226 824	.0180 726	.0143 870
.79	.0536 551	.0434 257	.0351 068	.0283 523	.0228 759	.0184 413
.80	.0643 805 <sup>+</sup>	.0527 534	.0431 781	.0353 052	.0288 410	.0235 491
.81	.0769 977	.0638 643	.0529 134	.0437 967	.0362 170	.0299 250
.82	.0917 866	.0770 493	.0646 093	.0541 252	.0453 020	.0378 860
.83	.1090 565 <sup>+</sup>	.0926 351	.0786 045 <sup>+</sup>	.0666 356	.0564 400	.0477 661
.84	.1291 464	.1109 856	.0952 818	.0817 242	.0700 362	.0599 728
.85	.1524 235	.1325 018	.1150 703	.0998 420	.0865 572	.0749 820
.86	.1792 807	.1576 210	.1384 457	.1214 969	.1065 373	.0933 505
.87	.2101 315	.1868 125	.1659 283	.1472 543	.1305 804	.1157 115
.88	.2454 012	.2205 709	.1980 776	.1777 337	.1593 590	.1427 867
.89	.2855 145	.2594 039	.2354 821	.2136 002	.1936 129	.1753 794
.90	.3308 764	.3038 139	.2787 414	.2555 493	.2341 271	.2143 951
.91	.3818 441	.3542 692	.3284 381	.3042 789	.2817 154	.2606 664
.92	.4386 867	.4111 618	.3850 940	.3604 455	.3371 721	.3152 253
.93	.5015 266	.4747 435	.4491 026	.4245 942	.4012 016	.3789 025
.94	.5702 521	.5450 313	.5206 264	.4970 486	.4743 022	.4523 861
.95	.6443 861	.6216 603	.5994 358	.5777 361	.5565 788	.5359 769
.96	.7228 777	.7036 505 <sup>+</sup>	.6846 495 <sup>+</sup>	.6659 018	.6474 305	.6292 546
.97	.8037 466	.7890 059	.7742 870	.7596 130	.7450 046	.7304 803
.98	.8833 983	.8739 436	.8644 054	.8547 984	.8451 357	.8354 296
.99	.9549 568	.9510 245 <sup>+</sup>	.9470 170	.9429 394	.9387 963	.9345 921
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_w(p, q)$  FUNCTION $p = 4.4$  to  $1.00$  $q = 1.5$  $p = 22$  to  $27$ 

	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$
$I(p, q)$	$8.445\ 3055 \times \frac{1}{10^4}$	$7.000\ 2997 \times \frac{1}{10^4}$	$7.422\ 2405 \times \frac{1}{10^4}$	$6.985\ 6381 \times \frac{1}{10^4}$	$6.590\ 2246 \times \frac{1}{10^4}$	$6.230\ 7578 \times \frac{1}{10^4}$
$x$						
.44	.0000 001					
.45	.0000 001					
.46	.0000 002	.0000 001				
.47	.0000 002	.0000 001	.0000 001			
.48	.0000 004	.0000 002	.0000 001			
.49	.0000 006	.0000 003	.0000 002	.0000 001		
.50	.0000 009	.0000 005	.0000 002	.0000 001	.0000 001	
.51	.0000 014	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
.52	.0000 022	.0000 011	.0000 006	.0000 003	.0000 002	.0000 001
.53	.0000 032	.0000 018	.0000 009	.0000 005 <sup>+</sup>	.0000 003	.0000 001
.54	.0000 049	.0000 027	.0000 015 <sup>+</sup>	.0000 008	.0000 004	.0000 002
.55	.0000 072	.0000 040	.0000 023	.0000 013	.0000 007	.0000 004
.56	.0000 106	.0000 060	.0000 035 <sup>+</sup>	.0000 020	.0000 011	.0000 006
.57	.0000 155 <sup>+</sup>	.0000 090	.0000 052	.0000 030	.0000 018	.0000 010
.58	.0000 224	.0000 133	.0000 078	.0000 046	.0000 027	.0000 016
.59	.0000 323	.0000 194	.0000 117	.0000 070	.0000 042	.0000 025 <sup>+</sup>
.60	.0000 462	.0000 283	.0000 173	.0000 106	.0000 065 <sup>+</sup>	.0000 039
.61	.0000 657	.0000 409	.0000 255 <sup>+</sup>	.0000 158	.0000 098	.0000 061
.62	.0000 920	.0000 588	.0000 372	.0000 235 <sup>+</sup>	.0000 148	.0000 093
.63	.0001 306	.0000 839	.0000 539	.0000 346	.0000 222	.0000 142
.64	.0001 824	.0001 191	.0000 777	.0000 507	.0000 330	.0000 215 <sup>+</sup>
.65	.0002 533	.0001 680	.0001 113	.0000 737	.0000 488	.0000 323
.66	.0003 499	.0002 336	.0001 585 <sup>+</sup>	.0001 066	.0000 716	.0000 481
.67	.0004 868	.0003 286	.0002 244	.0001 531	.0001 044	.0000 712
.68	.0006 570	.0004 557	.0003 158	.0002 187	.0001 514	.0001 047
.69	.0008 933	.0006 287	.0004 421	.0003 107	.0002 182	.0001 531
.70	.0012 085 <sup>+</sup>	.0008 628	.0006 155 <sup>+</sup>	.0004 387	.0003 126	.0002 225 <sup>+</sup>
.71	.0016 270	.0011 780	.0008 522	.0006 162	.0004 452	.0003 215 <sup>+</sup>
.72	.0021 797	.0016 003	.0011 740	.0008 607	.0006 306	.0004 617
.73	.0029 065 <sup>+</sup>	.0021 633	.0016 089	.0011 958	.0008 882	.0006 594
.74	.0038 576	.0029 102	.0021 940	.0016 520	.0012 444	.0009 364
.75	.0050 967	.0038 966	.0029 760	.0022 728	.0017 342	.0013 225 <sup>+</sup>
.76	.0067 035 <sup>+</sup>	.0051 928	.0040 197	.0031 096	.0024 041	.0018 576
.77	.0087 779	.0068 884	.0054 018	.0042 333	.0033 156	.0025 954
.78	.0114 438	.0090 950	.0072 248	.0057 349	.0045 495 <sup>+</sup>	.0036 071
.79	.0148 546	.0119 567	.0096 176	.0077 312	.0062 111	.0049 872
.80	.0191 990	.0156 460	.0127 434	.0103 723	.0084 374	.0068 598
.81	.0247 074	.0203 847	.0168 072	.0138 490	.0114 049	.0093 872
.82	.0316 602	.0264 390	.0220 645 <sup>+</sup>	.0184 027	.0153 399	.0127 802
.83	.0403 954	.0341 386	.0288 326	.0243 368	.0205 307	.0173 109
.84	.0513 182	.0438 832	.0375 020	.0320 301	.0273 418	.0233 279
.85	.0649 105 <sup>+</sup>	.0561 544	.0485 409	.0419 514	.0362 305 <sup>+</sup>	.0312 742
.86	.0817 398	.0715 276	.0625 543	.0546 764	.0477 660	.0417 087
.87	.1024 675 <sup>+</sup>	.0906 834	.0802 083	.0709 052	.0626 495 <sup>+</sup>	.0553 290
.88	.1278 551	.1144 166	.1023 337	.0914 792	.0817 362	.0729 975 <sup>+</sup>
.89	.1587 653	.1436 426	.1298 909	.1173 968	.1060 547	.0957 662
.90	.1961 559	.1793 951	.1639 825 <sup>+</sup>	.1498 222	.1368 230	.1248 987
.91	.2410 613	.2228 126	.2058 452	.1900 832	.1754 529	.1618 834
.92	.2945 535	.2751 032	.2568 199	.2396 489	.2235 356	.2084 262
.93	.3579 797	.3374 703	.3182 872	.3000 606	.2827 884	.2664 078
.94	.4312 944	.4110 177	.3915 432	.3728 558	.3549 383	.3377 720
.95	.5159 391	.4964 706	.4775 734	.4592 471	.4414 889	.4242 942
.96	.6113 904	.5938 500	.5766 470	.5597 872	.5432 780	.5271 244
.97	.7190 562	.7017 469	.6875 651	.6735 222	.6596 282	.6458 918
.98	.8256 911	.8159 305 <sup>+</sup>	.8061 571	.7963 797	.7866 062	.7768 440
.99	.9303 308	.9266 163	.9216 521	.9172 414	.9127 876	.9082 935 <sup>+</sup>
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $p = 28$  to 33 $x = .53$  to 1.00 $q = 1.5$ 

	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$	$p = 33$
$B(p, q) = .5902\ 8232 \times \frac{1}{10^4}$		$.5602\ 6797 \times \frac{1}{10^4}$	$.5327\ 1380 \times \frac{1}{10^4}$	$.5073\ 4648 \times \frac{1}{10^4}$	$.4839\ 3049 \times \frac{1}{10^4}$	$.4622\ 6196 \times \frac{1}{10^4}$
$x$						
.53	.0000 001					
.54	.0000 001	.0000 001				
.55	.0000 002	.0000 001	.0000 001	.0000 001		
.56	.0000 004	.0000 002	.0000 002	.0000 001	.0000 001	
.57	.0000 006	.0000 003	.0000 002	.0000 002	.0000 001	.0000 001
.58	.0000 010	.0000 006	.0000 003	.0000 002	.0000 002	.0000 001
.59	.0000 015 <sup>+</sup>	.0000 009	.0000 005 <sup>+</sup>	.0000 003	.0000 003	.0000 002
.60	.0000 024	.0000 015 <sup>-</sup>	.0000 009	.0000 005 <sup>+</sup>		
.61	.0000 038	.0000 023	.0000 015 <sup>-</sup>	.0000 009	.0000 006	.0000 003
.62	.0000 059	.0000 037	.0000 023	.0000 015 <sup>-</sup>	.0000 009	.0000 006
.63	.0000 091	.0000 058	.0000 037	.0000 024	.0000 015 <sup>+</sup>	.0000 010
.64	.0000 140	.0000 091	.0000 059	.0000 038	.0000 025 <sup>-</sup>	.0000 016
.65	.0000 213	.0000 141	.0000 093	.0000 061	.0000 040	.0000 027
.66	.0000 323	.0000 216	.0000 145 <sup>+</sup>	.0000 097	.0000 065 <sup>+</sup>	.0000 044
.67	.0000 485 <sup>-</sup>	.0000 330	.0000 225 <sup>-</sup>	.0000 153	.0000 104	.0000 071
.68	.0000 724	.0000 500 <sup>+</sup>	.0000 345 <sup>+</sup>	.0000 238	.0000 165 <sup>-</sup>	.0000 113
.69	.0001 074	.0000 753	.0000 528	.0000 370	.0000 250	.0000 181
.70	.0001 584	.0001 126	.0000 801	.0000 569	.0000 404	.0000 287
.71	.0002 320	.0001 674	.0001 207	.0000 870	.0000 626	.0000 451
.72	.0003 379	.0002 472	.0001 807	.0001 320	.0000 965 <sup>-</sup>	.0000 704
.73	.0004 892	.0003 628	.0002 689	.0001 992	.0001 475 <sup>+</sup>	.0001 092
.74	.0007 042	.0005 293	.0003 977	.0002 987	.0002 242	.0001 682
.75	.0010 079	.0007 678	.0005 846	.0004 449	.0003 385 <sup>+</sup>	.0002 574
.76	.0014 345 <sup>+</sup>	.0011 073	.0008 543	.0006 588	.0005 079	.0003 913
.77	.0020 305 <sup>+</sup>	.0015 878	.0012 410	.0009 696	.0007 572	.0005 911
.78	.0028 585 <sup>-</sup>	.0022 641	.0017 925 <sup>-</sup>	.0014 185 <sup>-</sup>	.0011 221	.0008 872
.79	.0040 024	.0032 105 <sup>-</sup>	.0025 741	.0020 630	.0016 527	.0013 245
.80	.0055 743	.0045 275 <sup>+</sup>	.0036 757	.0029 829	.0024 197	.0019 620
.81	.0077 225 <sup>+</sup>	.0063 501	.0052 193	.0042 880	.0035 215 <sup>+</sup>	.0028 910
.82	.0106 423	.0088 580	.0073 696	.0061 288	.0050 949	.0042 339
.83	.0145 890	.0122 894	.0103 479	.0087 096	.0073 278	.0061 631
.84	.0198 938	.0169 576	.0144 487	.0123 061	.0104 772	.0089 170
.85	.0269 834	.0232 710	.0200 611	.0172 872	.0148 914	.0128 231
.86	.0364 029	.0317 585 <sup>+</sup>	.0276 955 <sup>-</sup>	.0241 430	.0210 386	.0183 271
.87	.0488 422	.0430 979	.0380 142	.0335 177	.0295 426	.0260 303
.88	.0651 650 <sup>+</sup>	.0581 494	.0518 691	.0462 504	.0412 262	.0367 358
.89	.0864 397	.0779 910	.0703 420	.0634 209	.0571 618	.0515 042
.90	.1139 681	.1039 549	.0947 877	.0863 996	.0787 287	.0717 169
.91	.1493 064	.1376 569	.1268 732	.1168 964	.1076 711	.0991 449
.92	.1942 683	.1810 103	.1686 027	.1569 974	.1461 484	.1360 113
.93	.2508 917	.2362 041	.2223 090	.2091 713	.1967 562	.1850 297
.94	.3213 368	.3056 118	.2905 756	.2762 059	.2624 805 <sup>-</sup>	.2493 771
.95	.4076 567	.3915 688	.3760 219	.3610 061	.3465 110	.3325 255 <sup>+</sup>
.96	.5113 298	.4958 963	.4808 246	.4661 146	.4517 653	.4377 747
.97	.6323 207	.6189 217	.6057 006	.5926 623	.5798 113	.5671 509
.98	.7670 998	.7573 800	.7476 904	.7380 363	.7284 226	.7188 541
.99	.9037 620	.8991 957	.8945 970	.8899 685 <sup>+</sup>	.8853 124	.8806 308
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE 1. THE  $I_w(p, q)$  FUNCTION

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 $p$  59 to 1.00 $q = 1.5$  $p = 34$  to 39

$p$	$p = 34$	$p = 35$	$p = 36$	$p = 37$	$p = 38$	$p = 39$
$I_w(p, q)$	$1.4421 0301 \times 10^4$	$1.4234 8004 \times 10^4$	$1.4060 7733 \times 10^4$	$1.3898 3424 \times 10^4$	$1.3746 4589 \times 10^4$	$1.3604 1883 \times 10^4$
.59	.0000 001					
.60	.0000 001	.0000 001				
.61	.0000 002	.0000 001	.0000 001	.0000 001		
.62	.0000 004	.0000 002	.0000 001	.0000 001	.0000 001	
.63	.0000 006	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001
.64	.0000 011	.0000 007	.0000 004	.0000 003	.0000 002	.0000 001
.65	.0000 018	.0000 012	.0000 008	.0000 005 <sup>+</sup>	.0000 003	.0000 002
.66	.0000 029	.0000 020	.0000 013	.0000 009	.0000 006	.0000 004
.67	.0000 048	.0000 033	.0000 022	.0000 015 <sup>+</sup>	.0000 010	.0000 007
.68	.0000 078	.0000 054	.0000 037	.0000 026	.0000 018	.0000 012
.69	.0000 127	.0000 089	.0000 062	.0000 043	.0000 030	.0000 021
.70	.0000 204	.0000 144	.0000 102	.0000 073	.0000 051	.0000 036
.71	.0000 325 <sup>+</sup>	.0000 234	.0000 168	.0000 121	.0000 087	.0000 062
.72	.0000 514	.0000 375 <sup>+</sup>	.0000 274	.0000 199	.0000 145 <sup>+</sup>	.0000 106
.73	.0000 808	.0000 598	.0000 442	.0000 327	.0000 241	.0000 178
.74	.0001 202	.0000 946	.0000 709	.0000 531	.0000 398	.0000 298
.75	.0001 957	.0001 487	.0001 130	.0000 858	.0000 651	.0000 494
.76	.0003 014	.0002 321	.0001 787	.0001 375 <sup>-</sup>	.0001 058	.0000 813
.77	.0004 613	.0003 598	.0002 806	.0002 188	.0001 705 <sup>-</sup>	.0001 328
.78	.0007 013	.0005 542	.0004 377	.0003 457	.0002 729	.0002 154
.79	.0010 595 <sup>-</sup>	.0008 478	.0006 783	.0005 424	.0004 337	.0003 466
.80	.0015 904	.0012 887	.0010 439	.0008 454	.0006 844	.0005 539
.81	.0023 725 <sup>-</sup>	.0019 463	.0015 962	.0013 087	.0010 727	.0008 790
.82	.0035 171	.0029 208	.0024 247	.0020 124	.0016 696	.0013 849
.83	.0051 817	.0043 551	.0036 593	.0030 737	.0025 811	.0021 669
.84	.0075 866	.0064 526	.0054 864	.0046 636	.0039 630	.0033 668
.85	.0110 384	.0094 992	.0081 721	.0070 284	.0060 431	.0051 946
.86	.0159 599	.0138 941	.0120 922	.0105 210	.0091 515 <sup>+</sup>	.0079 583
.87	.0229 282	.0201 898	.0177 732	.0156 417	.0137 622	.0121 055 <sup>-</sup>
.88	.0327 244	.0291 424	.0259 453	.0230 927	.0205 485 <sup>+</sup>	.0182 802
.89	.0463 925 <sup>+</sup>	.0417 763	.0376 092	.0338 480	.0304 571	.0273 986
.90	.0653 107	.0594 603	.0541 198	.0492 465 <sup>+</sup>	.0448 013	.0407 480
.91	.0912 685 <sup>-</sup>	.0839 956	.0772 827	.0710 891	.0653 767	.0601 101
.92	.1205 439	.1177 058	.1094 586	.1017 658	.0945 927	.0879 067
.93	.1739 588	.1635 114	.1536 566	.1443 644	.1356 058	.1273 531
.94	.2368 732	.2249 467	.2135 757	.2027 385 <sup>-</sup>	.1924 138	.1825 809
.95	.3190 380	.3060 364	.2935 085 <sup>+</sup>	.2814 417	.2698 233	.2586 405 <sup>+</sup>
.96	.4241 402	.4108 586	.3979 261	.3853 385 <sup>+</sup>	.3730 911	.3611 789
.97	.5546 844	.5424 140	.5303 416	.5184 688	.5067 966	.4953 256
.98	.7093 348	.6998 687	.6904 595 <sup>-</sup>	.6811 103	.6718 243	.6626 043
.99	.8759 258	.8711 994	.8664 534	.8616 895 <sup>+</sup>	.8569 096	.8521 152
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .64$  to  $1.00$  $q = 1.5$  $p = 40$  to  $45$ 

	$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$B(p, q) = .3470\ 6998 \times \frac{1}{10^8}$	$.3345\ 2529 \times \frac{1}{10^8}$	$.3227\ 1851 \times \frac{1}{10^8}$	$.3115\ 9029 \times \frac{1}{10^8}$	$.3010\ 8724 \times \frac{1}{10^8}$	$.2911\ 6120 \times \frac{1}{10^8}$	
$x$						
.64	.0000 001	.0000.001				
.65	.0000 001	.0000 001	.0000 001			
.66	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001	.0000 001
.67	.0000 005 <sup>-</sup>	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.68	.0000 008	.0000 006	.0000 004	.0000 003	.0000 002	.0000 002
.69	.0000 015 <sup>-</sup>	.0000 010	.0000 007	.0000 005 <sup>+</sup>	.0000 004	.0000 002
.70	.0000 026	.0000 018	.0000 013	.0000 009	.0000 006	.0000 005
.71	.0000 045 <sup>-</sup>	.0000 032	.0000 023	.0000 017	.0000 012	.0000 009
.72	.0000 077	.0000 056	.0000 041	.0000 030	.0000 022	.0000 016
.73	.0000 132	.0000 097	.0000 072	.0000 053	.0000 039	.0000 029
.74	.0000 223	.0000 167	.0000 125 <sup>-</sup>	.0000 094	.0000 070	.0000 052
.75	.0000 375 <sup>+</sup>	.0000 284	.0000 216	.0000 164	.0000 124	.0000 094
.76	.0000 625 <sup>+</sup>	.0000 481	.0000 369	.0000 284	.0000 218	.0000 167
.77	.0001 035 <sup>-</sup>	.0000 806	.0000 627	.0000 488	.0000 380	.0000 296
.78	.0001 699	.0001 340	.0001 057	.0000 833	.0000 657	.0000 518
.79	.0002 770	.0002 213	.0001 767	.0001 411	.0001 126	.0000 899
.80	.0004 482	.0003 626	.0002 932	.0002 371	.0001 916	.0001 549
.81	.0007 200	.0005 897	.0004 828	.0003 953	.0003 235	.0002 647
.82	.0011 484	.0009 521	.0007 892	.0006 540	.0005 418	.0004 488
.83	.0018 187	.0015 261	.0012 802	.0010 737	.0009 004	.0007 548
.84	.0028 596	.0024 282	.0020 614	.0017 496	.0014 847	.0012 506
.85	.0044 641	.0038 355 <sup>-</sup>	.0032 946	.0028 294	.0024 293	.0020 855
.86	.0069 190	.0060 139	.0052 261	.0045 406	.0039 441	.0034 254
.87	.0106 457	.0093 598	.0082 274	.0072 305 <sup>+</sup>	.0063 531	.0055 811
.88	.0162 584	.0144 571	.0128 525 <sup>+</sup>	.0114 237	.0101 518	.0090 197
.89	.0246 417	.0221 573	.0199 193	.0179 037	.0160 800	.0144 556
.90	.0370 532	.0336 864	.0306 193	.0278 266	.0252 828	.0229 686
.91	.0552 560	.0507 836	.0466 641	.0428 707	.0393 786	.0361 646
.92	.0816 767	.0758 735 <sup>-</sup>	.0704 694	.0654 384	.0607 561	.0563 995
.93	.1195 797	.1122 601	.1053 698	.0988 856	.0927 852	.0870 473
.94	.1732 195 <sup>+</sup>	.1643 098	.1558 325 <sup>+</sup>	.1477 690	.1401 012	.1328 115
.95	.2478 808	.2375 312	.2275 792	.2180 123	.2088 180	.1999 842
.96	.3495 966	.3383 385 <sup>+</sup>	.3273 990	.3167 721	.3064 516	.2964 313
.97	.4840 560	.4729 879	.4621 208	.4514 543	.4409 875 <sup>+</sup>	.4307 194
.98	.6534 529	.6443 723	.6353 640	.6264 325 <sup>+</sup>	.6175 770	.6088 000
.99	.8473 079	.8424 892	.8376 605 <sup>+</sup>	.8328 232	.8279 786	.8231 270
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .68$  to  $1.00$  $q = 1.5$  $p = 46$  to  $50$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$B(p, q) \times 10^3$	$.2817\ 6899 \times \frac{1}{10^3}$	$.2728\ 7102 \times \frac{1}{10^3}$	$.2644\ 3171 \times \frac{1}{10^3}$	$.2564\ 1863 \times \frac{1}{10^3}$	$.2488\ 0224 \times \frac{1}{10^3}$
$x$					
.68	.0000 001	.0000 001			
.69	.0000 002	.0000 001	.0000 001	.0000 001	
.70	.0000 003	.0000 002	.0000 002	.0000 001	.0000 001
.71	.0000 006	.0000 004	.0000 003	.0000 002	.0000 002
.72	.0000 011	.0000 008	.0000 006	.0000 004	.0000 003
.73	.0000 021	.0000 016	.0000 012	.0000 009	.0000 006
.74	.0000 030	.0000 029	.0000 022	.0000 016	.0000 012
.75	.0000 071	.0000 054	.0000 041	.0000 031	.0000 023
.76	.0000 128	.0000 099	.0000 076	.0000 058	.0000 045 <sup>-</sup>
.77	.0000 230	.0000 179	.0000 139	.0000 108	.0000 084
.78	.0000 408	.0000 321	.0000 253	.0000 199	.0000 157
.79	.0000 717	.0000 572	.0000 457	.0000 364	.0000 290
.80	.0001 252	.0001 011	.0000 817	.0000 660	.0000 533
.81	.0002 165 <sup>+</sup>	.0001 771	.0001 448	.0001 184	.0000 968
.82	.0003 716	.0003 077	.0002 547	.0002 108	.0001 745 <sup>-</sup>
.83	.0006 327	.0005 302	.0004 443	.0003 722	.0003 117
.84	.0010 685 <sup>-</sup>	.0009 061	.0007 683	.0006 514	.0005 521
.85	.0017 899	.0015 359	.0013 178	.0011 304	.0009 695 <sup>+</sup>
.86	.0029 742	.0025 821	.0022 412	.0019 450 <sup>+</sup>	.0016 877
.87	.0049 020	.0043 048	.0037 796	.0033 180	.0029 123
.88	.0080 125 <sup>-</sup>	.0071 164	.0063 195 <sup>+</sup>	.0056 110	.0049 810
.89	.0120 856	.0116 632	.0104 737	.0094 040	.0084 423
.90	.0208 615 <sup>-</sup>	.0189 450 <sup>-</sup>	.0172 018	.0156 166	.0141 753
.91	.0332 074	.0304 871	.0279 853	.0256 849	.0235 702
.92	.0523 468	.0485 778	.0450 735 <sup>-</sup>	.0418 159	.0387 883
.93	.0816 518	.0765 794	.0718 119	.0673 319	.0631 231
.94	.1258 831	.1192 996	.1130 453	.1071 050 <sup>-</sup>	.1014 641
.95	.1914 988	.1833 501	.1755 264	.1680 165 <sup>+</sup>	.1608 093
.96	.2867 050 <sup>-</sup>	.2772 662	.2681 086	.2592 257	.2506 111
.97	.4206 486	.4107 737	.4010 932	.3916 053	.3823 081
.98	.6000 939	.5914 783	.5829 452	.5744 957	.5661 306
.99	.8182 723	.8134 131	.8085 513	.8036 879	.7988 241
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\dagger$   
= .01 to .60

$q = 2$

$p = 2$  to  $4.5$

	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$	$p = 4$	$p = 4.5$
$B(p, q) = .1666\ 6667$		$.1142\ 8571$	$.8333\ 3333 \times \frac{1}{10}$	$.6349\ 2063 \times \frac{1}{10}$	$.5000\ 0000 \times \frac{1}{10}$	$.4040\ 4040 \times \frac{1}{10}$
$x$						
.01	.0002 980 <sup>e</sup>	.0000 347	.0000 040	.0000 004	.0000 008	.0000 001
.02	.0011 840 <sup>e</sup>	.0001 952	.0000 315 <sup>+</sup>	.0000 050 <sup>+</sup>	.0000 040	.0000 008
.03	.0026 460 <sup>e</sup>	.0005 339	.0001 056	.0000 206	.0000 124	.0000 027
.04	.0046 720 <sup>e</sup>	.0010 880	.0002 483	.0000 558	.0000 300 <sup>e</sup>	.0000 074
.05	.0072 500 <sup>e</sup>	.0018 867	.0004 812	.0001 209	.0000 617	.0000 160
.06	.0103 680 <sup>e</sup>	.0029 541	.0008 251	.0002 270	.0001 133	.0000 329
.07	.0140 140 <sup>e</sup>	.0043 106	.0013 000	.0003 861	.0001 917	.0000 595 <sup>1</sup>
.08	.0181 760 <sup>e</sup>	.0059 736	.0019 251	.0006 111	.0003 044	.0001 003
.09	.0228 420 <sup>e</sup>	.0079 582	.0027 192	.0009 153	.0004 600 <sup>e</sup>	.0001 597
.10	.0280 000 <sup>e</sup>	.0102 774	.0037 000 <sup>e</sup>	.0013 123		
.11	.0336 380 <sup>e</sup>	.0129 423	.0048 848	.0018 165 <sup>+</sup>	.0006 676	.0002 439
.12	.0397 440 <sup>e</sup>	.0159 626	.0062 899	.0024 423	.0009 373	.0003 563
.13	.0463 060 <sup>e</sup>	.0193 465 <sup>-</sup>	.0079 312	.0032 042	.0012 795 <sup>+</sup>	.0005 061
.14	.0533 120 <sup>e</sup>	.0231 010	.0098 235 <sup>+</sup>	.0041 171	.0017 057	.0007 000
.15	.0607 500 <sup>e</sup>	.0272 319	.0119 812	.0051 958	.0022 275 <sup>e</sup>	.0009 460
.16	.0686 080 <sup>e</sup>	.0317 440 <sup>e</sup>	.0144 179	.0064 553	.0028 574	.0012 530
.17	.0768 740 <sup>e</sup>	.0366 410	.0171 464	.0079 103	.0036 081	.0016 300
.18	.0855 360 <sup>e</sup>	.0419 258	.0201 787	.0095 756	.0044 930	.0020 888
.19	.0945 820 <sup>e</sup>	.0476 003	.0235 264	.0114 658	.0055 250	.0026 386
.20	.1040 000 <sup>e</sup>	.0536 656	.0272 000 <sup>e</sup>	.0135 953	.0067 200 <sup>e</sup>	.0032 915
.21	.1137 780 <sup>e</sup>	.0601 222	.0312 096	.0159 784	.0080 904	.0040 595 <sup>1</sup>
.22	.1239 040 <sup>e</sup>	.0669 698	.0355 643	.0186 289	.0096 513	.0049 554
.23	.1343 660 <sup>e</sup>	.0742 071	.0402 728	.0215 607	.0114 175 <sup>+</sup>	.0059 923
.24	.1451 520 <sup>e</sup>	.0818 326	.0453 427	.0247 868	.0134 038	.0071 841
.25	.1562 500 <sup>e</sup>	.0898 437	.0507 812	.0283 203	.0156 250 <sup>e</sup>	.0085 449
.26	.1676 480 <sup>e</sup>	.0982 377	.0565 947	.0321 737	.0180 962	.0100 895 <sup>-</sup>
.27	.1793 340 <sup>e</sup>	.1070 109	.0627 888	.0363 591	.0208 325 <sup>-</sup>	.0118 328
.28	.1912 960 <sup>e</sup>	.1161 591	.0693 683	.0408 880	.0238 487	.0137 901
.29	.2035 220 <sup>e</sup>	.1256 776	.0763 376	.0457 716	.0271 596	.0159 780
.30	.2160 000 <sup>e</sup>	.1355 613	.0837 000 <sup>e</sup>	.0510 204	.0307 800 <sup>e</sup>	.0184 117
.31	.2287 180 <sup>e</sup>	.1458 044	.0914 584	.0566 444	.0347 244	.0211 977
.32	.2416 640 <sup>e</sup>	.1564 007	.0996 147	.0626 530	.0390 070	.0240 825
.33	.2548 260 <sup>e</sup>	.1673 434	.1081 704	.0690 550 <sup>-</sup>	.0436 419	.0273 526
.34	.2681 920 <sup>e</sup>	.1786 254	.1171 259	.0758 585 <sup>-</sup>	.0486 426	.0309 347
.35	.2817 500 <sup>e</sup>	.1902 389	.1264 812	.0830 710	.0540 225 <sup>e</sup>	.0348 454
.36	.2954 880 <sup>e</sup>	.2021 760 <sup>e</sup>	.1362 355 <sup>+</sup>	.0906 993	.0597 943	.0391 015
.37	.3093 940 <sup>e</sup>	.2144 280	.1463 872	.0987 493	.0659 705 <sup>-</sup>	.0437 103
.38	.3234 560 <sup>e</sup>	.2269 861	.1569 339	.1072 261	.0725 627	.0487 153
.39	.3376 620 <sup>e</sup>	.2398 407	.1678 728	.1161 351	.0795 824	.0541 056
.40	.3520 000 <sup>e</sup>	.2529 822	.1792 000 <sup>e</sup>	.1254 792	.0870 400 <sup>e</sup>	.0599 662
.41	.3664 580 <sup>e</sup>	.2664 004	.1909 112	.1352 614	.0949 456	.0661 325
.42	.3810 240 <sup>e</sup>	.2800 847	.2030 011	.1454 840	.1033 083	.0727 900
.43	.3956 860 <sup>e</sup>	.2940 241	.2154 640	.1561 480	.1121 367	.0799 222
.44	.4104 320 <sup>e</sup>	.3082 073	.2282 931	.1672 538	.1214 383	.0875 144
.45	.4252 500 <sup>e</sup>	.3226 227	.2414 812	.1788 009	.1312 200 <sup>e</sup>	.0955 897
.46	.4401 280 <sup>e</sup>	.3372 581	.2550 203	.1907 876	.1414 876	.1041 608
.47	.4550 540 <sup>e</sup>	.3521 013	.2689 016	.2032 117	.1522 460	.1132 398
.48	.4700 160 <sup>e</sup>	.3671 393	.2831 155 <sup>+</sup>	.2160 695 <sup>-</sup>	.1634 902	.1228 178
.49	.4850 020 <sup>e</sup>	.3823 592	.2976 520	.2293 567	.1752 500 <sup>-</sup>	.1329 651
.50	.5000 000 <sup>e</sup>	.3977 476	.3125 000 <sup>e</sup>	.2430 680	.1875 000 <sup>e</sup>	.1436 311
.51	.5149 980 <sup>e</sup>	.4132 905 <sup>+</sup>	.3276 480	.2571 967	.2002 499	.1548 448
.52	.5299 840 <sup>e</sup>	.4289 741	.3430 835 <sup>+</sup>	.2717 356	.2134 992	.1666 104
.53	.5449 460 <sup>e</sup>	.4447 838	.3587 936	.2866 759	.2272 459	.1780 307
.54	.5598 720 <sup>e</sup>	.4607 049	.3747 643	.3020 082	.2414 868	.1918 272
.55	.5747 500 <sup>e</sup>	.4767 225 <sup>+</sup>	.3909 812	.3177 215 <sup>+</sup>	.2562 175 <sup>e</sup>	.2052 851
.56	.5895 680 <sup>e</sup>	.4928 212	.4074 291	.3338 042	.2714 321	.2193 120
.57	.6043 140 <sup>e</sup>	.5089 853	.4240 920	.3502 432	.2871 232	.2339 079
.58	.6189 760 <sup>e</sup>	.5251 989	.4409 531	.3670 244	.3032 821	.2490 714
.59	.6335 420 <sup>e</sup>	.5414 459	.4579 952	.3841 325 <sup>-</sup>	.3198 983	.2647 900
.60	.6480 000 <sup>e</sup>	.5577 096	.4752 000 <sup>e</sup>	.4015 509	.3369 600 <sup>e</sup>	.2810 856

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$p = 2$  to 4.5

	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$	$p = 4$	$p = 4.5$
$\int (p, q)$	$\cdot 10660\ 6067$	$\cdot 11142\ 8571$	$\cdot 8333\ 3333 \times \frac{1}{10}$	$\cdot 6349\ 2063 \times \frac{1}{10}$	$\cdot 5000\ 0000 \times \frac{1}{10}$	$\cdot 4040\ 4040 \times \frac{1}{10}$
$x$						
$\cdot 61$	$\cdot 6623\ 380^a$	$\cdot 5739\ 733$	$\cdot 4925\ 488$	$\cdot 4192\ 621$	$\cdot 3544\ 535^+$	$\cdot 2979\ 243$
$\cdot 62$	$\cdot 6705\ 440^a$	$\cdot 5902\ 109$	$\cdot 5100\ 219$	$\cdot 4372\ 470$	$\cdot 3723\ 037$	$\cdot 3153\ 058$
$\cdot 63$	$\cdot 6906\ 060^a$	$\cdot 6064\ 320$	$\cdot 5275\ 992$	$\cdot 4554\ 856$	$\cdot 3906\ 734$	$\cdot 3332\ 189$
$\cdot 64$	$\cdot 7045\ 120^a$	$\cdot 6225\ 920^a$	$\cdot 5452\ 595^+$	$\cdot 4739\ 564$	$\cdot 4093\ 041$	$\cdot 3516\ 504$
$\cdot 65$	$\cdot 7182\ 500^a$	$\cdot 6386\ 820$	$\cdot 5629\ 812$	$\cdot 4926\ 367$	$\cdot 4284\ 150^a$	$\cdot 3705\ 846$
$\cdot 66$	$\cdot 7318\ 080^a$	$\cdot 6546\ 838$	$\cdot 5807\ 419$	$\cdot 5115\ 027$	$\cdot 4478\ 038$	$\cdot 3900\ 033$
$\cdot 67$	$\cdot 7451\ 740^a$	$\cdot 6705\ 789$	$\cdot 5985\ 184$	$\cdot 5305\ 289$	$\cdot 4675\ 060$	$\cdot 4098\ 859$
$\cdot 68$	$\cdot 7583\ 360^a$	$\cdot 6863\ 487$	$\cdot 6162\ 867$	$\cdot 5496\ 899$	$\cdot 4874\ 954$	$\cdot 4302\ 094$
$\cdot 69$	$\cdot 7712\ 820^a$	$\cdot 7019\ 741$	$\cdot 6340\ 224$	$\cdot 5689\ 540$	$\cdot 5077\ 435^+$	$\cdot 4509\ 480$
$\cdot 70$	$\cdot 7840\ 000^a$	$\cdot 7174\ 360$	$\cdot 6517\ 000^a$	$\cdot 5882\ 975^-$	$\cdot 5262\ 200^a$	$\cdot 4720\ 729$
$\cdot 71$	$\cdot 7964\ 780^a$	$\cdot 7327\ 148$	$\cdot 6692\ 936$	$\cdot 6076\ 861$	$\cdot 5488\ 923$	$\cdot 4935\ 527$
$\cdot 72$	$\cdot 8087\ 040^a$	$\cdot 7477\ 909$	$\cdot 6867\ 763$	$\cdot 6270\ 886$	$\cdot 5697\ 257$	$\cdot 5153\ 528$
$\cdot 73$	$\cdot 8206\ 660^a$	$\cdot 7626\ 442$	$\cdot 7041\ 208$	$\cdot 6464\ 718$	$\cdot 5906\ 834$	$\cdot 5374\ 358$
$\cdot 74$	$\cdot 8323\ 520^a$	$\cdot 7772\ 545^-$	$\cdot 7212\ 987$	$\cdot 6658\ 009$	$\cdot 6117\ 262$	$\cdot 5597\ 608$
$\cdot 75$	$\cdot 8437\ 500^a$	$\cdot 7916\ 013$	$\cdot 7382\ 812$	$\cdot 6850\ 396$	$\cdot 6328\ 125^a$	$\cdot 5822\ 837$
$\cdot 76$	$\cdot 8548\ 480^a$	$\cdot 8056\ 640$	$\cdot 7550\ 387$	$\cdot 7041\ 593$	$\cdot 6538\ 986$	$\cdot 6049\ 570$
$\cdot 77$	$\cdot 8656\ 340^a$	$\cdot 8194\ 215^+$	$\cdot 7715\ 408$	$\cdot 7230\ 940$	$\cdot 6749\ 384$	$\cdot 6277\ 297$
$\cdot 78$	$\cdot 8760\ 960^a$	$\cdot 8328\ 527$	$\cdot 7877\ 563$	$\cdot 7418\ 300$	$\cdot 6958\ 831$	$\cdot 6505\ 472$
$\cdot 79$	$\cdot 8862\ 220^a$	$\cdot 8459\ 361$	$\cdot 8036\ 536$	$\cdot 7603\ 163$	$\cdot 7166\ 815^-$	$\cdot 6733\ 510$
$\cdot 80$	$\cdot 8960\ 000^a$	$\cdot 8586\ 501$	$\cdot 8192\ 000^a$	$\cdot 7785\ 094$	$\cdot 7372\ 800^a$	$\cdot 6960\ 790$
$\cdot 81$	$\cdot 9054\ 180^a$	$\cdot 8709\ 727$	$\cdot 8343\ 624$	$\cdot 7963\ 643$	$\cdot 7576\ 223$	$\cdot 7186\ 650^+$
$\cdot 82$	$\cdot 9144\ 640^a$	$\cdot 8828\ 819$	$\cdot 8491\ 067$	$\cdot 8138\ 345^-$	$\cdot 7776\ 494$	$\cdot 7410\ 387$
$\cdot 83$	$\cdot 9231\ 260^a$	$\cdot 8943\ 553$	$\cdot 8633\ 984$	$\cdot 8308\ 718$	$\cdot 7972\ 998$	$\cdot 7631\ 258$
$\cdot 84$	$\cdot 9313\ 920^a$	$\cdot 9053\ 703$	$\cdot 8772\ 019$	$\cdot 8474\ 266$	$\cdot 8165\ 090$	$\cdot 7848\ 474$
$\cdot 85$	$\cdot 9392\ 500^a$	$\cdot 9159\ 041$	$\cdot 8904\ 812$	$\cdot 8634\ 478$	$\cdot 8352\ 100^a$	$\cdot 8061\ 205^+$



TABLE I. THE  $I_w(p, q)$  FUNCTION $x = .61$  to  $1.00$  $q = 2$ 

	$p = 2$	$p = 2.5$	$p = 3$	$p = 3.5$	$p = 4$
$B(p, q)$	$.1666\ 6667$	$.1142\ 8571$	$.8333\ 3333 \times \frac{1}{10}$	$.6349\ 2063 \times \frac{1}{10}$	$.5000\ 0000 \times \frac{1}{10}$
$x$					
.61	.6623 380 <sup>e</sup>	.5730 733	.4925 488	.4192 621	.3544 535 <sup>+</sup>
.62	.6705 440 <sup>e</sup>	.5902 109	.5100 219	.4372 470	.3723 637
.63	.6906 060 <sup>e</sup>	.6064 320	.5275 992	.4554 856	.3906 734
.64	.7045 120 <sup>e</sup>	.6225 920 <sup>e</sup>	.5452 595 <sup>+</sup>	.4739 504	.4093 641
.65	.7182 500 <sup>e</sup>	.6386 820	.5629 812	.4926 367	.4284 150 <sup>e</sup>
.66	.7318 080 <sup>e</sup>	.6546 838	.5807 419	.5115 027	.4478 038
.67	.7451 740 <sup>e</sup>	.6705 789	.5985 184	.5305 289	.4675 060
.68	.7583 360 <sup>e</sup>	.6863 487	.6162 867	.5496 890	.4874 954
.69	.7712 820 <sup>e</sup>	.7019 741	.6340 224	.5689 549	.5077 435 <sup>+</sup>
.70	.7840 000 <sup>e</sup>	.7174 360	.6517 000 <sup>e</sup>	.5882 975 <sup>-</sup>	.5282 200 <sup>e</sup>
.71	.7964 780 <sup>e</sup>	.7327 148	.6692 936	.6076 861	.5488 923
.72	.8087 040 <sup>e</sup>	.7477 909	.6867 763	.6270 886	.5697 257
.73	.8206 660 <sup>e</sup>	.7626 442	.7041 208	.6464 718	.5906 834
.74	.8323 520 <sup>e</sup>	.7772 545 <sup>-</sup>	.7212 987	.6658 009	.6117 262
.75	.8437 500 <sup>e</sup>	.7916 013	.7382 812	.6850 396	.6328 125 <sup>e</sup>
.76	.8548 480 <sup>e</sup>	.8056 040	.7550 387	.7041 503	.6538 986
.77	.8656 340 <sup>e</sup>	.8194 215 <sup>+</sup>	.7715 408	.7230 940	.6749 384
.78	.8760 960 <sup>e</sup>	.8328 527	.7877 563	.7418 300	.6958 831
.79	.8862 220 <sup>e</sup>	.8459 361	.8036 536	.7603 163	.7166 815 <sup>-</sup>
.80	.8960 000 <sup>e</sup>	.8586 501	.8192 000 <sup>e</sup>	.7785 094	.7372 800 <sup>e</sup>
.81	.9054 180 <sup>e</sup>	.8709 727	.8343 624	.7963 643	.7576 223
.82	.9144 640 <sup>e</sup>	.8828 819	.8491 067	.8138 345 <sup>-</sup>	.7776 494
.83	.9231 260 <sup>e</sup>	.8943 553	.8633 984	.8308 718	.7972 998
.84	.9313 920 <sup>e</sup>	.9053 703	.8772 019	.8474 266	.8165 090
.85	.9392 500 <sup>e</sup>	.9159 041	.8904 812	.8634 478	.8352 100 <sup>e</sup>
.86	.9466 880 <sup>e</sup>	.9259 337	.9031 995 <sup>+</sup>	.8788 826	.8533 327
.87	.9536 940 <sup>e</sup>	.9354 358	.9153 192	.8936 766	.8708 044
.88	.9602 560 <sup>e</sup>	.9443 871	.9268 019	.9077 739	.8875 491
.89	.9663 620 <sup>e</sup>	.9527 637	.9376 088	.9211 170	.9034 883
.90	.9720 000 <sup>e</sup>	.9605 418	.9477 000 <sup>e</sup>	.9336 467	.9185 400 <sup>e</sup>
.91	.9771 580 <sup>e</sup>	.9676 974	.9570 352	.9453 021	.9326 195 <sup>-</sup>
.92	.9818 240 <sup>e</sup>	.9742 060	.9655 731	.9560 209	.9456 387
.93	.9859 860 <sup>e</sup>	.9800 432	.9732 720	.9657 388	.9575 066
.94	.9896 320 <sup>e</sup>	.9851 843	.9800 891	.9743 991	.9681 287
.95	.9927 500 <sup>e</sup>	.9896 042	.9859 812	.9819 073	.9774 075 <sup>e</sup>
.96	.9953 280 <sup>e</sup>	.9932 779	.9909 043	.9882 212	.9852 420
.97	.9973 540 <sup>e</sup>	.9961 800	.9948 136	.9932 609	.9915 279
.98	.9988 160 <sup>e</sup>	.9982 849	.9976 635 <sup>+</sup>	.9969 538	.9961 576
.99	.9997 020 <sup>e</sup>	.9995 669	.9994 080	.9992 256	.9990 199
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$p = 5$  to

$q = 2$

03 to 60

	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$	$p = 7.5$
$b, q) = .3333\ 3333 \times \frac{1}{10}$	$.2797\ 2028 \times \frac{1}{10}$	$.2380\ 9524 \times \frac{1}{10}$	$.2051\ 2821 \times \frac{1}{10}$	$.1785\ 7143 \times \frac{1}{10}$	$.1508\ 6275 \times \frac{1}{10}$	
$\times$						
03	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001
04	.0000 006	.0000 004	.0000 003	.0000 002	.0000 002	.0000 001
05	.0000 018	.0000 012	.0000 008	.0000 005 <sup>+</sup>	.0000 004	.0000 002
06	.0000 044	.0000 027	.0000 017	.0000 011	.0000 007	.0000 005 <sup>+</sup>
07	.0000 095 <sup>-</sup>	.0000 056	.0000 034	.0000 022		.0000 009
08	.0000 184	.0000 106	.0000 064 <sup>e</sup>	.0000 040	.0000 014	.0000 017
09	.0000 328	.0000 188		.0000 070	.0000 026	.0000 044
10	.0000 550 <sup>e</sup>			.0000 116	.0000 044	.0000 074
				.0000 186	.0000 074	.0000 119
11	.0000 878	.0000 315 <sup>-</sup>	.0000 112	.0000 288	.0000 185 <sup>-</sup>	.0000 122
12	.0001 344	.0000 503	.0000 188	.0000 434	.0000 279	.0000 186
13	.0001 986	.0000 774	.0000 300	.0000 636	.0000 413	.0000 276
14	.0002 850 <sup>+</sup>	.0001 153	.0000 464	.0000 913	.0000 596	.0000 401
15	.0003 987	.0001 669	.0000 695 <sup>-</sup>	.0001 285 <sup>-</sup>	.0000 845 <sup>-</sup>	
16	.0005 453	.0002 357	.0001 013	.0001 775 <sup>-</sup>		
17	.0007 312	.0003 258	.0001 443			
18	.0009 637	.0004 417	.0002 014			
19	.0012 504	.0005 888	.0002 757			
20	.0016 000 <sup>e</sup>	.0007 728	.0003 712 <sup>e</sup>			
21	.0020 216	.0010 004	.0004 923	.0002 411	.0001 176	.0000 572
22	.0025 253	.0012 787	.0006 440	.0003 228	.0001 611	.0000 801
23	.0031 216	.0016 159	.0008 320	.0004 263	.0002 176	.0001 100
24	.0038 221	.0020 207	.0010 625 <sup>+</sup>	.0005 561	.0002 899	.0001 509 <sup>+</sup>
25	.0046 387	.0025 024	.0013 428	.0007 172	.0003 815 <sup>-</sup>	.0002 022
26	.0055 842	.0030 716	.0016 805 <sup>+</sup>	.0009 152	.0004 964	.0002 683
27	.0066 722	.0037 391	.0020 843	.0011 565 <sup>+</sup>	.0006 391	.0003 519
28	.0079 168	.0045 170	.0025 637	.0014 484	.0008 150 <sup>-</sup>	.0004 560
29	.0093 326	.0054 179	.0031 288	.0017 986	.0010 298	.0005 876
30	.0109 350 <sup>e</sup>	.0064 552	.0037 908 <sup>e</sup>	.0022 161	.0012 903	.0007 487
31	.0127 400	.0076 432	.0045 618	.0027 104	.0016 040	.0009 459
32	.0147 640	.0089 971	.0054 546	.0032 921	.0019 791	.0011 856
33	.0170 239	.0105 326	.0064 832	.0039 728	.0024 250 <sup>-</sup>	.0014 751
34	.0195 372	.0122 663	.0076 622	.0047 651	.0029 518	.0018 223
35	.0223 218	.0142 156	.0090 075 <sup>+</sup>	.0056 824	.0035 708	.0022 362
36	.0253 958	.0163 984	.0105 356	.0067 393	.0042 944	.0027 271
37	.0287 777	.0188 335 <sup>-</sup>	.0122 642	.0079 510	.0051 358	.0033 959
38	.0324 864	.0215 401	.0142 116	.0093 360	.0061 098	.0039 859
39	.0365 408	.0245 382	.0163 973	.0109 104	.0072 321	.0047 777
40	.0409 600 <sup>e</sup>	.0278 483	.0188 410 <sup>e</sup>	.0126 930	.0085 197	.0056 691
41	.0457 632	.0314 912	.0215 655 <sup>-</sup>	.0147 059	.0099 909	.0067 611
42	.0509 696	.0354 883	.0245 909	.0169 683	.0116 653	.0079 943
43	.0565 983	.0398 614	.0279 404	.0195 031	.0135 637	.0094 02
44	.0626 682	.0446 324	.0316 375 <sup>+</sup>	.0223 337	.0157 085 <sup>-</sup>	.0110 12
		.0488 226	.0357 062	.0254 843	.0181 230	.0128 466

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .61$  to  $1.00$  $q = 2$ 

	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$
$B(p, q)$	$3.333\ 3333 \times 10^{-1}$	$2.797\ 2028 \times 10^{-1}$	$2.386\ 0524 \times 10^{-1}$	$2.051\ 2821 \times 10^{-1}$	$1.785\ 7141$
$\lambda$					
.61	.2491 550	.2074 602	.1720 781	.1422 438	.1172 243
.62	.2656 785 <sup>†</sup>	.2220 014	.1863 048	.1551 942	.1288 911
.63	.2828 444	.2390 737	.2013 257	.1680 782	.1414 004
.64	.3006 477	.2559 801	.2171 535 <sup>†</sup>	.1836 184	.1548 112
.65	.3190 709	.2736 200	.2337 080	.1991 352	.1691 269
.66	.3381 208	.2910 937	.2512 680	.2155 463	.1843 844
.67	.3577 832	.3110 027	.2695 660	.2328 664	.2006 096
.68	.3780 227	.3309 086	.2886 030	.2511 069	.2178 248
.69	.3988 350	.3514 287	.3086 450	.2702 753	.2360 494
.70	.4201 750 <sup>c</sup>	.3726 362	.3294 172 <sup>c</sup>	.2903 750 <sup>+</sup>	.2552 983
.71	.4420 362	.3945 102	.3500 948	.3114 046	.2755 821
.72	.4643 802	.4170 253	.3733 617	.3333 576	.2969 061
.73	.4871 718	.4401 514	.3964 957	.3562 216	.3192 698
.74	.5103 715 <sup>†</sup>	.4638 534	.4203 686	.3799 780	.3426 661
.75	.5340 355 <sup>†</sup>	.4880 907	.4449 463	.4046 015 <sup>+</sup>	.3670 807
.76	.5578 156	.5128 174	.4701 878	.4300 593	.3924 912
.77	.5819 586	.5379 813	.4960 453	.4563 103	.4188 665 <sup>+</sup>
.78	.6063 066	.5635 242	.5224 631	.4833 049	.4461 654
.79	.6307 066	.5893 812	.5493 776	.5109 839	.4743 366
.80	.6553 600 <sup>c</sup>	.6154 804	.5767 168 <sup>c</sup>	.5392 781	.5033 165 <sup>-</sup>
.81	.6799 230	.6417 427	.6043 902	.5681 070	.5330 293
.82	.7044 057	.6680 812	.6323 330	.5973 788	.5633 852
.83	.7287 225 <sup>†</sup>	.6944 012	.6604 196	.6269 887	.5942 795 <sup>+</sup>
.84	.7527 815 <sup>-</sup>	.7205 094	.6885 441	.6568 187	.6255 915 <sup>+</sup>
.85	.7764 843	.7465 639	.7165 841	.6867 365 <sup>-</sup>	.6571 830
.86	.7997 259	.7721 732	.7444 037	.7165 912	.6888 971
.87	.8223 045 <sup>†</sup>	.7972 968	.7718 546	.7462 280	.7205 567
.88	.8443 711	.8217 937	.7987 750 <sup>+</sup>	.7754 564	.7519 631
.89	.8655 202	.8455 126	.8240 880	.8040 798	.7828 946
.90	.8857 350 <sup>c</sup>	.8662 914	.8503 051 <sup>c</sup>	.8318 792	.8131 047
.91	.9048 466	.8899 566	.8745 186	.8586 147	.8423 205 <sup>-</sup>
.92	.9227 141	.9103 231	.8974 054	.8840 248	.8702 407
.93	.9391 793	.9291 931	.9187 261	.9078 250 <sup>+</sup>	.8965 343
.94	.9540 752	.9463 566	.9382 220	.9297 065 <sup>-</sup>	.9208 382
.95	.9672 262	.9615 902	.9556 195 <sup>-</sup>	.9493 346	.9427 553
.96	.9784 472	.9746 566	.9706 107	.9663 480	.9618 528
.97	.9875 441	.9853 045 <sup>-</sup>	.9829 070	.9803 568	.9776 592
.98	.9943 129	.9921 679	.9921 435 <sup>-</sup>	.9909 413	.9896 631
.99	.9985 396	.9982 654	.9979 600	.9976 504	.9973 099
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q = 2$  $p = 8$

	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$
$p, q) = .1388\ 8889 \times \frac{1}{10}$	$.1238\ 3901 \times \frac{1}{10}$	$.1111\ 1111 \times \frac{1}{10}$	$.1002\ 5063 \times \frac{1}{10}$	$.9090\ 9091 \times \frac{1}{10}$	$.8281\ 5735$	
$\cdot 10$	$\cdot 0000\ 001$					
$\cdot 11$	$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 12$	$\cdot 0000\ 003$	$\cdot 0000\ 001$				
$\cdot 13$	$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 14$	$\cdot 0000\ 012$	$\cdot 0000\ 005$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 15$	$\cdot 0000\ 020$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	
$\cdot 16$	$\cdot 0000\ 033$	$\cdot 0000\ 014$	$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 17$	$\cdot 0000\ 053$	$\cdot 0000\ 023$	$\cdot 0000\ 010$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 18$	$\cdot 0000\ 083$	$\cdot 0000\ 037$	$\cdot 0000\ 017$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 19$	$\cdot 0000\ 127$	$\cdot 0000\ 058$	$\cdot 0000\ 027$	$\cdot 0000\ 012$	$\cdot 0000\ 006$	$\cdot 0000\ 003$
$\cdot 20$	$\cdot 0000\ 189$	$\cdot 0000\ 089$	$\cdot 0000\ 042$	$\cdot 0000\ 020$	$\cdot 0000\ 009$	$\cdot 0000\ 004$
$\cdot 21$	$\cdot 0000\ 277$	$\cdot 0000\ 134$	$\cdot 0000\ 064$	$\cdot 0000\ 031$	$\cdot 0000\ 015$	$\cdot 0000\ 007$
$\cdot 22$	$\cdot 0000\ 397$	$\cdot 0000\ 196$	$\cdot 0000\ 097$	$\cdot 0000\ 048$	$\cdot 0000\ 023$	$\cdot 0000\ 011$
$\cdot 23$	$\cdot 0000\ 561$	$\cdot 0000\ 283$	$\cdot 0000\ 143$	$\cdot 0000\ 072$	$\cdot 0000\ 036$	$\cdot 0000\ 018$
$\cdot 24$	$\cdot 0000\ 779$	$\cdot 0000\ 402$	$\cdot 0000\ 207$	$\cdot 0000\ 106$	$\cdot 0000\ 055$	$\cdot 0000\ 028$
$\cdot 25$	$\cdot 0001\ 068$	$\cdot 0000\ 563$	$\cdot 0000\ 296$	$\cdot 0000\ 155$	$\cdot 0000\ 081$	$\cdot 0000\ 042$
$\cdot 26$	$\cdot 0001\ 445^+$	$\cdot 0000\ 776$	$\cdot 0000\ 416$	$\cdot 0000\ 222$	$\cdot 0000\ 119$	$\cdot 0000\ 063$
$\cdot 27$	$\cdot 0001\ 932$	$\cdot 0001\ 057$	$\cdot 0000\ 577$	$\cdot 0000\ 314$	$\cdot 0000\ 171$	$\cdot 0000\ 093$
$\cdot 28$	$\cdot 0002\ 554$	$\cdot 0001\ 423$	$\cdot 0000\ 791$	$\cdot 0000\ 439$	$\cdot 0000\ 243$	$\cdot 0000\ 134$
$\cdot 29$	$\cdot 0003\ 342$	$\cdot 0001\ 895^+$	$\cdot 0001\ 072$	$\cdot 0000\ 605^+$	$\cdot 0000\ 341$	$\cdot 0000\ 192$
$\cdot 30$	$\cdot 0004\ 330$	$\cdot 0002\ 498$	$\cdot 0001\ 437$	$\cdot 0000\ 825$	$\cdot 0000\ 472$	$\cdot 0000\ 270$
$\cdot 31$	$\cdot 0005\ 561$	$\cdot 0003\ 260$	$\cdot 0001\ 906$	$\cdot 0001\ 112$	$\cdot 0000\ 648$	$\cdot 0000\ 376$
$\cdot 32$	$\cdot 0007\ 081$	$\cdot 0004\ 217$	$\cdot 0002\ 505^+$	$\cdot 0001\ 485$	$\cdot 0000\ 878$	$\cdot 0000\ 518$
$\cdot 33$	$\cdot 0008\ 945$	$\cdot 0005\ 409$	$\cdot 0003\ 263$	$\cdot 0001\ 964$	$\cdot 0001\ 179$	$\cdot 0000\ 707$
$\cdot 34$	$\cdot 0011\ 215$	$\cdot 0006\ 883$	$\cdot 0004\ 214$	$\cdot 0002\ 574$	$\cdot 0001\ 569$	$\cdot 0000\ 955$
$\cdot 35$	$\cdot 0013\ 962$	$\cdot 0008\ 693$	$\cdot 0005\ 399$	$\cdot 0003\ 346$	$\cdot 0002\ 069$	$\cdot 0001\ 277$
$\cdot 36$	$\cdot 0017\ 265^+$	$\cdot 0010\ 901$	$\cdot 0006\ 865^+$	$\cdot 0004\ 314$	$\cdot 0002\ 706$	$\cdot 0001\ 694$
$\cdot 37$	$\cdot 0021\ 215^+$	$\cdot 0013\ 578$	$\cdot 0008\ 668$	$\cdot 0005\ 522$	$\cdot 0003\ 510$	$\cdot 0002\ 227$
$\cdot 38$	$\cdot 0025\ 913$	$\cdot 0016\ 805$	$\cdot 0010\ 871$	$\cdot 0007\ 017$	$\cdot 0004\ 520$	$\cdot 0002\ 906$
$\cdot 39$	$\cdot 0031\ 470$	$\cdot 0020\ 672$	$\cdot 0013\ 546$	$\cdot 0008\ 857$	$\cdot 0005\ 780$	$\cdot 0003\ 764$
$\cdot 40$	$\cdot 0038\ 011$	$\cdot 0025\ 284$	$\cdot 0016\ 777$	$\cdot 0011\ 108$	$\cdot 0007\ 340$	$\cdot 0004\ 841$
$\cdot 41$	$\cdot 0045\ 674$	$\cdot 0030\ 754$	$\cdot 0020\ 658$	$\cdot 0013\ 846$	$\cdot 0009\ 262$	$\cdot 0006\ 184$
$\cdot 42$	$\cdot 0054\ 610$	$\cdot 0037\ 211$	$\cdot 0025\ 295$	$\cdot 0017\ 157$	$\cdot 0011\ 615$	$\cdot 0007\ 848$
$\cdot 43$	$\cdot 0064\ 986$	$\cdot 0044\ 799$	$\cdot 0030\ 809$	$\cdot 0021\ 142$	$\cdot 0014\ 480$	$\cdot 0009\ 860$
$\cdot 44$	$\cdot 0076\ 984$	$\cdot 0053\ 675$	$\cdot 0037\ 335$	$\cdot 0025\ 913$	$\cdot 0017\ 950^+$	$\cdot 0012\ 412$
$\cdot 45$	$\cdot 0090\ 802$	$\cdot 0064\ 014$	$\cdot 0045\ 022$	$\cdot 0031\ 598$	$\cdot 0022\ 133$	$\cdot 0015\ 475$
$\cdot 46$	$\cdot 0106\ 653$	$\cdot 0076\ 007$	$\cdot 0054\ 040$	$\cdot 0038\ 341$	$\cdot 0027\ 149$	$\cdot 0019\ 190$
$\cdot 47$	$\cdot 0124\ 771$	$\cdot 0089\ 865$	$\cdot 0064\ 574$	$\cdot 0046\ 303$	$\cdot 0033\ 137$	$\cdot 0023\ 673$
$\cdot 48$	$\cdot 0145\ 405^+$	$\cdot 0105\ 816$	$\cdot 0076\ 828$	$\cdot 0055\ 664$	$\cdot 0040\ 254$	$\cdot 0029\ 058$
$\cdot 49$	$\cdot 0168\ 823$	$\cdot 0124\ 108$	$\cdot 0091\ 028$	$\cdot 0066\ 627$	$\cdot 0048\ 673$	$\cdot 0035\ 496$
$\cdot 50$	$\cdot 0195\ 312$	$\cdot 0145\ 012$	$\cdot 0107\ 422$	$\cdot 0079\ 411$	$\cdot 0058\ 594$	$\cdot 0043\ 158$
$\cdot 51$	$\cdot 0225\ 178$	$\cdot 0168\ 817$	$\cdot 0126\ 278$	$\cdot 0094\ 265$		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $1.00$  $q = 2$ 

	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$B(p, q) = .1388\ 8889 \times \frac{x}{10}$	$.1238\ 3901 \times \frac{x}{10}$	$.1111\ 1111 \times \frac{x}{10}$	$.1002\ 5063 \times \frac{x}{10}$	$.9090\ 9091 \times \frac{x}{10}$	
$x$					
.71	.2143 902	.1885 381	.1655 131	.1450 655 <sup>+</sup>	.1269 5
.72	.2339 941	.2071 300	.1830 354	.1614 878	.1422 6
.73	.2548 414	.2270 386	.2019 295 <sup>+</sup>	.1793 192	.1590 1
.74	.2769 520	.2482 988	.2222 450 <sup>-</sup>	.1986 236	.1772 6
.75	.3003 387	.2709 385 <sup>+</sup>	.2440 252	.2194 602	.1970 9
.76	.3250 062	.2949 777	.2673 064	.2418 817	.2185 8
.77	.3509 491	.3204 267	.2921 157	.2659 324	.2417 8
.78	.3781 516	.3472 850 <sup>-</sup>	.3184 694	.2916 468	.2667 4
.79	.4065 852	.3755 394	.3463 711	.3190 467	.2935 1
.80	.4362 076	.4051 620	.3758 096	.3481 392	.3221 2
.81	.4669 611	.4361 083	.4067 565 <sup>-</sup>	.3789 139	.3525 7
.82	.4987 704	.4683 152	.4391 632	.4113 399	.3848 5
.83	.5315 410	.5016 983	.4729 588	.4453 624	.4189 3
.84	.5651 570	.5361 495 <sup>+</sup>	.5080 464	.4808 989	.4547 4
.85	.5994 792	.5715 343	.5442 998	.5178 352	.4921 8
.86	.6343 420	.6076 884	.5815 600	.5560 210	.5311 2
.87	.6695 518	.6444 152	.6196 308	.5952 651	.5713 7
.88	.7048 837	.6814 815 <sup>+</sup>	.6582 750 <sup>+</sup>	.6353 297	.6127 0
.89	.7400 787	.7186 146	.6972 092	.6759 248	.6548 1
.90	.7748 410	.7554 975 <sup>+</sup>	.7360 989	.7167 017	.6973 5
.91	.8088 343	.7917 654	.7745 529	.7572 463	.7398 9
.92	.8416 790	.8270 006	.8121 175 <sup>+</sup>	.7970 710	.7818 9
.93	.8729 476	.8607 276	.8482 701	.8356 073	.8227 6
.94	.9021 620	.8926 085 <sup>+</sup>	.8824 120	.8721 963	.8617 8
.95	.9287 886	.9214 367	.9138 616	.9060 794	.8981 0
.96	.9522 342	.9471 315 <sup>-</sup>	.9418 462	.9363 879	.9307 6
.97	.9718 418	.9687 316	.9654 934	.9621 319	.9586 5
.98	.9868 851	.9853 885 <sup>+</sup>	.9838 224	.9821 881	.9804 8
.99	.9965 643	.9961 595 <sup>+</sup>	.9957 338	.9952 873	.9948 2
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$p = 11$  to 16

80

$q = 2$

$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$
$\cdot 7575\ 7576 \times \frac{1}{10^8}$	$\cdot 6410\ 2564 \times \frac{1}{10^8}$	$\cdot 5494\ 5055 \times \frac{1}{10^8}$	$\cdot 4761\ 9048 \times \frac{1}{10^8}$	$\cdot 4166\ 6667 \times \frac{1}{10^8}$	$\cdot 3676\ 4706 \times \frac{1}{10^8}$
$\cdot 0000\ 001$					
$\cdot 0000\ 001$					
$\cdot 0000\ 002$					
$\cdot 0000\ 003$	$\cdot 0000\ 001$				
$\cdot 0000\ 006$	$\cdot 0000\ 001$				
$\cdot 0000\ 009$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 014$	$\cdot 0000\ 004$	$\cdot 0000\ 001$			
$\cdot 0000\ 022$	$\cdot 0000\ 006$	$\cdot 0000\ 002$			
$\cdot 0000\ 034$	$\cdot 0000\ 009$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 0000\ 050^+$	$\cdot 0000\ 015^-$	$\cdot 0000\ 004$	$\cdot 0000\ 001$		
$\cdot 0000\ 074$	$\cdot 0000\ 022$	$\cdot 0000\ 007$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 107$	$\cdot 0000\ 034$	$\cdot 0000\ 010$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 0000\ 154$	$\cdot 0000\ 050^-$	$\cdot 0000\ 016$	$\cdot 0000\ 005^+$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 218$	$\cdot 0000\ 073$	$\cdot 0000\ 024$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0000\ 306$	$\cdot 0000\ 106$	$\cdot 0000\ 036$	$\cdot 0000\ 012$	$\cdot 0000\ 004$	$\cdot 0000\ 001$
$\cdot 0000\ 423$	$\cdot 0000\ 151$	$\cdot 0000\ 053$	$\cdot 0000\ 019$	$\cdot 0000\ 007$	$\cdot 0000\ 002$
$\cdot 0000\ 580$	$\cdot 0000\ 213$	$\cdot 0000\ 078$	$\cdot 0000\ 028$	$\cdot 0000\ 010$	$\cdot 0000\ 004$
$\cdot 0000\ 787$	$\cdot 0000\ 297$	$\cdot 0000\ 112$	$\cdot 0000\ 042$	$\cdot 0000\ 016$	$\cdot 0000\ 006$
$\cdot 0001\ 058$	$\cdot 0000\ 411$	$\cdot 0000\ 159$	$\cdot 0000\ 061$	$\cdot 0000\ 023$	$\cdot 0000\ 009$
$\cdot 0001\ 411$	$\cdot 0000\ 564$	$\cdot 0000\ 224$	$\cdot 0000\ 088$	$\cdot 0000\ 035^-$	$\cdot 0000\ 014$
$\cdot 0001\ 866$	$\cdot 0000\ 765^+$	$\cdot 0000\ 312$	$\cdot 0000\ 127$	$\cdot 0000\ 051$	$\cdot 0000\ 021$
$\cdot 0002\ 448$	$\cdot 0001\ 030$	$\cdot 0000\ 431$	$\cdot 0000\ 180$	$\cdot 0000\ 075^-$	$\cdot 0000\ 031$
$\cdot 0003\ 188$	$\cdot 0001\ 376$	$\cdot 0000\ 591$	$\cdot 0000\ 252$	$\cdot 0000\ 107$	$\cdot 0000\ 046$
$\cdot 0004\ 122$	$\cdot 0001\ 823$	$\cdot 0000\ 802$	$\cdot 0000\ 351$	$\cdot 0000\ 153$	$\cdot 0000\ 067$
$\cdot 0005\ 294$	$\cdot 0002\ 398$	$\cdot 0001\ 081$	$\cdot 0000\ 485^-$	$\cdot 0000\ 217$	$\cdot 0000\ 096$
$\cdot 0006\ 756$	$\cdot 0003\ 133$	$\cdot 0001\ 445^+$	$\cdot 0000\ 663$	$\cdot 0000\ 303$	$\cdot 0000\ 138$
$\cdot 0008\ 568$	$\cdot 0004\ 065^-$	$\cdot 0001\ 918$	$\cdot 0000\ 901$	$\cdot 0000\ 422$	$\cdot 0000\ 197$
$\cdot 0010\ 803$	$\cdot 0005\ 240$	$\cdot 0002\ 529$	$\cdot 0001\ 215^-$	$\cdot 0000\ 581$	$\cdot 0000\ 277$
$\cdot 0013\ 542$	$\cdot 0006\ 714$	$\cdot 0003\ 312$	$\cdot 0001\ 626$	$\cdot 0000\ 795^+$	$\cdot 0000\ 387$
$\cdot 0016\ 885^-$	$\cdot 0008\ 552$	$\cdot 0004\ 309$	$\cdot 0002\ 161$	$\cdot 0001\ 080$	$\cdot 0000\ 537$
$\cdot 0020\ 942$	$\cdot 0010\ 830$	$\cdot 0005\ 572$	$\cdot 0002\ 854$	$\cdot 0001\ 456$	$\cdot 0000\ 740$
$\cdot 0025\ 844$	$\cdot 0013\ 641$	$\cdot 0007\ 163$	$\cdot 0003\ 744$	$\cdot 0001\ 950^-$	$\cdot 0001\ 012$
$\cdot 0031\ 738$	$\cdot 0017\ 090$	$\cdot 0009\ 155^+$	$\cdot 0004\ 883$	$\cdot 0002\ 594$	$\cdot 0001\ 373$
$\cdot 0038\ 795^-$	$\cdot 0021\ 302$	$\cdot 0011\ 638$	$\cdot 0006\ 330$	$\cdot 0003\ 430$	$\cdot 0001\ 852$
$\cdot 0047\ 206$	$\cdot 0026\ 423$	$\cdot 0014\ 716$	$\cdot 0008\ 160$	$\cdot 0004\ 597$	$\cdot 0002\ 481$
$\cdot 0057\ 190$	$\cdot 0032\ 620$	$\cdot 0018\ 512$	$\cdot 0010\ 460$	$\cdot 0005\ 888$	$\cdot 0003\ 303$
$\cdot 0068\ 993$	$\cdot 0040\ 084$	$\cdot 0023\ 173$	$\cdot 0013\ 338$	$\cdot 0007\ 648$	$\cdot 0004\ 370$
$\cdot 0082\ 891$	$\cdot 0049\ 038$	$\cdot 0028\ 867$	$\cdot 0016\ 920$	$\cdot 0009\ 880$	$\cdot 0005\ 749$
$\cdot 0099\ 193$	$\cdot 0059\ 733$	$\cdot 0035\ 794$	$\cdot 0021\ 357$	$\cdot 0012\ 695^+$	$\cdot 0007\ 521$
$\cdot 0118\ 244$	$\cdot 0072\ 457$	$\cdot 0044\ 183$	$\cdot 0026\ 828$	$\cdot 0016\ 229$	$\cdot 0009\ 784$
$\cdot 0140\ 425^-$	$\cdot 0087\ 533$	$\cdot 0054\ 300$	$\cdot 0033\ 541$	$\cdot 0020\ 642$	$\cdot 0012\ 661$
$\cdot 0166\ 159$	$\cdot 0105\ 328$	$\cdot 0066\ 448$	$\cdot 0041\ 743$	$\cdot 0026\ 127$	$\cdot 0016\ 299$
$\cdot 0195\ 010$	$\cdot 0126\ 253$	$\cdot 0080\ 976$	$\cdot 0051\ 720$	$\cdot 0032\ 913$	$\cdot 0020\ 876$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $1.00$  $q = 2$ 

	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .7575\ 7576 \times \frac{1}{10^8}$		$.6410\ 2564 \times \frac{1}{10^8}$	$.5494\ 5055 \times \frac{1}{10^8}$	$.4761\ 9048 \times \frac{1}{10^8}$	$.4166\ 6667 \times \frac{1}{10^8}$
$x$					
.81	.3042 942	.2616 339	.2241 995 <sup>+</sup>	.1915 452	.1632 060
.82	.3358 680	.2920 474	.2531 200	.2187 442	.1885 426
.83	.3696 076	.3249 456	.2847 870	.2488 915 <sup>-</sup>	.2169 700
.84	.4054 910	.3603 581	.3192 871	.2821 337	.2486 957
.85	.4434 596	.3982 769	.3566 712	.3185 860	.2839 012
.86	.4834 112	.4386 481	.3969 438	.3583 192	.3227 293
.87	.5251 919	.4813 611	.4400 505 <sup>+</sup>	.4013 457	.3652 673
.88	.5685 876	.5262 376	.4858 640	.4476 022	.4115 268
.89	.6133 132	.5730 177	.5341 661	.4969 284	.4614 195 <sup>+</sup>
.90	.6590 023	.6213 450 <sup>-</sup>	.5846 291	.5490 430	.5147 278
.91	.7051 936	.6707 490	.6367 923	.6035 148	.5710 692
.92	.7513 183	.7206 261	.6900 363	.6597 288	.6298 543
.93	.7966 833	.7702 172	.7435 526	.7168 470	.6902 368
.94	.8404 550 <sup>+</sup>	.8185 829	.7963 099	.7737 627	.7510 544
.95	.8816 401	.8645 761	.8470 144	.8290 475 <sup>-</sup>	.8107 597
.96	.9190 646	.9068 104	.8940 661	.8808 904	.8673 382
.97	.9513 509	.9436 256	.9355 077	.9270 275 <sup>+</sup>	.9182 142
.98	.9768 922	.9730 487	.9689 682	.9646 617	.9601 398
.99	.9938 255 <sup>-</sup>	.9927 511	.9915 988	.9903 702	.9890 671
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

00

$q = 2$

$p = 17$  to 22

$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$
$\cdot 3267\ 9739 \times \frac{1}{10^8}$	$\cdot 2923\ 9766 \times \frac{1}{10^8}$	$\cdot 2631\ 5789 \times \frac{1}{10^8}$	$\cdot 2380\ 9524 \times \frac{1}{10^8}$	$\cdot 2164\ 5022 \times \frac{1}{10^8}$	$\cdot 1976\ 2846 \times \frac{1}{10^8}$
$\cdot 0000\ 001$					
$\cdot 0000\ 001$					
$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 0000\ 003$	$\cdot 0000\ 001$				
$\cdot 0000\ 005^+$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$		
$\cdot 0000\ 013$	$\cdot 0000\ 005^+$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 0000\ 019$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	
$\cdot 0000\ 029$	$\cdot 0000\ 012$	$\cdot 0000\ 005^+$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 043$	$\cdot 0000\ 019$	$\cdot 0000\ 008$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 063$	$\cdot 0000\ 028$	$\cdot 0000\ 013$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0000\ 091$	$\cdot 0000\ 042$	$\cdot 0000\ 020$	$\cdot 0000\ 009$	$\cdot 0000\ 004$	$\cdot 0000\ 002$
$\cdot 0000\ 132$	$\cdot 0000\ 062$	$\cdot 0000\ 030$	$\cdot 0000\ 014$	$\cdot 0000\ 007$	$\cdot 0000\ 003$
$\cdot 0000\ 188$	$\cdot 0000\ 091$	$\cdot 0000\ 044$	$\cdot 0000\ 021$	$\cdot 0000\ 010$	$\cdot 0000\ 005^-$
$\cdot 0000\ 267$	$\cdot 0000\ 132$	$\cdot 0000\ 065^+$	$\cdot 0000\ 032$	$\cdot 0000\ 016$	$\cdot 0000\ 008$
$\cdot 0000\ 375^+$	$\cdot 0000\ 190$	$\cdot 0000\ 096$	$\cdot 0000\ 048$	$\cdot 0000\ 024$	$\cdot 0000\ 012$
$\cdot 0000\ 523$	$\cdot 0000\ 270$	$\cdot 0000\ 139$	$\cdot 0000\ 071$	$\cdot 0000\ 037$	$\cdot 0000\ 019$
$\cdot 0000\ 725^-$	$\cdot 0000\ 381$	$\cdot 0000\ 200$	$\cdot 0000\ 105^-$	$\cdot 0000\ 055^-$	$\cdot 0000\ 029$
$\cdot 0000\ 997$	$\cdot 0000\ 535^+$	$\cdot 0000\ 286$	$\cdot 0000\ 153$	$\cdot 0000\ 082$	$\cdot 0000\ 043$
$\cdot 0001\ 361$	$\cdot 0000\ 745^-$	$\cdot 0000\ 407$	$\cdot 0000\ 222$	$\cdot 0000\ 120$	$\cdot 0000\ 065^+$
$\cdot 0001\ 847$	$\cdot 0001\ 030$	$\cdot 0000\ 573$	$\cdot 0000\ 318$	$\cdot 0000\ 176$	$\cdot 0000\ 097$
$\cdot 0002\ 490$	$\cdot 0001\ 415^-$	$\cdot 0000\ 802$	$\cdot 0000\ 453$	$\cdot 0000\ 256$	$\cdot 0000\ 144$
$\cdot 0003\ 336$	$\cdot 0001\ 930$	$\cdot 0001\ 114$	$\cdot 0000\ 642$	$\cdot 0000\ 369$	$\cdot 0000\ 212$
$\cdot 0004\ 442$	$\cdot 0002\ 617$	$\cdot 0001\ 538$	$\cdot 0000\ 902$	$\cdot 0000\ 528$	$\cdot 0000\ 308$
$\cdot 0005\ 881$	$\cdot 0003\ 526$	$\cdot 0002\ 109$	$\cdot 0001\ 258$	$\cdot 0000\ 749$	$\cdot 0000\ 445^+$
$\cdot 0007\ 743$	$\cdot 0004\ 723$	$\cdot 0002\ 873$	$\cdot 0001\ 745^-$	$\cdot 0001\ 057$	$\cdot 0000\ 639$
$\cdot 0010\ 138$	$\cdot 0006\ 289$	$\cdot 0003\ 892$	$\cdot 0002\ 403$	$\cdot 0001\ 481$	$\cdot 0000\ 911$
$\cdot 0013\ 203$	$\cdot 0008\ 328$	$\cdot 0005\ 240$	$\cdot 0003\ 291$	$\cdot 0002\ 062$	$\cdot 0001\ 290$
$\cdot 0017\ 105^+$	$\cdot 0010\ 968$	$\cdot 0007\ 016$	$\cdot 0004\ 478$	$\cdot 0002\ 853$	$\cdot 0001\ 814$
$\cdot 0022\ 049$	$\cdot 0014\ 367$	$\cdot 0009\ 339$	$\cdot 0006\ 058$	$\cdot 0003\ 922$	$\cdot 0002\ 535^-$
$\cdot 0028\ 282$	$\cdot 0018\ 722$	$\cdot 0012\ 365^-$	$\cdot 0008\ 149$	$\cdot 0005\ 360$	$\cdot 0003\ 519$
$\cdot 0036\ 103$	$\cdot 0024\ 274$	$\cdot 0016\ 283$	$\cdot 0010\ 900$	$\cdot 0007\ 282$	$\cdot 0004\ 857$
$\cdot 0045\ 868$	$\cdot 0031\ 316$	$\cdot 0021\ 331$	$\cdot 0014\ 500^-$	$\cdot 0009\ 837$	$\cdot 0006\ 662$
$\cdot 0058\ 006$	$\cdot 0040\ 204$	$\cdot 0027\ 802$	$\cdot 0019\ 186$	$\cdot 0013\ 214$	$\cdot 0009\ 086$
$\cdot 0073\ 025^+$	$\cdot 0051\ 370$	$\cdot 0036\ 054$	$\cdot 0025\ 253$	$\cdot 0017\ 654$	$\cdot 0012\ 320$
$\cdot 0091\ 525^-$	$\cdot 0065\ 329$	$\cdot 0046\ 527$	$\cdot 0033\ 068$	$\cdot 0023\ 459$	$\cdot 0016\ 613$
$\cdot 0114\ 209$	$\cdot 0082\ 701$	$\cdot 0059\ 752$	$\cdot 0043\ 084$	$\cdot 0031\ 008$	$\cdot 0022\ 279$
$\cdot 0141\ 905^-$	$\cdot 0104\ 218$	$\cdot 0076\ 373$	$\cdot 0055\ 855^-$	$\cdot 0040\ 774$	$\cdot 0029\ 715^-$
$\cdot 0175\ 569$	$\cdot 0130\ 750^-$	$\cdot 0097\ 160$	$\cdot 0072\ 057$	$\cdot 0053\ 342$	$\cdot 0039\ 422$
$\cdot 0216\ 309$	$\cdot 0163\ 313$	$\cdot 0123\ 037$	$\cdot 0092\ 511$	$\cdot 0069\ 434$	$\cdot 0052\ 027$
$\cdot 0265\ 400$	$\cdot 0203\ 100$	$\cdot 0155\ 094$	$\cdot 0118\ 205^+$	$\cdot 0089\ 930$	$\cdot 0068\ 307$
$\cdot 0324\ 294$	$\cdot 0251\ 489$	$\cdot 0194\ 621$	$\cdot 0150\ 323$	$\cdot 0115\ 904$	$\cdot 0089\ 221$
$\cdot 0394\ 640$	$\cdot 0310\ 074$	$\cdot 0243\ 126$	$\cdot 0190\ 273$	$\cdot 0148\ 651$	$\cdot 0115\ 947$
	$\cdot 0280\ 677$	$\cdot 0202\ 366$	$\cdot 0239\ 718$	$\cdot 0189\ 724$	$\cdot 0149\ 920$

TABLE I. THE  $I_w(p, q)$  FUNCTION

3 to 1.00		$q = 2$		$p =$	
$p = 23$		$p = 24$		$p = 25$	
$p = 26$		$p = 27$		$p = 28$	
$q) = .1811\ 5942 \times \frac{1}{10^8}$		$.1666\ 6667 \times \frac{1}{10^8}$		$.1538\ 4615 \times \frac{1}{10^8}$	
$.1424\ 5014 \times \frac{1}{10^8}$		$.1322\ 7513 \times \frac{1}{10^8}$		$.1231\ 5014 \times \frac{1}{10^8}$	
.0000 001		.0000 001		.0000 001	
.0000 001		.0000 001		.0000 001	
.0000 002		.0000 001		.0000 001	
.0000 004		.0000 002		.0000 001	
.0000 006		.0000 003		.0000 002	
.0000 010		.0000 005 <sup>-</sup>		.0000 002	
.0000 015 <sup>-</sup>		.0000 008		.0000 004	
.0000 023		.0000 012		.0000 006	
.0000 035 <sup>+</sup>		.0000 019		.0000 010	
.0000 054		.0000 030		.0000 016	
.0000 081		.0000 046		.0000 026	
.0000 121		.0000 069		.0000 040	
.0000 180		.0000 105 <sup>-</sup>		.0000 061	
.0000 264		.0000 157		.0000 093	
.0000 386		.0000 233		.0000 140	
.0000 560		.0000 343		.0000 210	
.0000 806		.0000 502		.0000 313	
.0001 152		.0000 730		.0000 462	
.0001 635 <sup>+</sup>		.0001 053		.0000 678	
.0002 307		.0001 510		.0000 987	
.0003 234		.0002 150 <sup>-</sup>		.0001 427	
.0004 505 <sup>-</sup>		.0003 041		.0002 050 <sup>+</sup>	
.0006 237		.0004 275 <sup>+</sup>		.0002 926	
.0008 584		.0005 973		.0004 150 <sup>-</sup>	
.0011 747		.0008 293		.0005 847	
.0015 982		.0011 448		.0008 189	
.0021 621		.0015 710		.0011 399	
.0029 090		.0021 435 <sup>-</sup>		.0015 773	
.0038 924		.0029 080		.0021 697	
.0051 804		.0039 233		.0029 674	
.0068 578		.0052 638		.0040 351	
.0090 305 <sup>+</sup>		.0070 237		.0054 559	
.0118 293		.0093 212		.0073 356	
.0154 148		.0123 034		.0098 078	
.0199 824		.0161 521		.0130 400	
.0257 686		.0210 905 <sup>+</sup>		.0172 408	
.0330 566		.0273 897		.0226 674	
.0421 822		.0353 765 <sup>+</sup>		.0296 342	
.0535 404		.0454 406		.0385 220	
.0675 892		.0580 414		.0497 865 <sup>+</sup>	
.0848 534		.0737 137		.0639 664	
.1059 242		.0930 705 <sup>+</sup>		.0816 896	
.1314 549		.1168 017		.1036 749	
.1621 405 <sup>+</sup>		.1456 664		.1307 285 <sup>-</sup>	
.1981 822		.1807 218		.1572 137	
.2395 404		.2167 822		.1947 822	
.2875 892		.2547 404		.2335 404	
.3421 822		.2947 822		.2735 822	
.4045 404		.3367 404		.3145 404	
.4747 822		.3807 822		.3567 822	
.5529 404		.4267 404		.3997 404	
.6391 822		.4747 822		.4477 822	
.7333 404		.5247 404		.4977 404	
.8455 822		.5767 822		.5497 822	
.9757 404		.6307 404		.6037 404	
1.1239 822		.6867 822		.6507 822	
1.2991 404		.7447 404		.7087 404	
1.4993 822		.8047 822		.7687 822	
1.7235 404		.8667 404		.8297 404	
1.9707 822		.9307 822		.8917 822	
2.2409 404		1.0000 000		.9537 404	
2.5441 822		1.0000 000		1.0000 000	



	$p = 29$	$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$
$q = .1149\ 4253 \times \frac{1}{10^8}$	$.1075\ 2688 \times \frac{1}{10^8}$	$.1008\ 0645 \times \frac{1}{10^8}$	$.9469\ 6970 \times \frac{1}{10^8}$	$.8912\ 6560 \times \frac{1}{10^8}$	$.8403\ 3613$	
52	.0000 001					
53	.0000 001	.0000 001				
54	.0000 002	.0000 001	.0000 001			
55	.0000 004	.0000 002	.0000 001	.0000 001		
56	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001	
57	.0000 011	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
58	.0000 018	.0000 011	.0000 006	.0000 004	.0000 002	.0000 001
59	.0000 029	.0000 018	.0000 011	.0000 007	.0000 004	.0000 002
60	.0000 046	.0000 029	.0000 018	.0000 011	.0000 007	.0000 004
61	.0000 073	.0000 046	.0000 029	.0000 018	.0000 011	.0000 007
62	.0000 115-	.0000 073	.0000 047	.0000 030	.0000 019	.0000 012
63	.0000 178	.0000 116	.0000 075+	.0000 049	.0000 032	.0000 020
64	.0000 274	.0000 181	.0000 119	.0000 079	.0000 052	.0000 034
65	.0000 419	.0000 281	.0000 188	.0000 126	.0000 084	.0000 056
66	.0000 635-	.0000 432	.0000 294	.0000 200	.0000 136	.0000 092
67	.0000 956	.0000 660	.0000 456	.0000 314	.0000 217	.0000 149
68	.0001 428	.0001 001	.0000 701	.0000 491	.0000 343	.0000 240
69	.0002 119	.0001 508	.0001 072	.0000 761	.0000 540	.0000 383
70	.0003 123	.0002 254	.0001 625+	.0001 171	.0000 843	.0000 606
71	.0004 572	.0003 346	.0002 447	.0001 788	.0001 305-	.0000 952
72	.0006 647	.0004 933	.0003 657	.0002 710	.0002 006	.0001 484
73	.0009 601	.0007 223	.0005 429	.0004 078	.0003 060	.0002 295
74	.0013 778	.0010 506	.0008 004	.0006 093	.0004 635-	.0003 523
75	.0019 644	.0015 179	.0011 719	.0009 041	.0006 969	.0005 368
76	.0027 830	.0021 788	.0017 044	.0013 322	.0010 404	.0008 120
77	.0039 177	.0031 071	.0024 621	.0019 494	.0015 424	.0012 194
78	.0054 802	.0044 020	.0035 330	.0028 332	.0022 704	.0018 181
79	.0076 178	.0061 963	.0050 359	.0040 896	.0033 187	.0026 912
80	.0105 225-	.0086 656	.0071 305+	.0058 629	.0048 171	.0039 551
81	.0144 426	.0120 400	.0100 289	.0083 475-	.0069 429	.0057 707
82	.0196 966	.0166 186	.0140 106	.0118 029	.0099 361	.0083 589
83	.0266 882	.0227 862	.0194 396	.0165 724	.0141 182	.0120 195
84	.0359 235-	.0310 318	.0267 858	.0231 041	.0199 148	.0171 546
85	.0480 289	.0419 692	.0366 467	.0319 767	.0278 831	.0242 982
86	.0637 684	.0563 582	.0497 729	.0439 269	.0387 423	.0341 484
87	.0840 575+	.0751 231	.0670 911	.0598 778	.0534 061	.0476 051
88	.1099 703	.0993 660	.0897 232	.0809 638	.0730 146	.0658 074
89	.1427 329	.1303 673	.1189 950+	.1085 472	.0989 581	.0901 651
90	.1836 950+	.1695 646	.1564 234	.1442 147	.1328 836	.1223 765
91	.2342 651	.2184 960	.2036 678	.1897 389	.1766 674	.1644 119
92	.2957 911	.2786 851	.2624 230	.2469 793	.2323 270	.2184 384
93	.3693 585+	.3514 392	.3342 186	.3176 869	.3018 320	.2866 401
94	.4554 685-	.4375 157	.4200 776	.4031 570	.3867 545+	.3708 691
95	.5535 421	.5365 969	.5199 624	.5036 499	.4876 687	.4720 265
96	.6611 797	.6464 868	.6319 115-	.6174 678	.6031 685+	.5890 253
97	.7730 755-	.7619 134	.7507 253	.7395 283	.7283 168	.7171 175
98	.8794 543	.8727 749	.8660 109	.8591 688	.8522 531	.8452 708
99	.9638 520	.9616 105-	.9593 174	.9569 740	.9545 816	.9521 413
00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .58$  to  $1.00$  $q = 2$ 

	$p = 35$	$p = 36$	$p = 37$	$p = 38$	$p = 39$
$B(p, q) = .7936\ 5079 \times \frac{1}{103}$		$.7507\ 5075 \times \frac{1}{103}$	$.7112\ 3755 \times \frac{1}{103}$	$.6747\ 6383 \times \frac{1}{103}$	$.6410\ 2564$
$x$					
.58	.0000 001				
.59	.0000 001	.0000 001	.0000 001		
.60	.0000 003	.0000 002	.0000 001	.0000 001	
.61	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001
.62	.0000 008	.0000 005 <sup>-</sup>	.0000 003	.0000 002	.0000 001
.63	.0000 013	.0000 009	.0000 006	.0000 004	.0000 002
.64	.0000 022	.0000 015 <sup>-</sup>	.0000 010	.0000 006	.0000 004
.65	.0000 038	.0000 025 <sup>+</sup>	.0000 017	.0000 011	.0000 007
.66	.0000 062	.0000 042	.0000 029	.0000 019	.0000 013
.67	.0000 103	.0000 071	.0000 048	.0000 033	.0000 023
.68	.0000 168	.0000 117	.0000 082	.0000 057	.0000 040
.69	.0000 271	.0000 192	.0000 136	.0000 096	.0000 068
.70	.0000 436	.0000 313	.0000 225 <sup>-</sup>	.0000 161	.0000 116
.71	.0000 694	.0000 506	.0000 368	.0000 268	.0000 195 <sup>-</sup>
.72	.0001 097	.0000 810	.0000 598	.0000 441	.0000 325 <sup>+</sup>
.73	.0001 720	.0001 288	.0000 964	.0000 721	.0000 539
.74	.0002 676	.0002 031	.0001 541	.0001 168	.0000 885 <sup>-</sup>
.75	.0004 132	.0003 178	.0002 443	.0001 877	.0001 441
.76	.0006 333	.0004 936	.0003 845 <sup>-</sup>	.0002 993	.0002 329
.77	.0009 634	.0007 607	.0006 003	.0004 734	.0003 731
.78	.0014 549	.0011 635 <sup>+</sup>	.0009 299	.0007 428	.0005 930
.79	.0021 809	.0017 662	.0014 296	.0011 564	.0009 349
.80	.0032 452	.0026 611	.0021 808	.0017 862	.0014 622
.81	.0047 933	.0039 790	.0033 011	.0027 372	.0022 684
.82	.0070 276	.0059 047	.0049 584	.0041 614	.0034 907
.83	.0102 263	.0086 954	.0073 895 <sup>+</sup>	.0062 763	.0053 281
.84	.0147 679	.0127 058	.0109 255 <sup>-</sup>	.0093 896	.0080 655
.85	.0211 613	.0184 188	.0160 229	.0139 314	.0121 068
.86	.0300 814	.0264 839	.0233 040	.0204 955 <sup>-</sup>	.0180 166
.87	.0424 099	.0377 608	.0336 038	.0298 895 <sup>-</sup>	.0265 729
.88	.0592 785 <sup>-</sup>	.0533 688	.0480 239	.0431 933	.0388 305 <sup>-</sup>
.89	.0821 092	.0747 346	.0679 889	.0618 230	.0561 909
.90	.1126 420	.1036 306	.0952 951	.0875 904	.0804 737
.91	.1529 315 <sup>-</sup>	.1421 857	.1321 355 <sup>-</sup>	.1227 426	.1139 701
.92	.2052 852	.1928 384	.1810 693	.1699 491	.1594 493
.93	.2720 961	.2581 837	.2448 858	.2321 845 <sup>-</sup>	.2200 614
.94	.3554 975 <sup>+</sup>	.3406 354	.3262 770	.3124 153	.2990 424
.95	.4567 293	.4417 818	.4271 872	.4129 477	.3990 641
.96	.5750 484	.5612 472	.5476 301	.5342 043	.5209 764
.97	.7059 347	.6947 775 <sup>+</sup>	.6836 544	.6725 734	.6615 419
.98	.8382 269	.8311 266	.8239 750 <sup>+</sup>	.8167 771	.8095 375 <sup>-</sup>
.99	.9496 544	.9471 220	.9445 452	.9419 253	.9392 634
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .62$  to  $1.00$  $q = 2$  $p = 41$  to  $45$ 

	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$B(p, q) = .5807\ 2009 \times \frac{1}{10^8}$	$.5537\ 0986 \times \frac{1}{10^8}$	$.5285\ 4123 \times \frac{1}{10^8}$	$.5050\ 5051 \times \frac{1}{10^8}$	$.4830\ 9179 \times \frac{1}{10^8}$	
$x$					
.62	.0000 001				
.63	.0000 001	.0000 001			
.64	.0000 002	.0000 001	.0000 001		.0000 001
.65	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.66	.0000 006	.0000 004	.0000 003	.0000 002	.0000 002
.67	.0000 011	.0000 007	.0000 005 <sup>+</sup>	.0000 003	.0000 004
.68	.0000 019	.0000 013	.0000 009	.0000 006	.0000 008
.69	.0000 034	.0000 024	.0000 017	.0000 012	.0000 016
.70	.0000 059	.0000 042	.0000 030	.0000 022	
.71	.0000 103	.0000 075 <sup>-</sup>	.0000 054	.0000 039	.0000 028
.72	.0000 177	.0000 130	.0000 096	.0000 070	.0000 052
.73	.0000 301	.0000 224	.0000 167	.0000 125 <sup>-</sup>	.0000 093
.74	.0000 507	.0000 384	.0000 290	.0000 219	.0000 166
.75	.0000 849	.0000 651	.0000 499	.0000 382	.0000 292
.76	.0001 407	.0001 093	.0000 849	.0000 659	.0000 511
.77	.0002 314	.0001 821	.0001 433	.0001 126	.0000 885 <sup>+</sup>
.78	.0003 774	.0003 008	.0002 397	.0001 909	.0001 519
.79	.0006 101	.0004 925 <sup>+</sup>	.0003 974	.0003 205 <sup>+</sup>	.0002 584
.80	.0009 783	.0007 997	.0006 533	.0005 336	.0004 356
.81	.0015 555 <sup>+</sup>	.0012 872	.0010 647	.0008 803	.0007 275 <sup>-</sup>
.82	.0024 525 <sup>+</sup>	.0020 543	.0017 199	.0014 394	.0012 041
.83	.0038 341	.0032 502	.0027 540	.0023 326	.0019 748
.84	.0059 425 <sup>+</sup>	.0050 974	.0043 705 <sup>+</sup>	.0037 458	.0032 091
.85	.0091 302	.0079 235 <sup>+</sup>	.0068 734	.0059 600	.0051 660
.86	.0139 026	.0122 046	.0107 095 <sup>+</sup>	.0093 939	.0082 367
.87	.0209 746	.0186 226	.0165 277	.0146 628	.0130 034
.88	.0313 409	.0281 390	.0252 543	.0226 567	.0203 189
.89	.0403 598	.0420 840	.0381 878	.0346 396	.0314 100
.90	.0678 443	.0622 571	.0571 089	.0523 678	.0480 038
.91	.0981 454	.0910 262	.0843 935 <sup>-</sup>	.0782 173	.0724 693
.92	.1401 989	.1313 939	.1231 004	.1152 930	.1079 469
.93	.1974 749	.1869 735 <sup>+</sup>	.1769 747	.1674 595 <sup>+</sup>	.1584 093
.94	.2737 273	.2617 656	.2502 538	.2391 811	.2285 363
.95	.3723 640	.3595 449	.3470 768	.3349 567	.3231 808
.96	.4951 360	.4825 325 <sup>+</sup>	.4701 451	.4579 766	.4460 294
.97	.6396 552	.6288 127	.6180 449	.6073 574	.5967 548
.98	.7949 512	.7876 132	.7782 508	.7728 677	.7654 679
.99	.9338 177	.9310 361	.9282 168	.9253 607	.9224 690
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .66$  to  $1.00$  $q = 2$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) = .4625\ 3469 \times \frac{1}{10^8}$		$.4432\ 6241 \times \frac{1}{10^8}$	$.4251\ 7007 \times \frac{1}{10^8}$	$.4081\ 6327 \times \frac{1}{10^8}$
$x$				
.66	.0000 001	.0000 001		
.67	.0000 002	.0000 001	.0000 001	.0000 001
.68	.0000 003	.0000 002	.0000 001	.0000 001
.69	.0000 006	.0000 004	.0000 003	.0000 002
.70	.0000 011	.0000 008	.0000 006	.0000 004
.71	.0000 021	.0000 015 <sup>-</sup>	.0000 011	.0000 008
.72	.0000 038	.0000 028	.0000 020	.0000 015 <sup>+</sup>
.73	.0000 069	.0000 052	.0000 038	.0000 029
.74	.0000 125 <sup>+</sup>	.0000 094	.0000 071	.0000 054
.75	.0000 224	.0000 171	.0000 131	.0000 100
.76	.0000 396	.0000 307	.0000 238	.0000 184
.77	.0000 695 <sup>+</sup>	.0000 546	.0000 429	.0000 336
.78	.0001 209	.0000 962	.0000 765 <sup>-</sup>	.0000 608
.79	.0002 083	.0001 678	.0001 351	.0001 087
.80	.0003 554	.0002 899	.0002 364	.0001 927
.81	.0006 010	.0004 963	.0004 097	.0003 381
.82	.0010 069	.0008 417	.0007 033	.0005 875 <sup>-</sup>
.83	.0016 713	.0014 139	.0011 958	.0010 109
.84	.0027 482	.0023 527	.0020 134	.0017 224
.85	.0044 761	.0038 769	.0033 568	.0029 055 <sup>-</sup>
.86	.0072 194	.0063 255 <sup>+</sup>	.0055 404	.0048 512
.87	.0115 277	.0102 159	.0090 503	.0080 151
.88	.0182 159	.0163 250 <sup>+</sup>	.0146 256	.0130 990
.89	.0284 717	.0257 997	.0233 711	.0211 647
.90	.0439 889	.0402 970	.0369 036	.0337 859
.91	.0671 224	.0621 509	.0575 305 <sup>+</sup>	.0532 385 <sup>-</sup>
.92	.1010 383	.0945 442	.0884 425 <sup>+</sup>	.0827 120
.93	.1498 056	.1416 302	.1338 653	.1264 935 <sup>-</sup>
.94	.2183 077	.2084 839	.1990 530	.1900 033
.95	.3117 452	.3006 452	.2898 758	.2794 318
.96	.4343 052	.4228 053	.4115 305 <sup>+</sup>	.4004 812
.97	.5862 417	.5758 224	.5655 006	.5552 799
.98	.7580 550 <sup>-</sup>	.7506 324	.7432 034	.7357 714
.99	.9195 425 <sup>+</sup>	.9165 823	.9135 894	.9105 647
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$p$  to  $\cdot 60$

$q = 2.5$

$p =$

	$p = 2.5$	$p = 3$	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$
$(p, q) = .7363\ 1078 \times \frac{1}{10}$		$.5079\ 3651 \times \frac{1}{10}$	$.3681\ 5539 \times \frac{1}{10}$	$.2770\ 5628 \times \frac{1}{10}$	$.2147\ 5731 \times \frac{1}{10}$	$.1704\ 9$
$x$						
$\cdot 01$	$\cdot 0000\ 537$	$\cdot 0000\ 065^-$	$\cdot 0000\ 008$	$\cdot 0000\ 001$		
$\cdot 02$	$\cdot 0003\ 007$	$\cdot 0000\ 513$	$\cdot 0000\ 086$	$\cdot 0000\ 014$	$\cdot 0000\ 002$	
$\cdot 03$	$\cdot 0008\ 198$	$\cdot 0001\ 712$	$\cdot 0000\ 350^+$	$\cdot 0000\ 070$	$\cdot 0000\ 014$	$\cdot 0000\ 00$
$\cdot 04$	$\cdot 0016\ 645^-$	$\cdot 0004\ 013$	$\cdot 0000\ 947$	$\cdot 0000\ 220$	$\cdot 0000\ 050^+$	$\cdot 0000\ 03$
$\cdot 05$	$\cdot 0028\ 758$	$\cdot 0007\ 746$	$\cdot 0002\ 044$	$\cdot 0000\ 530$	$\cdot 0000\ 136$	$\cdot 0000\ 03$
$\cdot 06$	$\cdot 0044\ 861$	$\cdot 0013\ 230$	$\cdot 0003\ 822$	$\cdot 0001\ 086$	$\cdot 0000\ 305^-$	$\cdot 0000\ 03$
$\cdot 07$	$\cdot 0065\ 218$	$\cdot 0020\ 762$	$\cdot 0006\ 476$	$\cdot 0001\ 987$	$\cdot 0000\ 602$	$\cdot 0000\ 13$
$\cdot 08$	$\cdot 0090\ 042$	$\cdot 0030\ 625^-$	$\cdot 0010\ 207$	$\cdot 0003\ 347$	$\cdot 0001\ 083$	$\cdot 0000\ 33$
$\cdot 09$	$\cdot 0119\ 506$	$\cdot 0043\ 085^+$	$\cdot 0015\ 224$	$\cdot 0005\ 293$	$\cdot 0001\ 816$	$\cdot 0000\ 63$
$\cdot 10$	$\cdot 0153\ 747$	$\cdot 0058\ 392$	$\cdot 0021\ 738$	$\cdot 0007\ 964$	$\cdot 0002\ 879$	$\cdot 0001\ 03$
$\cdot 11$	$\cdot 0192\ 876$	$\cdot 0076\ 779$	$\cdot 0029\ 963$	$\cdot 0011\ 508$	$\cdot 0004\ 362$	$\cdot 0001\ 63$
$\cdot 12$	$\cdot 0236\ 975^-$	$\cdot 0098\ 465^-$	$\cdot 0040\ 114$	$\cdot 0016\ 085^+$	$\cdot 0006\ 366$	$\cdot 0002\ 43$
$\cdot 13$	$\cdot 0286\ 103$	$\cdot 0123\ 651$	$\cdot 0052\ 404$	$\cdot 0021\ 862$	$\cdot 0009\ 003$	$\cdot 0003\ 63$
$\cdot 14$	$\cdot 0340\ 299$	$\cdot 0152\ 523$	$\cdot 0067\ 046$	$\cdot 0029\ 014$	$\cdot 0012\ 305^+$	$\cdot 0005\ 23$
$\cdot 15$	$\cdot 0399\ 583$	$\cdot 0185\ 255^-$	$\cdot 0084\ 247$	$\cdot 0037\ 721$	$\cdot 0016\ 675^-$	$\cdot 0007\ 23$
$\cdot 16$	$\cdot 0463\ 959$	$\cdot 0222\ 001$	$\cdot 0104\ 212$	$\cdot 0048\ 169$	$\cdot 0021\ 984$	$\cdot 0009\ 93$
$\cdot 17$	$\cdot 0533\ 411$	$\cdot 0262\ 903$	$\cdot 0127\ 139$	$\cdot 0060\ 549$	$\cdot 0028\ 473$	$\cdot 0013\ 23$
$\cdot 18$	$\cdot 0607\ 913$	$\cdot 0308\ 087$	$\cdot 0153\ 223$	$\cdot 0075\ 052$	$\cdot 0036\ 303$	$\cdot 0017\ 33$
$\cdot 19$	$\cdot 0687\ 422$	$\cdot 0357\ 667$	$\cdot 0182\ 650^+$	$\cdot 0091\ 875^-$	$\cdot 0045\ 641$	$\cdot 0022\ 44$
$\cdot 20$	$\cdot 0771\ 886$	$\cdot 0411\ 741$	$\cdot 0215\ 599$	$\cdot 0111\ 213$	$\cdot 0056\ 660$	$\cdot 0028\ 53$
$\cdot 21$	$\cdot 0861\ 238$	$\cdot 0470\ 391$	$\cdot 0252\ 242$	$\cdot 0133\ 263$	$\cdot 0069\ 543$	$\cdot 0035\ 93$
$\cdot 22$	$\cdot 0955\ 402$	$\cdot 0533\ 689$	$\cdot 0292\ 740$	$\cdot 0158\ 220$	$\cdot 0084\ 475^+$	$\cdot 0044\ 63$
$\cdot 23$	$\cdot 1054\ 291$	$\cdot 0601\ 690$	$\cdot 0337\ 248$	$\cdot 0186\ 278$	$\cdot 0101\ 648$	$\cdot 0054\ 93$
$\cdot 24$	$\cdot 1157\ 809$	$\cdot 0674\ 439$	$\cdot 0385\ 909$	$\cdot 0217\ 628$	$\cdot 0121\ 258$	$\cdot 0066\ 83$
$\cdot 25$	$\cdot 1265\ 850^-$	$\cdot 0751\ 965^+$	$\cdot 0438\ 857$	$\cdot 0252\ 457$	$\cdot 0143\ 502$	$\cdot 0080\ 73$
$\cdot 26$	$\cdot 1378\ 301$	$\cdot 0834\ 285^+$	$\cdot 0496\ 214$	$\cdot 0290\ 949$	$\cdot 0168\ 582$	$\cdot 0096\ 73$
$\cdot 27$	$\cdot 1495\ 041$	$\cdot 0921\ 404$	$\cdot 0558\ 093$	$\cdot 0333\ 282$	$\cdot 0196\ 609$	$\cdot 0114\ 93$
$\cdot 28$	$\cdot 1615\ 940$	$\cdot 1013\ 313$	$\cdot 0624\ 595^-$	$\cdot 0379\ 626$	$\cdot 0228\ 056$	$\cdot 0135\ 63$
$\cdot 29$	$\cdot 1740\ 864$	$\cdot 1109\ 992$	$\cdot 0695\ 808$	$\cdot 0430\ 149$	$\cdot 0262\ 856$	$\cdot 0159\ 03$
$\cdot 30$	$\cdot 1869\ 670$	$\cdot 1211\ 409$	$\cdot 0771\ 809$	$\cdot 0485\ 005^+$	$\cdot 0301\ 298$	$\cdot 0185\ 33$
$\cdot 31$	$\cdot 2002\ 209$	$\cdot 1317\ 520$	$\cdot 0852\ 664$	$\cdot 0544\ 346$	$\cdot 0343\ 580$	$\cdot 0214\ 73$
$\cdot 32$	$\cdot 2138\ 328$	$\cdot 1428\ 268$	$\cdot 0938\ 425^+$	$\cdot 0608\ 310$	$\cdot 0389\ 898$	$\cdot 0247\ 53$
$\cdot 33$	$\cdot 2277\ 868$	$\cdot 1543\ 587$	$\cdot 1029\ 132$	$\cdot 0677\ 028$	$\cdot 0440\ 442$	$\cdot 0283\ 83$
$\cdot 34$	$\cdot 2420\ 664$	$\cdot 1663\ 399$	$\cdot 1124\ 811$	$\cdot 0750\ 619$	$\cdot 0495\ 396$	$\cdot 0323\ 83$
$\cdot 35$	$\cdot 2566\ 548$	$\cdot 1787\ 614$	$\cdot 1225\ 476$	$\cdot 0829\ 192$	$\cdot 0554\ 939$	$\cdot 0367\ 93$
$\cdot 36$	$\cdot 2715\ 347$	$\cdot 1916\ 134$	$\cdot 1331\ 128$	$\cdot 0912\ 844$	$\cdot 0619\ 243$	$\cdot 0416\ 23$
$\cdot 37$	$\cdot 2866\ 884$	$\cdot 2048\ 850^-$	$\cdot 1441\ 754$	$\cdot 1001\ 659$	$\cdot 0688\ 471$	$\cdot 0468\ 93$
$\cdot 38$	$\cdot 3020\ 977$	$\cdot 2185\ 640$	$\cdot 1557\ 328$	$\cdot 1095\ 708$	$\cdot 0762\ 775^+$	$\cdot 0526\ 23$
$\cdot 39$	$\cdot 3177\ 444$	$\cdot 2326\ 377$	$\cdot 1677\ 811$	$\cdot 1195\ 051$	$\cdot 0842\ 301$	$\cdot 0588\ 33$
$\cdot 40$	$\cdot 3336\ 096$	$\cdot 2470\ 920$	$\cdot 1803\ 149$	$\cdot 1299\ 730$	$\cdot 0927\ 180$	$\cdot 0655\ 53$
$\cdot 41$	$\cdot 3496\ 744$	$\cdot 2619\ 123$	$\cdot 1933\ 277$	$\cdot 1409\ 776$	$\cdot 1017\ 533$	$\cdot 0728\ 03$
$\cdot 42$	$\cdot 3659\ 195^-$	$\cdot 2770\ 829$	$\cdot 2068\ 115^-$	$\cdot 1525\ 204$	$\cdot 1113\ 467$	$\cdot 0805\ 83$
$\cdot 43$	$\cdot 3823\ 255^-$	$\cdot 2925\ 871$	$\cdot 2207\ 568$	$\cdot 1646\ 013$	$\cdot 1215\ 075^-$	$\cdot 0885\ 93$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = \cdot 61$  to  $1\cdot 00$  $q = 2\cdot 5$ 

	$p = 2\cdot 5$	$p = 3$	$p = 3\cdot 5$	$p = 4$	$p =$
$B(p, q) = \cdot 7363\ 1078 \times \frac{1}{10}$		$\cdot 5079\ 3651 \times \frac{1}{10}$	$\cdot 3681\ 5539 \times \frac{1}{10}$	$\cdot 2770\ 5628 \times \frac{1}{10}$	$\cdot 2147$
$x$					
$\cdot 61$	$\cdot 6822\ 556$	$\cdot 6055\ 273$	$\cdot 5322\ 924$	$\cdot 4640\ 391$	$\cdot 4016$
$\cdot 62$	$\cdot 6979\ 023$	$\cdot 6233\ 144$	$\cdot 5515\ 373$	$\cdot 4840\ 939$	$\cdot 4218$
$\cdot 63$	$\cdot 7133\ 116$	$\cdot 6409\ 736$	$\cdot 5707\ 986$	$\cdot 5043\ 281$	$\cdot 4425$
$\cdot 64$	$\cdot 7284\ 653$	$\cdot 6584\ 781$	$\cdot 5900\ 433$	$\cdot 5247\ 060$	$\cdot 4634$
$\cdot 65$	$\cdot 7433\ 452$	$\cdot 6758\ 011$	$\cdot 6092\ 379$	$\cdot 5451\ 902$	$\cdot 4847$
$\cdot 66$	$\cdot 7579\ 336$	$\cdot 6929\ 160$	$\cdot 6283\ 482$	$\cdot 5657\ 420$	$\cdot 5061$
$\cdot 67$	$\cdot 7722\ 132$	$\cdot 7097\ 960$	$\cdot 6473\ 396$	$\cdot 5863\ 211$	$\cdot 5278$
$\cdot 68$	$\cdot 7861\ 672$	$\cdot 7264\ 146$	$\cdot 6661\ 769$	$\cdot 6068\ 862$	$\cdot 5496$
$\cdot 69$	$\cdot 7997\ 791$	$\cdot 7427\ 455^-$	$\cdot 6848\ 247$	$\cdot 6273\ 946$	$\cdot 5715$
$\cdot 70$	$\cdot 8130\ 330$	$\cdot 7587\ 624$	$\cdot 7032\ 470$	$\cdot 6478\ 024$	$\cdot 5934$
$\cdot 71$	$\cdot 8259\ 136$	$\cdot 7744\ 398$	$\cdot 7214\ 080$	$\cdot 6680\ 648$	$\cdot 6154$
$\cdot 72$	$\cdot 8384\ 060$	$\cdot 7897\ 521$	$\cdot 7392\ 714$	$\cdot 6881\ 361$	$\cdot 6373$
$\cdot 73$	$\cdot 8504\ 959$	$\cdot 8046\ 744$	$\cdot 7568\ 011$	$\cdot 7079\ 698$	$\cdot 6590$
$\cdot 74$	$\cdot 8621\ 699$	$\cdot 8191\ 823$	$\cdot 7739\ 612$	$\cdot 7275\ 185^-$	$\cdot 6807$
$\cdot 75$	$\cdot 8734\ 150^+$	$\cdot 8332\ 520$	$\cdot 7907\ 157$	$\cdot 7467\ 346$	$\cdot 7021$
$\cdot 76$	$\cdot 8842\ 191$	$\cdot 8468\ 603$	$\cdot 8070\ 291$	$\cdot 7655\ 701$	$\cdot 7232$
$\cdot 77$	$\cdot 8945\ 709$	$\cdot 8599\ 848$	$\cdot 8228\ 666$	$\cdot 7839\ 765^+$	$\cdot 7439$
$\cdot 78$	$\cdot 9044\ 598$	$\cdot 8726\ 042$	$\cdot 8381\ 936$	$\cdot 8019\ 058$	$\cdot 7643$
$\cdot 79$	$\cdot 9138\ 762$	$\cdot 8846\ 979$	$\cdot 8529\ 765^+$	$\cdot 8193\ 098$	$\cdot 7842$
$\cdot 80$	$\cdot 9228\ 114$	$\cdot 8962\ 464$	$\cdot 8671\ 827$	$\cdot 8361\ 409$	$\cdot 8036$
$\cdot 81$	$\cdot 9312\ 578$	$\cdot 9072\ 316$	$\cdot 8807\ 805^+$	$\cdot 8523\ 523$	$\cdot 8223$
$\cdot 82$	$\cdot 9392\ 087$	$\cdot 9176\ 365^-$	$\cdot 8937\ 398$	$\cdot 8678\ 980$	$\cdot 8404$
$\cdot 83$	$\cdot 9466\ 589$	$\cdot 9274\ 456$	$\cdot 9060\ 318$	$\cdot 8827\ 334$	$\cdot 8578$
$\cdot 84$	$\cdot 9536\ 041$	$\cdot 9366\ 451$	$\cdot 9176\ 295^-$	$\cdot 8968\ 154$	$\cdot 8744$
$\cdot 85$	$\cdot 9600\ 417$	$\cdot 9452\ 230$	$\cdot 9285\ 080$	$\cdot 9101\ 030$	$\cdot 8902$
$\cdot 86$	$\cdot 9659\ 701$	$\cdot 9531\ 692$	$\cdot 9386\ 448$	$\cdot 9225\ 576$	$\cdot 9050$
$\cdot 87$	$\cdot 9713\ 897$	$\cdot 9604\ 757$	$\cdot 9480\ 199$	$\cdot 9341\ 436$	$\cdot 9189$
$\cdot 88$	$\cdot 9763\ 025^+$	$\cdot 9671\ 370$	$\cdot 9566\ 165^-$	$\cdot 9448\ 285^+$	$\cdot 9318$
$\cdot 89$	$\cdot 9807\ 124$	$\cdot 9731\ 505^-$	$\cdot 9644\ 211$	$\cdot 9545\ 844$	$\cdot 9437$
$\cdot 90$	$\cdot 9846\ 253$	$\cdot 9785\ 163$	$\cdot 9714\ 243$	$\cdot 9633\ 877$	$\cdot 9544$
$\cdot 91$	$\cdot 9880\ 494$	$\cdot 9832\ 380$	$\cdot 9776\ 213$	$\cdot 9712\ 209$	$\cdot 9640$
$\cdot 92$	$\cdot 9909\ 958$	$\cdot 9873\ 232$	$\cdot 9830\ 123$	$\cdot 9780\ 729$	$\cdot 9725$
$\cdot 93$	$\cdot 9934\ 782$	$\cdot 9907\ 838$	$\cdot 9876\ 039$	$\cdot 9839\ 406$	$\cdot 9797$
$\cdot 94$	$\cdot 9955\ 139$	$\cdot 9936\ 370$	$\cdot 9914\ 100$	$\cdot 9888\ 305^+$	$\cdot 9858$
$\cdot 95$	$\cdot 9971\ 242$	$\cdot 9959\ 061$	$\cdot 9944\ 529$	$\cdot 9927\ 608$	$\cdot 9908$
$\cdot 96$	$\cdot 9983\ 355^+$	$\cdot 9976\ 218$	$\cdot 9967\ 658$	$\cdot 9957\ 638$	$\cdot 9946$
$\cdot 97$	$\cdot 9991\ 802$	$\cdot 9988\ 244$	$\cdot 9983\ 955^-$	$\cdot 9978\ 907$	$\cdot 9973$
$\cdot 98$	$\cdot 9996\ 993$	$\cdot 9995\ 672$	$\cdot 9994\ 071$	$\cdot 9992\ 178$	$\cdot 9989$
$\cdot 99$	$\cdot 9999\ 463$	$\cdot 9999\ 224$	$\cdot 9998\ 933$	$\cdot 9998\ 587$	$\cdot 9998$
$1\cdot 00$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000$

$x = .03$  to  $.60$  $q = 2.5$ 

	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$	$p = 7.5$
$B(p, q) = .1380\ 5827 \times \frac{x}{10}$		$.1136\ 6411 \times \frac{x}{10}$	$.9491\ 5061 \times \frac{x}{100}$	$.8023\ 3492 \times \frac{x}{100}$	$.6854\ 9767 \times \frac{x}{100}$
$x$					
.03	.0000 001				
.04	.0000 003	.0000 001			
.05	.0000 009	.0000 002	.0000 001		
.06	.0000 023	.0000 006	.0000 002		
.07	.0000 053	.0000 016	.0000 005 <sup>-</sup>	.0000 001	
.08	.0000 110	.0000 035 <sup>-</sup>	.0000 011	.0000 003	.0000 001
.09	.0000 207	.0000 069	.0000 023	.0000 008	.0000 002
.10	.0000 365 <sup>-</sup>	.0000 128	.0000 045 <sup>-</sup>	.0000 016	.0000 005 <sup>+</sup>
.11	.0000 608	.0000 224	.0000 082	.0000 030	.0000 011
.12	.0000 967	.0000 372	.0000 142	.0000 054	.0000 020
.13	.0001 481	.0000 593	.0000 236	.0000 093	.0000 037
.14	.0002 194	.0000 912	.0000 376	.0000 154	.0000 063
.15	.0003 160	.0001 359	.0000 580	.0000 246	.0000 104
.16	.0004 442	.0001 972	.0000 870	.0000 381	.0000 166
.17	.0006 109	.0002 795 <sup>+</sup>	.0001 270	.0000 574	.0000 258
.18	.0008 242	.0003 880	.0001 814	.0000 843	.0000 390
.19	.0010 932	.0005 285 <sup>+</sup>	.0002 538	.0001 212	.0000 576
.20	.0014 277	.0007 080	.0003 488	.0001 708	.0000 832
.21	.0018 387	.0009 342	.0004 715 <sup>-</sup>	.0002 366	.0001 181
.22	.0023 384	.0012 157	.0006 278	.0003 224	.0001 647
.23	.0029 398	.0015 623	.0008 248	.0004 329	.0002 261
.24	.0036 569	.0019 846	.0010 701	.0005 737	.0003 060
.25	.0045 050 <sup>+</sup>	.0024 947	.0013 725 <sup>-</sup>	.0007 508	.0004 086
.26	.0055 002	.0031 052	.0017 418	.0009 715 <sup>-</sup>	.0005 391
.27	.0066 597	.0038 304	.0021 889	.0012 438	.0007 033
.28	.0080 015 <sup>+</sup>	.0046 852	.0027 258	.0015 770	.0009 078
.29	.0095 448	.0056 861	.0033 659	.0019 813	.0011 605 <sup>+</sup>
.30	.0113 093	.0068 504	.0041 233	.0024 681	.0014 700
.31	.0133 159	.0081 966	.0050 138	.0030 500 <sup>+</sup>	.0018 463
.32	.0155 860	.0097 444	.0060 543	.0037 409	.0023 002
.33	.0181 418	.0115 143	.0072 628	.0045 562	.0028 443
.34	.0210 061	.0135 282	.0086 589	.0055 122	.0034 921
.35	.0242 022	.0158 087	.0102 632	.0066 271	.0042 587
.36	.0277 539	.0183 793	.0120 976	.0079 203	.0051 606
.37	.0316 851	.0212 644	.0141 852	.0094 125 <sup>-</sup>	.0062 160
.38	.0360 202	.0244 891	.0165 503	.0111 260	.0074 444
.39	.0407 836	.0280 794	.0192 183	.0130 847	.0088 670
.40	.0459 996	.0320 615 <sup>+</sup>	.0222 157	.0153 135 <sup>+</sup>	.0105 068
.41	.0516 926	.0364 624	.0255 700	.0178 390	.0123 881
.42	.0578 864	.0413 092	.0293 094	.0206 891	.0145 373
.43	.0646 045 <sup>+</sup>	.0466 292	.0334 631	.0238 928	.0169 821
.44	.0718 701	.0524 500 <sup>+</sup>	.0380 609	.0274 804	.0197 519
.45	.0797 052	.0587 988	.0431 331	.0314 835 <sup>-</sup>	.0228 777

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .61$  to  $1.00$  $q = 2.5$ 

	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$	$p = 7$
$B(p, q) = .1380\ 5827 \times \frac{x}{10}$		$.1136\ 6411 \times \frac{x}{10}$	$.9491\ 5061 \times \frac{x}{108}$	$.8023\ 3492 \times \frac{x}{108}$	$.6854\ 9$
$\frac{x}{10}$					
.61	.2953 218	.2512 579	.2128 035 <sup>-</sup>	.1795 007	.1508 5
.62	.3147 328	.2697 478	.2301 683	.1956 109	.1656 3
.63	.3347 971	.2890 146	.2484 093	.2126 710	.1814 2
.64	.3554 906	.3090 440	.2675 233	.2306 899	.1982 3
.65	.3767 853	.3298 169	.2875 023	.2496 720	.2160 7
.66	.3986 489	.3513 094	.3083 331	.2696 161	.2349 6
.67	.4210 448	.3734 926	.3299 967	.2905 154	.2549 1
.68	.4439 321	.3963 322	.3524 683	.3123 565 <sup>+</sup>	.2759 2
.69	.4672 651	.4197 886	.3757 171	.3351 196	.2979 7
.70	.4909 940	.4438 163	.3997 053	.3587 775 <sup>+</sup>	.3210 5
.71	.5150 641	.4683 641	.4243 884	.3832 953	.3451 5
.72	.5394 161	.4933 749	.4497 148	.4086 297	.3702 2
.73	.5639 862	.5187 856	.4756 253	.4347 290	.3962 3
.74	.5887 061	.5445 268	.5020 532	.4615 324	.4231 3
.75	.6135 028	.5705 231	.5289 239	.4889 696	.4508 5
.76	.6382 992	.5966 929	.5561 548	.5169 605 <sup>-</sup>	.4793 1
.77	.6630 138	.6229 485 <sup>-</sup>	.5836 553	.5454 150 <sup>-</sup>	.5084 4
.78	.6875 610	.6491 961	.6113 265 <sup>-</sup>	.5742 326	.5381 4
.79	.7118 517	.6753 364	.6390 617	.6033 026	.5682 8
.80	.7357 930	.7012 642	.6667 460	.6325 035 <sup>-</sup>	.5987 6
.81	.7592 891	.7268 693	.6942 572	.6617 035 <sup>+</sup>	.6294 2
.82	.7822 413	.7520 366	.7214 653	.6907 607	.6601 2
.83	.8045 490	.7766 466	.7482 337	.7195 230	.6907 0
.84	.8261 099	.8005 765 <sup>-</sup>	.7744 193	.7478 291	.7209 7
.85	.8468 208	.8237 002	.7998 739	.7755 091	.7507 6
.86	.8665 788	.8458 900	.8244 445 <sup>-</sup>	.8023 853	.7798 4
.87	.8852 820	.8670 175 <sup>+</sup>	.8479 751	.8282 740	.8080 2
.88	.9028 305 <sup>-</sup>	.8869 549	.8703 080	.8529 866	.8350 8
.89	.9191 282	.9055 767	.8912 861	.8763 320	.8607 8
.90	.9340 845 <sup>+</sup>	.9227 620	.9107 548	.8981 197	.8849 1
.91	.9476 159	.9383 964	.9285 653	.9181 625 <sup>+</sup>	.9072 2
.92	.9596 488	.9523 760	.9445 781	.9362 815 <sup>-</sup>	.9275 1
.93	.9701 222	.9646 100	.9586 677	.9523 111	.9455 5
.94	.9789 920	.9750 265 <sup>-</sup>	.9707 286	.9661 064	.9611 6
.95	.9862 353	.9835 781	.9806 829	.9775 526	.9741 9
.96	.9918 574	.9902 506	.9884 906	.9865 778	.9845 1
.97	.9959 016	.9950 752	.9941 653	.9931 712	.9920 9
.98	.9984 636	.9981 471	.9977 969	.9974 123	.9969 9
.99	.9997 195 <sup>-</sup>	.9996 605 <sup>-</sup>	.9995 948	.9995 224	.9994 4
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q = 2.5$

$p = 8.5 \text{ to } 11$

$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$
$\frac{1}{10^3} \cdot 4504 \ 3364 \times \frac{1}{10^3}$	$\cdot 3972 \ 7706 \times \frac{1}{10^3}$	$\cdot 3525 \ 1328 \times \frac{1}{10^3}$	$\cdot 3145 \ 1100 \times \frac{1}{10^3}$	$\cdot 2820 \ 1063 \times \frac{1}{10^3}$
$\cdot 0000 \ 001$				
$\cdot 0000 \ 002$	$\cdot 0000 \ 001$			
$\cdot 0000 \ 004$	$\cdot 0000 \ 002$	$\cdot 0000 \ 001$		
$\cdot 0000 \ 008$	$\cdot 0000 \ 003$	$\cdot 0000 \ 001$	$\cdot 0000 \ 001$	
$\cdot 0000 \ 013$	$\cdot 0000 \ 006$	$\cdot 0000 \ 002$	$\cdot 0000 \ 001$	
$\cdot 0000 \ 023$	$\cdot 0000 \ 010$	$\cdot 0000 \ 004$	$\cdot 0000 \ 002$	$\cdot 0000 \ 001$
$\cdot 0000 \ 038$	$\cdot 0000 \ 017$	$\cdot 0000 \ 008$	$\cdot 0000 \ 004$	$\cdot 0000 \ 002$
$\cdot 0000 \ 060$	$\cdot 0000 \ 028$	$\cdot 0000 \ 013$	$\cdot 0000 \ 006$	$\cdot 0000 \ 003$
$\cdot 0000 \ 094$	$\cdot 0000 \ 045^-$	$\cdot 0000 \ 022$	$\cdot 0000 \ 010$	$\cdot 0000 \ 005^-$
$\cdot 0000 \ 143$	$\cdot 0000 \ 070$			
$\cdot 0000 \ 214$	$\cdot 0000 \ 108$	$\cdot 0000 \ 034$	$\cdot 0000 \ 017$	$\cdot 0000 \ 008$
$\cdot 0000 \ 314$	$\cdot 0000 \ 161$	$\cdot 0000 \ 054$	$\cdot 0000 \ 027$	$\cdot 0000 \ 013$
$\cdot 0000 \ 453$	$\cdot 0000 \ 238$	$\cdot 0000 \ 083$	$\cdot 0000 \ 042$	$\cdot 0000 \ 022$
$\cdot 0000 \ 642$	$\cdot 0000 \ 344$	$\cdot 0000 \ 124$	$\cdot 0000 \ 065^-$	$\cdot 0000 \ 034$
$\cdot 0000 \ 898$	$\cdot 0000 \ 491$	$\cdot 0000 \ 184$	$\cdot 0000 \ 098$	$\cdot 0000 \ 052$
$\cdot 0001 \ 239$	$\cdot 0000 \ 690$	$\cdot 0000 \ 267$	$\cdot 0000 \ 145^+$	$\cdot 0000 \ 079$
$\cdot 0001 \ 689$	$\cdot 0000 \ 957$	$\cdot 0000 \ 383$	$\cdot 0000 \ 212$	$\cdot 0000 \ 117$
$\cdot 0002 \ 274$	$\cdot 0001 \ 312$	$\cdot 0000 \ 541$	$\cdot 0000 \ 305^-$	$\cdot 0000 \ 171$
$\cdot 0003 \ 030$	$\cdot 0001 \ 777$	$\cdot 0000 \ 754$	$\cdot 0000 \ 433$	$\cdot 0000 \ 247$
		$\cdot 0001 \ 039$	$\cdot 0000 \ 606$	$\cdot 0000 \ 353$
$\cdot 0003 \ 995^+$	$\cdot 0002 \ 382$	$\cdot 0001 \ 416$	$\cdot 0000 \ 839$	$\cdot 0000 \ 496$
$\cdot 0005 \ 218$	$\cdot 0003 \ 160$	$\cdot 0001 \ 908$	$\cdot 0001 \ 149$	$\cdot 0000 \ 690$
$\cdot 0006 \ 753$	$\cdot 0004 \ 152$	$\cdot 0002 \ 546$	$\cdot 0001 \ 557$	$\cdot 0000 \ 950^-$
$\cdot 0008 \ 605^+$	$\cdot 0005 \ 408$	$\cdot 0003 \ 365^+$	$\cdot 0002 \ 088$	$\cdot 0001 \ 293$
$\cdot 0011 \ 031$	$\cdot 0006 \ 984$	$\cdot 0004 \ 409$	$\cdot 0002 \ 775^+$	$\cdot 0001 \ 743$
$\cdot 0013 \ 936$	$\cdot 0008 \ 947$	$\cdot 0005 \ 727$	$\cdot 0003 \ 656$	$\cdot 0002 \ 328$
$\cdot 0017 \ 480$	$\cdot 0011 \ 375^-$	$\cdot 0007 \ 380$	$\cdot 0004 \ 776$	$\cdot 0003 \ 083$
$\cdot 0021 \ 775^-$	$\cdot 0014 \ 357$	$\cdot 0009 \ 439$	$\cdot 0006 \ 190$	$\cdot 0004 \ 049$
$\cdot 0026 \ 949$	$\cdot 0017 \ 998$	$\cdot 0011 \ 986$	$\cdot 0007 \ 961$	$\cdot 0005 \ 275^-$
$\cdot 0033 \ 147$	$\cdot 0022 \ 416$	$\cdot 0015 \ 115^+$	$\cdot 0010 \ 166$	$\cdot 0006 \ 821$
$\cdot 0040 \ 530$	$\cdot 0027 \ 744$	$\cdot 0018 \ 937$	$\cdot 0012 \ 893$	$\cdot 0008 \ 756$
$\cdot 0049 \ 278$	$\cdot 0034 \ 134$	$\cdot 0023 \ 578$	$\cdot 0016 \ 244$	$\cdot 0011 \ 164$
$\cdot 0059 \ 591$	$\cdot 0041 \ 758$	$\cdot 0029 \ 180$	$\cdot 0020 \ 338$	$\cdot 0014 \ 142$
$\cdot 0071 \ 689$	$\cdot 0050 \ 806$	$\cdot 0035 \ 906$	$\cdot 0025 \ 311$	$\cdot 0017 \ 800$
$\cdot 0085 \ 815^+$	$\cdot 0061 \ 492$	$\cdot 0043 \ 941$	$\cdot 0031 \ 320$	$\cdot 0022 \ 271$
$\cdot 0102 \ 235^+$	$\cdot 0074 \ 052$	$\cdot 0053 \ 490$	$\cdot 0038 \ 540$	$\cdot 0027 \ 704$
$\cdot 0121 \ 238$	$\cdot 0088 \ 746$	$\cdot 0064 \ 784$	$\cdot 0047 \ 173$	$\cdot 0034 \ 270$
$\cdot 0143 \ 138$	$\cdot 0105 \ 860$	$\cdot 0078 \ 079$	$\cdot 0057 \ 445^+$	$\cdot 0042 \ 167$
$\cdot 0168 \ 273$	$\cdot 0125 \ 709$	$\cdot 0093 \ 659$	$\cdot 0069 \ 608$	$\cdot 0051 \ 615^-$
$\cdot 0197 \ 007$	$\cdot 0148 \ 633$	$\cdot 0111 \ 838$	$\cdot 0083 \ 945^-$	$\cdot 0062 \ 865^+$
$\cdot 0229 \ 728$	$\cdot 0175 \ 000^-$	$\cdot 0132 \ 957$	$\cdot 0100 \ 768$	$\cdot 0076 \ 200$
$\cdot 0266 \ 852$	$\cdot 0205 \ 209$	$\cdot 0157 \ 391$	$\cdot 0120 \ 424$	$\cdot 0091 \ 933$
$\cdot 0308 \ 817$	$\cdot 0239 \ 687$	$\cdot 0185 \ 548$	$\cdot 0143 \ 293$	$\cdot 0110 \ 415^+$
$\cdot 0356 \ 085^-$	$\cdot 0278 \ 889$	$\cdot 0217 \ 866$	$\cdot 0169 \ 791$	$\cdot 0132 \ 033$
$\cdot 0409 \ 140$	$\cdot 0323 \ 302$	$\cdot 0254 \ 820$	$\cdot 0200 \ 371$	$\cdot 0157 \ 213$
$\cdot 0468 \ 489$	$\cdot 0373 \ 436$	$\cdot 0296 \ 915^+$	$\cdot 0235 \ 524$	$\cdot 0186 \ 422$
$\cdot 0534 \ 656$	$\cdot 0429 \ 830$	$\cdot 0344 \ 692$	$\cdot 0275 \ 778$	$\cdot 0220 \ 170$
$\cdot 0608 \ 180$	$\cdot 0493 \ 047$	$\cdot 0398 \ 720$	$\cdot 0321 \ 701$	$\cdot 0259 \ 010$
$\cdot 0689 \ 615^+$	$\cdot 0563 \ 672$	$\cdot 0459 \ 601$	$\cdot 0373 \ 898$	$\cdot 0303 \ 537$
$\cdot 0779 \ 524$	$\cdot 0642 \ 308$	$\cdot 0527 \ 966$	$\cdot 0433 \ 008$	$\cdot 0354 \ 391$
$\cdot 0878 \ 473$	$\cdot 0729 \ 576$	$\cdot 0604 \ 469$	$\cdot 0499 \ 708$	$\cdot 0412 \ 256$
$\cdot 0987 \ 029$	$\cdot 0826 \ 105^-$	$\cdot 0689 \ 787$	$\cdot 0574 \ 706$	$\cdot 0477 \ 853$
$\cdot 1105 \ 754$	$\cdot 0932 \ 529$	$\cdot 0784 \ 613$	$\cdot 0658 \ 736$	$\cdot 0551 \ 946$
$\cdot 1235 \ 195^+$	$\cdot 1049 \ 484$	$\cdot 0889 \ 652$	$\cdot 0752 \ 558$	$\cdot 0635 \ 332$
$\cdot 1375 \ 882$	$\cdot 1177 \ 597$	$\cdot 1005 \ 613$	$\cdot 0856 \ 948$	$\cdot 0728 \ 838$
$\cdot 1528 \ 317$	$\cdot 1317 \ 479$	$\cdot 1133 \ 205^+$	$\cdot 0972 \ 695^+$	$\cdot 0833 \ 316$
$\cdot 1692 \ 964$	$\cdot 1469 \ 718$	$\cdot 1273 \ 124$	$\cdot 1100 \ 588$	$\cdot 0949 \ 636$
$\cdot 1870 \ 245^+$	$\cdot 1634 \ 864$	$\cdot 1426 \ 042$	$\cdot 1241 \ 411$	$\cdot 1078 \ 675^-$
$\cdot 2060 \ 523$	$\cdot 1813 \ 426$	$\cdot 1592 \ 602$	$\cdot 1395 \ 928$	$\cdot 1221 \ 306$
$\cdot 2264 \ 094$	$\cdot 2005 \ 851$	$\cdot 1773 \ 399$	$\cdot 1564 \ 873$	$\cdot 1378 \ 389$

TABLE I. THE  $I_{\pi}(p, q)$  FUNCTION $x = .71$  to  $1.00$  $q = 2.5$  $p = 8.5$  to  $11$ 

	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$
$B(p, q) = .5141\ 2325 \times \frac{1}{10^8}$	$.4504\ 3364 \times \frac{1}{10^8}$	$.3972\ 7706 \times \frac{1}{10^8}$	$.3525\ 1328 \times \frac{1}{10^8}$	$.3145\ 1100 \times \frac{1}{10^8}$	$.2820\ 1063 \times \frac{1}{10^8}$	
.71	.2776 422	.2481 177	.2212 519	.1968 969	.1748 931	.1550 751
.72	.3015 470	.2711 898	.2433 722	.2179 772	.1948 728	.1739 172
.73	.3266 918	.2956 278	.2669 653	.2406 178	.2164 806	.1944 368
.74	.3530 509	.3214 219	.2920 387	.2648 442	.2397 609	.2166 965 <sup>+</sup>
.75	.3805 856	.3485 491	.3185 866	.2906 691	.2647 454	.2407 477
.76	.4092 433	.3769 714	.3465 881	.3180 901	.2914 515 <sup>-</sup>	.2666 279
.77	.4389 562	.4066 346	.3760 049	.3470 872	.3198 788	.2943 580
.78	.4696 400	.4374 667	.4067 799	.3776 208	.3500 074	.3239 389
.79	.5011 932	.4693 763	.4388 352	.4096 289	.3817 940	.3553 484
.80	.5334 960	.5022 511	.4720 697	.4430 252	.4151 697	.3885 374
.81	.5664 089	.5359 568	.5063 578	.4776 961	.4500 363	.4234 262
.82	.5997 726	.5703 357	.5415 472	.5134 985 <sup>-</sup>	.4862 636	.4599 010
.83	.6334 072	.6052 055 <sup>+</sup>	.5774 572	.5502 575 <sup>+</sup>	.5236 864	.4978 097
.84	.6671 117	.6403 589	.6138 779	.5877 644	.5621 010	.5369 581
.85	.7006 642	.6755 625 <sup>+</sup>	.6505 682	.6257 744	.6012 633	.5771 066
.86	.7338 222	.7105 573	.6872 560	.6640 058	.6408 858	.6179 665 <sup>+</sup>
.87	.7663 232	.7450 587	.7236 372	.7021 388	.6806 366	.6591 975 <sup>-</sup>
.88	.7978 869	.7787 579	.7593 773	.7398 154	.7201 380	.7004 056
.89	.8282 167	.8113 242	.7941 123	.7766 410	.7589 668	.7411 428
.90	.8570 033	.8424 073	.8274 521	.8121 863	.7966 566	.7809 075 <sup>+</sup>
.91	.8839 290	.8716 429	.8589 846	.8459 918	.8327 012	.8191 480
.92	.9086 741	.8986 587	.8882 832	.8775 752	.8665 616	.8552 689
.93	.9309 248	.9230 833	.9149 158	.9064 408	.8976 767	.8886 417
.94	.9503 848	.9445 594	.9384 593	.9320 955 <sup>-</sup>	.9254 792	.9186 219
.95	.9667 899	.9627 604	.9585 183	.9540 695 <sup>-</sup>	.9494 196	.9445 747
.96	.9799 300	.9774 154	.9747 543	.9719 488	.9690 011	.9659 136
.97	.9896 796	.9883 454	.9869 262	.9854 221	.9838 336	.9821 612
.98	.9960 473	.9955 204	.9949 570	.9943 570	.9937 199	.9930 458
.99	.9992 626	.9991 614	.9990 525 <sup>+</sup>	.9989 360	.9988 117	.9986 795 <sup>-</sup>
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$x = .33$  to  $.90$  $q = 2.5$  $p = 18$  to  $23$ 

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$
$B(p, q) = .8745\ 9936 \times \frac{1}{10^3}$		$.7679\ 4090 \times \frac{1}{10^3}$	$.6786\ 4545 \times \frac{1}{10^3}$	$.6032\ 4040 \times \frac{1}{10^3}$	$.5390\ 6589 \times \frac{1}{10^3}$	$.4840\ 5916 \times \frac{1}{10^3}$
$x$						
.33	.0000 001					
.34	.0000 001					
.35	.0000 002	.0000 001				
.36	.0000 004	.0000 001	.0000 001			
.37	.0000 006	.0000 002	.0000 001			
.38	.0000 009	.0000 004	.0000 001	.0000 001		
.39	.0000 014	.0000 006	.0000 002	.0000 001		
.40	.0000 021	.0000 009	.0000 004	.0000 002	.0000 001	
.41	.0000 033	.0000 014	.0000 006	.0000 003	.0000 001	.0000 001
.42	.0000 049	.0000 022	.0000 010	.0000 004	.0000 002	.0000 001
.43	.0000 073	.0000 034	.0000 016	.0000 007	.0000 003	.0000 002
.44	.0000 108	.0000 051	.0000 024	.0000 011	.0000 005 <sup>+</sup>	.0000 002
.45	.0000 158	.0000 077	.0000 037	.0000 018	.0000 009	.0000 004
.46	.0000 229	.0000 113	.0000 056	.0000 027	.0000 013	.0000 007
.47	.0000 329	.0000 166	.0000 084	.0000 042	.0000 021	.0000 011
.48	.0000 468	.0000 242	.0000 124	.0000 064	.0000 033	.0000 017
.49	.0000 661	.0000 348	.0000 183	.0000 096	.0000 050 <sup>-</sup>	.0000 026
.50	.0000 926	.0000 498	.0000 267	.0000 142	.0000 076	.0000 040
.51	.0001 287	.0000 705 <sup>+</sup>	.0000 385 <sup>+</sup>	.0000 210	.0000 114	.0000 062
.52	.0001 776	.0000 992	.0000 553	.0000 307	.0000 170	.0000 094
.53	.0002 432	.0001 385 <sup>+</sup>	.0000 786	.0000 445 <sup>-</sup>	.0000 251	.0000 141
.54	.0003 309	.0001 919	.0001 110	.0000 640	.0000 368	.0000 211
.55	.0004 471	.0002 641	.0001 555 <sup>-</sup>	.0000 913	.0000 534	.0000 312
.56	.0006 002	.0003 609	.0002 163	.0001 293	.0000 770	.0000 458
.57	.0008 006	.0004 899	.0002 988	.0001 817	.0001 102	.0000 667
.58	.0010 614	.0006 607	.0004 100	.0002 537	.0001 566	.0000 964
.59	.0013 986	.0008 855 <sup>+</sup>	.0005 589	.0003 517	.0002 207	.0001 382
.60	.0018 323	.0011 795 <sup>+</sup>	.0007 569	.0004 843	.0003 091	.0001 968
.61	.0023 868	.0015 617	.0010 186	.0006 625 <sup>+</sup>	.0004 298	.0002 781
.62	.0030 918	.0020 556	.0013 625 <sup>-</sup>	.0009 005 <sup>+</sup>	.0005 936	.0003 904
.63	.0039 832	.0026 903	.0018 115 <sup>+</sup>	.0012 164	.0008 146	.0005 443
.64	.0051 042	.0035 013	.0023 944	.0016 329	.0011 107	.0007 537
.65	.0065 065 <sup>+</sup>	.0045 317	.0031 467	.0021 790	.0015 050 <sup>-</sup>	.0010 370
.66	.0082 514	.0058 337	.0041 120	.0028 905 <sup>-</sup>	.0020 267	.0014 177
.67	.0104 113	.0074 700	.0053 437	.0038 122	.0027 128	.0019 260
.68	.0130 712	.0095 152	.0069 063	.0049 992	.0036 097	.0026 004
.69	.0163 298	.0120 580	.0088 778	.0065 189	.0047 750 <sup>-</sup>	.0034 896
.70	.0203 015 <sup>-</sup>	.0152 025 <sup>-</sup>	.0113 514	.0084 534	.0062 800	.0046 548
.71	.0251 176	.0190 703	.0144 377	.0109 019	.0082 122	.0061 723
.72	.0309 278	.0238 026	.0182 674	.0139 832	.0106 782	.0081 304
.73	.0379 009	.0295 614	.0229 930	.0178 384	.0138 068	.0106 630
.74	.0462 262	.0365 318	.0287 915 <sup>+</sup>	.0226 342	.0177 523	.0138 933
.75	.0561 136	.0449 225 <sup>+</sup>	.0358 666	.0285 653	.0226 981	.0179 977
.76	.0677 931	.0549 671	.0444 498	.0358 571	.0288 603	.0231 801
.77	.0815 142	.0669 237	.0548 020	.0447 683	.0364 905 <sup>+</sup>	.0296 820
.78	.0975 433	.0810 739	.0672 136	.0555 918	.0458 792	.0377 869
.79	.1161 605 <sup>+</sup>	.0977 208	.0820 034	.0686 553	.0573 572	.0478 231
.80	.1376 540	.1171 843	.0995 156	.0843 206	.0712 963	.0601 664
.81	.1623 133	.1397 951	.1201 155 <sup>-</sup>	.1029 794	.0881 077	.0752 403
.82	.1904 191	.1658 862	.1441 810	.1250 479	.1082 381	.0935 143
.83	.2222 313	.1957 800	.1720 922	.1509 568	.1321 613	.1154 975 <sup>-</sup>
.84	.2579 727	.2297 730	.2042 151	.1811 363	.1603 654	.1417 283
.85	.2978 104	.2681 151	.2408 814	.2159 967	.1933 337	.1727 568
.86	.3418 314	.3109 846	.2823 620	.2559 003	.2315 179	.2091 192
.87	.3900 159	.3584 569	.3288 329	.3011 271	.2753 014	.2513 011
.88	.4422 048	.4104 683	.3803 353	.3518 294	.3249 525 <sup>+</sup>	.2996 882
.89	.4980 644	.4667 733	.4367 257	.4079 776	.3805 642	.3545 028
.90	.5570 466	.5268 969	.4976 199	.4692 945 <sup>-</sup>	.4419 806	.4157 221

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .91$  to  $1.00$  $q = 2.5$  $p = 18$  to  $23$ 

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$
$B(p, q) = .8745\ 9936 \times \frac{1}{10^8}$	$.7679\ 4090 \times \frac{1}{10^8}$	$.6786\ 4545 \times \frac{1}{10^8}$	$.6032\ 4040 \times \frac{1}{10^8}$	$.5390\ 6589 \times \frac{1}{10^8}$	$.4840\ 5916 \times \frac{1}{10^8}$	
$x$						
.91	.6183 494	.5900 830	.5623 299	.5351 805 <sup>-</sup>	.5087 098	.4829 790
.92	.6808 762	.6552 418	.6297 962	.6046 305 <sup>+</sup>	.5798 244	.5554 467
.93	.7432 035 <sup>+</sup>	.7209 009	.6985 219	.6761 486	.6538 551	.6317 086
.94	.8035 600	.7851 693	.7665 173	.7476 694	.7286 869	.7096 268
.95	.8598 329	.8457 281	.8312 706	.8165 059	.8014 776	.7862 272
.96	.9096 215 <sup>-</sup>	.8998 727	.8897 750 <sup>+</sup>	.8793 542	.8686 356	.8576 442
.97	.9503 748	.9446 519	.9386 625 <sup>+</sup>	.9324 171	.9259 262	.9192 008
.98	.9796 898	.9771 920	.9745 509	.9717 685 <sup>-</sup>	.9688 469	.9657 886
.99	.9959 486	.9954 186	.9948 523	.9942 497	.9936 105 <sup>+</sup>	.9929 346
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$x = .43$  to  $1.00$  $q = 2.5$  $p = 24$  to  $29$ 

	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$
$B(p, q) = .4366\ 0238 \times \frac{1}{10^3}$	$.3954\ 1348 \times \frac{1}{10^3}$	$.3594\ 6680 \times \frac{1}{10^3}$	$.3279\ 3462 \times \frac{1}{10^3}$	$.3001\ 4355 \times \frac{1}{10^3}$	$.2755\ 4162 \times \frac{1}{10^3}$	
$x$						
.43	.0000 001					
.44	.0000 001	.0000 001				
.45	.0000 002	.0000 001				
.46	.0000 003	.0000 002	.0000 001			
.47	.0000 005 <sup>+</sup>	.0000 003	.0000 001	.0000 001		
.48	.0000 008	.0000 004	.0000 002	.0000 001	.0000 001	
.49	.0000 014	.0000 007	.0000 004	.0000 002	.0000 001	
.50	.0000 021	.0000 011	.0000 006	.0000 003	.0000 002	.0000 001
.51	.0000 033	.0000 018	.0000 010	.0000 005 <sup>+</sup>	.0000 003	.0000 001
.52	.0000 052	.0000 028	.0000 016	.0000 009	.0000 005 <sup>-</sup>	.0000 003
.53	.0000 079	.0000 044	.0000 025 <sup>-</sup>	.0000 014	.0000 008	.0000 004
.54	.0000 121	.0000 069	.0000 039	.0000 022	.0000 013	.0000 007
.55	.0000 182	.0000 106	.0000 061	.0000 036	.0000 021	.0000 012
.56	.0000 272	.0000 161	.0000 095 <sup>-</sup>	.0000 056	.0000 033	.0000 019
.57	.0000 403	.0000 243	.0000 146	.0000 088	.0000 052	.0000 031
.58	.0000 592	.0000 363	.0000 222	.0000 136	.0000 083	.0000 050 <sup>+</sup>
.59	.0000 863	.0000 538	.0000 335 <sup>-</sup>	.0000 208	.0000 129	.0000 080
.60	.0001 250 <sup>-</sup>	.0000 792	.0000 501	.0000 317	.0000 200	.0000 126
.61	.0001 796	.0001 157	.0000 744	.0000 478	.0000 306	.0000 196
.62	.0002 562	.0001 677	.0001 096	.0000 715 <sup>+</sup>	.0000 466	.0000 303
.63	.0003 628	.0002 414	.0001 603	.0001 062	.0000 703	.0000 465 <sup>-</sup>
.64	.0005 104	.0003 449	.0002 326	.0001 566	.0001 053	.0000 706
.65	.0007 130	.0004 893	.0003 351	.0002 291	.0001 564	.0001 066
.66	.0009 896	.0006 894	.0004 793	.0003 327	.0002 305 <sup>+</sup>	.0001 595 <sup>+</sup>
.67	.0013 645 <sup>-</sup>	.0009 647	.0006 808	.0004 796	.0003 374	.0002 369
.68	.0018 693	.0013 412	.0009 604	.0006 866	.0004 901	.0003 492
.69	.0025 449	.0018 523	.0013 457	.0009 760	.0007 067	.0005 110
.70	.0034 430	.0025 417	.0018 730	.0013 778	.0010 120	.0007 422
.71	.0046 295 <sup>-</sup>	.0034 656	.0025 897	.0019 319	.0014 389	.0010 702
.72	.0061 869	.0046 956	.0035 574	.0026 906	.0020 318	.0015 321
.73	.0082 184	.0063 223	.0048 551	.0037 223	.0028 494	.0021 780
.74	.0108 515 <sup>+</sup>	.0084 599	.0065 838	.0051 155 <sup>+</sup>	.0039 686	.0030 744
.75	.0142 425 <sup>+</sup>	.0112 501	.0088 711	.0069 840	.0054 899	.0043 094
.76	.0185 816	.0148 683	.0118 769	.0094 723	.0075 432	.0059 986
.77	.0240 976	.0195 288	.0157 998	.0127 628	.0102 943	.0082 918
.78	.0310 632	.0254 910	.0208 838	.0170 830	.0139 536 <sup>+</sup>	.0113 819
.79	.0397 999	.0330 655 <sup>+</sup>	.0274 201	.0227 139	.0187 845 <sup>+</sup>	.0155 140
.80	.0506 818	.0426 199	.0357 834	.0299 987	.0251 138	.0209 966
.81	.0641 380	.0545 831	.0463 793	.0393 508	.0333 414	.0282 131
.82	.0806 533	.0694 482	.0597 087	.0512 616	.0439 501	.0376 335 <sup>+</sup>
.83	.1007 644	.0877 717	.0763 408	.0663 057	.0575 139	.0498 257
.84	.1250 521	.1101 685 <sup>+</sup>	.0969 163	.0851 421	.0747 024	.0654 632
.85	.1541 258	.1372 996	.1221 393	.1085 095 <sup>+</sup>	.0962 806	.0853 292
.86	.1886 001	.1698 508	.1527 589	.1372 119	.1230 986	.1103 110
.87	.2290 590	.2084 990	.1895 386	.1720 913	.1560 689	.1413 827
.88	.2760 057	.2538 625 <sup>+</sup>	.2332 073	.2139 822	.1961 241	.1795 671
.89	.3297 955 <sup>+</sup>	.3064 317	.2843 902	.2636 416	.2441 494	.2258 719
.90	.3905 481	.3664 755 <sup>+</sup>	.3435 103	.3216 491	.3008 810	.2811 884
.91	.4580 370	.4339 211	.4106 590	.3882 692	.3667 625 <sup>+</sup>	.3461 430
.92	.5315 565 <sup>+</sup>	.5082 039	.4854 304	.4632 701	.4417 500 <sup>-</sup>	.4208 908
.93	.6097 691	.5880 901	.5667 190	.5456 972	.5250 608	.5048 409
.94	.6905 417	.6714 806	.6524 880	.6336 051	.6148 690	.5963 136
.95	.7707 945 <sup>+</sup>	.7552 172	.7395 308	.7237 689	.7079 633	.6921 436
.96	.8464 044	.8349 397	.8232 732	.8114 272	.7994 234	.7872 823
.97	.9122 518	.9050 899	.8977 260	.8901 710	.8824 355 <sup>+</sup>	.8745 301
.98	.9625 960	.9592 716	.9558 183	.9522 388	.9485 361	.9447 129
.99	.9922 218	.9914 720	.9906 851	.9898 612	.9890 002	.9881 022
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$p = 30$  to  $36$

	$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$	$p = 36$
$B(p, q) = 2536\ 7324 \times \frac{1}{10^8}$	$2341\ 5991 \times \frac{1}{10^8}$	$2166\ 8529 \times \frac{1}{10^8}$	$2009\ 8346 \times \frac{1}{10^8}$	$1868\ 2970 \times \frac{1}{10^8}$	$1740\ 3314 \times \frac{1}{10^8}$	$1624\ 3093 \times \frac{1}{10^8}$	
$x$							
.51	.0000 001						
.52	.0000 001	.0000 001					
.53	.0000 002	.0000 001	.0000 001				
.54	.0000 004	.0000 002	.0000 001	.0000 001			
.55	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001		
.56	.0000 011	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001	
.57	.0000 019	.0000 011	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
.58	.0000 031	.0000 019	.0000 011	.0000 007	.0000 004	.0000 002	.0000 001
.59	.0000 049	.0000 030	.0000 019	.0000 012	.0000 007	.0000 004	.0000 003
.60	.0000 079	.0000 050 <sup>-</sup>	.0000 031	.0000 019	.0000 012	.0000 008	.0000 005 <sup>-</sup>
.61	.0000 125 <sup>+</sup>	.0000 080	.0000 051	.0000 032	.0000 021	.0000 013	.0000 008
.62	.0000 197	.0000 128	.0000 083	.0000 053	.0000 035 <sup>-</sup>	.0000 022	.0000 014
.63	.0000 306	.0000 202	.0000 133	.0000 087	.0000 057	.0000 038	.0000 025 <sup>-</sup>
.64	.0000 473	.0000 317	.0000 212	.0000 141	.0000 094	.0000 063	.0000 042
.65	.0000 725 <sup>+</sup>	.0000 493	.0000 335 <sup>-</sup>	.0000 227	.0000 154	.0000 104	.0000 070
.66	.0001 102	.0000 760	.0000 524	.0000 361	.0000 248	.0000 170	.0000 117
.67	.0001 662	.0001 164	.0000 814	.0000 569	.0000 397	.0000 277	.0000 193
.68	.0002 485 <sup>+</sup>	.0001 766	.0001 254	.0000 889	.0000 629	.0000 445 <sup>+</sup>	.0000 315 <sup>-</sup>
.69	.0003 689	.0002 660	.0001 916	.0001 378	.0000 990	.0000 711	.0000 510
.70	.0005 435 <sup>+</sup>	.0003 975 <sup>+</sup>	.0002 904	.0002 119	.0001 544	.0001 124	.0000 818
.71	.0007 948	.0005 895 <sup>+</sup>	.0004 367	.0003 232	.0002 389	.0001 764	.0001 301
.72	.0011 537	.0008 677	.0006 517	.0004 890	.0003 664	.0002 743	.0002 052
.73	.0016 625 <sup>+</sup>	.0012 674	.0009 651	.0007 340	.0005 576	.0004 232	.0003 209
.74	.0023 784	.0018 377	.0014 182	.0010 932	.0008 418	.0006 475 <sup>+</sup>	.0004 976
.75	.0033 782	.0026 449	.0020 683	.0016 156	.0012 606	.0009 826	.0007 652
.76	.0047 640	.0037 787	.0029 937	.0023 691	.0018 729	.0014 791	.0011 670
.77	.0066 702	.0053 590	.0043 006	.0034 474	.0027 606	.0022 084	.0017 650 <sup>+</sup>
.78	.0092 722	.0075 444	.0061 315 <sup>-</sup>	.0049 777	.0040 369	.0032 707	.0026 475 <sup>-</sup>
.79	.0127 966	.0105 425 <sup>+</sup>	.0086 757	.0071 317	.0058 565 <sup>+</sup>	.0048 047	.0039 382
.80	.0175 324	.0146 225 <sup>-</sup>	.0121 819	.0101 379	.0084 284	.0070 005 <sup>-</sup>	.0058 092
.81	.0238 442	.0201 285 <sup>+</sup>	.0169 732	.0142 976	.0120 318	.0101 156	.0084 969
.82	.0321 860	.0274 957	.0234 636	.0200 022	.0170 349	.0144 944	.0123 218
.83	.0431 145 <sup>-</sup>	.0372 656	.0321 762	.0277 540	.0239 167	.0205 912	.0177 128
.84	.0573 009	.0501 017	.0437 618	.0381 866	.0332 906	.0289 965 <sup>-</sup>	.0252 349
.85	.0755 390	.0668 012	.0590 147	.0520 860	.0459 289	.0404 644	.0356 205 <sup>+</sup>
.86	.0987 448	.0883 006	.0788 839	.0704 059	.0627 835 <sup>-</sup>	.0559 390	.0498 004
.87	.1279 448	.1156 693	.1044 728	.0942 753	.0850 002	.0765 748	.0689 306
.88	.1642 434	.1500 842	.1370 213	.1249 870	.1139 155 <sup>-</sup>	.1037 427	.0944 070
.89	.2087 636	.1927 759	.1778 584	.1639 594	.1510 271	.1390 095 <sup>-</sup>	.1278 555 <sup>-</sup>
.90	.2625 483	.2449 335 <sup>-</sup>	.2283 130	.2126 534	.1979 191	.1840 732	.1710 779
.91	.3264 087	.3075 526	.2895 633	.2724 256	.2561 215 <sup>-</sup>	.2406 302	.2259 289
.92	.4007 077	.3812 109	.3624 059	.3442 943	.3268 743	.3101 408	.2940 863
.93	.4850 636	.4657 512	.4469 215 <sup>+</sup>	.4285 891	.4107 650 <sup>-</sup>	.3934 572	.3766 712
.94	.5779 693	.5598 635 <sup>-</sup>	.5420 205 <sup>-</sup>	.5244 619	.5072 066	.4902 710	.4736 693
.95	.6763 375 <sup>+</sup>	.6605 709	.6448 679	.6292 508	.6137 402	.5983 550 <sup>-</sup>	.5831 127
.96	.7750 240	.7626 676	.7502 315 <sup>-</sup>	.7377 332	.7251 895 <sup>+</sup>	.7126 163	.7000 287
.97	.8664 652	.8582 511	.8498 979	.8414 155 <sup>+</sup>	.8328 136	.8241 016	.8152 888
.98	.9407 725 <sup>-</sup>	.9367 178	.9325 519	.9282 779	.9238 990	.9194 184	.9148 392
.99	.9871 671	.9861 951	.9851 863	.9841 408	.9830 586	.9819 400	.9807 851
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

## TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

 $x = .58 \text{ to } 1.00$ 
$$q = 2.5$$

$p = 37$  to  $43$

	$p = 37$	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$B(p, q) = \cdot 1518\ 8347 \times \frac{1}{10^8}$	$\cdot 1422\ 7059 \times \frac{1}{10^8}$	$\cdot 1334\ 8846 \times \frac{1}{10^8}$	$\cdot 1254\ 4698 \times \frac{1}{10^8}$	$\cdot 1180\ 6775 \times \frac{1}{10^8}$	$\cdot 1112\ 8225 \times \frac{1}{10^8}$	$\cdot 1050\ 3043 \times \frac{1}{10^8}$	
$\cdot 58$	$\cdot 0000\ 001$	$\cdot 0000\ 001$					
$\cdot 59$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 60$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$			
$\cdot 61$	$\cdot 0000\ 005^+$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$		
$\cdot 62$	$\cdot 0000\ 009$	$\cdot 0000\ 006$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 63$	$\cdot 0000\ 016$	$\cdot 0000\ 011$	$\cdot 0000\ 007$	$\cdot 0000\ 004$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	
$\cdot 64$	$\cdot 0000\ 028$	$\cdot 0000\ 018$	$\cdot 0000\ 012$	$\cdot 0000\ 008$	$\cdot 0000\ 005^+$	$\cdot 0000\ 004$	
$\cdot 65$	$\cdot 0000\ 047$	$\cdot 0000\ 032$	$\cdot 0000\ 022$	$\cdot 0000\ 014$	$\cdot 0000\ 010$	$\cdot 0000\ 007$	
$\cdot 66$	$\cdot 0000\ 080$	$\cdot 0000\ 055^-$	$\cdot 0000\ 038$	$\cdot 0000\ 026$	$\cdot 0000\ 018$	$\cdot 0000\ 012$	
$\cdot 67$	$\cdot 0000\ 134$	$\cdot 0000\ 093$	$\cdot 0000\ 065^-$	$\cdot 0000\ 045^-$	$\cdot 0000\ 031$	$\cdot 0000\ 022$	
$\cdot 68$	$\cdot 0000\ 222$	$\cdot 0000\ 157$	$\cdot 0000\ 110$	$\cdot 0000\ 078$	$\cdot 0000\ 055^-$	$\cdot 0000\ 038$	
$\cdot 69$	$\cdot 0000\ 365^+$	$\cdot 0000\ 261$	$\cdot 0000\ 187$	$\cdot 0000\ 133$	$\cdot 0000\ 095^+$	$\cdot 0000\ 068$	
$\cdot 70$	$\cdot 0000\ 594$	$\cdot 0000\ 431$	$\cdot 0000\ 313$	$\cdot 0000\ 227$	$\cdot 0000\ 164$	$\cdot 0000\ 119$	
$\cdot 71$	$\cdot 0000\ 959$	$\cdot 0000\ 706$	$\cdot 0000\ 519$	$\cdot 0000\ 382$	$\cdot 0000\ 280$	$\cdot 0000\ 206$	
$\cdot 72$	$\cdot 0001\ 533$	$\cdot 0001\ 145^-$	$\cdot 0000\ 854$	$\cdot 0000\ 636$	$\cdot 0000\ 474$	$\cdot 0000\ 353$	
$\cdot 73$	$\cdot 0002\ 431$	$\cdot 0001\ 840$	$\cdot 0001\ 391$	$\cdot 0001\ 051$	$\cdot 0000\ 794$	$\cdot 0000\ 599$	
$\cdot 74$	$\cdot 0003\ 820$	$\cdot 0002\ 931$	$\cdot 0002\ 246$	$\cdot 0001\ 720$	$\cdot 0001\ 316$	$\cdot 0001\ 007$	
$\cdot 75$	$\cdot 0005\ 954$	$\cdot 0004\ 628$	$\cdot 0003\ 595^-$	$\cdot 0002\ 790$	$\cdot 0002\ 163$	$\cdot 0001\ 676$	
$\cdot 76$	$\cdot 0009\ 199$	$\cdot 0007\ 245^+$	$\cdot 0005\ 701$	$\cdot 0004\ 483$	$\cdot 0003\ 523$	$\cdot 0002\ 766$	
$\cdot 77$	$\cdot 0014\ 094$	$\cdot 0011\ 244$	$\cdot 0008\ 964$	$\cdot 0007\ 140$	$\cdot 0005\ 683$	$\cdot 0004\ 520$	
$\cdot 78$	$\cdot 0021\ 411$	$\cdot 0017\ 301$	$\cdot 0013\ 968$	$\cdot 0011\ 269$	$\cdot 0009\ 085^-$	$\cdot 0007\ 319$	
$\cdot 79$	$\cdot 0032\ 251$	$\cdot 0026\ 389$	$\cdot 0021\ 575^-$	$\cdot 0017\ 626$	$\cdot 0014\ 389$	$\cdot 0011\ 738$	
$\cdot 80$	$\cdot 0048\ 164$	$\cdot 0039\ 900$	$\cdot 0033\ 028$	$\cdot 0027\ 319$	$\cdot 0022\ 580$	$\cdot 0018\ 651$	
$\cdot 81$	$\cdot 0071\ 311$	$\cdot 0059\ 800$	$\cdot 0050\ 108$	$\cdot 0041\ 956$	$\cdot 0035\ 105^-$	$\cdot 0029\ 352$	
$\cdot 82$	$\cdot 0104\ 661$	$\cdot 0088\ 828$	$\cdot 0075\ 332$	$\cdot 0063\ 839$	$\cdot 0054\ 062$	$\cdot 0045\ 751$	
$\cdot 83$	$\cdot 0152\ 242$	$\cdot 0130\ 748$	$\cdot 0112\ 205^-$	$\cdot 0096\ 222$	$\cdot 0082\ 458$	$\cdot 0070\ 617$	
$\cdot 84$	$\cdot 0219\ 435^-$	$\cdot 0190\ 666$	$\cdot 0165\ 547$	$\cdot 0143\ 634$	$\cdot 0124\ 538$	$\cdot 0107\ 910$	
$\cdot 85$	$\cdot 0313\ 317$	$\cdot 0275\ 384$	$\cdot 0241\ 869$	$\cdot 0212\ 285^-$	$\cdot 0186\ 195^+$	$\cdot 0163\ 208$	
$\cdot 86$	$\cdot 0443\ 013$	$\cdot 0393\ 803$	$\cdot 0349\ 813$	$\cdot 0310\ 525^+$	$\cdot 0275\ 471$	$\cdot 0244\ 222$	
$\cdot 87$	$\cdot 0620\ 030$	$\cdot 0557\ 317$	$\cdot 0500\ 603$	$\cdot 0449\ 304$	$\cdot 0403\ 115^-$	$\cdot 0361\ 405^+$	
$\cdot 88$	$\cdot 0858\ 493$	$\cdot 0780\ 132$	$\cdot 0708\ 452$	$\cdot 0642\ 948$	$\cdot 0583\ 143$	$\cdot 0528\ 589$	
$\cdot 89$	$\cdot 1175\ 147$	$\cdot 1079\ 384$	$\cdot 0990\ 790$	$\cdot 0908\ 909$	$\cdot 0833\ 302$	$\cdot 0763\ 550^+$	
$\cdot 90$	$\cdot 1588\ 947$	$\cdot 1474\ 854$	$\cdot 1368\ 117$	$\cdot 1268\ 358$	$\cdot 1175\ 209$	$\cdot 1088\ 307$	
$\cdot 91$	$\cdot 2119\ 933$	$\cdot 1987\ 978$	$\cdot 1863\ 159$	$\cdot 1745\ 205^+$	$\cdot 1633\ 842$	$\cdot 1528\ 794$	
$\cdot 92$	$\cdot 2787\ 006$	$\cdot 2639\ 720$	$\cdot 2498\ 868$	$\cdot 2364\ 300$	$\cdot 2235\ 854$	$\cdot 2113\ 362$	
$\cdot 93$	$\cdot 3604\ 097$	$\cdot 3446\ 734$	$\cdot 3294\ 610$	$\cdot 3147\ 693$	$\cdot 3005\ 937$	$\cdot 2869\ 280$	
$\cdot 94$	$\cdot 4574\ 134$	$\cdot 4415\ 134$	$\cdot 4259\ 775^-$	$\cdot 4108\ 120$	$\cdot 3960\ 219$	$\cdot 3816\ 105^+$	
$\cdot 95$	$\cdot 5680\ 291$	$\cdot 5531\ 188$	$\cdot 5383\ 949$	$\cdot 5238\ 692$	$\cdot 5095\ 523$	$\cdot 4954\ 536$	
$\cdot 96$	$\cdot 6874\ 411$	$\cdot 6748\ 671$	$\cdot 6623\ 195^+$	$\cdot 6498\ 106$	$\cdot 6373\ 517$	$\cdot 6249\ 536$	
$\cdot 97$	$\cdot 8063\ 842$	$\cdot 7973\ 966$	$\cdot 7883\ 344$	$\cdot 7792\ 061$	$\cdot 7700\ 195^+$	$\cdot 7607\ 826$	
$\cdot 98$	$\cdot 9101\ 647$	$\cdot 9053\ 979$	$\cdot 9005\ 420$	$\cdot 8956\ 002$	$\cdot 8905\ 757$	$\cdot 8854\ 714$	
$\cdot 99$	$\cdot 9795\ 942$	$\cdot 9783\ 673$	$\cdot 9771\ 048$	$\cdot 9758\ 069$	$\cdot 9744\ 737$	$\cdot 9731\ 055^+$	
$\cdot 100$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	$\cdot 10000\ 000$	

$x = .63 \text{ to } 1.00$ 
$$p = 4$$

	$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$B(p, q) = .9925\ 9531 \times \frac{1}{10^4}$		$.9392\ 2997 \times \frac{1}{10^4}$	$.8897\ 9681 \times \frac{1}{10^4}$	$.8439\ 3100 \times \frac{1}{10^4}$	$.8013\ 0822 \times \frac{1}{10^4}$	$.7616\ 3950 \times \frac{1}{10^4}$	$.7246\ 667$
$\cdot 63$	.0000 001	.0000 001					
$\cdot 64$	.0000 002	.0000 001	.0000 001				
$\cdot 65$	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001		
$\cdot 66$	.0000 006	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
$\cdot 67$	.0000 010	.0000 007	.0000 005	.0000 003	.0000 002	.0000 002	.0000 001
$\cdot 68$	.0000 019	.0000 013	.0000 009	.0000 007	.0000 005	.0000 003	.0000 002
$\cdot 69$	.0000 035	.0000 025	.0000 018	.0000 012	.0000 009	.0000 006	.0000 004
$\cdot 70$	.0000 062	.0000 045	.0000 032	.0000 023	.0000 017	.0000 012	.0000 009
$\cdot 71$	.0000 111	.0000 081	.0000 059	.0000 043	.0000 032	.0000 023	.0000 017
$\cdot 72$	.0000 195	.0000 145	.0000 107	.0000 080	.0000 059	.0000 044	.0000 032
$\cdot 73$	.0000 340	.0000 256	.0000 193	.0000 145	.0000 109	.0000 082	.0000 061
$\cdot 74$	.0000 587	.0000 448	.0000 342	.0000 261	.0000 199	.0000 151	.0000 115
$\cdot 75$	.0001 005	.0000 777	.0000 601	.0000 464	.0000 358	.0000 276	.0000 213
$\cdot 76$	.0001 702	.0001 333	.0001 044	.0000 817	.0000 639	.0000 500	.0000 391
$\cdot 77$	.0002 854	.0002 265 <sup>+</sup>	.0001 797	.0001 425 <sup>+</sup>	.0001 129	.0000 894	.0000 708
$\cdot 78$	.0004 740	.0003 811	.0003 062	.0002 459	.0001 974	.0001 584	.0001 270
$\cdot 79$	.0007 797	.0006 348	.0005 166	.0004 201	.0003 415	.0002 774	.0002 253
$\cdot 80$	.0012 699	.0010 469	.0008 625 <sup>+</sup>	.0007 103	.0005 808	.0004 808	.0003 953
$\cdot 81$	.0020 481	.0017 092	.0014 256	.0011 884	.0009 901	.0008 245 <sup>+</sup>	.0006 862
$\cdot 82$	.0032 704	.0027 625	.0023 321	.0019 677	.0016 594	.0013 987	.0011 783
$\cdot 83$	.0051 694	.0044 189	.0037 752	.0032 236	.0027 511	.0023 467	.0020 007
$\cdot 84$	.0080 868	.0069 945	.0060 463	.0052 239	.0045 110	.0038 935	.0033 589
$\cdot 85$	.0125 169	.0109 522	.0095 778	.0083 716	.0073 135	.0063 860	.0055 736
$\cdot 86$	.0191 615 <sup>+</sup>	.0169 585	.0150 007	.0132 621	.0117 191	.0103 598	.0091 379
$\cdot 87$	.0289 984	.0259 542	.0232 174	.0207 588	.0185 515	.0165 711	.0147 955
$\cdot 88$	.0433 585	.0392 378	.0354 908	.0320 859	.0289 939	.0261 879	.0236 431
$\cdot 89$	.0640 034	.0585 531	.0535 407	.0489 342	.0447 035 <sup>+</sup>	.0408 206	.0372 590
$\cdot 90$	.0931 855 <sup>+</sup>	.0861 638	.0796 335 <sup>+</sup>	.0735 647	.0679 284	.0626 972	.0578 450
$\cdot 91$	.1336 546	.1248 805 <sup>+</sup>	.1166 299	.1088 771	.1015 968	.0947 647	.0883 572
$\cdot 92$	.1885 517	.1779 795 <sup>+</sup>	.1679 289	.1583 808	.1493 162	.1407 162	.1325 620
$\cdot 93$	.2610 964	.2489 130	.2372 047	.2259 609	.2151 705 <sup>+</sup>	.2048 218	.1949 029
$\cdot 94$	.3539 313	.3406 641	.3277 772	.3152 686	.3031 352	.2913 734	.2799 788
$\cdot 95$	.4679 434	.4545 454	.4413 931	.4284 910	.4158 427	.4034 514	.3913 193
$\cdot 96$	.6003 794	.5882 214	.5761 608	.5642 050 <sup>+</sup>	.5523 611	.5406 357	.5290 347
$\cdot 97$	.7421 873	.7328 432	.7234 774	.7140 963	.7047 062	.6953 132	.6859 230
$\cdot 98$	.8750 364	.8697 117	.8643 193	.8588 625	.8533 441	.8477 672	.8421 344
$\cdot 99$	.9702 655	.9687 941	.9672 889	.9657 501	.9641 781	.9625 732	.9609 357
I .00	I .0000 000	I .0000 000	I .0000 000	I .0000 000	I .0000 000	I .0000 000	I .0000 000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .01$  to  $.60$  $q = 3$  $p = 3$  to  $5$ 

	$p = 3$	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$
$B(p, q) = .3333\ 3333 \times \frac{x}{10}$	$.2308\ 8023 \times \frac{x}{10}$	$.1666\ 6667 \times \frac{x}{10}$	$.1243\ 2012 \times \frac{x}{10}$	$.09523\ 8005 \times \frac{x}{10}$	$.07450\ 2015 \times \frac{x}{10}$	
$x$						
.01	.0000 099	.0000 012	.0000 001			
.02	.0000 776	.0000 136	.0000 023	.0000 004	.0000 001	
.03	.0002 580	.0000 552	.0000 110	.0000 024	.0000 003	.0000 001
.04	.0006 022	.0001 487	.0000 360	.0000 080	.0000 020	.0000 005
.05	.0011 581	.0003 195 <sup>+</sup>	.0000 864	.0000 230	.0000 060	.0000 016
.06	.0019 703	.0005 951	.0001 762	.0000 513	.0000 147	.0000 042
.07	.0030 799	.0010 042	.0003 210	.0001 009	.0000 313	.0000 103
.08	.0045 253	.0015 764	.0005 384	.0001 809	.0000 600	.0000 196
.09	.0063 413	.0023 415 <sup>-</sup>	.0008 477	.0003 020	.0001 061	.0000 376
.10	.0085 600 <sup>o</sup>	.0033 295 <sup>-</sup>	.0012 700 <sup>o</sup>	.0004 767	.0001 765 <sup>o</sup>	.0000 616
.11	.0112 105 <sup>-</sup>	.0045 702	.0018 273	.0007 190	.0002 791	.0001 073
.12	.0143 189	.0060 928	.0025 431	.0010 447	.0004 234	.0001 697
.13	.0179 086	.0079 258	.0034 413	.0014 707	.0006 202	.0002 436
.14	.0220 003	.0100 970	.0045 469	.0020 156	.0008 817	.0003 814
.15	.0266 119	.0126 330	.0058 852	.0026 691	.0012 216	.0005 357
.16	.0317 587	.0155 592	.0074 816	.0034 229	.0016 551	.0007 197
.17	.0374 538	.0188 998	.0093 619	.0043 663	.0021 684	.0009 367
.18	.0437 073	.0226 774	.0115 516	.0054 948	.0028 695	.0012 953
.19	.0505 275 <sup>+</sup>	.0269 133	.0140 760	.0067 208	.0036 854	.0017 915
.20	.0579 200 <sup>o</sup>	.0316 269	.0169 600 <sup>o</sup>	.0080 586	.0046 720 <sup>o</sup>	.0024 009
.21	.0658 883	.0368 364	.0202 280	.0100 427	.0058 470	.0030 882
.22	.0744 338	.0425 576	.0239 035 <sup>-</sup>	.0124 279	.0072 285 <sup>-</sup>	.0040 075 <sup>o</sup>
.23	.0835 557	.0488 052	.0280 093	.0158 393	.0088 458	.0050 884
.24	.0932 512	.0555 915 <sup>+</sup>	.0325 671	.0188 019	.0107 209	.0063 481
.25	.1035 156	.0629 272	.0375 977	.0221 405 <sup>+</sup>	.0127 784	.0077 120
.26	.1143 424	.0708 212	.0431 203	.0258 297	.0153 430	.0092 017
.27	.1257 232	.0792 800	.0491 530	.0300 436	.0181 420	.0108 414
.28	.1376 478	.0883 088	.0557 124	.0346 555 <sup>+</sup>	.0212 690	.0126 555
.29	.1501 045 <sup>+</sup>	.0979 103	.0628 136	.0397 481	.0248 321	.0146 100
.30	.1630 800 <sup>o</sup>	.1080 855 <sup>+</sup>	.0704 700 <sup>o</sup>	.0453 138	.0287 955 <sup>o</sup>	.0168 126
.31	.1765 593	.1188 335 <sup>+</sup>	.0786 932	.0513 027	.0331 895	.0192 087
.32	.1905 263	.1301 513	.0874 932	.0576 245 <sup>+</sup>	.0380 373	.0218 649
.33	.2049 631	.1420 341	.0968 779	.0651 075	.0434 757	.0248 520
.34	.2198 509	.1544 752	.1068 534	.0729 384	.0492 247	.0281 929
.35	.2351 694	.1674 658	.1174 239	.0812 625 <sup>o</sup>	.0553 029 <sup>+</sup>	.0317 820
.36	.2508 973	.1809 954	.1285 914	.0901 814	.0617 362	.0356 360
.37	.2670 122	.1950 518	.1403 559	.0997 124	.0685 617	.0397 507
.38	.2834 907	.2096 209	.1527 154	.1098 594	.0758 234	.0441 225
.39	.3003 084	.2246 868	.1656 655 <sup>+</sup>	.1206 124	.0836 693	.0487 439
.40	.3174 400 <sup>o</sup>	.2402 319	.1792 000 <sup>o</sup>	.1320 365	.0920 560 <sup>o</sup>	.0536 216
.41	.3348 596	.2562 371	.1933 103	.1440 754	.1008 975 <sup>+</sup>	.0587 607
.42	.3525 403	.2726 816	.2079 858	.1567 600 <sup>+</sup>	.1100 163	.0641 684
.43	.3704 540	.2895 431	.2232 135 <sup>+</sup>	.1700 592	.1195 428	.0698 510
.44	.3885 753	.3067 978	.2389 786	.1840 991	.1292 448	.0758 317
.45	.4068 731	.3244 205 <sup>-</sup>	.2552 639	.1988 635 <sup>+</sup>	.1392 277	.0820 515
.46	.4253 194	.3423 846	.2720 502	.2137 438	.1495 045 <sup>+</sup>	.0885 693
.47	.4438 840	.3606 624	.2893 163	.2295 284	.1601 043	.0953 460
.48	.4625 400	.3792 249	.3070 388	.2460 046	.1709 200	.1023 302
.49	.4812 550 <sup>-</sup>	.3980 420	.3251 924	.2628 528	.1819 664	.1095 625
.50	.5000 000 <sup>o</sup>	.4170 825 <sup>+</sup>	.3437 500 <sup>o</sup>	.2803 568	.1932 625 <sup>o</sup>	.1170 103
.51	.5187 450 <sup>+</sup>	.4363 144	.3626 824	.2984 038	.2048 040	.1247 923
.52	.5374 600	.4557 046	.3819 588	.3169 394	.2166 620	.1328 883
.53	.5561 151	.4752 105 <sup>+</sup>	.4015 466	.3359 666	.2288 639	.1412 608
.54	.5746 806	.4948 247	.4214 115 <sup>-</sup>	.3553 459	.2407 634	.1498 603
.55	.5931 269	.5144 852	.4415 177	.3753 452	.2529 401	.1586 602
.56	.6114 247	.5341 656	.4618 279	.3956 600	.2654 602	.1676 602
.57	.6295 451	.5538 299	.4823 037	.4162 635	.2782 123	.1768 602
.58	.6474 597	.5734 422	.5020 051	.4372 091	.2912 420	.1861 212
.59	.6651 404	.5929 660	.5235 912	.4584 175 <sup>+</sup>	.3044 201	.1955 523
.60	.6825 600 <sup>o</sup>	.6123 651	.5443 200 <sup>o</sup>	.4798 544	.3179 040 <sup>o</sup>	.2050 608

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION $x = .61$  to  $1.00$  $q = 3$  $p = 3$  to  $5.5$ 

	$p = 3$	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$
$B(p, q) \times 10^4$	3333 3333 $\times 10^4$	2308 8023 $\times 10^4$	1666 6667 $\times 10^4$	1243 2012 $\times 10^4$	9523 8095 $\times 10^4$	7459 2075 $\times 10^4$
.61	6696 916	6316 032	5650 488	5014 684	4418 500	3868 052
.62	7165 093	6506 442	5857 340	5232 156	4641 129	4090 962
.63	7329 878	6604 521	6003 315	5450 459	4866 413	4318 361
.64	7491 027	6879 617	6267 668	5660 688	5093 841	4549 748
.65	7648 306	7062 279	6470 852	5887 528	5322 833	4784 566
.66	7801 401	7241 266	6671 517	6105 247	5552 843	5022 241
.67	7950 369	7416 512	6869 517	6321 707	5783 261	5262 150
.68	8094 737	7587 783	7064 407	6536 362	6013 460	5503 634
.69	8234 907	7754 672	7255 745 <sup>†</sup>	6748 662	6242 841	5746 006
.70	8369 206 <sup>†</sup>	7916 966	7443 100 <sup>†</sup>	6958 053	6470 605 <sup>†</sup>	5988 546
.71	8498 953 <sup>†</sup>	8074 195	7626 045 <sup>†</sup>	7163 981	6696 397	6230 506
.72	8624 522	8226 263	7804 168	7366 807	6919 265 <sup>†</sup>	6471 117
.73	8742 768	8372 849	7977 666	7563 257	7138 622	6709 586
.74	8856 576	8513 711	8144 354	7755 524	7353 788	6945 107
.75	8964 843	8648 623 <sup>†</sup>	8305 664	7942 178	7564 687	7176 866
.76	9067 488	8777 387	8460 698	8122 711	7768 850	7404 922
.77	9164 443	8899 815 <sup>†</sup>	8608 980	8296 636	7967 419	7628 766
.78	9255 662	9015 749	8750 359	8463 488	8159 155 <sup>†</sup>	7841 274
.79	9341 117	9125 055 <sup>†</sup>	8884 513	8622 831	8343 448	8049 737
.80	9420 800 <sup>†</sup>	9227 626	9011 200 <sup>†</sup>	8774 259	8519 680 <sup>†</sup>	8250 368
.81	9494 725	9324 382	9130 210	8917 403	8687 321	8444 407
.82	9562 927	9412 259	9241 369	9051 941	8845 853	8625 130
.83	9625 362	9494 273	9344 513	9177 560	8994 299	8797 857
.84	9682 113	9569 402	9440 643	9294 654	9141 739	8959 964
.85	9734 591	9647 265 <sup>†</sup>	9536 614	9401 230	9282 448	9116 801
.86	9781 997	9719 265 <sup>†</sup>	9625 463	9498 669	9416 315	9259 156
.87	9824 913	9784 264	9707 211	9587 213	9548 744	9397 361
.88	9862 811	9842 680	9784 654	9665 927	9681 612	9532 212
.89	9895 365 <sup>†</sup>	9894 865 <sup>†</sup>	9854 654	9735 451	9808 299	9664 526
.90	9924 400 <sup>†</sup>	9931 601	9921 500 <sup>†</sup>	9795 510	9924 685 <sup>†</sup>	9784 250 <sup>†</sup>
.91	9949 587	9961 564	9981 652	9846 213	9986 665 <sup>†</sup>	9901 470
.92	9971 747	9986 612	9994 859	9889 419	9989 860	9986 442
.93	9990 291	9996 799	9993 611	9923 284	9994 124	9989 554
.94	9996 297	9997 198	9992 357	9950 673	9997 069	9994 447
.95	9998 419	9998 595	9992 702	9970 668	9992 439	9992 930
.96	9999 628	9999 436	9993 316	9984 971	9993 162	9995 051
.97	9999 720	9999 317	9993 956	9993 314	9993 370	9999 166
.98	9999 224	9999 888	9993 471	9997 965 <sup>†</sup>	9997 363	9999 660
.99	9999 991	9999 858	9999 804	9999 739	9999 660	9999 568
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .04$  to  $.60$  $q = 3$  $p = 6$  to  $8.5$ 

	$p = 6$	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$
$B(p, q) = .5952\ 3810 \times \frac{1}{10^3}$	$.4826\ 5460 \times \frac{1}{10^3}$	$.3968\ 2540 \times \frac{1}{10^3}$	$.3302\ 3736 \times \frac{1}{10^3}$	$.2777\ 7778 \times \frac{1}{10^3}$	$.2358\ 8383 \times \frac{1}{10^3}$	
$x$						
.04	.0000 001					
.05	.0000 004	.0000 001				
.06	.0000 012	.0000 003	.0000 001			
.07	.0000 029	.0000 009	.0000 003	.0000 001		
.08	.0000 064	.0000 020	.0000 007	.0000 002	.0000 001	
.09	.0000 127	.0000 043	.0000 015	.0000 005	.0000 002	.0000 001
.10	.0000 234	.0000 084	.0000 030	.0000 011	.0000 004	.0000 001
.11	.0000 407	.0000 153	.0000 057	.0000 021	.0000 008	.0000 003
.12	.0000 673	.0000 265	.0000 103	.0000 040	.0000 015 <sup>+</sup>	.0000 006
.13	.0001 067	.0000 437	.0000 177	.0000 072	.0000 029	.0000 011
.14	.0001 633	.0000 694	.0000 292	.0000 122	.0000 051	.0000 021
.15	.0002 423	.0001 065	.0000 464	.0000 201	.0000 087	.0000 037
.16	.0003 499	.0001 588	.0000 715 <sup>+</sup>	.0000 320	.0000 142	.0000 063
.17	.0004 935 <sup>+</sup>	.0002 308	.0001 071	.0000 494	.0000 226	.0000 103
.18	.0006 816	.0003 278	.0001 565 <sup>+</sup>	.0000 742	.0000 350 <sup>+</sup>	.0000 164
.19	.0009 239	.0004 564	.0002 238	.0001 090	.0000 528	.0000 254
.20	.0012 314	.0006 240	.0003 139	.0001 568	.0000 779	.0000 385 <sup>+</sup>
.21	.0016 164	.0008 390	.0004 323	.0002 213	.0001 127	.0000 570
.22	.0020 926	.0011 114	.0005 861	.0003 070	.0001 599	.0000 829
.23	.0026 751	.0014 524	.0007 828	.0004 192	.0002 232	.0001 182
.24	.0033 805 <sup>+</sup>	.0018 742	.0010 316	.0005 642	.0003 068	.0001 660
.25	.0042 267	.0023 909	.0013 428	.0007 493	.0004 158	.0002 295 <sup>+</sup>
.26	.0052 329	.0030 177	.0017 279	.0009 831	.0005 562	.0003 130
.27	.0064 199	.0037 714	.0021 999	.0012 752	.0007 350 <sup>+</sup>	.0004 215 <sup>-</sup>
.28	.0078 097	.0046 704	.0027 735 <sup>-</sup>	.0016 367	.0009 605 <sup>-</sup>	.0005 608
.29	.0094 256	.0057 345 <sup>+</sup>	.0034 646	.0020 802	.0012 420	.0007 378
.30	.0112 922	.0069 851	.0042 909	.0026 196	.0015 904	.0009 607
.31	.0134 351	.0084 448	.0052 716	.0032 706	.0020 179	.0012 388
.32	.0158 811	.0101 381	.0064 277	.0040 504	.0025 384	.0015 829
.33	.0186 577	.0120 906	.0077 818	.0049 782	.0031 673	.0020 052
.34	.0217 935 <sup>-</sup>	.0143 292	.0093 580	.0060 746	.0039 219	.0025 196
.35	.0253 175 <sup>+</sup>	.0168 822	.0111 822	.0073 623	.0048 213	.0031 419
.36	.0292 594	.0197 791	.0132 818	.0088 658	.0058 864	.0038 893
.37	.0336 492	.0230 502	.0156 858	.0106 113	.0071 403	.0047 816
.38	.0385 171	.0267 269	.0184 246	.0126 269	.0086 079	.0058 401
.39	.0438 932	.0308 411	.0215 299	.0149 425 <sup>+</sup>	.0103 163	.0070 886
.40	.0498 074	.0354 256	.0250 348	.0175 898	.0122 946	.0085 529
.41	.0562 892	.0405 133	.0289 732	.0206 019	.0145 738	.0102 612
.42	.0633 676	.0461 373	.0333 803	.0240 137	.0171 871	.0122 440
.43	.0710 705 <sup>-</sup>	.0523 308	.0382 916	.0278 616	.0201 696	.0145 340
.44	.0794 247	.0591 265 <sup>+</sup>	.0437 436	.0321 828	.0235 583	.0171 662
.45	.0884 559	.0665 568	.0497 728	.0370 162	.0273 918	.0201 780
.46	.0981 878	.0746 531	.0564 157	.0424 011	.0317 105 <sup>+</sup>	.0236 089
.47	.1086 426	.0834 458	.0637 089	.0483 776	.0365 560	.0275 003
.48	.1198 402	.0929 639	.0716 881	.0549 862	.0419 713	.0318 958
.49	.1317 981	.1032 346	.0803 884	.0622 675 <sup>+</sup>	.0480 003	.0368 406
.50	.1445 312	.1142 834	.0898 437	.0702 618	.0546 875 <sup>e</sup>	.0423 815 <sup>+</sup>
.51	.1580 516	.1261 331	.1000 864	.0790 088	.0620 778	.0485 665 <sup>+</sup>
.52	.1723 681	.1388 041	.1111 469	.0885 472	.0702 161	.0554 447
.53	.1874 861	.1523 138	.1230 533	.0989 143	.0791 470	.0630 655 <sup>-</sup>
.54	.2034 075 <sup>+</sup>	.1666 763	.1358 313	.1101 457	.0889 141	.0714 788
.55	.2201 303	.1819 019	.1495 031	.1222 746	.0995 597	.0807 342
.56	.2376 483	.1979 971	.1640 878	.1353 313	.1111 243	.0908 804
.57	.2559 510	.2149 641	.1796 003	.1493 433	.1236 462	.1019 650 <sup>-</sup>
.58	.2750 235 <sup>-</sup>	.2328 005 <sup>+</sup>	.1960 513	.1643 337	.1371 607	.1140 335 <sup>-</sup>
.59	.2948 461	.2514 990	.2134 467	.1803 219	.1516 993	.1271 290
.60	.3153 946	.2710 469	.2317 870	.1973 221	.1672 898	.1412 913

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .61$  to  $1.00$  $q = 3$  $p = 6$  to  $8.5$ 

	$p = 6$	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$
$B(p, q) = .5952\ 3810 \times \frac{1}{10^8}$	$.4826\ 5460 \times \frac{1}{10^8}$	$.3968\ 2540 \times \frac{1}{10^8}$	$.3302\ 3736 \times \frac{1}{10^8}$	$.2777\ 7778 \times \frac{1}{10^8}$	$.2358\ 8383 \times \frac{1}{10^8}$	
$\cdot 61$	.3366 393	.2914 263	.2510 674	.2153 432	.1839 547	.1565 564
$\cdot 62$	.3585 458	.3126 134	.2712 770	.2343 882	.2017 113	.1729 553
$\cdot 63$	.3810 745 <sup>-</sup>	.3345 787	.2923 984	.2544 535 <sup>+</sup>	.2205 708	.1905 137
$\cdot 64$	.4041 805 <sup>-</sup>	.3572 863	.3144 075 <sup>+</sup>	.2755 288	.2405 373	.2092 507
$\cdot 65$	.4278 137	.3806 941	.3372 733	.2975 960	.2616 074	.2291 786
$\cdot 66$	.4519 187	.4047 537	.3609 571	.3206 292	.2837 696	.2503 011
$\cdot 67$	.4764 353	.4294 099	.3854 128	.3445 939	.3070 034	.2726 133
$\cdot 68$	.5012 977	.4546 013	.4105 864	.3694 467	.3312 788	.2961 002
$\cdot 69$	.5264 356	.4802 598	.4364 159	.3951 353	.3565 555 <sup>+</sup>	.3207 365 <sup>+</sup>
$\cdot 70$	.5517 738	.5063 107	.4628 312	.4215 975 <sup>-</sup>	.3827 828	.3464 851
$\cdot 71$	.5772 327	.5326 733	.4897 540	.4487 614	.4098 985 <sup>+</sup>	.3732 967
$\cdot 72$	.6027 284	.5592 605 <sup>+</sup>	.5170 982	.4765 453	.4378 290	.4011 090
$\cdot 73$	.6281 732	.5859 796	.5447 693	.5048 573	.4664 888	.4298 463
$\cdot 74$	.6534 761	.6127 322	.5726 655 <sup>+</sup>	.5335 958	.4957 800	.4594 186
$\cdot 75$	.6785 431	.6394 149	.6006 775 <sup>-</sup>	.5626 490	.5255 928	.4897 214
$\cdot 76$	.7032 777	.6659 199	.6286 889	.5918 960	.5558 051	.5206 356
$\cdot 77$	.7275 817	.6921 354	.6565 772	.6212 065 <sup>-</sup>	.5862 827	.5520 271
$\cdot 78$	.7513 559	.7179 404	.6842 140	.6504 419	.6168 803	.5837 474
$\cdot 79$	.7745 009	.7432 358	.7114 664	.6794 559	.6474 414	.6156 334
$\cdot 80$	.7969 178	.7678 851	.7381 975 <sup>+</sup>	.7080 955 <sup>+</sup>	.6777 995 <sup>+</sup>	.6475 088
$\cdot 81$	.8185 090	.7917 753	.7642 679	.7362 023	.7077 796	.6791 845 <sup>-</sup>
$\cdot 82$	.8391 800	.8147 888	.7895 369	.7636 138	.7371 990	.7104 601
$\cdot 83$	.8588 397	.8368 102	.8138 644	.7901 653	.7658 695 <sup>-</sup>	.7411 259
$\cdot 84$	.8774 020	.8577 280	.8371 123	.8156 916	.7935 995 <sup>-</sup>	.7709 649
$\cdot 85$	.8947 872	.8774 362	.8591 466	.8400 298	.8201 965 <sup>-</sup>	.7997 554
$\cdot 86$	.9109 236	.8958 366	.8798 399	.8630 214	.8454 702	.8272 748
$\cdot 87$	.9257 486	.9128 400	.8990 736	.8845 160	.8692 358	.8533 029
$\cdot 88$	.9392 108	.9283 694	.9167 411	.9043 738	.8913 182	.8776 267
$\cdot 89$	.9512 719	.9423 616	.9327 504	.9224 703	.9115 565 <sup>+</sup>	.9000 462
$\cdot 90$	.9619 082	.9547 704	.9470 279	.9387 000	.9298 092	.9203 798
$\cdot 91$	.9711 132	.9655 691	.9595 219	.9529 814	.9459 600	.9384 720
$\cdot 92$	.9788 995 <sup>+</sup>	.9747 537	.9702 068	.9652 621	.9599 246	.9542 011
$\cdot 93$	.9853 013	.9823 462	.9790 877	.9755 247	.9716 579	.9674 889
$\cdot 94$	.9903 771	.9883 984	.9862 047	.9837 933	.9811 622	.9783 102
$\cdot 95$	.9942 118	.9929 950 <sup>+</sup>	.9916 390	.9901 403	.9884 964	.9867 051
$\cdot 96$	.9969 203	.9962 588	.9955 176	.9946 942	.9937 863	.9927 917
$\cdot 97$	.9986 501	.9983 540	.9980 204	.9976 479	.9972 350 <sup>+</sup>	.9967 804
$\cdot 98$	.9995 845 <sup>+</sup>	.9994 915 <sup>-</sup>	.9993 861	.9992 679	.9991 361	.9989 903
$\cdot 99$	.9999 461	.9999 337	.9999 197	.9999 039	.9998 862	.9998 664
$1.00$	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .11$  to  $.70$  $q = 3$  $p = 9$  to  $12$ 

	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$
$B(p, q) = .2020\ 2020 \times \frac{1}{10^8}$	$.1743\ 4892 \times \frac{1}{10^8}$	$.1515\ 1515 \times \frac{1}{10^8}$	$.1325\ 0518 \times \frac{1}{10^8}$	$.1165\ 5012 \times \frac{1}{10^8}$	$.9157\ 5092 \times \frac{1}{10^8}$	
$x$						
.11	.0000 001					
.12	.0000 002	.0000 001				
.13	.0000 005 <sup>-</sup>	.0000 002	.0000 001			
.14	.0000 009	.0000 004	.0000 001	.0000 001		
.15	.0000 016	.0000 007	.0000 003	.0000 001	.0000 001	
.16	.0000 028	.0000 012	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
.17	.0000 047	.0000 021	.0000 010	.0000 004	.0000 002	
.18	.0000 077	.0000 036	.0000 016	.0000 008	.0000 003	.0000 001
.19	.0000 122	.0000 058	.0000 028	.0000 013	.0000 006	.0000 001
.20	.0000 189	.0000 093	.0000 045 <sup>+</sup>	.0000 022	.0000 011	.0000 002
.21	.0000 287	.0000 144	.0000 072	.0000 036	.0000 018	.0000 004
.22	.0000 427	.0000 219	.0000 112	.0000 057	.0000 029	.0000 007
.23	.0000 623	.0000 327	.0000 171	.0000 089	.0000 046	.0000 012
.24	.0000 894	.0000 479	.0000 256	.0000 136	.0000 072	.0000 020
.25	.0001 261	.0000 690	.0000 376	.0000 204	.0000 111	.0000 032
.26	.0001 754	.0000 978	.0000 544	.0000 301	.0000 166	.0000 050 <sup>+</sup>
.27	.0002 406	.0001 368	.0000 774	.0000 437	.0000 246	.0000 077
.28	.0003 259	.0001 886	.0001 087	.0000 625 <sup>-</sup>	.0000 358	.0000 116
.29	.0004 363	.0002 569	.0001 507	.0000 881	.0000 513	.0000 173
.30	.0005 777	.0003 459	.0002 064	.0001 227	.0000 727	.0000 253
.31	.0007 571	.0004 608	.0002 794	.0001 688	.0001 017	.0000 366
.32	.0009 826	.0006 075 <sup>-</sup>	.0003 742	.0002 297	.0001 405 <sup>+</sup>	.0000 521
.33	.0012 638	.0007 933	.0004 961	.0003 091	.0001 920	.0000 735 <sup>-</sup>
.34	.0016 116	.0010 266	.0006 515 <sup>-</sup>	.0004 120	.0002 598	.0001 024
.35	.0020 384	.0013 171	.0008 479	.0005 440	.0003 479	.0001 411
.36	.0025 585 <sup>-</sup>	.0016 763	.0010 942	.0007 118	.0004 616	.0001 925 <sup>+</sup>
.37	.0031 880	.0021 171	.0014 007	.0009 236	.0006 072	.0002 601
.38	.0039 450 <sup>+</sup>	.0026 543	.0017 794	.0011 888	.0007 918	.0003 483
.39	.0048 497	.0033 048	.0022 439	.0015 185 <sup>+</sup>	.0010 245 <sup>-</sup>	.0004 624
.40	.0059 245 <sup>-</sup>	.0040 877	.0028 102	.0019 255 <sup>+</sup>	.0013 153	.0006 087
.41	.0071 941	.0050 240	.0034 960	.0024 247	.0016 766	.0007 950 <sup>-</sup>
.42	.0086 857	.0061 376	.0043 216	.0030 330	.0021 221	.0010 304
.43	.0104 290	.0074 547	.0053 098	.0037 698	.0026 683	.0013 259
.44	.0124 564	.0090 043	.0064 860	.0046 570	.0033 337	.0016 945 <sup>-</sup>
.45	.0148 026	.0108 180	.0078 785 <sup>-</sup>	.0057 192	.0041 394	.0021 510
.46	.0175 050 <sup>+</sup>	.0129 305 <sup>-</sup>	.0095 184	.0069 843	.0051 097	.0027 130
.47	.0206 038	.0153 792	.0114 400	.0084 829	.0062 717	.0034 009
.48	.0241 413	.0182 045 <sup>+</sup>	.0136 810	.0102 492	.0076 559	.0042 380
.49	.0281 626	.0214 499	.0162 820	.0123 207	.0092 962	.0052 508
.50	.0327 148	.0251 613	.0192 871	.0147 386	.0112 305 <sup>-</sup>	.0064 697
.51	.0378 473	.0293 879	.0227 437	.0175 476	.0135 002	.0079 289
.52	.0436 112	.0341 812	.0267 022	.0207 963	.0161 510	.0096 669
.53	.0500 591	.0395 950 <sup>+</sup>	.0312 165 <sup>+</sup>	.0245 369	.0192 327	.0117 264
.54	.0572 449	.0456 857	.0363 433	.0288 252	.0227 991	.0141 554
.55	.0652 235 <sup>+</sup>	.0525 112	.0421 420	.0337 206	.0269 082	.0170 062
.56	.0740 499	.0601 308	.0486 745 <sup>-</sup>	.0392 859	.0316 222	.0203 367
.57	.0837 790	.0686 051	.0560 047	.0455 868	.0370 072	.0242 097
.58	.0944 650 <sup>-</sup>	.0779 948	.0641 984	.0526 919	.0431 329	.0286 935 <sup>+</sup>
.59	.1061 607	.0883 607	.0733 223	.0606 720	.0500 726	.0338 613
.60	.1189 168	.0997 626	.0834 433	.0695 996	.0579 024	.0397 916
.61	.1327 812	.1122 588	.0946 285 <sup>+</sup>	.0795 485 <sup>-</sup>	.0667 007	.0465 674
.62	.1477 979	.1259 049	.1069 435 <sup>+</sup>	.0905 924	.0765 479	.0542 761
.63	.1640 065 <sup>+</sup>	.1407 534	.1204 520	.1028 046	.0875 249	.0630 091
.64	.1814 410	.1568 521	.1352 146	.1162 569	.0997 128	.0728 604
.65	.2001 289	.1742 434	.1512 876	.1310 181	.1131 914	.0839 266
.66	.2200 899	.1929 630	.1687 217	.1471 529	.1280 380	.0963 048
.67	.2413 356	.2130 387	.1875 609	.1647 207	.1443 261	.1100 919
.68	.2638 673	.2344 892	.2078 409	.1837 736	.1621 233	.1253 830
.69	.2876 760	.2573 225 <sup>+</sup>	.2295 875 <sup>+</sup>	.2043 551	.1814 903	.1422 692
.70	.3127 405 <sup>-</sup>	.2815 350 <sup>-</sup>	.2528 153	.2264 983	.2024 783	.1608 358

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .71$  to  $1.00$  $q = 3$  $p = 9$  to  $12$ 

	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$
$B(p, q) = .2020\ 2020 \times \frac{1}{10^8}$		$.1743\ 4892 \times \frac{1}{10^8}$	$.1515\ 1515 \times \frac{1}{10^8}$	$.1325\ 0518 \times \frac{1}{10^8}$	$.1165\ 5012 \times \frac{1}{10^8}$	$.9157\ 5092 \times \frac{1}{10^8}$
$x$						
.71	.3390 267	.3071 096	.2775 258	.2502 236	.2251 270	.1811 596
.72	.3664 868	.3340 151	.3037 057	.2755 374	.2494 628	.2033 070
.73	.3950 578	.3622 040	.3313 255 <sup>-</sup>	.3024 295 <sup>+</sup>	.2754 960	.2273 303
.74	.4246 609	.3916 119	.3603 376	.3308 713	.3032 185 <sup>+</sup>	.2532 653
.75	.4552 009	.4221 561	.3906 750 <sup>+</sup>	.3608 139	.3326 017	.2811 276
.76	.4865 654	.4537 346	.4222 494	.3921 858	.3635 933	.3109 094
.77	.5186 243	.4862 253	.4549 502	.4248 912	.3961 154	.3425 756
.78	.5512 299	.5194 852	.4886 432	.4588 083	.4300 620	.3760 608
.79	.5842 166	.5533 502	.5231 696	.4937 878	.4652 970	.4112 651
.80	.6174 015 <sup>+</sup>	.5876 349	.5583 457	.5296 517	.5016 522	.4480 510
.81	.6505 852	.6221 331	.5939 627	.5661 927	.5389 257	.4862 403
.82	.6835 525 <sup>+</sup>	.6566 185 <sup>-</sup>	.6297 869	.6031 736	.5768 815 <sup>-</sup>	.5256 113
.83	.7160 746	.6908 461	.6655 606	.6403 281	.6152 486	.5658 971
.84	.7479 110	.7245 543	.7010 041	.6773 621	.6537 220	.6067 838
.85	.7788 120	.7574 674	.7358 181	.7139 552	.6919 643	.6479 112
.86	.8085 227	.7892 993	.7696 869	.7497 648	.7296 083	.6888 739
.87	.8367 871	.8197 580	.8022 834	.7844 299	.7662 615 <sup>+</sup>	.7292 245 <sup>-</sup>
.88	.8633 530	.8485 511	.8332 749	.8175 779	.8015 124	.7684 795 <sup>-</sup>
.89	.8879 783	.8753 928	.8623 305 <sup>-</sup>	.8488 321	.8349 386	.8061 273
.90	.9104 381	.9000 120	.8891 300	.8778 218	.8661 172	.8416 400
.91	.9305 334	.9221 616	.9133 755 <sup>+</sup>	.9041 945 <sup>+</sup>	.8946 391	.8744 890
.92	.9481 000	.9416 307	.9348 040	.9276 313	.9201 251	.9041 652
.93	.9630 207	.9582 572	.9532 032	.9478 642	.9422 468	.9302 047
.94	.9752 372	.9719 434	.9684 300	.9646 986	.9607 515 <sup>-</sup>	.9522 214
.95	.9847 647	.9826 739	.9804 317	.9780 379	.9754 922	.9699 464
.96	.9917 087	.9905 356	.9892 710	.9879 137	.9864 627	.9832 766
.97	.9962 828	.9957 410	.9951 539	.9945 204	.9938 398	.9923 333
.98	.9988 298	.9986 542	.9984 630	.9982 556	.9980 316	.9975 319
.99	.9998 446	.9998 206	.9997 944	.9997 658	.9997 347	.9996 649
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$x = .20$  to  $.80$  $q = 3$  $p = 13$ 

	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$B(p, q) = .7326\ 0073 \times \frac{1}{10^4}$	$.5952\ 3810 \times \frac{1}{10^4}$	$.4901\ 9608 \times \frac{1}{10^4}$	$.4084\ 9673 \times \frac{1}{10^4}$	$.3439\ 9725 \times \frac{1}{10^4}$	$.2923\ 9760 \times \frac{1}{10^4}$	
$x$						
.20	.0000 001					
.21	.0000 001					
.22	.0000 002					
.23	.0000 003	.0000 001				
.24	.0000 006	.0000 002				
.25	.0000 009	.0000 003	.0000 001			
.26	.0000 015 <sup>-</sup>	.0000 004	.0000 001			
.27	.0000 024	.0000 007	.0000 002	.0000 001		
.28	.0000 037	.0000 012	.0000 004	.0000 001		
.29	.0000 058	.0000 019	.0000 006	.0000 002	.0000 001	
.30	.0000 087	.0000 030	.0000 010	.0000 003	.0000 001	
.31	.0000 130	.0000 046	.0000 016	.0000 006	.0000 002	.0000 001
.32	.0000 192	.0000 070	.0000 025 <sup>+</sup>	.0000 009	.0000 003	.0000 001
.33	.0000 278	.0000 104	.0000 039	.0000 014	.0000 005 <sup>+</sup>	.0000 002
.34	.0000 399	.0000 154	.0000 059	.0000 023	.0000 009	.0000 003
.35	.0000 566	.0000 225 <sup>+</sup>	.0000 089	.0000 035 <sup>-</sup>	.0000 014	.0000 005 <sup>R</sup>
.36	.0000 795 <sup>-</sup>	.0000 325 <sup>+</sup>	.0000 132	.0000 053	.0000 021	.0000 009
.37	.0001 104	.0000 464	.0000 194	.0000 080	.0000 033	.0000 014
.38	.0001 517	.0000 655 <sup>+</sup>	.0000 281	.0000 119	.0000 051	.0000 021
.39	.0002 066	.0000 915 <sup>+</sup>	.0000 403	.0000 176	.0000 076	.0000 033
.40	.0002 789	.0001 267	.0000 571	.0000 256	.0000 114	.0000 050 <sup>R</sup>
.41	.0003 733	.0001 738	.0000 803	.0000 368	.0000 168	.0000 076
.42	.0004 954	.0002 362	.0001 118	.0000 525 <sup>+</sup>	.0000 245 <sup>+</sup>	.0000 114
.43	.0006 525 <sup>+</sup>	.0003 184	.0001 542	.0000 742	.0000 355 <sup>-</sup>	.0000 160
.44	.0008 530	.0004 258	.0002 110	.0001 038	.0000 508	.0000 247
.45	.0011 070	.0005 650 <sup>-</sup>	.0002 862	.0001 440	.0000 721	.0000 350
.46	.0014 268	.0007 441	.0003 852	.0001 981	.0001 013	.0000 515 <sup>R</sup>
.47	.0018 268	.0009 731	.0005 146	.0002 703	.0001 412	.0000 734
.48	.0023 240	.0012 639	.0006 824	.0003 660	.0001 952	.0001 035 <sup>R</sup>
.49	.0029 382	.0016 307	.0008 985 <sup>-</sup>	.0004 918	.0002 677	.0001 440
.50	.0036 926	.0020 905 <sup>-</sup>	.0011 749	.0006 561	.0003 643	.0002 012
.51	.0046 140	.0026 633	.0015 263	.0008 692	.0004 921	.0002 772
.52	.0057 331	.0033 729	.0019 702	.0011 436	.0006 600	.0003 790
.53	.0070 851	.0042 467	.0025 275 <sup>-</sup>	.0014 948	.0008 790	.0005 143
.54	.0087 098	.0053 169	.0032 229	.0019 414	.0011 629	.0006 930
.55	.0106 524	.0066 202	.0040 857	.0025 059	.0015 283	.0009 274
.56	.0129 635 <sup>-</sup>	.0081 993	.0051 502	.0032 150 <sup>+</sup>	.0019 959	.0012 328
.57	.0156 994	.0101 023	.0064 561	.0041 007	.0025 903	.0016 281
.58	.0189 228	.0123 840	.0080 496	.0052 006	.0033 415 <sup>+</sup>	.0021 364
.59	.0227 025 <sup>+</sup>	.0151 060	.0099 837	.0065 586	.0042 853	.0027 861
.60	.0271 140	.0183 372	.0123 188	.0082 264	.0054 639	.0036 115
.61	.0322 391	.0221 542	.0151 236	.0102 631	.0069 276	.0046 536
.62	.0381 662	.0266 415 <sup>-</sup>	.0184 754	.0127 373	.0087 350 <sup>+</sup>	.0059 617
.63	.0449 900	.0318 915 <sup>-</sup>	.0224 606	.0157 269	.0109 544	.0075 940
.64	.0528 107	.0380 048	.0271 753	.0193 203	.0136 647	.0096 192
.65	.0617 341	.0450 897	.0327 254	.0236 169	.0169 564	.0121 177
.66	.0718 702	.0532 623	.0392 266	.0287 279	.0209 326	.0151 825
.67	.0833 323	.0626 450 <sup>-</sup>	.0468 044	.0347 762	.0257 099	.0189 208
.68	.0962 358	.0733 663	.0555 935 <sup>+</sup>	.0418 966	.0314 185 <sup>-</sup>	.0234 551
.69	.1106 963	.0855 593	.0657 370	.0502 360	.0382 033	.0289 240
.70	.1268 277	.0993 597	.0773 853	.0599 522	.0462 237	.0354 831
.71	.1447 400	.1149 039	.0906 940	.0712 132	.0556 528	.0433 053
.72	.1645 361	.1323 265 <sup>-</sup>	.1058 225 <sup>+</sup>	.0841 953	.0666 773	.0525 804
.73	.1863 092	.1517 569	.1229 303	.0990 812	.0794 950 <sup>+</sup>	.0635 151
.74	.2101 390	.1733 158	.1421 739	.1160 562	.0943 132	.0763 305 <sup>R</sup>
.75	.2360 878	.1971 111	.1637 024	.1353 050 <sup>+</sup>	.1113 448	.0912 604
.76	.2641 963	.2232 326	.1876 526	.1570 064	.1308 039	.1085 472
.77	.2944 791	.2517 472	.2141 432	.1813 273	.1520 006	.1284 360
.78	.3269 198	.2826 928	.2432 676	.2084 157	.1778 332	.1511 725
.79	.3614 657	.3160 716	.2750 872	.2383 925 <sup>+</sup>	.2057 801	.1769 853
.80	.3980 232	.3518 437	.3096 225 <sup>-</sup>	.2713 419	.2368 893	.2060 847

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $1.00$  $q = 3$  $p = 13$  to  $18$ 

	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$B(p, q) = .7326\ 0073 \times \frac{1}{10^8}$	$.5952\ 3810 \times \frac{1}{10^8}$	$.4901\ 9608 \times \frac{1}{10^8}$	$.4084\ 9673 \times \frac{1}{10^8}$	$.3439\ 9725 \times \frac{1}{10^8}$	$.2923\ 9766 \times \frac{1}{10^8}$	
$x$						
.81	.4364 525 <sup>+</sup>	.3899 202	.3468 445 <sup>-</sup>	.3073 010	.2712 670	.2386 456
.82	.4765 629	.4301 555 <sup>+</sup>	.3866 652	.3462 482	.3089 635 <sup>+</sup>	.2747 932
.83	.5181 084	.4723 415 <sup>-</sup>	.4289 283	.3880 910	.3499 592	.3145 863
.84	.5607 843	.5162 002	.4733 995 <sup>+</sup>	.4326 532	.3941 480	.3579 983
.85	.6042 252	.5613 793	.5197 576	.4796 620	.4413 206	.4048 963
.86	.6480 036	.6074 478	.5675 872	.5287 364	.4911 483	.4550 195 <sup>-</sup>
.87	.6916 319	.6538 947	.6163 731	.5793 769	.5431 668	.5079 578
.88	.7345 656	.7001 300	.6654 976	.6309 576	.5967 629	.5631 315 <sup>+</sup>
.89	.7762 116	.7454 904	.7142 426	.6827 240	.6511 660	.6197 751
.90	.8159 389	.7892 493	.7617 972	.7337 960	.7054 448	.6769 268
.91	.8530 963	.8306 340	.8072 732	.7831 804	.7585 154	.7334 296
.92	.8870 349	.8688 504	.8497 307	.8297 952	.8091 620	.7879 462
.93	.9171 390	.9031 186	.8882 169	.8725 105 <sup>-</sup>	.8560 776	.8389 971
.94	.9428 667	.9327 204	.9218 205 <sup>-</sup>	.9102 084	.8979 286	.8850 276
.95	.9637 998	.9570 621	.9497 470	.9418 711	.9334 536	.9245 163
.96	.9797 082	.9757 555 <sup>-</sup>	.9714 188	.9667 005 <sup>-</sup>	.9616 047	.9561 372
.97	.9906 286	.9887 205 <sup>+</sup>	.9866 054	.9842 801	.9817 426	.9789 916
.98	.9969 606	.9963 146	.9955 912	.9947 876	.9939 017	.9929 313
.99	.9995 842	.9994 921	.9993 878	.9992 708	.9991 406	.9989 964
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



$x = .33$  to  $.90$  $q = 3$  $p = 19$  to  $24$ 

	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$
$B(p, q) = .2506\ 2657 \times \frac{1}{10^3}$	$.2164\ 5022 \times \frac{1}{10^3}$	$.1882\ 1758 \times \frac{1}{10^3}$	$.1646\ 9038 \times \frac{1}{10^3}$	$.1449\ 2754 \times \frac{1}{10^3}$	$.1282\ 0513 \times \frac{1}{10^3}$	
$x$						
.33	.0000 001					
.34	.0000 001					
.35	.0000 002	.0000 001				
.36	.0000 003	.0000 001	.0000 001			
.37	.0000 006	.0000 002	.0000 001			
.38	.0000 009	.0000 004	.0000 002	.0000 001		
.39	.0000 014	.0000 006	.0000 003	.0000 001		
.40	.0000 022	.0000 010	.0000 004	.0000 002	.0000 001	
.41	.0000 034	.0000 015 <sup>+</sup>	.0000 007	.0000 003	.0000 001	.0000 001
.42	.0000 053	.0000 024	.0000 011	.0000 005 <sup>+</sup>	.0000 002	.0000 001
.43	.0000 080	.0000 038	.0000 018	.0000 008	.0000 004	.0000 002
.44	.0000 120	.0000 058	.0000 028	.0000 013	.0000 006	.0000 003
.45	.0000 178	.0000 088	.0000 043	.0000 021	.0000 010	.0000 005
.46	.0000 261	.0000 131	.0000 066	.0000 033	.0000 016	.0000 008
.47	.0000 379	.0000 195 <sup>+</sup>	.0000 100	.0000 051	.0000 026	.0000 013
.48	.0000 547	.0000 287	.0000 150 <sup>+</sup>	.0000 079	.0000 041	.0000 021
.49	.0000 781	.0000 419	.0000 224	.0000 119	.0000 063	.0000 041
.50	.0001 106	.0000 606	.0000 330	.0000 179	.0000 097	.0000 054
.51	.0001 554	.0000 868	.0000 482	.0000 267	.0000 148	.0000 081
.52	.0002 166	.0001 233	.0000 699	.0000 395	.0000 222	.0000 125
.53	.0002 995 <sup>+</sup>	.0001 737	.0001 093	.0000 578	.0000 331	.0000 190
.54	.0004 111	.0002 429	.0001 429	.0000 838	.0000 490	.0000 285 <sup>+</sup>
.55	.0005 602	.0003 379	.0002 019	.0001 206	.0000 718	.0000 426
.56	.0007 580	.0004 642	.0002 831	.0001 721	.0001 043	.0000 630
.57	.0010 187	.0006 348	.0003 950	.0002 438	.0001 593	.0000 924
.58	.0013 598	.0008 620	.0005 443	.0003 426	.0002 149	.0001 344
.59	.0018 034	.0011 625 <sup>+</sup>	.0007 466	.0004 779	.0003 049	.0001 940
.60	.0023 765 <sup>+</sup>	.0015 575 <sup>+</sup>	.0010 170	.0006 618	.0004 293	.0002 777
.61	.0031 123	.0020 731	.0013 759	.0009 100	.0006 000	.0003 945
.62	.0040 511	.0027 419	.0018 490	.0012 427	.0008 326	.0005 562
.63	.0052 417	.0036 038	.0024 687	.0016 855 <sup>+</sup>	.0011 472	.0007 786
.64	.0067 425 <sup>+</sup>	.0047 076	.0032 750 <sup>+</sup>	.0022 708	.0015 697	.0010 820
.65	.0086 231	.0061 125 <sup>+</sup>	.0043 174	.0030 395 <sup>+</sup>	.0021 313	.0014 931
.66	.0109 657	.0078 565	.0056 565 <sup>+</sup>	.0040 422	.0028 799	.0020 461
.67	.0138 667	.0101 240	.0073 657	.0053 416	.0048 622	.0027 847
.68	.0174 383	.0129 162	.0095 337	.0070 146	.0065 145 <sup>+</sup>	.0037 645 <sup>+</sup>
.69	.0218 099	.0163 844	.0122 665	.0091 545 <sup>+</sup>	.0088 120	.0050 552
.70	.0271 294	.0206 662	.0156 896	.0118 741	.0089 695 <sup>+</sup>	.0067 437
.71	.0335 644	.0259 204	.0199 504	.0153 080	.0117 123	.0089 373
.72	.0413 029	.0323 284	.0252 206	.0196 156	.0152 141	.0117 677
.73	.0505 535 <sup>+</sup>	.0400 956	.0316 979	.0249 838	.0196 368	.0153 942
.74	.0615 447	.0494 515 <sup>+</sup>	.0396 076	.0316 294	.0251 888	.0200 083
.75	.0745 235 <sup>+</sup>	.0606 494	.0492 033	.0398 012	.0321 085 <sup>+</sup>	.0258 373
.76	.0897 527	.0739 653	.0607 670	.0497 810	.0406 726	.0331 483
.77	.1075 060	.0896 943	.0746 071	.0618 835 <sup>+</sup>	.0511 957	.0422 505
.78	.1280 665 <sup>+</sup>	.1081 467	.0910 555 <sup>+</sup>	.0764 548	.0640 309	.0534 975 <sup>+</sup>
.79	.1517 098	.1296 493	.1104 620	.0938 681	.0795 672	.0672 871
.80	.1787 028	.1544 915 <sup>+</sup>	.1331 855 <sup>+</sup>	.1145 174	.0982 252	.0840 581
.81	.2092 864	.1830 021	.1595 820	.1388 066	.1204 480	.1042 844
.82	.2436 602	.2154 439	.1899 928	.1671 353	.1466 882	.1284 637
.83	.2819 647	.2520 386	.2247 161	.1998 786	.1773 894	.1571 003
.84	.3242 587	.2920 341	.2639 901	.2373 617	.2129 604	.1906 800
.85	.3704 955 <sup>+</sup>	.3381 769	.3079 590	.2798 276	.2537 421	.2296 413
.86	.4204 964	.3876 813	.3566 382	.3273 985 <sup>+</sup>	.2999 664	.2743 234
.87	.4739 224	.4411 948	.4098 744	.3800 305 <sup>+</sup>	.3517 060	.3249 268
.88	.5302 475 <sup>+</sup>	.4982 634	.4673 028	.4374 627	.4088 162	.3814 152
.89	.5887 329	.5581 972	.5283 028	.4991 629	.4708 706	.4435 010
.90	.6484 088	.6200 409	.5919 567	.5642 737	.5370 940	.5105 052

TABLE I. THE  $I_x(p, q)$  FUNCTION

$x = .91$  to  $1.00$

$q = 3$

$p = 19$  to  $24$

	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$
$B(p, q) = .2506\ 2657 \times \frac{1}{10^8}$		$.2164\ 5022 \times \frac{1}{10^8}$	$.1822\ 1758 \times \frac{1}{10^8}$	$.1646\ 9038 \times \frac{1}{10^8}$	$.1449\ 2754 \times \frac{1}{10^8}$	$.1282\ 0513 \times \frac{1}{10^8}$
$x$						
.91	.7080 651	.6825 538	.6570 169	.6315 652	.6062 985 <sup>+</sup>	.5813 066
.92	.7662 590	.7442 065 <sup>-</sup>	.7218 894	.6994 022	.6768 332	.6542 643
.93	.8213 473	.8032 051	.7846 456	.7657 415 <sup>+</sup>	.7465 624	.7271 749
.94	.8715 532	.8575 540	.8430 789	.8281 764	.8128 946	.7972 805 <sup>+</sup>
.95	.9150 825 <sup>-</sup>	.9051 770	.8948 257	.8840 554	.8728 935 <sup>+</sup>	.8613 676
.96	.9503 052	.9441 172	.9375 826	.9307 120	.9235 166	.9160 083
.97	.9760 268	.9728 481	.9694 564	.9658 532	.9620 404	.9580 203
.98	.9918 747	.9907 302	.9894 965 <sup>-</sup>	.9881 723	.9867 566	.9852 485 <sup>+</sup>
.99	.9988 379	.9986 644	.9984 754	.9982 706	.9980 493	.9978 112
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .43$  to  $1.00$  $q = 3$  $p = 25$  to  $30$ 

	$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$	$p = 30$
$B(p, q) = .1139\ 6011 \times \frac{1}{10^3}$	$.1017\ 5010 \times \frac{1}{10^3}$	$.9122\ 4229 \times \frac{1}{10^3}$	$.8210\ 1806 \times \frac{1}{10^3}$	$.7415\ 6470 \times \frac{1}{10^3}$	$.6720\ 4301 \times \frac{1}{10^3}$	
$x$						
.43	.0000 001					
.44	.0000 001	.0000 001				
.45	.0000 002	.0000 001	.0000 001			
.46	.0000 004	.0000 002	.0000 001			
.47	.0000 007	.0000 003	.0000 002	.0000 001		
.48	.0000 011	.0000 006	.0000 003	.0000 001	.0000 001	
.49	.0000 018	.0000 009	.0000 005 <sup>-</sup>	.0000 003	.0000 001	.0000 001
.50	.0000 028	.0000 015 <sup>+</sup>	.0000 008	.0000 004	.0000 002	.0000 001
.51	.0000 045 <sup>-</sup>	.0000 024	.0000 013	.0000 007	.0000 004	.0000 002
.52	.0000 070	.0000 039	.0000 022	.0000 012	.0000 007	.0000 004
.53	.0000 108	.0000 062	.0000 035 <sup>-</sup>	.0000 020	.0000 011	.0000 006
.54	.0000 166	.0000 096	.0000 056	.0000 032	.0000 018	.0000 011
.55	.0000 252	.0000 149	.0000 088	.0000 051	.0000 030	.0000 018
.56	.0000 380	.0000 228	.0000 137	.0000 082	.0000 049	.0000 029
.57	.0000 567	.0000 346	.0000 211	.0000 129	.0000 078	.0000 047
.58	.0000 838	.0000 522	.0000 324	.0000 201	.0000 124	.0000 076
.59	.0001 230	.0000 779	.0000 492	.0000 310	.0000 195 <sup>-</sup>	.0000 122
.60	.0001 791	.0001 152	.0000 740	.0000 474	.0000 303	.0000 193
.61	.0002 587	.0001 692	.0001 104	.0000 719	.0000 467	.0000 303
.62	.0003 706	.0002 463	.0001 633	.0001 081	.0000 714	.0000 470
.63	.0005 270	.0003 559	.0002 397	.0001 612	.0001 081	.0000 724
.64	.0007 439	.0005 102	.0003 491	.0002 383	.0001 624	.0001 104
.65	.0010 423	.0007 258	.0005 043	.0003 496	.0002 419	.0001 671
.66	.0014 499	.0010 250 <sup>-</sup>	.0007 229	.0005 088	.0003 574	.0002 506
.67	.0020 027	.0014 368	.0010 285 <sup>+</sup>	.0007 347	.0005 238	.0003 727
.68	.0027 470	.0019 997	.0014 525 <sup>-</sup>	.0010 528	.0007 616	.0005 499
.69	.0037 419	.0027 633	.0020 361	.0014 972	.0010 987	.0008 049
.70	.0050 625 <sup>+</sup>	.0037 916	.0028 335 <sup>+</sup>	.0021 132	.0015 729	.0011 687
.71	.0068 029	.0051 663	.0039 148	.0029 605 <sup>+</sup>	.0022 345 <sup>+</sup>	.0016 836
.72	.0090 803	.0069 905 <sup>+</sup>	.0053 701	.0041 171	.0031 504	.0024 064
.73	.0120 389	.0093 937	.0073 141	.0056 835 <sup>+</sup>	.0044 082	.0034 130
.74	.0158 553	.0125 362	.0098 911	.0077 887	.0061 219	.0048 034
.75	.0207 420	.0166 147	.0132 812	.0105 959	.0084 380	.0067 080
.76	.0269 532	.0218 683	.0177 066	.0143 093	.0115 430	.0092 956
.77	.0347 887	.0285 834	.0234 378	.0191 823	.0156 714	.0127 816
.78	.0445 969	.0370 990	.0308 007	.0255 242	.0211 145 <sup>+</sup>	.0174 378
.79	.0567 774	.0478 105 <sup>-</sup>	.0401 817	.0337 086	.0282 295 <sup>+</sup>	.0236 026
.80	.0717 800	.0611 716	.0520 322	.0441 790	.0374 477	.0316 913
.81	.0901 009	.0776 931	.0668 697	.0574 534	.0492 813	.0422 055 <sup>-</sup>
.82	.1122 742	.0979 367	.0852 757	.0741 246	.0643 276	.0557 400
.83	.1388 569	.1225 036	.1078 862	.0948 551	.0832 668	.0729 851
.84	.1704 066	.1520 137	.1353 753	.1203 637	.1068 532	.0947 218
.85	.2074 486	.1870 756	.1684 265 <sup>+</sup>	.1514 006	.1358 949	.1218 060
.86	.2504 326	.2282 428	.2076 917	.1887 085 <sup>+</sup>	.1712 169	.1551 367
.87	.2996 758	.2759 556	.2537 316	.2329 645 <sup>+</sup>	.2136 066	.1956 038
.88	.3552 929	.3304 663	.3069 383	.2847 000	.2637 324	.2440 084
.89	.4171 120	.3917 470	.3674 356	.3441 957	.3220 348	.3009 514
.90	.4845 811	.4593 829	.4349 600	.4113 512	.3885 856	.3666 835 <sup>+</sup>
.91	.5566 686	.5324 545 <sup>-</sup>	.5087 246	.4855 308	.4629 169	.4409 190
.92	.6317 706	.6094 208	.5872 774	.5653 964	.5438 279	.5226 165 <sup>+</sup>
.93	.7076 419	.6880 230	.6683 739	.6487 466	.6291 895 <sup>-</sup>	.6097 469
.94	.7813 802	.7652 382	.7488 975 <sup>+</sup>	.7323 995 <sup>+</sup>	.7157 837	.6990 876
.95	.8495 056	.8373 351	.8248 838	.8121 788	.7992 470	.7861 145 <sup>-</sup>
.96	.9081 997	.9001 038	.8917 338	.8831 034	.8742 265 <sup>-</sup>	.8651 169
.97	.9537 958	.9493 703	.9447 474	.9399 309	.9349 252	.9297 349
.98	.9836 475 <sup>+</sup>	.9819 530	.9801 646	.9782 822	.9763 056	.9742 350 <sup>-</sup>
.99	.9975 559	.9972 829	.9969 918	.9966 823	.9963 540	.9960 066
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .50$  to 1.00 $q = 3$  $p = 31$  to 36

$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$	$p = 36$
$B(p, q) = .6100\ 4810 \times \frac{1}{100}$ $.5570\ 4100 \times \frac{1}{100}$ $.5002\ 0403 \times \frac{1}{100}$ $.4668\ 5341 \times \frac{1}{100}$ $.4290\ 0043 \times \frac{1}{100}$ $.3951\ 3197 \times \frac{1}{100}$					
$x$					
.50	.0000 0001				
.51	.0000 0001	.0000 0001			
.52	.0000 0002	.0000 0001	.0000 0001		
.53	.0000 0003	.0000 0002	.0000 0001	.0000 0001	
.54	.0000 0006	.0000 0003	.0000 0002	.0000 0001	.0000 0001
.55	.0000 0100	.0000 0006	.0000 0004	.0000 0002	.0000 0001
.56	.0000 0117	.0000 0100	.0000 0006	.0000 0003	.0000 0002
.57	.0000 0129	.0000 0117	.0000 0100	.0000 0006	.0000 0003
.58	.0000 0147	.0000 0129	.0000 0108	.0000 0101	.0000 0007
.59	.0000 0166	.0000 0148	.0000 0130	.0000 0119	.0000 0107
.60	.0000 0183	.0000 0178	.0000 0150	.0000 0131	.0000 0123
.61	.0000 0196	.0000 0187	.0000 0182	.0000 0153	.0000 0122
.62	.0000 0199	.0000 0203	.0000 0193	.0000 0187	.0000 0137
.63	.0000 0184	.0000 0213	.0000 0215	.0000 0193	.0000 0164
.64	.0000 0150	.0000 0208	.0000 0214	.0000 0212	.0000 0196
.65	.0000 0152	.0000 0203	.0000 0215	.0000 0214	.0000 0198
.66	.0000 0154	.0000 0205	.0000 0215	.0000 0214	.0000 0208
.67	.0000 0148	.0000 0208	.0000 0210	.0000 0210	.0000 0208
.68	.0000 0164	.0000 0213	.0000 0210	.0000 0211	.0000 0214
.69	.0000 0186	.0000 0217	.0000 0212	.0000 0210	.0000 0213
.70	.0000 0208	.0000 0213	.0000 0216	.0000 0214	.0000 0212
.71	.0000 0204	.0000 0210	.0000 0213	.0000 0218	.0000 0210
.72	.0000 0201	.0000 0211	.0000 0211	.0000 0212	.0000 0212
.73	.0000 0201	.0000 0211	.0000 0215	.0000 0212	.0000 0210
.74	.0000 0202	.0000 0212	.0000 0218	.0000 0212	.0000 0210
.75	.0000 0203	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.76	.0000 0212	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.77	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.78	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.79	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.80	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.81	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.82	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.83	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.84	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.85	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.86	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.87	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.88	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.89	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.90	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.91	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.92	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.93	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.94	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.95	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.96	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.97	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.98	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
.99	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210
1.00	.0000 0213	.0000 0210	.0000 0215	.0000 0211	.0000 0210

## TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

.56 to 1.00

$$q = 3$$

$p = 37$  to  $43$

	$p = 37$	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$(p, q) = .3647\ 3721 \times \frac{1}{10^4}$		$.3373\ 8192 \times \frac{1}{10^4}$	$.3126\ 9543 \times \frac{1}{10^4}$	$.2903\ 6005 \times \frac{1}{10^4}$	$.2701\ 0237 \times \frac{1}{10^4}$	$.2516\ 8630 \times \frac{1}{10^4}$	$.2349\ 0721 \times \frac{1}{10^4}$
$x$							
.56	.0000 001						
.57	.0000 001	.0000 001					
.58	.0000 002	.0000 002	.0000 001	.0000 001			
.59	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001		
.60	.0000 008	.0000 005 <sup>+</sup>	.0000 003	.0000 002	.0000 001	.0000 001	
.61	.0000 014	.0000 009	.0000 006	.0000 004	.0000 002	.0000 001	.0000 001
.62	.0000 024	.0000 016	.0000 010	.0000 007	.0000 004	.0000 003	.0000 002
.63	.0000 042	.0000 028	.0000 018	.0000 012	.0000 008	.0000 005 <sup>+</sup>	.0000 003
.64	.0000 071	.0000 048	.0000 032	.0000 021	.0000 014	.0000 010	.0000 006
.65	.0000 120	.0000 082	.0000 056	.0000 038	.0000 026	.0000 018	.0000 012
.66	.0000 200	.0000 138	.0000 096	.0000 066	.0000 046	.0000 032	.0000 022
.67	.0000 330	.0000 232	.0000 163	.0000 114	.0000 080	.0000 056	.0000 039
.68	.0000 539	.0000 385 <sup>-</sup>	.0000 274	.0000 195 <sup>+</sup>	.0000 139	.0000 099	.0000 070
.69	.0000 872	.0000 632	.0000 457	.0000 330	.0000 238	.0000 172	.0000 124
.70	.0001 399	.0001 028	.0000 754	.0000 553	.0000 405 <sup>-</sup>	.0000 296	.0000 216
.71	.0002 223	.0001 656	.0001 232	.0000 916	.0000 680	.0000 504	.0000 374
.72	.0003 499	.0002 643	.0001 994	.0001 503	.0001 131	.0000 851	.0000 640
.73	.0005 458	.0004 179	.0003 196	.0002 442	.0001 864	.0001 421	.0001 083
.74	.0008 435 <sup>-</sup>	.0006 545 <sup>+</sup>	.0005 074	.0003 929	.0003 039	.0002 349	.0001 814
.75	.0012 917	.0010 157	.0007 978	.0006 260	.0004 907	.0003 843	.0003 007
.76	.0019 602	.0015 616	.0012 427	.0009 879	.0007 846	.0006 225 <sup>+</sup>	.0004 935 <sup>-</sup>
.77	.0029 476	.0023 785 <sup>-</sup>	.0019 173	.0015 439	.0012 420	.0009 982	.0008 016
.78	.0043 917	.0035 890	.0029 298	.0023 894	.0019 467	.0015 846	.0012 887
.79	.0064 831	.0053 645 <sup>-</sup>	.0044 343	.0036 617	.0030 209	.0024 899	.0020 505 <sup>+</sup>
.80	.0094 811	.0079 421	.0066 461	.0055 562	.0046 406	.0038 724	.0032 286
.81	.0137 341	.0116 447	.0098 632	.0083 462	.0070 559	.0059 599	.0050 298
.82	.0197 023	.0169 050 <sup>-</sup>	.0144 904	.0124 089	.0106 168	.0090 755 <sup>+</sup>	.0077 515 <sup>+</sup>
.83	.0279 836	.0242 934	.0210 693	.0182 560	.0158 043	.0136 701	.0118 144
.84	.0393 392	.0345 473	.0303 102	.0265 685 <sup>+</sup>	.0232 684	.0203 610	.0178 025 <sup>+</sup>
.85	.0547 164	.0485 987	.0431 249	.0382 336	.0338 681	.0299 764	.0265 109
.86	.0752 622	.0675 949	.0606 539	.0543 786	.0487 119	.0436 009	.0389 961
.87	.1023 193	.0929 034	.0842 804	.0763 937	.0691 892	.0626 156	.0566 242
.88	.1373 908	.1266 871	.1156 163	.1059 295 <sup>-</sup>	.0969 789	.0887 181	.0811 024
.89	.1820 588	.1688 329	.1564 422	.1448 491	.1340 153	.1239 028	.1144 742
.90	.2378 323	.2228 081	.2085 747	.1951 077	.1823 813	.1703 689	.1590 429
.91	.3059 043	.2894 197	.2736 293	.2585 231	.2440 891	.2303 134	.2171 806
.92	.3867 933	.3694 457	.3526 460	.3363 977	.3207 018	.3055 571	.2909 606
.93	.4798 601	.4625 228	.4455 505 <sup>+</sup>	.4289 569	.4127 531	.3969 485 <sup>+</sup>	.3815 504
.94	.5827 186	.5665 004	.5504 529	.5345 947	.5189 426	.5035 120	.4883 165 <sup>+</sup>
.95	.6906 193	.6767 358	.6628 522	.6489 864	.6351 553	.6213 748	.6076 599
.96	.7060 068	.7855 347	.7749 524	.7642 724	.7535 069	.7426 680	.7317 670
.97	.8886 535 <sup>+</sup>	.8821 711	.8755 523	.8688 027	.8619 283	.8549 348	.8478 281
.98	.9571 370	.9543 298	.9514 339	.9484 505 <sup>-</sup>	.9453 805 <sup>-</sup>	.9422 251	.9389 856
.99	.9930 135 <sup>+</sup>	.9925 026	.9919 702	.9914 161	.9908 402	.9902 421	.9896 219
.00	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000

$p = 44$  to 50

	$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$q) = .2195\ 8718 \times \frac{1}{10^4}$		$.2055\ 7097 \times \frac{1}{10^4}$	$.1927\ 2279 \times \frac{1}{10^4}$	$.1809\ 2343 \times \frac{1}{10^4}$	$.1700\ 6803 \times \frac{1}{10^4}$	$.1600\ 6403 \times \frac{1}{10^4}$	$.1508\ 2956 \times \frac{1}{10^4}$
1	.0000 001						
2	.0000 001	.0000 001					
3	.0000 002	.0000 001	.0000 001	.0000 001			
4	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001	
5	.0000 008	.0000 005+	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001
6	.0000 015-	.0000 010	.0000 007	.0000 005-	.0000 003	.0000 002	.0000 002
7	.0000 027	.0000 019	.0000 013	.0000 009	.0000 007	.0000 005-	.0000 003
8	.0000 050-	.0000 035+	.0000 025-	.0000 018	.0000 012	.0000 009	.0000 006
9	.0000 089	.0000 064	.0000 046	.0000 033	.0000 024	.0000 017	.0000 012
0	.0000 158	.0000 115+	.0000 084	.0000 061	.0000 045-	.0000 032	.0000 024
1	.0000 277	.0000 205-	.0000 151	.0000 112	.0000 083	.0000 061	.0000 045-
2	.0000 480	.0000 360	.0000 270	.0000 202	.0000 151	.0000 113	.0000 085-
3	.0000 824	.0000 627	.0000 476	.0000 361	.0000 274	.0000 208	.0000 158
4	.0001 399	.0001 078	.0000 830	.0000 639	.0000 491	.0000 378	.0000 290
5	.0002 351	.0001 836	.0001 433	.0001 118	.0000 871	.0000 678	.0000 528
6	.0003 909	.0003 093	.0002 446	.0001 933	.0001 526	.0001 204	.0000 949
7	.0006 431	.0005 156	.0004 130	.0003 306	.0002 644	.0002 113	.0001 688
8	.0010 472	.0008 502	.0006 898	.0005 592	.0004 530	.0003 667	.0002 967
9	.0016 872	.0013 872	.0011 396	.0009 355+	.0007 674	.0006 291	.0005 154
0	.0026 896	.0022 388	.0018 621	.0015 477	.0012 854	.0010 669	.0008 849
1	.0042 414	.0035 737	.0030 089	.0025 315+	.0021 284	.0017 882	.0015 014
2	.0066 153	.0056 413	.0048 071	.0040 933	.0034 831	.0029 619	.0025 171
3	.0102 025-	.0088 038	.0075 912	.0065 411	.0056 324	.0048 467	.0041 680
4	.0155 535-	.0135 784	.0118 455+	.0103 267	.0089 966	.0078 327	.0068 151
5	.0234 283	.0206 889	.0182 570	.0161 000	.0141 885+	.0124 961	.0109 987
6	.0348 518	.0311 257	.0277 788	.0247 754	.0220 825-	.0196 701	.0175 108
7	.0511 692	.0462 076	.0416 992	.0376 064	.0338 941	.0305 298	.0274 835-
8	.0740 889	.0676 365+	.0617 061	.0562 603	.0512 642	.0466 844	.0424 896
9	.1056 924	.0975 213	.0899 258	.0828 720	.0763 269	.0702 590	.0646 382
0	.1483 754	.1383 382	.1289 033	.1200 427	.1117 288	.1039 345-	.0966 333
1	.2046 739	.1927 755-	.1814 667	.1707 283	.1605 405-	.1508 833	.1417 366
2	.2769 072	.2633 904	.2504 022	.2379 336	.2259 743	.2145 133	.2035 388
3	.3665 640	.3519 932	.3378 401	.3241 054	.3107 886	.2978 879	.2854 006
4	.4733 684	.4586 785-	.4442 562	.4301 099	.4162 465-	.4026 719	.3893 909
5	.5940 247	.5804 825+	.5670 456	.5537 256	.5405 331	.5274 781	.5145 695+
6	.7208 154	.7098 240	.6988 032	.6877 633	.6767 140	.6656 547	.6546 244
7	.8406 140	.8332 982	.8258 865+	.8183 846	.8107 981	.8031 325+	.7953 935-
8	.9356 633	.9322 594	.9287 753	.9252 124	.9215 723	.9178 562	.9140 659
9	.9889 793	.9883 142	.9876 264	.9869 160	.9861 827	.9854 265+	.9846 474
0	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$x = .01$  to  $.60$  $q = 3.5$  $p = 3.5$  to  $6$ 

	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$
$B(p, q) = .1533\ 9808 \times \frac{x}{10}$		$.1065\ 6011 \times \frac{x}{10}$	$.7669\ 9039 \times \frac{x}{10^3}$	$.5683\ 2057 \times \frac{x}{10^3}$	$.4314\ 3210 \times \frac{x}{10^2}$	$.3343\ 0622 \times \frac{x}{10^2}$
$x$						
.01	.0000 018	.0000 002				
.02	.0000 203	.0000 036	.0000 006	.0000 001		
.03	.0000 821	.0000 179	.0000 038	.0000 008	.0000 002	
.04	.0002 203	.0000 554	.0000 137	.0000 033	.0000 008	.0000 002
.05	.0004 715 <sup>+</sup>	.0001 324	.0000 365 <sup>-</sup>	.0000 099	.0000 026	.0000 007
.06	.0008 747	.0002 689	.0000 811	.0000 241	.0000 070	.0000 020
.07	.0014 700	.0004 879	.0001 589	.0000 509	.0000 161	.0000 050 <sup>+</sup>
.08	.0022 981	.0008 148	.0002 835 <sup>-</sup>	.0000 971	.0000 328	.0000 109
.09	.0033 994	.0012 776	.0004 712	.0001 711	.0000 613	.0000 217
.10	.0048 140	.0019 058	.0007 405 <sup>+</sup>	.0002 832	.0001 069	.0000 399
.11	.0065 804	.0027 304	.0011 121	.0004 459	.0001 764	.0000 690
.12	.0087 361	.0037 833	.0016 086	.0006 733	.0002 781	.0001 135 <sup>+</sup>
.13	.0113 166	.0050 974	.0022 544	.0009 818	.0004 219	.0001 792
.14	.0143 557	.0067 055 <sup>+</sup>	.0030 758	.0013 893	.0006 193	.0002 729
.15	.0178 848	.0086 407	.0041 001	.0019 160	.0008 836	.0004 029
.16	.0219 331	.0109 358	.0053 559	.0025 836	.0012 301	.0005 790
.17	.0265 272	.0136 229	.0068 729	.0034 155 <sup>+</sup>	.0016 755 <sup>-</sup>	.0008 126
.18	.0316 912	.0167 334	.0086 813	.0044 368	.0022 385 <sup>+</sup>	.0011 168
.19	.0374 464	.0202 976	.0108 117	.0056 739	.0029 397	.0015 062
.20	.0438 114	.0243 445 <sup>+</sup>	.0132 951	.0071 543	.0038 011	.0019 973
.21	.0508 020	.0289 016	.0161 623	.0089 068	.0048 467	.0026 084
.22	.0584 312	.0339 947	.0194 439	.0109 608	.0061 016	.0033 596
.23	.0667 088	.0396 476	.0231 699	.0133 465 <sup>+</sup>	.0075 927	.0042 726
.24	.0756 421	.0458 824	.0273 697	.0160 946	.0093 480	.0053 710
.25	.0852 333	.0527 186	.0320 715 <sup>-</sup>	.0192 360	.0113 966	.0066 800
.26	.0954 899	.0601 737	.0373 024	.0228 014	.0137 688	.0082 262
.27	.1064 045 <sup>-</sup>	.0682 626	.0430 882	.0268 216	.0164 954	.0100 380
.28	.1179 748	.0769 977	.0494 530	.0313 267	.0196 079	.0121 448
.29	.1301 940	.0863 889	.0564 190	.0363 461	.0231 383	.0145 775 <sup>-</sup>
.30	.1430 526	.0964 432	.0640 066	.0419 084	.0271 185 <sup>+</sup>	.0173 678
.31	.1565 382	.1071 650 <sup>+</sup>	.0722 339	.0480 410	.0315 805 <sup>-</sup>	.0205 484
.32	.1706 363	.1185 560	.0811 167	.0547 699	.0365 558	.0241 525 <sup>+</sup>
.33	.1853 298	.1306 150 <sup>-</sup>	.0906 684	.0621 193	.0420 756	.0282 140
.34	.2005 991	.1433 378	.1008 998	.0701 119	.0481 700	.0327 667
.35	.2164 227	.1567 177	.1118 190	.0787 680	.0548 681	.0378 445 <sup>-</sup>
.36	.2327 766	.1707 450 <sup>-</sup>	.1234 311	.0881 060	.0621 977	.0434 809
.37	.2496 350 <sup>-</sup>	.1854 072	.1357 386	.0981 414	.0701 849	.0497 089
.38	.2669 701	.2006 891	.1487 407	.1088 874	.0788 540	.0565 605 <sup>-</sup>
.39	.2847 524	.2165 728	.1624 338	.1203 544	.0882 272	.0640 666
.40	.3029 506	.2330 378	.1768 111	.1325 496	.0983 242	.0722 568
.41	.3215 320	.2500 607	.1918 625 <sup>+</sup>	.1454 773	.1091 623	.0811 585 <sup>+</sup>
.42	.3404 622	.2676 161	.2075 752	.1591 384	.1207 557	.0907 975 <sup>-</sup>
.43	.3597 059	.2856 759	.2239 328	.1735 305 <sup>+</sup>	.1331 157	.1011 968
.44	.3792 263	.3042 096	.2409 160	.1886 478	.1462 503	.1123 770
.45	.3989 858	.3231 847	.2585 024	.2044 808	.1601 639	.1243 557
.46	.4189 460	.3425 666	.2766 664	.2210 166	.1748 575 <sup>-</sup>	.1371 472
.47	.4390 676	.3623 187	.2953 797	.2382 386	.1903 279	.1507 621
.48	.4593 106	.3824 026	.3146 108	.2561 263	.2065 682	.1652 073
.49	.4796 350 <sup>-</sup>	.4027 783	.3343 255 <sup>-</sup>	.2746 558	.2235 674	.1804 858
.50	.5000 000 <sup>0</sup>	.4234 041	.3544 869	.2937 995 <sup>+</sup>	.2413 101	.1965 961
.51	.5203 650 <sup>+</sup>	.4442 373	.3750 556	.3135 262	.2597 767	.2135 322
.52	.5406 894	.4652 335 <sup>+</sup>	.3959 895 <sup>+</sup>	.3338 009	.2789 434	.2312 835 <sup>+</sup>
.53	.5609 324	.4863 479	.4172 446	.3545 855 <sup>+</sup>	.2987 819	.2498 345 <sup>-</sup>
.54	.5810 540	.5075 344	.4387 744	.3758 383	.3192 595 <sup>+</sup>	.2691 645 <sup>+</sup>
.55	.6010 142	.5287 465 <sup>-</sup>	.4605 307	.3975 144	.3403 393	.2892 480
.56	.6207 737	.5499 371	.4824 634	.4195 659	.3619 798	.3100 540
.57	.6402 941	.5710 590	.5045 211	.4419 417	.3841 356	.3315 465 <sup>-</sup>
.58	.6595 378	.5920 649	.5266 508	.4645 885 <sup>-</sup>	.4067 572	.3536 840
.59	.6784 680	.6129 075 <sup>+</sup>	.5487 986	.4874 499	.4297 909	.3764 200
.60	.6970 494	.6335 401	.5709 098	.5104 678	.4531 795 <sup>+</sup>	.3997 027

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$p = 3.5$  to 6

	$p = 3.5$	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$
$B(p, q) = 1533\ 9808 \times \frac{1}{10}$	$1065\ 6011 \times \frac{1}{10}$	$7669\ 9039 \times \frac{1}{100}$	$5683\ 2057 \times \frac{1}{100}$	$4314\ 3210 \times \frac{1}{100}$	$3343\ 0622 \times \frac{1}{100}$	
.61	.7152 476	.6539 163	.5929 289	.5335 816	.4768 622	.4234 754
.62	.7330 299	.6739 907	.6148 005-	.5507 294	.5007 748	.4476 764
.63	.7503 650+	.6937 187	.6364 686	.5798 476	.5248 501	.4722 394
.64	.7672 234	.7130 571	.6578 780	.6028 717	.5490 185+	.4970 937
.65	.7835 773	.7319 638	.6789 737	.6257 363	.5732 077	.5221 645+
.66	.7994 009	.7503 987	.6997 016	.6483 759	.5973 437	.5473 733
.67	.8146 702	.7683 231	.7200 089	.6707 246	.6213 507	.5726 380
.68	.8293 637	.7857 007	.7398 441	.6927 172	.6451 522	.5978 740
.69	.8434 618	.8024 972	.7591 575-	.7142 893	.6686 709	.6229 941
.70	.8569 474	.8186 809	.7779 015+	.7353 776	.6918 293	.6479 092
.71	.8698 060	.8342 225+	.7960 310	.7559 207	.7145 506	.6725 293
.72	.8820 252	.8490 958	.8135 034	.7758 592	.7307 589	.6907 636
.73	.8935 955+	.8632 774	.8302 793	.7951 362	.7583 801	.7205 215+
.74	.9045 101	.8767 472	.8463 226	.8136 980	.7793 423	.7437 135+
.75	.9147 647	.8894 882	.8616 008	.8314 946	.7995 763	.7662 517
.76	.9243 579	.9014 873	.8760 855+	.8484 796	.8190 168	.7880 508
.77	.9332 912	.9127 346	.8897 524	.8646 113	.8376 025-	.8090 288
.78	.9415 688	.9232 243	.9025 816	.8798 529	.8552 771	.8291 084
.79	.9491 980	.9329 543	.9145 582	.8941 731	.8719 899	.8482 174
.80	.9561 886	.9419 266	.9256 723	.9075 463	.8876 964	.8662 899
.81	.9625 536	.9501 472	.9359 190	.9199 530	.9023 593	.8832 671
.82	.9683 088	.9576 262	.9452 990	.9313 807	.9159 486	.8990 988
.83	.9734 728	.9643 779	.9538 186	.9418 236	.9284 427	.9137 435-
.84	.9780 669	.9704 207	.9614 898	.9512 833	.9398 290	.9271 702
.85	.9821 152	.9757 774	.9683 305+	.9597 693	.9501 040	.9393 589
.86	.9856 443	.9804 746	.9743 644	.9672 985+	.9592 745-	.9503 013
.87	.9886 834	.9845 432	.9796 213	.9738 963	.9673 571	.9600 020
.88	.9912 639	.9880 178	.9841 365-	.9795 959	.9743 797	.9684 790
.89	.9934 196	.9909 368	.9879 513	.9844 388	.9803 808	.9757 640
.90	.9951 860	.9933 422	.9911 126	.9884 746	.9854 098	.9819 034
.91	.9966 006	.9952 791	.9936 723	.9917 606	.9895 272	.9869 579
.92	.9977 019	.9967 956	.9956 873	.9943 616	.9928 043	.9910 028
.93	.9985 300	.9979 419	.9972 189	.9963 492	.9953 221	.9941 275+
.94	.9991 253	.9987 704	.9983 317	.9978 013	.9971 713	.9964 348
.95	.9995 285-	.9993 345-	.9990 934	.9988 004	.9984 505+	.9980 392
.96	.9997 797	.9996 878	.9995 730	.9994 327	.9992 644	.9990 655-
.97	.9999 179	.9998 832	.9998 396	.9997 860	.9997 214	.9996 447
.98	.9999 797	.9999 711	.9999 601	.9999 466	.9999 302	.9999 106
.99	.9999 982	.9999 974	.9999 964	.9999 951	.9999 936	.9999 918
I.00	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .05$  to  $.60$  $q = 3.5$  $p = 6.5$  to  $9$ 

	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$
$B(p, q) = .2636\ 5295 \times \frac{1}{10^3}$	$.2111\ 4077 \times \frac{1}{10^3}$	$.1713\ 7442 \times \frac{1}{10^3}$	$.1407\ 6051 \times \frac{1}{10^3}$	$.1168\ 4619 \times \frac{1}{10^3}$	$.9792\ 0356 \times \frac{1}{10^3}$	
.05	.0000 002					
.06	.0000 006	.0000 002				
.07	.0000 016	.0000 005 <sup>-</sup>	.0000 001			
.08	.0000 036	.0000 012	.0000 004	.0000 001		
.09	.0000 076	.0000 026	.0000 009	.0000 003	.0000 001	
.10	.0000 147	.0000 054	.0000 020	.0000 007	.0000 003	.0000 001
.11	.0000 267	.0000 102	.0000 039	.0000 015 <sup>-</sup>	.0000 006	.0000 002
.12	.0000 459	.0000 184	.0000 073	.0000 029	.0000 011	.0000 004
.13	.0000 754	.0000 314	.0000 130	.0000 053	.0000 022	.0000 009
.14	.0001 191	.0000 515 <sup>-</sup>	.0000 221	.0000 094	.0000 040	.0000 017
.15	.0001 819	.0000 814	.0000 361	.0000 159	.0000 070	.0000 030
.16	.0002 699	.0001 247	.0000 571	.0000 260	.0000 118	.0000 053
.17	.0003 903	.0001 858	.0000 878	.0000 412	.0000 192	.0000 089
.18	.0005 517	.0002 702	.0001 313	.0000 633	.0000 304	.0000 145 <sup>-</sup>
.19	.0007 641	.0003 843	.0001 918	.0000 951	.0000 468	.0000 229
.20	.0010 393	.0005 301	.0002 744	.0001 395 <sup>+</sup>	.0000 705 <sup>-</sup>	.0000 354
.21	.0013 903	.0007 347	.0003 852	.0002 006	.0001 038	.0000 534
.22	.0018 321	.0009 906	.0005 315 <sup>+</sup>	.0002 832	.0001 500 <sup>+</sup>	.0000 790
.23	.0023 814	.0013 161	.0007 218	.0003 932	.0002 129	.0001 146
.24	.0030 568	.0017 250 <sup>+</sup>	.0009 662	.0005 375 <sup>-</sup>	.0002 972	.0001 634
.25	.0038 785 <sup>+</sup>	.0022 331	.0012 761	.0007 243	.0004 086	.0002 292
.26	.0048 688	.0028 577	.0016 648	.0009 634	.0005 541	.0003 169
.27	.0060 517	.0036 183	.0021 473	.0012 659	.0007 418	.0004 323
.28	.0074 529	.0045 360	.0027 404	.0016 447	.0009 811	.0005 821
.29	.0090 999	.0056 342	.0034 629	.0021 144	.0012 833	.0007 747
.30	.0110 219	.0069 380	.0043 356	.0026 916	.0016 611	.0010 196
.31	.0132 495 <sup>-</sup>	.0084 745 <sup>-</sup>	.0053 813	.0033 948	.0021 291	.0013 281
.32	.0158 148	.0102 727	.0066 249	.0042 448	.0027 039	.0017 131
.33	.0187 511	.0123 632	.0080 935 <sup>+</sup>	.0052 644	.0034 042	.0021 897
.34	.0220 927	.0147 787	.0098 163	.0064 786	.0042 510	.0027 746
.35	.0258 749	.0175 532	.0118 244	.0079 148	.0052 675 <sup>-</sup>	.0034 872
.36	.0301 336	.0207 221	.0141 509	.0096 028	.0064 792	.0043 489
.37	.0349 051	.0243 221	.0168 310	.0115 744	.0079 144	.0053 837
.38	.0402 257	.0283 911	.0199 013	.0138 638	.0096 036	.0066 183
.39	.0461 318	.0329 676	.0234 003	.0165 074	.0115 799	.0080 818
.40	.0526 591	.0380 908	.0273 677	.0195 436	.0138 789	.0098 063
.41	.0598 428	.0438 000	.0318 446	.0230 127	.0165 388	.0118 265 <sup>-</sup>
.42	.0677 168	.0501 346	.0368 727	.0269 568	.0196 000	.0141 800
.43	.0763 137	.0571 335 <sup>-</sup>	.0424 947	.0314 194	.0231 051	.0169 070
.44	.0856 642	.0648 349	.0487 533	.0364 455 <sup>-</sup>	.0270 990	.0200 506
.45	.0957 970	.0732 760	.0556 914	.0420 808	.0316 280	.0236 562
.46	.1067 380	.0824 922	.0633 512	.0483 719	.0367 406	.0277 718
.47	.1185 107	.0925 173	.0717 742	.0553 654	.0424 861	.0324 474
.48	.1311 349	.1033 824	.0810 007	.0631 079	.0489 149	.0377 350 <sup>-</sup>
.49	.1446 270	.1151 161	.0910 691	.0716 453	.0560 779	.0436 881
.50	.1589 996	.1277 437	.1020 155 <sup>-</sup>	.0810 224	.0640 260	.0503 615 <sup>+</sup>
.51	.1742 608	.1412 866	.1138 734	.0912 823	.0728 099	.0578 107
.52	.1904 140	.1557 624	.1266 728	.1024 660	.0824 790	.0660 912
.53	.2074 580	.1711 839	.1404 403	.1146 116	.0930 811	.0752 586
.54	.2253 861	.1875 590	.1551 976	.1277 539	.1046 619	.0853 670
.55	.2441 861	.2048 904	.1709 618	.1419 235 <sup>-</sup>	.1172 640	.0964 692
.56	.2638 404	.2231 746	.1877 446	.1571 463	.1309 264	.1086 154
.57	.2843 251	.2424 023	.2055 516	.1734 429	.1456 838	.1218 526
.58	.3056 106	.2625 575 <sup>-</sup>	.2243 821	.1908 279	.1615 653	.1362 237
.59	.3276 606	.2836 174	.2442 281	.2093 091	.1785 945 <sup>-</sup>	.1517 666
.60	.3504 331	.3055 524	.2650 745 <sup>+</sup>	.2288 871	.1967 877	.1685 132

TABLE I. THE  $I_n(p, q)$  FUNCTION $x = .61$  to  $1.00$  $q = 3.5$  $p = 6.5$  to  $9$ 

	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$
$B(p, q) = .2636\ 5295 \times 10^3$	$.2111\ 4077 \times 10^3$	$.1713\ 7442 \times 10^3$	$.1407\ 6051 \times 10^3$	$.1168\ 4610 \times 10^3$	$.9792\ 0356 \times 10^2$	
$x$						
.61	.3738 703	.3283 253	.2868 083	.2495 545 <sup>+</sup>	.2161 530	.1864 887
.62	.3970 443	.3518 915 <sup>+</sup>	.3096 083	.2712 954	.2366 935 <sup>+</sup>	.2057 102
.63	.4225 670	.3761 991	.3333 447	.2940 846	.2583 979	.2261 862
.64	.4476 803	.4011 884	.3578 700	.3178 877	.2812 485 <sup>+</sup>	.2479 152
.65	.4732 108	.4267 022	.3832 146	.3420 601	.3052 161	.2708 849
.66	.4990 801	.4529 358	.4092 822	.3683 460	.3302 602	.2950 717
.67	.5252 049	.4795 475 <sup>+</sup>	.4360 092	.3948 826	.3563 286	.3204 392
.68	.5514 911	.5065 084	.4633 160	.4221 910	.3833 570	.3469 377
.69	.5778 552	.5337 530	.4910 913	.4501 851	.4112 685 <sup>+</sup>	.3745 939
.70	.6041 921	.5601 697	.5192 514	.4787 672	.4399 734	.4030 599
.71	.6304 945 <sup>+</sup>	.5880 514	.5476 809 <sup>+</sup>	.5078 291	.4693 604	.4325 128
.72	.6564 866	.6160 861	.5762 615 <sup>+</sup>	.5372 527	.4997 415 <sup>+</sup>	.4627 551
.73	.6820 393	.6433 575 <sup>+</sup>	.6048 704	.5669 101	.5297 622	.4930 699
.74	.7072 471	.6703 494	.6333 723	.5966 649	.5604 925 <sup>+</sup>	.5251 017
.75	.7319 142	.6969 314	.6616 477	.6263 728	.5913 822	.5569 168
.76	.7559 299	.7220 891	.6895 441	.6538 826	.6222 712	.5886 449
.77	.7791 946	.7483 976	.7169 229	.6850 380	.6529 905 <sup>+</sup>	.6210 054
.78	.8016 086	.7730 358	.7436 453	.7130 788	.6833 641	.6529 122
.79	.8230 749	.7967 853	.7695 695 <sup>+</sup>	.7416 427	.7132 105 <sup>+</sup>	.6844 668
.80	.8435 064	.8195 326	.7945 573	.7687 675 <sup>+</sup>	.7423 451	.7154 642
.81	.8628 109	.8411 700	.8184 749	.7948 943	.7705 824	.7456 054
.82	.8809 416	.8615 979	.8411 952	.8195 648	.7977 991	.7749 496
.83	.8979 072	.8804 260	.8622 969	.8415 412	.8236 471	.8030 153
.84	.9134 645	.8964 257	.8825 850	.8657 648	.8491 668	.8296 699
.85	.9276 695 <sup>+</sup>	.9147 514	.9010 481	.8889 296	.8739 995 <sup>+</sup>	.8567 995
.86	.9404 983	.9295 941	.9179 211	.9054 246	.8921 472	.8781 473
.87	.9518 475	.9428 774	.9331 422	.9226 581	.9114 558	.8995 702
.88	.9618 911	.9546 199	.9466 743	.9380 682	.9288 198	.9189 509
.89	.9705 894	.9648 295	.9585 944	.9516 155 <sup>+</sup>	.9441 717	.9361 545
.90	.9779 443	.9735 250	.9686 411	.9642 914	.9574 774	.9512 030
.91	.9840 497	.9807 664	.9774 276	.9741 192	.9687 497	.9649 865
.92	.9889 492	.9866 251	.9843 115 <sup>+</sup>	.9811 592	.9780 020	.9749 591
.93	.9927 593	.9912 694	.9897 523	.9874 959	.9854 554	.9829 963
.94	.9955 848	.9949 154	.9945 198	.9932 946	.9920 417	.9904 296
.95	.9975 621	.9979 149	.9976 946	.9975 943	.9970 144	.9969 476
.96	.9988 144	.9995 659	.9998 266	.99979 152	.99978 274	.99979 952
.97	.9995 548	.99994 595 <sup>+</sup>	.99993 409	.99991 949	.99990 314	.99988 694
.98	.9998 875 <sup>+</sup>	.99998 606	.99998 296	.99997 942	.99997 540	.99997 087
.99	.9999 896	.99999 871	.99999 842	.99999 808	.99999 769	.99999 726
1.00	1.00000 000	1.00000 000	1.00000 000	1.00000 000	1.00000 000	1.00000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .11$  to  $.70$  $q = 3.5$  $p = 9.5$  to  $13$ 

	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$B(p, q) = .8276\ 6053 \times \frac{1}{10^8}$	$.7050\ 2657 \times \frac{1}{10^8}$	$.6048\ 2885 \times \frac{1}{10^8}$	$.5222\ 4190 \times \frac{1}{10^8}$	$.3961\ 8351 \times \frac{1}{10^8}$	$.3067\ 2272 \times \frac{1}{10^8}$	
$x$						
.11	.0000 001					
.12	.0000 002	.0000 001				
.13	.0000 004	.0000 001	.0000 001			
.14	.0000 007	.0000 003	.0000 001			
.15	.0000 013	.0000 006	.0000 002	.0000 001		
.16	.0000 024	.0000 011	.0000 005 <sup>-</sup>	.0000 002		
.17	.0000 041	.0000 019	.0000 009	.0000 004	.0000 001	
.18	.0000 069	.0000 032	.0000 015 <sup>+</sup>	.0000 007	.0000 002	
.19	.0000 112	.0000 054	.0000 026	.0000 013	.0000 003	.0000 001
.20	.0000 177	.0000 088	.0000 044	.0000 022	.0000 005 <sup>+</sup>	.0000 001
.21	.0000 274	.0000 139	.0000 071	.0000 036	.0000 009	.0000 002
.22	.0000 414	.0000 216	.0000 112	.0000 058	.0000 015 <sup>+</sup>	.0000 004
.23	.0000 614	.0000 327	.0000 174	.0000 092	.0000 025 <sup>+</sup>	.0000 007
.24	.0000 894	.0000 487	.0000 264	.0000 142	.0000 041	.0000 012
.25	.0001 280	.0000 711	.0000 393	.0000 217	.0000 065 <sup>+</sup>	.0000 019
.26	.0001 804	.0001 022	.0000 576	.0000 324	.0000 101	.0000 031
.27	.0002 506	.0001 447	.0000 831	.0000 476	.0000 154	.0000 049
.28	.0003 436	.0002 019	.0001 182	.0000 689	.0000 232	.0000 077
.29	.0004 653	.0002 782	.0001 657	.0000 983	.0000 342	.0000 118
.30	.0006 227	.0003 786	.0002 293	.0001 383	.0000 498	.0000 177
.31	.0008 244	.0005 094	.0003 135 <sup>-</sup>	.0001 922	.0000 715 <sup>-</sup>	.0000 262
.32	.0010 801	.0006 780	.0004 238	.0002 639	.0001 013	.0000 384
.33	.0014 016	.0008 932	.0005 668	.0003 584	.0001 418	.0000 554
.34	.0018 023	.0011 655 <sup>-</sup>	.0007 506	.0004 816	.0001 962	.0000 790
.35	.0022 976	.0015 071	.0009 846	.0006 408	.0002 687	.0001 113
.36	.0029 051	.0019 321	.0012 798	.0008 446	.0003 641	.0001 551
.37	.0036 449	.0024 570	.0016 495 <sup>+</sup>	.0011 033	.0004 886	.0002 138
.38	.0045 395 <sup>+</sup>	.0031 002	.0021 088	.0014 291	.0006 498	.0002 919
.39	.0056 141	.0038 830	.0026 751	.0018 362	.0008 564	.0003 947
.40	.0068 965 <sup>+</sup>	.0048 294	.0033 686	.0023 410	.0011 194	.0005 290
.41	.0084 178	.0059 662	.0042 120	.0029 628	.0014 515 <sup>-</sup>	.0007 028
.42	.0102 118	.0073 231	.0052 311	.0037 232	.0018 677	.0009 260
.43	.0123 154	.0089 333	.0064 549	.0046 474	.0023 856	.0012 104
.44	.0147 687	.0108 330	.0079 157	.0057 634	.0030 257	.0015 702
.45	.0176 147	.0130 621	.0096 492	.0071 028	.0038 117	.0020 222
.46	.0208 995 <sup>-</sup>	.0156 635 <sup>-</sup>	.0116 949	.0087 012	.0047 707	.0025 860
.47	.0246 720	.0186 838	.0140 960	.0105 977	.0059 335 <sup>+</sup>	.0032 846
.48	.0289 839	.0221 729	.0168 994	.0128 357	.0073 352	.0041 449
.49	.0338 893	.0261 838	.0201 558	.0154 624	.0090 150 <sup>-</sup>	.0051 975 <sup>+</sup>
.50	.0394 446	.0307 726	.0239 196	.0185 296	.0110 168	.0064 778
.51	.0457 080	.0359 983	.0282 487	.0220 929	.0133 894	.0080 259
.52	.0527 391	.0419 222	.0332 047	.0262 124	.0161 866	.0098 871
.53	.0605 982	.0486 078	.0388 518	.0309 516	.0194 671	.0121 123
.54	.0693 463	.0561 200	.0452 574	.0363 783	.0232 950 <sup>-</sup>	.0147 581
.55	.0790 438	.0645 250 <sup>-</sup>	.0524 908	.0425 633	.0277 392	.0178 873
.56	.0897 500 <sup>-</sup>	.0738 889	.0606 230	.0495 804	.0328 738	.0215 689
.57	.1015 224	.0842 777	.0697 262	.0575 056	.0387 773	.0258 782
.58	.1144 159	.0957 560	.0798 727	.0664 169	.0455 328	.0308 965 <sup>-</sup>
.59	.1284 813	.1083 860	.0911 337	.0763 927	.0532 267	.0367 113
.60	.1437 649	.1222 266	.1035 793	.0875 116	.0619 488	.0434 157
.61	.1603 073	.1373 324	.1172 761	.0998 507	.0717 906	.0511 079
.62	.1781 419	.1537 523	.1322 868	.1134 848	.0828 450 <sup>-</sup>	.0598 906
.63	.1972 943	.1715 280	.1486 686	.1284 846	.0952 043	.0698 696
.64	.2177 805 <sup>-</sup>	.1906 932	.1664 716	.1449 156	.1089 593	.0811 532
.65	.2396 063	.2112 717	.1857 373	.1628 359	.1241 974	.0938 500 <sup>+</sup>
.66	.2627 660	.2332 763	.2064 972	.1822 951	.1410 003	.1080 678
.67	.2872 408	.2567 073	.2287 708	.2033 319	.1594 427	.1239 108
.68	.3129 984	.2815 510	.2525 641	.2259 723	.1795 893	.1414 780
.69	.3399 915 <sup>-</sup>	.3077 784	.2778 679	.2502 276	.2014 925 <sup>+</sup>	.1608 596
.70	.3681 569	.3353 439	.3046 560	.2760 924	.2251 900	.1821 350 <sup>-</sup>

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .71$  to  $1.00$  $q = 3.5$  $p = 9.5$  to  $13$ 

	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$B(p, q) = .8276\ 6053 \times \frac{1}{10^3}$	$.7050\ 2657 \times \frac{1}{10^3}$	$.6048\ 2885 \times \frac{1}{10^3}$	$.5222\ 4190 \times \frac{1}{10^3}$	$.3961\ 8351 \times \frac{1}{10^3}$	$.3067\ 2272 \times \frac{1}{10^3}$	
$.71$	.3974 153	.3641 842	.3328 839	.3035 425 <sup>+</sup>	.2507 016	.2053 685 <sup>+</sup>
$.72$	.4276 701	.3942 173	.3624 868	.3325 333	.2780 270	.2306 066
$.73$	.4588 070	.4253 414	.3933 789	.3629 972	.3071 424	.2578 739
$.74$	.4906 945 <sup>-</sup>	.4574 348	.4254 517	.3948 428	.3379 980	.2871 692
$.75$	.5231 833	.4903 549	.4585 738	.4279 531	.3705 151	.3184 619
$.76$	.5561 072	.5239 390	.4925 898	.4621 843	.4045 841	.3516 878
$.77$	.5892 838	.5580 041	.5273 207	.4973 655 <sup>+</sup>	.4400 620	.3867 459
$.78$	.6225 156	.5923 481	.5625 638	.5332 980	.4767 711	.4234 944
$.79$	.6555 917	.6267 508	.5980 942	.5697 563	.5144 976	.4617 485 <sup>-</sup>
$.80$	.6882 900	.6609 764	.6336 659	.6064 886	.5529 915 <sup>-</sup>	.5012 776
$.81$	.7203 797	.6947 755 <sup>+</sup>	.6690 145 <sup>-</sup>	.6432 191	.5919 674	.5418 048
$.82$	.7516 246	.7278 884	.7038 596	.6796 506	.6311 059	.5830 062
$.83$	.7817 866	.7600 490	.7379 095 <sup>+</sup>	.7154 683	.6700 568	.6245 128
$.84$	.8106 301	.7909 895 <sup>-</sup>	.7708 655 <sup>-</sup>	.7503 445 <sup>-</sup>	.7084 432	.6659 133
$.85$	.8379 272	.8204 457	.8024 276	.7839 449	.7458 678	.7067 595 <sup>+</sup>
$.86$	.8634 624	.8481 632	.8323 017	.8159 357	.7819 210	.7465 740
$.87$	.8870 394	.8739 041	.8602 070	.8459 922	.8161 900	.7848 603
$.88$	.9084 867	.8974 547	.8858 848	.8738 088	.8482 712	.8211 163
$.89$	.9276 652	.9186 338	.9091 084	.8991 098	.8777 843	.8548 504
$.90$	.9444 748	.9373 013	.9296 930	.9216 620	.9043 875 <sup>+</sup>	.8856 016
$.91$	.9588 618	.9533 672	.9475 073	.9412 875 <sup>-</sup>	.9277 966	.9129 623
$.92$	.9708 254	.9668 004	.9624 843	.9578 779	.9478 036	.9366 043
$.93$	.9804 246	.9776 374	.9746 323	.9714 077	.9642 974	.9563 071
$.94$	.9877 834	.9859 895 <sup>+</sup>	.9840 451	.9819 475 <sup>-</sup>	.9772 844	.9719 880
$.95$	.9930 936	.9920 487	.9909 099	.9896 750 <sup>-</sup>	.9869 074	.9837 306
$.96$	.9966 165 <sup>-</sup>	.9960 893	.9955 118	.9948 823	.9934 602	.9918 106
$.97$	.9986 779	.9984 660	.9982 326	.9979 768	.9973 945 <sup>+</sup>	.9967 120
$.98$	.9996 580	.9996 017	.9995 393	.9994 705 <sup>+</sup>	.9993 128	.9991 261
$.99$	.9999 677	.9999 622	.9999 561	.9999 494	.9999 338	.9999 152
$1.00$	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .21$  to  $.80$  $q = 3.5$  $p = 14$  to  $19$ 

	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$
$B(p, q) = .2416\ 6032 \times \frac{1}{10^3}$	$.1933\ 2826 \times \frac{1}{10^3}$	$.1567\ 5264 \times \frac{1}{10^3}$	$.1286\ 1755 \times \frac{1}{10^3}$	$.1066\ 5846 \times \frac{1}{10^3}$	$.8929\ 5453 \times \frac{1}{10^4}$	
$x$						
.21	.0000 001					
.22	.0000 001					
.23	.0000 002					
.24	.0000 003	.0000 001				
.25	.0000 006	.0000 002				
.26	.0000 010	.0000 003	.0000 001			
.27	.0000 016	.0000 005	.0000 002			
.28	.0000 025 <sup>+</sup>	.0000 008	.0000 003	.0000 001		
.29	.0000 040	.0000 013	.0000 005	.0000 001		
.30	.0000 062	.0000 022	.0000 007	.0000 003	.0000 001	
.31	.0000 095 <sup>+</sup>	.0000 034	.0000 012	.0000 004	.0000 002	.0000 001
.32	.0000 144	.0000 053	.0000 020	.0000 007	.0000 003	.0000 001
.33	.0000 214	.0000 082	.0000 031	.0000 012	.0000 004	.0000 002
.34	.0000 315	.0000 124	.0000 049	.0000 019	.0000 007	.0000 003
.35	.0000 456	.0000 185 <sup>+</sup>	.0000 075	.0000 030	.0000 012	.0000 005
.36	.0000 654	.0000 273	.0000 113	.0000 046	.0000 019	.0000 008
.37	.0000 926	.0000 397	.0000 169	.0000 071	.0000 030	.0000 012
.38	.0001 298	.0000 572	.0000 250	.0000 108	.0000 047	.0000 020
.39	.0001 801	.0000 814	.0000 365	.0000 162	.0000 072	.0000 032
.40	.0002 474	.0001 146	.0000 527	.0000 240	.0000 109	.0000 049
.41	.0003 368	.0001 599	.0000 753	.0000 352	.0000 164	.0000 076
.42	.0004 544	.0002 210	.0001 066	.0000 510	.0000 243	.0000 115
.43	.0006 079	.0003 026	.0001 494	.0000 732	.0000 357	.0000 173
.44	.0008 067	.0004 107	.0002 074	.0001 040	.0000 518	.0000 257
.45	.0010 620	.0005 528	.0002 855	.0001 464	.0000 746	.0000 378
.46	.0013 878	.0007 381	.0003 895 <sup>+</sup>	.0002 041	.0001 063	.0000 550
.47	.0018 003	.0009 780	.0005 271	.0002 821	.0001 500 <sup>+</sup>	.0000 793
.48	.0023 191	.0012 862	.0007 078	.0003 867	.0002 100	.0001 134
.49	.0029 673	.0016 793	.0009 430	.0005 258	.0002 914	.0001 605 <sup>+</sup>
.50	.0037 719	.0021 773	.0012 472	.0007 094	.0004 010	.0002 254
.51	.0047 645	.0028 041	.0016 377	.0009 499	.0005 475 <sup>+</sup>	.0003 138
.52	.0059 814	.0035 878	.0021 356	.0012 625 <sup>+</sup>	.0007 418	.0004 333
.53	.0074 645 <sup>+</sup>	.0045 614	.0027 663	.0016 662	.0009 974	.0005 937
.54	.0092 616	.0057 635 <sup>+</sup>	.0035 597	.0021 837	.0013 314	.0008 073
.55	.0114 268	.0072 390	.0045 518	.0028 429	.0017 648	.0010 895
.56	.0140 208	.0090 391	.0057 843	.0036 768	.0023 231	.0014 597
.57	.0171 117	.0112 226	.0073 063	.0047 252	.0030 376	.0019 421
.58	.0207 749	.0138 561	.0091 744	.0060 347	.0039 459	.0025 661
.59	.0250 933	.0170 147	.0114 539	.0076 604	.0050 931	.0033 680
.60	.0301 574	.0207 819	.0142 191	.0096 661	.0065 326	.0043 914
.61	.0360 653	.0252 507	.0175 543	.0121 259	.0083 277	.0056 889
.62	.0429 220	.0305 228	.0215 541	.0151 246	.0105 522	.0073 235
.63	.0508 394	.0367 095	.0263 240	.0187 588	.0132 918	.0093 693
.64	.0599 350	.0439 304	.0319 802	.0231 372	.0166 453	.0119 135
.65	.0703 308	.0523 134	.0386 502	.0283 815 <sup>+</sup>	.0207 253	.0150 575 <sup>+</sup>
.66	.0821 523	.0619 939	.0464 718	.0346 266	.0256 590	.0189 184
.67	.0955 263	.0731 126	.0555 926	.0420 201	.0315 892	.0236 298
.68	.1105 785	.0858 147	.0661 688	.0507 222	.0386 739	.0293 431
.69	.1274 313	.1002 469	.0783 635	.0609 046	.0470 864	.0362 276
.70	.1462 007	.1165 548	.0923 441	.0727 485	.0570 144	.0444 708
.71	.1669 923	.1348 796	.1082 796	.0864 427	.0686 584	.0542 779
.72	.1898 980	.1553 538	.1263 367	.1021 800	.0822 294	.0658 699
.73	.2149 913	.1780 969	.1466 752	.1201 534	.0979 452	.0794 816
.74	.2423 225 <sup>+</sup>	.2032 099	.1694 426	.1405 505	.1160 262	.0953 576
.75	.2719 143	.2307 695 <sup>+</sup>	.1947 679	.1635 477	.1366 892	.1137 475 <sup>+</sup>
.76	.3037 557	.2608 222	.2227 545 <sup>+</sup>	.1893 025 <sup>+</sup>	.1601 403	.1348 987
.77	.3377 976	.2933 770	.2534 726	.2179 451	.1865 660	.1590 483
.78	.3739 471	.3283 990	.2869 502	.2495 686	.2161 231	.1864 122
.79	.4120 629	.3658 020	.3231 650	.2842 187	.2489 265	.2171 733
.80	.4519 505 <sup>+</sup>	.4054 421	.3620 343	.3218 820	.2850 364	.2514 660



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION

$x = .33 \text{ to } .90$

$q = 3.5$

$p = 20 \text{ to } 25$

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$B(p, q) = .7540\ 5050 \times \frac{x}{10^4}$	$.6417\ 4510 \times \frac{x}{10^4}$	$.5500\ 6723 \times \frac{x}{10^4}$	$.4745\ 6781 \times \frac{x}{10^4}$	$.4118\ 8904 \times \frac{x}{10^4}$	$.3594\ 6680 \times \frac{x}{10^4}$	
$x$						
.33	.0000 001					
.34	.0000 001					
.35	.0000 002	.0000 001				
.36	.0000 003	.0000 001				
.37	.0000 005 <sup>+</sup>	.0000 002	.0000 001			
.38	.0000 009	.0000 004	.0000 002	.0000 001		
.39	.0000 014	.0000 006	.0000 003	.0000 001		
.40	.0000 022	.0000 010	.0000 004	.0000 002	.0000 001	
.41	.0000 035 <sup>-</sup>	.0000 016	.0000 007	.0000 003	.0000 001	.0000 001
.42	.0000 054	.0000 025 <sup>+</sup>	.0000 012	.0000 005 <sup>+</sup>	.0000 003	.0000 001
.43	.0000 083	.0000 040	.0000 019	.0000 009	.0000 004	.0000 002
.44	.0000 126	.0000 062	.0000 030	.0000 015 <sup>-</sup>	.0000 007	.0000 003
.45	.0000 190	.0000 095 <sup>+</sup>	.0000 048	.0000 024	.0000 012	.0000 006
.46	.0000 283	.0000 145 <sup>+</sup>	.0000 074	.0000 038	.0000 019	.0000 010
.47	.0000 417	.0000 218	.0000 114	.0000 059	.0000 031	.0000 016
.48	.0000 609	.0000 325 <sup>+</sup>	.0000 173	.0000 092	.0000 048	.0000 025 <sup>+</sup>
.49	.0000 880	.0000 480	.0000 261	.0000 141	.0000 076	.0000 041
.50	.0001 260	.0000 701	.0000 389	.0000 214	.0000 118	.0000 065 <sup>-</sup>
.51	.0001 789	.0001 015 <sup>+</sup>	.0000 574	.0000 323	.0000 181	.0000 101
.52	.0002 519	.0001 457	.0000 839	.0000 481	.0000 275 <sup>+</sup>	.0000 157
.53	.0003 516	.0002 073	.0001 216	.0000 711	.0000 414	.0000 240
.54	.0004 870	.0002 924	.0001 748	.0001 041	.0000 618	.0000 365 <sup>+</sup>
.55	.0006 692	.0004 091	.0002 491	.0001 510	.0000 912	.0000 549
.56	.0009 126	.0005 680	.0003 520	.0002 172	.0001 336	.0000 819
.57	.0012 355 <sup>-</sup>	.0007 824	.0004 934	.0003 099	.0001 940	.0001 210
.58	.0016 606	.0010 697	.0006 862	.0004 385 <sup>-</sup>	.0002 792	.0001 772
.59	.0022 162	.0014 518	.0009 471	.0006 154	.0003 985 <sup>+</sup>	.0002 572
.60	.0029 376	.0019 563	.0012 974	.0008 571	.0005 643	.0003 703
.61	.0038 675 <sup>+</sup>	.0026 176	.0017 643	.0011 847	.0007 927	.0005 287
.62	.0050 583	.0034 783	.0023 822	.0016 253	.0011 050 <sup>+</sup>	.0007 489
.63	.0065 729	.0045 910	.0031 937	.0022 134	.0015 287	.0010 524
.64	.0084 866	.0060 193	.0042 522	.0029 927	.0020 991	.0014 676
.65	.0108 887	.0078 493	.0056 229	.0040 179	.0028 612	.0020 310
.66	.0138 842	.0101 464	.0073 857	.0053 566	.0038 718	.0027 898
.67	.0175 954	.0130 409	.0096 367	.0070 922	.0052 020	.0038 037
.68	.0221 633	.0166 708	.0124 912	.0093 262	.0069 400	.0051 484
.69	.0277 491	.0211 678	.0160 860	.0121 811	.0091 939	.0069 180
.70	.0345 350 <sup>-</sup>	.0267 105 <sup>+</sup>	.0205 813	.0158 034	.0120 953	.0092 292
.71	.0427 242	.0334 958	.0261 635 <sup>+</sup>	.0203 661	.0158 024	.0122 248
.72	.0525 412	.0417 449	.0330 403	.0260 715 <sup>-</sup>	.0205 038	.0160 775 <sup>-</sup>
.73	.0642 297	.0517 040	.0414 718	.0331 534	.0264 210	.0209 943
.74	.0780 505 <sup>-</sup>	.0636 422	.0517 108	.0418 783	.0338 113	.0272 199
.75	.0942 773	.0778 493	.0640 615 <sup>-</sup>	.0525 455 <sup>+</sup>	.0429 698	.0350 399
.76	.1131 910	.0946 308	.0788 459	.0654 861	.0542 290	.0447 824
.77	.1350 716	.1143 019	.0964 052	.0810 589	.0679 578	.0568 191
.78	.1601 886	.1371 773	.1170 917	.0996 446	.0845 567	.0715 623
.79	.1887 876	.1635 598	.1412 573	.1216 361	.1044 505 <sup>+</sup>	.0894 597
.80	.2210 759	.1937 248	.1692 391	.1474 246	.1280 760	.1109 848
.81	.2572 041	.2279 014	.2013 406	.1773 815 <sup>+</sup>	.1558 653	.1366 217
.82	.2972 458	.2662 502	.2378 080	.2118 352	.1882 224	.1668 431
.83	.3411 757	.3088 385 <sup>-</sup>	.2788 034	.2510 414	.2254 943	.2020 815 <sup>-</sup>
.84	.3888 453	.3556 119	.3243 724	.2951 494	.2679 338	.2426 914
.85	.4399 596	.4063 652	.3744 105 <sup>-</sup>	.3441 624	.3156 569	.2889 033
.86	.4940 548	.4607 132	.4286 258	.3978 949	.3685 936	.3407 696
.87	.5504 792	.5180 635 <sup>-</sup>	.4865 044	.4559 280	.4264 350 <sup>-</sup>	.3981 033
.88	.6083 819	.5775 956	.5472 784	.5175 676	.4885 802	.4604 140
.89	.6667 103	.6382 497	.6099 043	.5818 102	.5540 885 <sup>+</sup>	.5268 463
.90	.7242 238	.6987 308	.6730 558	.6473 224	.6216 450 <sup>-</sup>	.5961 280

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION

95

 $x = .91$  to  $1.00$  $q = 3.5$  $p = 20$  to  $25$ 

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$B(p, q) = .7540\ 5050 \times \frac{x}{101}$		$.6417\ 4510 \times \frac{x}{101}$	$.5500\ 6723 \times \frac{x}{101}$	$.4745\ 6781 \times \frac{x}{101}$	$.4118\ 8904 \times \frac{x}{101}$	$.3594\ 6680 \times \frac{x}{101}$
$x$						
.91	.7795 256	.7575 345 <sup>+</sup>	.7351 403	.7124 458	.6895 490	.6665 425 <sup>-</sup>
.92	.8311 216	.8130 023	.7943 481	.7752 360	.7557 416	.7359 386
.93	.8775 088	.8634 136	.8487 445 <sup>-</sup>	.8335 520	.8178 872	.8018 014
.94	.9173 002	.9071 224	.8964 162	.8852 089	.8735 288	.8614 060
.95	.9493 881	.9427 440	.9356 807	.9282 080	.9203 373	.9120 813
.96	.9731 415 <sup>+</sup>	.9693 900	.9653 599	.9610 512	.9564 654	.9516 043
.97	.9886 263	.9869 400	.9851 096	.9831 323	.9810 059	.9787 284
.98	.9968 103	.9963 095 <sup>-</sup>	.9957 602	.9951 608	.9945 095 <sup>-</sup>	.9938 047
.99	.9996 732	.9996 189	.9995 588	.9994 926	.9994 199	.9993 404
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\mu = .55$  to  $1.00$

$q = 3.5$

$p = 38$  to  $43$

	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$(p, q) = .8782\ 1352 \times 10^5$		$.8041\ 4732 \times 10^5$	$.7379\ 2343 \times 10^5$	$.6785\ 5028 \times 10^5$	$.6251\ 8115 \times 10^5$	$.5770\ 9030 \times 10^5$
$\mu$						
.55	.0000 001					
.56	.0000 001	.0000 001				
.57	.0000 002	.0000 001	.0000 001			
.58	.0000 004	.0000 002	.0000 001	.0000 001	.0000 001	
.59	.0000 007	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001
.60	.0000 012	.0000 008	.0000 005	.0000 003	.0000 002	.0000 001
.61	.0000 022	.0000 014	.0000 009	.0000 006	.0000 004	.0000 002
.62	.0000 038	.0000 025 <sup>†</sup>	.0000 017	.0000 011	.0000 007	.0000 005 <sup>---</sup>
.63	.0000 066	.0000 044	.0000 029	.0000 020	.0000 013	.0000 009
.64	.0000 113	.0000 077	.0000 052	.0000 035 <sup>†</sup>	.0000 024	.0000 016
.65	.0000 190	.0000 131	.0000 090	.0000 062	.0000 043	.0000 029
.66	.0000 318	.0000 223	.0000 156	.0000 100	.0000 076	.0000 053
.67	.0000 526	.0000 374	.0000 265 <sup>†</sup>	.0000 188	.0000 133	.0000 094
.68	.0000 860	.0000 620	.0000 447	.0000 321	.0000 231	.0000 166
.69	.0001 303	.0001 010	.0000 743	.0000 543	.0000 396	.0000 288
.70	.0002 232	.0001 656	.0001 228	.0000 909	.0000 672	.0000 496
.71	.0003 542	.0002 606	.0002 003	.0001 504	.0001 127	.0000 844
.72	.0005 565 <sup>†</sup>	.0004 246	.0003 235 <sup>†</sup>	.0002 162	.0001 871	.0001 421
.73	.0008 657	.0006 695 <sup>†</sup>	.0005 171	.0003 989	.0003 074	.0002 366
.74	.0013 344	.0010 451	.0008 181	.0006 396	.0004 995 <sup>†</sup>	.0003 896
.75	.0020 346	.0016 151	.0012 810	.0010 139	.0008 031	.0006 348
.76	.0030 708	.0024 707	.0019 853	.0015 934	.0012 773	.0010 229
.77	.0045 911	.0037 413	.0030 450 <sup>†</sup>	.0024 754	.0020 100	.0016 303
.78	.0067 953	.0056 676	.0046 219	.0038 040	.0031 288	.0025 701
.79	.0099 557	.0083 181	.0069 414	.0057 859	.0048 174	.0040 067
.80	.0144 358	.0122 092	.0103 137	.0087 026	.0073 351	.0061 759
.81	.0207 120	.0177 287	.0151 574	.0129 445 <sup>†</sup>	.0110 428	.0094 166
.82	.0293 969	.0254 614	.0220 274	.0190 356	.0164 327	.0141 713
.83	.0412 609	.0361 549	.0316 436	.0276 660	.0241 633	.0210 830
.84	.0572 485 <sup>†</sup>	.0507 375 <sup>†</sup>	.0449 177	.0397 234	.0350 943	.0309 743
.85	.0784 823	.0703 380	.0629 716	.0563 188	.0503 191	.0449 158
.86	.1062 465 <sup>†</sup>	.0962 693	.0871 399	.0787 961	.0711 838	.0642 475 <sup>---</sup>
.87	.1419 372	.1299 934	.1180 358	.1087 147	.0993 860	.0906 543
.88	.1869 663	.1730 318	.1599 843	.1477 830	.1363 921	.1257 715
.89	.2426 009	.2268 135 <sup>†</sup>	.2118 620	.1977 215 <sup>†</sup>	.1843 730	.1717 838
.90	.3097 262	.2924 318	.2758 749	.2600 195	.2449 186	.2304 998
.91	.3885 226	.3703 230	.3522 017	.3356 641	.3192 126	.3033 515 <sup>†</sup>
.92	.4780 674	.4598 129	.4419 423	.4244 747	.4074 222	.3908 020
.93	.5759 023	.5586 514	.5415 796	.5247 106	.5080 658	.4916 648
.94	.6776 599	.6625 580	.6474 543	.6323 682	.6173 227	.6023 382
.95	.7769 223	.7649 960	.7529 396	.7407 702	.7285 944	.7161 583
.96	.8855 894	.8754 587	.8649 157	.8540 807	.8429 516	.8322 746
.97	.9351 409	.9307 368	.9261 908	.9215 057	.9166 840	.9117 286
.98	.9792 071	.9776 338	.9759 931	.9742 848	.9725 085 <sup>†</sup>	.9706 642
.99	.9975 577	.9973 531	.9971 377	.9969 110	.9966 730	.9964 233
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

99

$p = 44$  to 50

	$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$p, q) = .5336\ 5339 \times \frac{1}{10^8}$		$.4943\ 3156 \times \frac{1}{10^8}$	$.4586\ 5815 \times \frac{1}{10^8}$	$.4262\ 2778 \times \frac{1}{10^8}$	$.3966\ 8724 \times \frac{1}{10^8}$	$.3697\ 2791 \times \frac{1}{10^8}$	$.3450\ 7938 \times \frac{1}{10^8}$
60	.0000 001	.0000 001					
61	.0000 002	.0000 001	.0000 001				
62	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001		
63	.0000 006	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	
64	.0000 011	.0000 007	.0000 005	.0000 003	.0000 002	.0000 001	.0000 001
65	.0000 020	.0000 014	.0000 009	.0000 006	.0000 004	.0000 003	.0000 002
66	.0000 037	.0000 026	.0000 018	.0000 012	.0000 009	.0000 006	.0000 004
67	.0000 066	.0000 047	.0000 033	.0000 023	.0000 016	.0000 011	.0000 008
68	.0000 119	.0000 085	.0000 061	.0000 043	.0000 031	.0000 022	.0000 016
69	.0000 210	.0000 152	.0000 110	.0000 080	.0000 058	.0000 042	.0000 030
70	.0000 366	.0000 270	.0000 198	.0000 146	.0000 107	.0000 079	.0000 058
71	.0000 631	.0000 472	.0000 352	.0000 263	.0000 196	.0000 146	.0000 108
72	.0001 078	.0000 816	.0000 618	.0000 467	.0000 353	.0000 266	.0000 201
73	.0001 819	.0001 397	.0001 072	.0000 821	.0000 629	.0000 481	.0000 368
74	.0003 036	.0002 363	.0001 838	.0001 428	.0001 108	.0000 859	.0000 666
75	.0005 012	.0003 953	.0003 115	.0002 452	.0001 929	.0001 516	.0001 190
76	.0008 182	.0006 538	.0005 220	.0004 163	.0003 317	.0002 641	.0002 101
77	.0013 210	.0010 693	.0008 647	.0006 986	.0005 639	.0004 548	.0003 664
78	.0021 090	.0017 288	.0014 159	.0011 585	.0009 470	.0007 735 <sup>+</sup>	.0006 313
79	.0033 290	.0027 632	.0022 914	.0018 984	.0015 715	.0012 997	.0010 741
80	.0051 947	.0043 652	.0036 646	.0030 738	.0025 759	.0021 569	.0018 046
81	.0080 117	.0068 143	.0057 904	.0049 160	.0041 701	.0035 345	.0029 933
82	.0122 091	.0105 087	.0090 370	.0077 645	.0066 656	.0057 175 <sup>+</sup>	.0049 005
83	.0183 776	.0160 047	.0139 257	.0121 063	.0105 159	.0091 272	.0079 157
84	.0273 123	.0240 614	.0211 790	.0186 262	.0163 678	.0143 719	.0126 098
85	.0400 559	.0356 904	.0317 735	.0282 632	.0251 207	.0223 105 <sup>+</sup>	.0198 000
86	.0579 354	.0521 987	.0469 909	.0422 689	.0379 919	.0341 222	.0306 244
87	.0825 773	.0752 163	.0684 564	.0622 557	.0565 742	.0513 738	.0466 186
88	.1158 819	.1066 846	.0981 414	.0902 147	.0828 682	.0760 666	.0697 758
89	.1599 279	.1487 767	.1383 007	.1284 704	.1192 560	.1106 281	.1025 575 <sup>+</sup>
90	.2167 683	.2037 079	.1913 005	.1795 269	.1683 670	.1578 000	.1478 045 <sup>+</sup>
91	.2880 788	.2733 909	.2592 825	.2457 460	.2327 725 <sup>+</sup>	.2203 518	.2084 723
92	.3746 220	.3588 906	.3436 137	.3287 951	.3144 367	.3005 391	.2871 009
93	.4755 250	.4596 622	.4440 901	.4288 211	.4138 656	.3992 325	.3849 294
94	.5874 338	.5726 276	.5579 366	.5433 765	.5289 620	.5147 067	.5006 230
95	.7037 475 <sup>+</sup>	.6912 874	.6787 927	.6662 776	.6537 559	.6412 407	.6287 446
96	.8143 588	.8053 133	.7961 472	.7868 695 <sup>+</sup>	.7774 892	.7680 150 <sup>+</sup>	.7584 558
97	.9066 423	.9014 283	.8960 898	.8906 300	.8850 523	.8793 601	.8735 570
98	.9687 516	.9667 708	.9647 218	.9626 046	.9604 194	.9581 663	.9558 457
99	.9961 617	.9958 880	.9956 020	.9953 035 <sup>+</sup>	.9949 923	.9946 681	.9943 307
00	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000	I.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .01$  to  $.60$  $q = 4$  $p = 4$  to  $6.5$ 

	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$
$B(p, q) = .7142\ 8571 \times \frac{1}{10^2}$	$.4972\ 8050 \times \frac{1}{10^2}$	$.3571\ 4286 \times \frac{1}{10^2}$	$.2632\ 6615 \times \frac{1}{10^2}$	$.1984\ 1270 \times \frac{1}{10^2}$	$.1524\ 1724 \times \frac{1}{10^2}$	
.01	.0000 003					
.02	.0000 053	.0000 010	.0000 002			
.03	.0000 264	.0000 058	.0000 013	.0000 003	.0000 001	
.04	.0000 813	.0000 207	.0000 052	.0000 013	.0000 003	.0000 001
.05	.0001 936	.0000 551	.0000 154	.0000 042	.0000 012	.0000 003
.06	.0003 915	.0001 220	.0000 373	.0000 113	.0000 033	.0000 010
.07	.0007 072	.0002 379	.0000 786	.0000 256	.0000 082	.0000 026
.08	.0011 763	.0004 228	.0001 493	.0000 519	.0000 178	.0000 060
.09	.0018 366	.0006 997	.0002 619	.0000 965 <sup>+</sup>	.0000 351	.0000 126
.10	.0027 280 <sup>e</sup>	.0010 948	.0004 316	.0001 676	.0000 642	.0000 243
.11	.0038 916	.0016 369	.0006 765 <sup>+</sup>	.0002 754	.0001 106	.0000 439
.12	.0053 693	.0023 572	.0010 169	.0004 322	.0001 813	.0000 752
.13	.0072 028	.0032 889	.0014 759	.0006 525 <sup>+</sup>	.0002 847	.0001 228
.14	.0094 339	.0044 670	.0020 790	.0009 533	.0004 315 <sup>+</sup>	.0001 931
.15	.0121 032	.0059 276	.0028 539	.0013 539	.0006 340	.0002 936
.16	.0152 503	.0077 080	.0038 303	.0018 757	.0009 068	.0004 335 <sup>-</sup>
.17	.0189 131	.0098 458	.0050 399	.0025 425 <sup>+</sup>	.0012 664	.0006 238
.18	.0231 276	.0123 790	.0065 160	.0033 805 <sup>+</sup>	.0017 318	.0008 773
.19	.0279 276	.0153 452	.0082 929	.0044 178	.0023 240	.0012 091
.20	.0333 440 <sup>e</sup>	.0187 815 <sup>+</sup>	.0104 064 <sup>e</sup>	.0056 843	.0030 664	.0016 360
.21	.0394 053	.0227 243	.0128 926	.0072 118	.0039 844	.0021 773
.22	.0461 368	.0272 084	.0157 883	.0090 337	.0051 056	.0028 544
.23	.0535 606	.0322 673	.0191 302	.0111 847	.0064 598	.0036 908
.24	.0616 955 <sup>-</sup>	.0379 325 <sup>-</sup>	.0229 548	.0137 004	.0080 784	.0047 125 <sup>-</sup>
.25	.0705 566	.0442 333	.0272 980	.0166 173	.0099 945 <sup>+</sup>	.0059 475 <sup>-</sup>
.26	.0801 558	.0511 967	.0321 948	.0199 724	.0122 430	.0074 259
.27	.0905 009	.0588 469	.0376 789	.0238 028	.0148 598	.0091 799
.28	.1015 962	.0672 052	.0437 826	.0281 456	.0178 821	.0112 435 <sup>-</sup>
.29	.1134 424	.0762 897	.0505 362	.0330 373	.0213 477	.0136 523
.30	.1260 360 <sup>e</sup>	.0861 154	.0579 676	.0385 136	.0252 948	.0164 436
.31	.1393 702	.0966 937	.0661 027	.0446 090	.0297 621	.0196 557
.32	.1534 344	.1080 324	.0749 644	.0513 568	.0347 877	.0233 281
.33	.1682 141	.1201 357	.0845 724	.0587 880	.0404 096	.0275 007
.34	.1836 917	.1330 038	.0949 435 <sup>-</sup>	.0669 319	.0466 645 <sup>-</sup>	.0322 141
.35	.1998 457	.1466 333	.1060 909	.0758 150 <sup>-</sup>	.0535 882	.0375 087
.36	.2166 517	.1610 168	.1180 242	.0854 611	.0612 147	.0434 246
.37	.2340 816	.1761 428	.1307 490	.0958 908	.0695 762	.0500 012
.38	.2521 046	.1919 964	.1442 673	.1071 215 <sup>+</sup>	.0787 022	.0572 769
.39	.2706 869	.2085 583	.1585 766	.1191 669	.0886 197	.0652 882
.40	.2897 920 <sup>e</sup>	.2258 059	.1736 704 <sup>e</sup>	.1320 365 <sup>-</sup>	.0993 526	.0740 700
.41	.3093 807	.2437 124	.1895 380	.1457 360	.1109 212	.0836 546
.42	.3294 116	.2622 478	.2061 644	.1602 666	.1233 422	.0940 716
.43	.3498 411	.2813 784	.2235 301	.1756 252	.1366 281	.1053 473
.44	.3706 237	.3010 672	.2416 115 <sup>+</sup>	.1918 036	.1507 869	.1175 044
.45	.3917 122	.3212 740	.2603 807	.2087 894	.1658 220	.1305 615 <sup>-</sup>
.46	.4130 579	.3419 558	.2798 056	.2265 650 <sup>-</sup>	.1817 320	.1445 326
.47	.4346 107	.3630 666	.2998 501	.2451 079	.1985 102	.1594 270
.48	.4563 199	.3845 578	.3204 741	.2643 908	.2161 445 <sup>-</sup>	.1752 488
.49	.4781 337	.4053 787	.3416 336	.2843 817	.2346 175 <sup>+</sup>	.1919 967
.50	.5000 000 <sup>e</sup>	.4284 763	.3632 812	.3050 434	.2539 062	.2096 634
.51	.5218 663	.4507 960	.3853 661	.3263 342	.2739 820	.2282 357
.52	.5436 801	.4732 815 <sup>+</sup>	.4078 342	.3482 078	.2948 105 <sup>-</sup>	.2476 940
.53	.5653 893	.4958 754	.4306 287	.3706 135 <sup>+</sup>	.3163 517	.2680 126
.54	.5869 421	.5185 191	.4536 899	.3934 963	.3385 600	.2891 591
.55	.6082 878	.5411 538	.4769 563	.4167 973	.3613 846	.3110 944
.56	.6293 763	.5637 198	.5003 641	.4404 540	.3847 691	.3337 729
.57	.6501 589	.5861 581	.5238 478	.4644 003	.4086 522	.3571 427
.58	.6705 884	.6084 094	.5473 412	.4885 673	.4329 677	.3811 453
.59	.6906 193	.6304 154	.5707 766	.5128 835 <sup>+</sup>	.4576 451	.4057 158
.60	.7102 080 <sup>e</sup>	.6521 187	.5940 864 <sup>e</sup>	.5372 751	.4826 097	.4307 838

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION

101

 $x = .61$  to  $1.00$  $q = 4$  $p = 4$  to  $6.5$ 

	$p = 4$	$p = 4.5$	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$
$B(p, q) = .7142\ 8571 \times \frac{1}{10^8}$	$.4972\ 8050 \times \frac{1}{10^8}$	$.3571\ 4286 \times \frac{1}{10^8}$	$.2632\ 6615 \times \frac{1}{10^8}$	$.1984\ 1270 \times \frac{1}{10^8}$	$.1524\ 1724 \times \frac{1}{10^8}$	
.61	.7293 131	.6734 633	.6172 027	.5616 666	.5077 830	.4562 729
.62	.7478 954	.6943 947	.6400 580	.5859 812	.5330 834	.4821 015-
.63	.7659 184	.7148 604	.6625 859	.6101 414	.5584 267	.5081 832
.64	.7833 483	.7348 104	.6847 209	.6340 695-	.5837 263	.5344 275+
.65	.8001 543	.7541 970	.7063 994	.6576 878	.6088 944	.5607 400
.66	.8163 083	.7729 754	.7275 601	.6809 200	.6338 421	.5870 234
.67	.8317 859	.7911 043	.7481 442	.7036 908	.6584 802	.6131 781
.68	.8465 656	.8085 455-	.7680 957	.7259 272	.6827 203	.6391 029
.69	.8606 298	.8252 646	.7873 623	.7475 588	.7064 750+	.6646 961
.70	.8739 640 <sup>e</sup>	.8412 313	.8058 956	.7685 183	.7296 591	.6898 560
.71	.8865 576	.8564 193	.8236 514	.7887 424	.7521 900	.7144 824
.72	.8984 038	.8708 067	.8405 901	.8081 721	.7739 888	.7384 769
.73	.9094 991	.8843 762	.8566 771	.8267 535-	.7949 811	.7617 445+
.74	.9198 442	.8971 151	.8718 832	.8444 379	.8150 974	.7841 944
.75	.9294 434	.9090 155-	.8861 847	.8611 831	.8342 743	.8057 411
.76	.9383 045+	.9200 744	.8995 638	.8769 531	.8524 552	.8263 051
.77	.9464 394	.9302 939	.9120 090	.8917 190	.8695 907	.8458 147
.78	.9538 632	.9396 809	.9235 147	.9054 591	.8856 398	.8642 063
.79	.9605 947	.9482 472	.9340 820	.9181 598	.9005 700	.8814 257
.80	.9666 560 <sup>e</sup>	.9560 096	.9437 184 <sup>e</sup>	.9298 150+	.9143 583	.8974 290
.81	.9720 724	.9629 896	.9524 378	.9404 273	.9269 914	.9121 834
.82	.9768 724	.9692 134	.9602 607	.9500 073	.9384 662	.9256 680
.83	.9810 869	.9747 115+	.9672 137	.9585 741	.9487 901	.9378 743
.84	.9847 497	.9795 188	.9733 297	.9661 552	.9579 813	.9488 068
.85	.9878 968	.9836 738	.9786 475+	.9727 861	.9660 685-	.9584 836
.86	.9905 661	.9872 188	.9832 113	.9785 104	.9730 910	.9669 360
.87	.9927 972	.9901 990	.9870 703	.9833 789	.9790 985-	.9742 088
.88	.9946 307	.9926 624	.9902 784	.9874 494	.9841 503	.9803 598
.89	.9961 084	.9946 588	.9928 932	.9907 862	.9883 149	.9854 595-
.90	.9972 720 <sup>e</sup>	.9962 399	.9949 756	.9934 584	.9916 689	.9895 896
.91	.9981 634	.9974 578	.9965 887	.9955 398	.9942 959	.9928 425-
.92	.9988 237	.9983 649	.9977 967	.9971 072	.9962 849	.9953 189
.93	.9992 928	.9990 128	.9986 641	.9982 388	.9977 287	.9971 263
.94	.9996 085+	.9994 512	.9992 544	.9990 129	.9987 217	.9983 760
.95	.9998 064	.9997 275+	.9996 282	.9995 058	.9993 574	.9991 803
.96	.9999 187	.9998 851	.9998 426	.9997 899	.9997 257	.9996 487
.97	.9999 736	.9999 626	.9999 485+	.9999 310	.9999 096	.9998 837
.98	.9999 947	.9999 924	.9999 895-	.9999 859	.9999 814	.9999 760
.99	.9999 997	.9999 995+	.9999 993	.9999 991	.9999 988	.9999 984
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .05 \text{ to } .70$  $q = 4$  $p = 7 \text{ to } 9.5$ 

$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$
$B(p, q) = .11904762 \times 10^5$					
$x$					
.05	.0000 001				
.06	.0000 003	.0000 001			
.07	.0000 008	.0000 003	.0000 001		
.08	.0000 020	.0000 007	.0000 002	.0000 001	
.09	.0000 045	.0000 016	.0000 006	.0000 002	.0000 001
.10	.0000 091	.0000 034	.0000 012	.0000 005	.0000 002
.11	.0000 173	.0000 067	.0000 026	.0000 010	.0000 003
.12	.0000 308	.0000 125	.0000 051	.0000 020	.0000 008
.13	.0000 525	.0000 222	.0000 091	.0000 034	.0000 016
.14	.0000 856	.0000 376	.0000 153	.0000 057	.0000 028
.15	.0001 346	.0000 611	.0000 250	.0000 100	.0000 048
.16	.0002 051	.0000 962	.0000 418	.0000 167	.0000 079
.17	.0003 042	.0001 470	.0000 705	.0000 280	.0000 125
.18	.0004 401	.0002 188	.0001 170	.0000 528	.0000 225
.19	.0006 220	.0003 181	.0001 611	.0000 810	.0000 361
.20	.0008 644	.0004 527	.0002 352	.0001 214	.0000 622
.21	.0011 783	.0006 321	.0003 464	.0001 708	.0000 987
.22	.0015 804	.0008 674	.0004 724	.0002 255	.0001 414
.23	.0020 885	.0011 717	.0006 252	.0003 005	.0002 051
.24	.0027 228	.0015 508	.0008 095	.0004 005	.0002 807
.25	.0035 057	.0020 480	.0011 884	.0005 344	.0003 627
.26	.0044 618	.0026 582	.0015 716	.0007 228	.0004 684
.27	.0056 181	.0034 005	.0020 534	.0010 282	.0006 001
.28	.0070 030	.0043 266	.0026 526	.0015 152	.0007 774
.29	.0086 507	.0054 361	.0033 905	.0021 004	.0010 230
.30	.0105 921	.0067 668	.0042 900	.0027 025	.0013 617
.31	.0128 637	.0083 500	.0053 801	.0034 414	.0017 854
.32	.0155 020	.0102 103	.0066 870	.0043 465	.0023 030
.33	.0185 480	.0124 166	.0082 442	.0053 420	.0029 264
.34	.0220 422	.0149 620	.0100 828	.0064 501	.0036 618
.35	.0260 243	.0179 136	.0122 424	.0077 120	.0045 098
.36	.0305 376	.0213 070	.0147 660	.0091 597	.0054 704
.37	.0356 252	.0251 856	.0176 797	.0107 311	.0065 500
.38	.0413 301	.0295 939	.0210 424	.0124 665	.0077 650
.39	.0476 949	.0345 773	.0248 930	.0143 002	.0091 070
.40	.0547 619	.0401 818	.0292 815	.0162 045	.0106 444
.41	.0625 719	.0464 535	.0342 530	.0182 001	.0123 852
.42	.0711 643	.0534 480	.0408 575	.0203 455	.0142 660
.43	.0805 763	.0611 804	.0481 445	.0225 605	.0162 217
.44	.0908 427	.0697 243	.0561 643	.0249 205	.0183 605
.45	.1019 949	.0791 116	.0649 642	.0274 204	.0206 641
.46	.1140 612	.0893 816	.0745 918	.0300 644	.0231 600
.47	.1270 655	.1005 708	.0849 955	.0328 414	.0258 174
.48	.1410 272	.1127 123	.0969 181	.0358 827	.0286 495
.49	.1559 607	.1258 348	.1099 000	.0390 478	.0316 988
.50	.1718 750	.1399 626	.1232 812	.0421 721	.0349 607
.51	.1887 732	.1551 145	.1366 924	.0452 060	.0383 821
.52	.2066 520	.1713 030	.1511 628	.0482 013	.0419 602
.53	.2255 015	.1885 364	.1667 149	.0512 051	.0457 292
.54	.2453 048	.2068 120	.1731 651	.0542 592	.0496 498
.55	.2660 370	.2261 255	.1911 227	.0573 094	.0537 681
.56	.2876 693	.2464 580	.2099 895	.0603 092	.0580 701
.57	.3101 590	.2677 890	.2299 520	.0632 846	.0625 016
.58	.3334 628	.2900 844	.2510 150	.0662 430	.0671 645
.59	.3575 238	.3133 043	.2731 357	.0691 724	.0719 416
.60	.3822 806	.3373 991	.2962 844	.0720 360	.0768 286
.61	.4076 639	.3623 103	.3204 173	.0749 663	.0818 413
.62	.4335 970	.3879 704	.3454 804	.0778 647	.0869 602
.63	.4599 962	.4143 932	.3714 088	.0807 011	.0921 244
.64	.4867 716	.4412 240	.3981 272	.0835 736	.0974 315
.65	.5138 270	.4686 396	.4255 501	.0864 283	.1028 627
.66	.5410 612	.4964 494	.4535 820	.0892 590	.1084 204
.67	.5683 680	.5245 453	.4821 177	.0920 677	.1141 070
.68	.5950 374	.5528 130	.5110 427	.0948 639	.1199 270
.69	.6227 567	.5811 322	.5402 343	.0976 296	.1258 882
.70	.6496 107	.6093 785	.5695 623	.1003 105	.1319 826

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION

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 $x = .71$  to 1.00 $q = 4$  $p = 7$  to 9.5

	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$
$H(p, q) = .11004762 \times \frac{1}{100}$		$.01353531 \times \frac{1}{100}$	$.75757576 \times \frac{1}{100}$	$.01534911 \times \frac{1}{100}$	$.50505051 \times \frac{1}{100}$	$.41843740 \times \frac{1}{100}$
.71	.6760836	.6474233	.5988890	.5608266	.5235206	.4872487
.72	.6720595	.6661400	.6280749	.5912085	.5548303	.5191943
.73	.67274239	.6624850	.6360714	.60214995	.5662547	.5314897
.74	.67520691	.67100188	.6554310	.6315375	.6176308	.5839560
.75	.6758754	.67449681	.67141945	.6811569	.6487786	.6164063
.76	.67687513	.67700460	.6749442	.7101882	.6705133	.6486393
.77	.6769066	.67641548	.67667052	.7384654	.7006466	.6804502
.78	.67614291	.67617581	.7019480	.7658234	.7389900	.7116290
.79	.67668352	.67600121	.8160407	.7921026	.7673576	.7419646
.80	.67601291	.6759664	.8488608	.8171516	.7945689	.7712483
.81	.67600739	.67587470	.8602080	.8408301	.8204525	.7992777
.82	.67616589	.67604682	.8662561	.8630114	.8418495	.8258606
.83	.67598558	.67627647	.8686556	.8845853	.8670168	.8508106
.84	.67630424	.67625084	.9154354	.9024614	.8886315	.8730963
.85	.67600402	.67607156	.9408551	.9105715	.9077937	.8952559
.86	.67600458	.67521880	.9449962	.9348721	.9250303	.9144910
.87	.67686952	.67625487	.9552635	.9483468	.9402983	.9316302
.88	.67606612	.67612415	.9638920	.9600677	.9535873	.9466329
.89	.67624038	.67685407	.9744412	.9668974	.9649218	.9595010
.90	.67672048	.67645007	.9814652	.9780886	.9743625	.9702809
.91	.67611662	.67612550	.9830007	.9846845	.9820669	.9790574
.92	.67641082	.67620144	.9914567	.9908173	.9879882	.9850624
.93	.67609249	.67607147	.9945002	.9946454	.9924734	.9911682
.94	.67607905	.67607010	.9969622	.9969496	.9956587	.9948852
.95	.67609215	.67607264	.9983477	.9981271	.9977636	.9973545
.96	.67609534	.67609455	.9992456	.9991841	.9990218	.9988382
.97	.67609529	.67609516	.9997344	.9997256	.9996606	.9996060
.98	.67609565	.67609618	.9999528	.9999424	.9999304	.9999166
.99	.67609595	.67609675	.9999969	.9999962	.9999954	.9999944
1.000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000	1.000000000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .11$  to  $.70$  $q = 4$  $p = 10$  to  $14$ 

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$B(p, q) = .3496\ 5035 \times \frac{1}{10^8}$	$.2944\ 5595 \times \frac{1}{10^8}$	$.2497\ 5025 \times \frac{1}{10^8}$	$.1831\ 5018 \times \frac{1}{10^8}$	$.1373\ 6264 \times \frac{1}{10^8}$	$.1050\ 4202 \times \frac{1}{10^8}$	
$x$						
.11	.0000 001					
.12	.0000 001					
.13	.0000 003	.0000 001				
.14	.0000 006	.0000 002	.0000 001			
.15	.0000 011	.0000 005	.0000 002			
.16	.0000 020	.0000 009	.0000 004	.0000 001		
.17	.0000 035	.0000 016	.0000 008	.0000 002		
.18	.0000 060	.0000 029	.0000 014	.0000 003	.0000 001	
.19	.0000 099	.0000 049	.0000 024	.0000 006	.0000 001	
.20	.0000 161	.0000 081	.0000 041	.0000 010	.0000 002	.0000 001
.21	.0000 253	.0000 131	.0000 067	.0000 018	.0000 005	.0000 001
.22	.0000 390	.0000 206	.0000 108	.0000 030	.0000 008	.0000 002
.23	.0000 587	.0000 317	.0000 171	.0000 049	.0000 014	.0000 004
.24	.0000 868	.0000 479	.0000 263	.0000 079	.0000 023	.0000 007
.25	.0001 261	.0000 710	.0000 398	.0000 124	.0000 038	.0000 011
.26	.0001 802	.0001 035	.0000 592	.0000 191	.0000 061	.0000 019
.27	.0002 536	.0001 484	.0000 864	.0000 290	.0000 096	.0000 031
.28	.0003 520	.0002 096	.0001 243	.0000 432	.0000 148	.0000 050
.29	.0004 820	.0002 921	.0001 762	.0000 634	.0000 225	.0000 079
.30	.0006 520	.0004 018	.0002 465	.0000 917	.0000 336	.0000 122
.31	.0008 717	.0005 459	.0003 404	.0001 307	.0000 495	.0000 185
.32	.0011 530	.0007 334	.0004 645	.0001 841	.0000 719	.0000 278
.33	.0015 095	.0009 748	.0006 268	.0002 561	.0001 031	.0000 410
.34	.0019 572	.0012 826	.0008 369	.0003 521	.0001 461	.0000 599
.35	.0025 146	.0016 715	.0011 062	.0004 789	.0002 044	.0000 862
.36	.0032 028	.0021 585	.0014 485	.0006 447	.0002 829	.0001 227
.37	.0040 459	.0027 635	.0018 795	.0008 593	.0003 875	.0001 726
.38	.0050 712	.0035 092	.0024 180	.0011 348	.0005 253	.0002 402
.39	.0063 089	.0044 213	.0030 854	.0014 854	.0007 053	.0003 309
.40	.0077 930	.0055 291	.0039 064	.0019 278	.0009 385	.0004 514
.41	.0095 608	.0068 652	.0049 091	.0024 818	.0012 377	.0006 100
.42	.0116 532	.0084 661	.0061 252	.0031 702	.0016 189	.0008 169
.43	.0141 149	.0103 720	.0075 903	.0040 196	.0021 004	.0010 847
.44	.0169 939	.0126 270	.0093 442	.0050 603	.0027 042	.0014 283
.45	.0203 418	.0152 794	.0114 305	.0063 268	.0034 559	.0018 659
.46	.0242 138	.0183 811	.0138 976	.0078 579	.0043 851	.0024 190
.47	.0286 678	.0219 881	.0167 979	.0096 975	.0055 260	.0031 130
.48	.0337 648	.0261 599	.0201 881	.0118 941	.0069 176	.0039 777
.49	.0395 680	.0309 594	.0241 294	.0145 013	.0086 041	.0050 477
.50	.0461 426	.0364 526	.0286 865	.0175 781	.0106 354	.0063 629
.51	.0535 551	.0427 082	.0339 282	.0211 886	.0130 670	.0079 692
.52	.0618 728	.0497 968	.0399 264	.0254 018	.0159 608	.0099 186
.53	.0711 627	.0577 903	.0467 556	.0302 919	.0193 847	.0122 698
.54	.0814 908	.0667 614	.0544 926	.0359 375	.0234 128	.0150 886
.55	.0929 213	.0767 824	.0632 154	.0424 213	.0281 253	.0184 480
.56	.1055 154	.0879 241	.0730 024	.0498 295	.0336 084	.0224 284
.57	.1193 300	.1002 553	.0839 312	.0582 509	.0399 538	.0271 176
.58	.1344 168	.1138 406	.0960 776	.0677 763	.0472 578	.0326 108
.59	.1508 209	.1287 401	.1095 141	.0784 965	.0556 210	.0390 098
.60	.1685 797	.1450 070	.1243 088	.0905 019	.0651 467	.0464 229
.61	.1877 210	.1626 871	.1405 231	.1038 804	.0759 403	.0549 637
.62	.2082 624	.1818 164	.1582 109	.1187 157	.0881 069	.0647 500
.63	.2302 093	.2024 201	.1774 161	.1350 855	.1017 502	.0759 024
.64	.2535 542	.2245 107	.1981 713	.1530 594	.1169 699	.0885 424
.65	.2782 749	.2480 866	.2204 957	.1726 965	.1338 596	.1027 902
.66	.3043 339	.2731 307	.2443 933	.1940 432	.1525 044	.1187 620
.67	.3316 770	.2996 086	.2698 512	.2171 306	.1729 772	.1365 675
.68	.3602 327	.3274 678	.2968 377	.2419 722	.1953 366	.1563 061
.69	.3899 116	.3566 364	.3253 010	.2685 612	.2196 230	.1780 633
.70	.4206 056	.3870 219	.3551 674	.2968 679	.2458 559	.2019 070

TABLE I. THE  $I_a(p, q)$  FUNCTION $x = .71$  to  $1.00$  $q = 4$  $p = 10$  to  $14$ 

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$H(p, q) = .34965035 \times 10^5$		$.29445595 \times 10^5$	$.24975925 \times 10^5$	$.18315018 \times 10^5$	$.13736204 \times 10^5$	$.10504202 \times 10^5$
$x$						
.71	.4521885	.4185114	.3863496	.3268381	.2749297	.2278832
.72	.4845154	.4509704	.4187006	.3583004	.3041112	.2560115
.73	.5174238	.4842435	.4521044	.3914146	.3360362	.2862807
.74	.5509734	.5181546	.4863204	.4257705	.3697063	.3186448
.75	.5842527	.5525082	.5214390	.4612860	.4049871	.3530181
.76	.6172609	.5860005	.5557675	.4972616	.4417059	.3892723
.77	.6510664	.6216716	.5924277	.5349617	.4796507	.4272329
.78	.6856137	.6569054	.6286664	.5724625	.5185700	.4666770
.79	.7199051	.6920847	.6644141	.6104628	.5581734	.5073320
.80	.7542174	.7272969	.6998180	.6498162	.5981344	.5488762
.81	.7777405	.7519915	.7247057	.6755492	.6256029	.5769940
.82	.8006145	.7757770	.7474820	.7001804	.6497615	.6014104
.83	.8142674	.7893661	.7606041	.7137049	.6637445	.6149362
.84	.8286111	.8035548	.7743820	.7279817	.6774082	.6289370
.85	.8436974	.8184608	.7888024	.7429552	.6918007	.6436140
.86	.8594282	.8339401	.8037679	.7573537	.7056456	.6568639
.87	.8758164	.8499942	.8193641	.7729049	.7206507	.6718053
.88	.8928699	.8666964	.8356344	.7894149	.7366476	.6867478
.89	.9105968	.8839684	.8524800	.8085702	.7552365	.7043145
.90	.9289983	.9019150	.8699671	.8274444	.7735938	.7217394
.91	.9479780	.9194408	.8868520	.8440008	.7895441	.7360512
.92	.9675347	.9375209	.9044448	.8612064	.8062844	.7522056
.93	.9876724	.9561720	.9225010	.8790674	.8235944	.7680600
.94	.10040270	.9719240	.9376286	.8940640	.8380617	.7818560
.95	.10286010	.9986450	.9636268	.9194542	.8629061	.8051104
.96	.10508647	.10204074	.9851450	.9412555	.8841846	.8259044
.97	.10709442	.10398545	.10081644	.9601524	.9025967	.8438914
.98	.10889010	.10572844	.10248646	.9769840	.9189750	.8596010
.99	.11048994	.10730271	.10399008	.9929975	.9353845	.8756786
1.00	1.119999999	1.086666667	1.053333333	1.020000000	1.000000000	1.000000000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .22$  to  $.80$  $q = 4$  $p = 15$  to  $20$ 

	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$
$B(p, q) = .8169\ 9346 \times 10^6$	$.6449\ 9484 \times 10^6$	$.5159\ 9587 \times 10^6$	$.4177\ 1004 \times 10^6$	$.3417\ 6350 \times 10^6$	$.2823\ 2702 \times 10^6$	$.2323\ 2702 \times 10^6$
$x$						
.22	.0000 001					
.23	.0000 001					
.24	.0000 002	.0000 001				
.25	.0000 003	.0000 001				
.26	.0000 006	.0000 002	.0000 001			
.27	.0000 010	.0000 003	.0000 001			
.28	.0000 017	.0000 006	.0000 002	.0000 001		
.29	.0000 027	.0000 009	.0000 003	.0000 001		
.30	.0000 041	.0000 015	.0000 005	.0000 002	.0000 001	
.31	.0000 068	.0000 025	.0000 009	.0000 003	.0000 001	
.32	.0000 106	.0000 040	.0000 015	.0000 005	.0000 002	.0000 001
.33	.0000 161	.0000 063	.0000 024	.0000 009	.0000 003	.0000 001
.34	.0000 243	.0000 097	.0000 039	.0000 015	.0000 005	.0000 002
.35	.0000 360	.0000 149	.0000 061	.0000 025	.0000 009	.0000 003
.36	.0000 526	.0000 224	.0000 084	.0000 039	.0000 015	.0000 005
.37	.0000 761	.0000 332	.0000 124	.0000 062	.0000 025	.0000 009
.38	.0001 087	.0000 487	.0000 216	.0000 105	.0000 039	.0000 015
.39	.0001 530	.0000 706	.0000 322	.0000 151	.0000 062	.0000 025
.40	.0002 138	.0001 013	.0000 473	.0000 220	.0000 105	.0000 039
.41	.0002 975	.0001 437	.0000 688	.0000 327	.0000 151	.0000 062
.42	.0004 079	.0002 018	.0000 990	.0000 482	.0000 220	.0000 105
.43	.0005 543	.0002 866	.0001 399	.0000 702	.0000 327	.0000 151
.44	.0007 466	.0003 866	.0001 986	.0001 012	.0000 482	.0000 220
.45	.0009 971	.0005 279	.0002 772	.0001 445	.0000 702	.0000 327
.46	.0013 268	.0007 145	.0003 834	.0002 132	.0001 012	.0000 482
.47	.0017 358	.0009 591	.0005 256	.0003 253	.0001 445	.0000 702
.48	.0022 641	.0012 771	.0007 145	.0004 968	.0002 132	.0001 012
.49	.0029 316	.0016 873	.0009 633	.0007 459	.0003 253	.0001 445
.50	.0037 689	.0022 125	.0012 883	.0010 448	.0004 968	.0002 132
.51	.0048 122	.0028 801	.0017 100	.0013 929	.0007 459	.0003 253
.52	.0061 934	.0037 227	.0022 526	.0019 533	.0010 448	.0004 968
.53	.0076 909	.0047 787	.0029 459	.0026 180	.0013 929	.0007 459
.54	.0096 304	.0060 935	.0038 254	.0034 843	.0019 533	.0009 633
.55	.0119 850	.0077 104	.0049 333	.0045 102	.0026 180	.0012 883
.56	.0148 260	.0097 172	.0064 198	.0059 815	.0034 843	.0017 100
.57	.0182 332	.0121 562	.0086 329	.0082 815	.0045 102	.0022 526
.58	.0222 051	.0151 154	.0101 701	.0106 976	.0059 815	.0029 459
.59	.0271 001	.0186 814	.0127 802	.0136 226	.0082 815	.0037 227
.60	.0327 813	.0229 593	.0159 612	.0171 213	.0106 976	.0048 122
.61	.0394 261	.0280 525	.0198 138	.0219 011	.0136 226	.0061 934
.62	.0471 657	.0340 829	.0244 507	.0274 249	.0171 213	.0076 909
.63	.0561 289	.0411 802	.0299 986	.0337 677	.0219 011	.0096 304
.64	.0664 502	.0494 835	.0365 887	.0408 797	.0274 249	.0119 850
.65	.0782 675	.0591 398	.0443 756	.0494 834	.0337 677	.0148 260
.66	.0917 200	.0703 027	.0535 169	.0594 832	.0408 797	.0177 104
.67	.1069 457	.0831 298	.0641 812	.0709 493	.0494 834	.0219 011
.68	.1240 781	.0977 800	.0765 443	.0843 598	.0594 832	.0269 459
.69	.1432 421	.1144 102	.0907 861	.0996 089	.0709 493	.0327 813
.70	.1645 505	.1331 710	.1070 868	.0116 037	.0843 598	.0394 261
.71	.1880 983	.1542 017	.1256 225	.1017 505	.0996 089	.0471 657
.72	.2139 586	.1776 249	.1465 596	.1203 434	.1144 102	.0561 289
.73	.2421 761	.2035 405	.1700 482	.1412 813	.1299 986	.0664 502
.74	.2727 623	.2320 187	.1962 153	.1650 433	.1494 835	.0782 675
.75	.3056 892	.2630 931	.2251 560	.1916 821	.1703 027	.0917 200
.76	.3408 836	.2967 531	.2569 253	.2213 146	.1962 153	.1069 457
.77	.3782 223	.3329 364	.2915 282	.2540 172	.2251 560	.1240 781
.78	.4175 260	.3715 225	.3289 108	.2898 084	.2569 253	.1432 421
.79	.4585 606	.4123 253	.3689 508	.3286 381	.2915 282	.1645 505
.80	.5010 255	.4550 887	.4114 489	.3793 760	.3329 364	.1880 983



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .33$  to  $.90$  $q = 4$  $p = 21$  to  $26$ 

	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$B(p, q) = .23527197 \times \frac{1}{10^6}$	$.19762840 \times \frac{1}{10^6}$	$.16722408 \times \frac{1}{10^6}$	$.14242011 \times \frac{1}{10^6}$	$.12210012 \times \frac{1}{10^6}$	$.10322573 \times \frac{1}{10^6}$	
$x$						
.33	.0000 001					
.34	.0000 001					
.35	.0000 002	.0000 001				
.36	.0000 003	.0000 001				
.37	.0000 005	.0000 002	.0000 001			
.38	.0000 008	.0000 003	.0000 001	.0000 001		
.39	.0000 013	.0000 006	.0000 003	.0000 001		
.40	.0000 021	.0000 010	.0000 003	.0000 002	.0000 001	
.41	.0000 034	.0000 016	.0000 007	.0000 003	.0000 002	.0000 001
.42	.0000 053	.0000 025 <sup>+</sup>	.0000 012	.0000 006	.0000 004	.0000 002
.43	.0000 084	.0000 041	.0000 020	.0000 009	.0000 005	.0000 003
.44	.0000 129	.0000 064	.0000 032	.0000 016	.0000 008	.0000 004
.45	.0000 197	.0000 100	.0000 051	.0000 026	.0000 013	.0000 006
.46	.0000 297	.0000 154	.0000 080	.0000 041	.0000 022	.0000 011
.47	.0000 433	.0000 235 <sup>+</sup>	.0000 124	.0000 065	.0000 033	.0000 018
.48	.0000 654	.0000 355	.0000 192	.0000 103	.0000 052	.0000 027
.49	.0000 957	.0000 530	.0000 292	.0000 150	.0000 080	.0000 043
.50	.0001 386	.0000 783	.0000 440	.0000 236	.0000 117	.0000 070
.51	.0001 988	.0001 145 <sup>+</sup>	.0000 656	.0000 375	.0000 211	.0000 121
.52	.0002 827	.0001 650	.0000 970	.0000 564	.0000 327	.0000 180
.53	.0003 984	.0002 383	.0001 419	.0000 841	.0000 507	.0000 292
.54	.0005 567	.0003 394	.0002 057	.0001 242	.0000 747	.0000 438
.55	.0007 714	.0004 785 <sup>+</sup>	.0002 935	.0001 817	.0001 111	.0000 670
.56	.0010 603	.0006 605 <sup>+</sup>	.0004 208	.0002 633	.0001 642	.0001 020
.57	.0014 460	.0009 290	.0005 942	.0003 783	.0002 401	.0001 810
.58	.0019 567	.0012 788	.0008 319	.0005 790	.0003 128	.0002 247
.59	.0026 280	.0017 464	.0011 554	.0007 612	.0004 906	.0002 767
.60	.0035 034	.0023 668	.0015 918	.0010 661	.0007 111	.0003 722
.61	.0046 368	.0031 833	.0021 758	.0014 811	.0010 033	.0005 207
.62	.0060 932	.0042 500 <sup>+</sup>	.0029 514	.0020 412	.0014 063	.0007 656
.63	.0079 512	.0056 329	.0039 732	.0027 912	.0019 433	.0010 621
.64	.0103 042	.0074 122	.0053 080	.0037 872	.0026 916	.0014 063
.65	.0132 629	.0096 847	.0070 417	.0050 697	.0036 796	.0020 458
.66	.0169 565	.0125 056	.0092 725	.0068 155	.0049 922	.0026 422
.67	.0215 344	.0161 908	.0121 223	.0090 309	.0067 183	.0034 253
.68	.0271 679	.0207 188	.0157 355	.0119 093	.0086 031	.0045 428
.69	.0340 503	.0263 326	.0202 811	.0155 602	.0118 922	.0057 662
.70	.0423 975 <sup>+</sup>	.0334 405 <sup>+</sup>	.0259 565 <sup>+</sup>	.0200 927	.0156 516	.0104 250
.71	.0524 470	.0416 767	.0329 820	.0260 126	.0203 318	.0150 122
.72	.0644 557	.0519 005	.0416 280	.0332 671	.0261 933	.0200 137
.73	.0786 971	.0641 945	.0521 630	.0422 361	.0340 829	.0264 168
.74	.0954 552	.0788 605 <sup>+</sup>	.0649 059	.0534 325	.0435 143	.0354 802
.75	.1150 184	.0962 141	.0801 877	.0666 001	.0551 350	.0455 044
.76	.1376 690	.1165 759	.0983 591	.0827 085	.0691 222	.0579 122
.77	.1636 721	.1402 607	.1197 758	.1019 450	.0864 960	.0731 763
.78	.1932 608	.1675 635	.1447 863	.1247 028	.1070 705	.0926 613
.79	.2266 190	.1987 413	.1717 147	.1513 630	.1313 015 <sup>+</sup>	.1132 263
.80	.2638 622	.2339 933	.2068 397	.1822 833	.1601 827	.1403 805
.81	.3050 165	.2734 366	.2443 688	.2177 527	.1941 959	.1734 950
.82	.3490 954	.3170 806	.2864 100	.2579 796	.2301 126	.2076 158
.83	.3985 783	.3647 994	.3320 397	.3030 470	.2751 337	.2491 873
.84	.4503 891	.4163 047	.3837 696	.3528 754	.3246 803	.2962 337
.85	.5048 790	.4711 213	.4385 144	.4071 833	.3772 232	.3487 081
.86	.5613 157	.5285 673	.4965 632	.4654 496	.4353 422	.4061 326
.87	.6187 813	.5877 437	.5570 588	.5268 800	.4971 412	.4685 612
.88	.6761 835 <sup>+</sup>	.6475 370	.6188 905 <sup>+</sup>	.5903 935 <sup>+</sup>	.5621 813	.5341 256
.89	.7322 824	.7066 393	.6807 047	.6546 123	.6284 873	.6024 458
.90	.7857 378	.7635 914	.7409 416	.7178 980	.6945 663	.6710 480

TABLE I. THE  $I_x(p, q)$  FUNCTION

109

 $x = .91$  to  $1.00$  $q = 4$  $p = 21$  to  $26$ 

	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$B(p, q) = .2352\ 7197 \times \frac{x}{10^4}$	$.1976\ 2846 \times \frac{x}{10^4}$	$.1672\ 2408 \times \frac{x}{10^4}$	$.1424\ 5014 \times \frac{x}{10^4}$	$.1221\ 0012 \times \frac{x}{10^4}$	$.1052\ 5873 \times \frac{x}{10^4}$	
.91	.8351 790	.8168 538	.7979 038	.7784 100	.7584 533	.7381 134
.92	.8792 996	.8649 078	.8498 618	.8342 140	.8180 185 <sup>+</sup>	.8013 307
.93	.9169 745 <sup>+</sup>	.9063 882	.8952 004	.8834 386	.8711 328	.8583 152
.94	.9473 964	.9402 432	.9326 023	.9244 830	.9158 968	.9068 573
.95	.9702 175 <sup>+</sup>	.9659 094	.9612 586	.9562 641	.9509 261	.9452 466
.96	.9856 770	.9834 784	.9810 800	.9784 771	.9756 660	.9726 435 <sup>+</sup>
.97	.9946 790	.9938 142	.9928 610	.9918 158	.9906 752	.9894 360
.98	.9987 660	.9985 541	.9983 182	.9980 568	.9977 686	.9974 523
.99	.9999 095 <sup>-</sup>	.9998 931	.9998 746	.9998 540	.9998 310	.9998 055 <sup>+</sup>
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .42$  to  $1.00$  $q = 4$  $p = 27$  to  $32$ 

	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$
$B(p, q) = .9122\ 4229 \times \frac{1}{10^8}$	$.7945\ 3361 \times \frac{1}{10^8}$	$.6952\ 1691 \times \frac{1}{10^8}$	$.6109\ 4819 \times \frac{1}{10^8}$	$.5390\ 7193 \times \frac{1}{10^8}$	$.4774\ 6371 \times \frac{1}{10^8}$	
$x$						
.42	.0000 001					
.43	.0000 001					
.44	.0000 002	.0000 001				
.45	.0000 003	.0000 002	.0000 001			
.46	.0000 006	.0000 003	.0000 001	.0000 001		
.47	.0000 009	.0000 005 <sup>-</sup>	.0000 003	.0000 001	.0000 001	
.48	.0000 016	.0000 008	.0000 004	.0000 002	.0000 001	.0000 001
.49	.0000 026	.0000 014	.0000 008	.0000 004	.0000 002	.0000 001
.50	.0000 042	.0000 023	.0000 013	.0000 007	.0000 004	.0000 002
.51	.0000 068	.0000 038	.0000 021	.0000 012	.0000 007	.0000 004
.52	.0000 109	.0000 062	.0000 036	.0000 020	.0000 012	.0000 007
.53	.0000 171	.0000 100	.0000 058	.0000 034	.0000 020	.0000 011
.54	.0000 267	.0000 159	.0000 094	.0000 056	.0000 033	.0000 019
.55	.0000 413	.0000 250 <sup>+</sup>	.0000 151	.0000 091	.0000 055 <sup>-</sup>	.0000 033
.56	.0000 631	.0000 390	.0000 240	.0000 147	.0000 090	.0000 055 <sup>-</sup>
.57	.0000 956	.0000 600	.0000 376	.0000 235 <sup>-</sup>	.0000 146	.0000 091
.58	.0001 433	.0000 915 <sup>+</sup>	.0000 583	.0000 370	.0000 235 <sup>-</sup>	.0000 148
.59	.0002 129	.0001 383	.0000 896	.0000 579	.0000 373	.0000 240
.60	.0003 133	.0002 070	.0001 363	.0000 895 <sup>-</sup>	.0000 586	.0000 383
.61	.0004 570	.0003 068	.0002 054	.0001 371	.0000 913	.0000 606
.62	.0006 607	.0004 507	.0003 066	.0002 079	.0001 407	.0000 949
.63	.0009 470	.0006 562	.0004 534	.0003 124	.0002 147	.0001 472
.64	.0013 457	.0009 470	.0006 646	.0004 651	.0003 246	.0002 261
.65	.0018 963	.0013 549	.0009 654	.0006 860	.0004 862	.0003 438
.66	.0026 500 <sup>-</sup>	.0019 220	.0013 900	.0010 026	.0007 213	.0005 177
.67	.0036 730	.0027 033	.0019 841	.0014 523	.0010 604	.0007 725 <sup>-</sup>
.68	.0050 496	.0037 707	.0028 078	.0020 853	.0015 448	.0011 418
.69	.0068 865 <sup>+</sup>	.0052 158	.0039 395 <sup>+</sup>	.0029 678	.0022 302	.0016 721
.70	.0093 166	.0071 556	.0054 808	.0041 871	.0031 910	.0024 263
.71	.0125 037	.0097 362	.0075 607	.0058 563	.0045 252	.0034 887
.72	.0166 478	.0131 392	.0103 424	.0081 203	.0063 604	.0049 707
.73	.0219 891	.0175 866	.0140 284	.0111 623	.0088 607	.0070 180
.74	.0288 121	.0233 460	.0188 678	.0152 110	.0122 344	.0098 186
.75	.0374 493	.0307 360	.0251 615 <sup>-</sup>	.0205 481	.0167 421	.0136 113
.76	.0482 818	.0401 284	.0332 679	.0275 145 <sup>+</sup>	.0227 047	.0186 955 <sup>+</sup>
.77	.0617 381	.0519 502	.0436 061	.0365 165 <sup>-</sup>	.0305 115 <sup>+</sup>	.0254 403
.78	.0782 895 <sup>-</sup>	.0666 811	.0566 565 <sup>-</sup>	.0480 284	.0406 254	.0342 924
.79	.0984 400	.0848 464	.0729 569	.0625 925 <sup>-</sup>	.0535 858	.0457 820
.80	.1227 108	.1070 044	.0930 931	.0808 127	.0700 060	.0605 240
.81	.1516 168	.1337 258	.1176 813	.1033 409	.0905 636	.0792 124
.82	.1856 356	.1655 636	.1473 411	.1308 529	.1159 805 <sup>+</sup>	.1026 050 <sup>+</sup>
.83	.2251 662	.2030 133	.1826 564	.1640 123	.1469 902	.1314 946
.84	.2704 796	.2464 610	.2241 238	.2034 195 <sup>-</sup>	.1842 887	.1666 637
.85	.3216 599	.2961 210	.2720 871	.2495 449	.2284 680	.2088 189
.86	.3785 401	.3519 637	.3266 591	.3026 460	.2799 295 <sup>+</sup>	.2585 028
.87	.4406 355 <sup>+</sup>	.4136 383	.3876 342	.3626 702	.3387 797	.3159 836
.88	.5070 831	.4803 971	.4543 974	.4291 507	.4047 119	.3811 246
.89	.5765 947	.5510 308	.5258 413	.5011 034	.4768 850 <sup>-</sup>	.4532 447
.90	.6474 392	.6238 304	.6003 059	.5769 437	.5538 150 <sup>+</sup>	.5309 849
.91	.7174 684	.6965 940	.6755 631	.6544 451	.6333 060	.6122 079
.92	.7842 064	.7667 016	.7488 717	.7307 713	.7124 537	.6939 706
.93	.8450 193	.8312 802	.8171 338	.8026 168	.7877 658	.7726 178
.94	.8973 797	.8874 809	.8771 791	.8664 936	.8554 448	.8440 538
.95	.9392 284	.9328 760	.9261 945 <sup>+</sup>	.9191 905 <sup>+</sup>	.9118 713	.9042 452
.96	.9694 071	.9659 550 <sup>-</sup>	.9622 858	.9583 991	.9542 947	.9499 731
.97	.9880 954	.9866 504	.9850 987	.9834 378	.9816 656	.9797 802
.98	.9971 065 <sup>+</sup>	.9967 300	.9963 215 <sup>+</sup>	.9958 798	.9954 036	.9948 918
.99	.9997 774	.9997 465 <sup>-</sup>	.9997 125 <sup>+</sup>	.9996 755 <sup>-</sup>	.9996 351	.9995 913
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION

III

 $x = .49$  to  $1.00$  $q = 4$  $p = 33$  to  $38$ 

	$p = 33$	$p = 34$	$p = 35$	$p = 36$	$p = 37$	$p = 38$
$B(p, q) =$	$.4244\ 1219 \times \frac{1}{10^8}$	$.3785\ 2979 \times \frac{1}{10^8}$	$.3386\ 8455 \times \frac{1}{10^8}$	$.3039\ 4767 \times \frac{1}{10^8}$	$.2735\ 5291 \times \frac{1}{10^8}$	$.2468\ 6482 \times \frac{1}{10^8}$
$x$						
.49	.0000 001					
.50	.0000 001	.0000 001				
.51	.0000 002	.0000 001	.0000 001			
.52	.0000 004	.0000 002	.0000 001	.0000 001		
.53	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001	
.54	.0000 011	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
.55	.0000 020	.0000 012	.0000 007	.0000 004	.0000 002	.0000 001
.56	.0000 033	.0000 020	.0000 012	.0000 007	.0000 004	.0000 003
.57	.0000 056	.0000 035	.0000 021	.0000 013	.0000 008	.0000 005
.58	.0000 093	.0000 059	.0000 037	.0000 023	.0000 014	.0000 009
.59	.0000 154	.0000 098	.0000 063	.0000 040	.0000 025 <sup>+</sup>	.0000 016
.60	.0000 250	.0000 162	.0000 105 <sup>+</sup>	.0000 068	.0000 044	.0000 028
.61	.0000 402	.0000 266	.0000 175 <sup>+</sup>	.0000 115 <sup>+</sup>	.0000 076	.0000 050
.62	.0000 639	.0000 429	.0000 288	.0000 193	.0000 129	.0000 086
.63	.0001 007	.0000 687	.0000 468	.0000 318	.0000 216	.0000 146
.64	.0001 571	.0001 089	.0000 753	.0000 520	.0000 358	.0000 247
.65	.0002 425 <sup>+</sup>	.0001 707	.0001 199	.0000 841	.0000 588	.0000 411
.66	.0003 708	.0002 650	.0001 890	.0001 345	.0000 955 <sup>+</sup>	.0000 678
.67	.0005 614	.0004 072	.0002 947	.0002 129	.0001 535 <sup>+</sup>	.0001 105 <sup>+</sup>
.68	.0008 420	.0006 196	.0004 550 <sup>+</sup>	.0003 336	.0002 441	.0001 783
.69	.0012 508	.0009 338	.0006 957	.0005 173	.0003 840	.0002 845 <sup>+</sup>
.70	.0018 408	.0013 937	.0010 531	.0007 942	.0005 979	.0004 494
.71	.0026 837	.0020 603	.0015 785 <sup>+</sup>	.0012 072	.0009 216	.0007 023
.72	.0038 763	.0030 167	.0023 431	.0018 166	.0014 059	.0010 863
.73	.0055 467	.0043 751	.0034 443	.0027 066	.0021 231	.0016 627
.74	.0078 633	.0062 848	.0050 137	.0039 924	.0031 737	.0025 187
.75	.0110 431	.0089 418	.0072 268	.0058 303	.0046 957	.0037 757
.76	.0153 629	.0125 998	.0103 146	.0084 290	.0068 765	.0056 009
.77	.0211 693	.0175 817	.0145 756	.0120 624	.0099 660	.0082 208
.78	.0288 895	.0242 921	.0203 897	.0170 850	.0142 924	.0119 377
.79	.0390 389	.0332 277	.0282 317	.0239 466	.0202 793	.0171 472
.80	.0522 272	.0449 864	.0386 826	.0332 073	.0284 621	.0243 581
.81	.0691 560	.0602 697	.0524 368	.0455 482	.0395 035 <sup>+</sup>	.0342 104
.82	.0906 088	.0798 777	.0703 018	.0617 764	.0542 031	.0474 894
.83	.1174 275 <sup>+</sup>	.1046 904	.0931 858	.0828 185	.0734 966	.0651 320
.84	.1504 707	.1356 320	.1220 678	.1096 971	.0984 399	.0882 171
.85	.1905 513	.1736 123	.1579 437	.1434 839	.1301 688	.1179 332
.86	.2383 483	.2194 397	.2017 435 <sup>+</sup>	.1852 203	.1698 262	.1555 138
.87	.2942 916	.2737 039	.2542 123	.2358 010	.2184 484	.2021 275 <sup>+</sup>
.88	.3584 217	.3366 270	.3157 553	.2958 140	.2768 032	.2587 173
.89	.4302 324	.4078 895 <sup>+</sup>	.3862 498	.3653 396	.3451 787	.3257 807
.90	.5085 114	.4864 466	.4648 360	.4437 194	.4231 307	.4030 984
.91	.5912 086	.5703 621	.5497 178	.5293 214	.5092 138	.4894 323
.92	.6753 720	.6567 058	.6380 176	.6193 508	.6007 462	.5822 422
.93	.7572 094	.7415 770	.7257 560	.7097 813	.6936 868	.6775 953
.94	.8323 425 <sup>+</sup>	.8203 331	.8080 481	.7955 105	.7827 430	.7697 684
.95	.8963 212	.8881 090	.8796 191	.8708 624	.8618 502	.8525 945 <sup>+</sup>
.96	.9454 355	.9406 833	.9357 188	.9305 442	.9251 627	.9195 776
.97	.9777 799	.9756 632	.9734 289	.9710 759	.9686 032	.9660 102
.98	.9943 432	.9937 567	.9931 312	.9924 656	.9917 591	.9910 105
.99	.9995 438	.9994 926	.9994 374	.9993 780	.9993 144	.9992 463
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

$x = .55$  to  $1.00$  $q = 4$  $p = 1.00$  to  $.44$ 

	$p = .39$	$p = .40$	$p = .41$	$p = .42$	$p = .43$	$p = .44$
$H(p, q) = .2233\ 5388 \times 10^4$	$.2025\ 7678 \times 10^4$	$.1841\ 6071 \times 10^4$	$.1677\ 3208 \times 10^4$	$.1532\ 0117 \times 10^4$	$.1401\ 6413 \times 10^4$	$.1281\ 2413 \times 10^4$
$x$						
.55	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001
.56	.0000 002	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001
.57	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001	.0000 001
.58	.0000 006	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.59	.0000 010	.0000 006	.0000 003	.0000 001	.0000 001	.0000 001
.60	.0000 018	.0000 012	.0000 008	.0000 005	.0000 003	.0000 002
.61	.0000 033	.0000 021	.0000 014	.0000 009	.0000 005	.0000 003
.62	.0000 057	.0000 038	.0000 025	.0000 016	.0000 010	.0000 006
.63	.0000 089	.0000 060	.0000 040	.0000 026	.0000 016	.0000 010
.64	.0000 169	.0000 116	.0000 078	.0000 050	.0000 031	.0000 019
.65	.0000 287	.0000 200	.0000 132	.0000 085	.0000 052	.0000 032
.66	.0000 480	.0000 339	.0000 229	.0000 149	.0000 093	.0000 057
.67	.0000 794	.0000 570	.0000 398	.0000 268	.0000 166	.0000 103
.68	.0001 300	.0000 946	.0000 658	.0000 440	.0000 280	.0000 173
.69	.0002 105	.0001 555	.0001 137	.0000 768	.0000 503	.0000 324
.70	.0003 375	.0002 526	.0001 890	.0001 317	.0000 875	.0000 569
.71	.0005 344	.0004 064	.0002 958	.0002 133	.0001 475	.0001 035
.72	.0008 380	.0006 454	.0004 974	.0003 542	.0002 524	.0001 740
.73	.0013 001	.0010 100	.0007 913	.0005 924	.0004 103	.0002 850
.74	.0019 957	.0015 790	.0012 477	.0009 690	.0006 755	.0004 750
.75	.0030 312	.0024 591	.0020 492	.0015 598	.0011 250	.0007 950
.76	.0045 549	.0036 888	.0030 894	.0023 994	.0017 598	.0012 950
.77	.0067 710	.0054 688	.0045 974	.0037 542	.0028 524	.0021 740
.78	.0099 560	.0082 913	.0070 894	.0057 542	.0044 103	.0034 750
.79	.0144 775	.0122 062	.0102 274	.0082 994	.0064 755	.0050 950
.80	.0208 157	.0177 638	.0151 974	.0123 994	.0097 598	.0076 950
.81	.0295 844	.0255 491	.0220 994	.0183 994	.0147 598	.0116 950
.82	.0415 496	.0361 041	.0316 994	.0270 994	.0224 598	.0183 950
.83	.0576 414	.0509 459	.0453 994	.0400 994	.0347 598	.0300 950
.84	.0789 520	.0705 706	.0639 994	.0576 994	.0513 598	.0459 950
.85	.1067 120	.0964 402	.0889 994	.0816 994	.0743 598	.0679 950
.86	.1422 334	.1299 347	.1189 994	.1096 994	.1013 598	.0939 950
.87	.1868 074	.1724 536	.1599 994	.1496 994	.1403 598	.1329 950
.88	.2415 454	.2252 711	.2109 994	.2006 994	.1913 598	.1839 950
.89	.3071 534	.2899 100	.2756 994	.2643 994	.2550 598	.2476 950
.90	.3836 460	.3647 922	.3499 994	.3386 994	.3293 598	.3219 950
.91	.4700 101	.4500 762	.4349 994	.4236 994	.4143 598	.4069 950
.92	.5638 745	.5436 764	.5286 994	.5173 994	.5080 598	.5006 950
.93	.6612 685	.6409 067	.6259 994	.6146 994	.6053 598	.5979 950
.94	.7566 005	.7432 886	.7289 994	.7176 994	.7083 598	.7009 950
.95	.8431 074	.8334 013	.8219 994	.8106 994	.8013 598	.7939 950
.96	.9137 926	.9078 118	.8969 994	.8856 994	.8763 598	.8689 950
.97	.9632 965	.9604 617	.9575 047	.9546 994	.9513 598	.9489 950
.98	.9962 190	.9963 836	.9989 047	.9999 994	.9999 598	.9999 950
.99	.9991 735	.9999 950	.9999 994	.9999 994	.9999 598	.9999 950
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .59$  to  $1.00$  $q = 4$  $p = 45$  to  $50$ 

	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$B(p, q) =$	$.1284\ 8186 \times \frac{1}{10^5}$	$.1179\ 9354 \times \frac{1}{10^5}$	$.1085\ 5406 \times \frac{1}{10^5}$	$.1000\ 4002 \times \frac{1}{10^5}$	$.9234\ 4630 \times \frac{1}{10^5}$	$.8537\ 5224 \times \frac{1}{10^5}$
$x$						
.59	.0000 001					
.60	.0000 001	.0000 001	.0000 001			
.61	.0000 002	.0000 002	.0000 001	.0000 001		
.62	.0000 005-	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.63	.0000 009	.0000 006	.0000 004	.0000 003	.0000 002	.0000 001
.64	.0000 017	.0000 012	.0000 008	.0000 005+	.0000 004	.0000 002
.65	.0000 032	.0000 022	.0000 015+	.0000 010	.0000 007	.0000 005-
.66	.0000 059	.0000 041	.0000 029	.0000 020	.0000 014	.0000 010
.67	.0000 106	.0000 075+	.0000 054	.0000 038	.0000 027	.0000 019
.68	.0000 190	.0000 137	.0000 099	.0000 071	.0000 051	.0000 037
.69	.0000 335-	.0000 245+	.0000 179	.0000 131	.0000 096	.0000 070
.70	.0000 584	.0000 434	.0000 322	.0000 239	.0000 177	.0000 131
.71	.0001 006	.0000 758	.0000 571	.0000 429	.0000 322	.0000 242
.72	.0001 713	.0001 309	.0000 999	.0000 762	.0000 580	.0000 441
.73	.0002 884	.0002 234	.0001 728	.0001 336	.0001 031	.0000 795+
.74	.0004 796	.0003 765-	.0002 952	.0002 312	.0001 809	.0001 414
.75	.0007 882	.0006 270	.0004 982	.0003 954	.0003 135+	.0002 483
.76	.0012 802	.0010 316	.0008 304	.0006 678	.0005 364	.0004 304
.77	.0020 544	.0016 769	.0013 672	.0011 136	.0009 060	.0007 365-
.78	.0032 571	.0026 923	.0022 230	.0018 336	.0015 109	.0012 437
.79	.0051 008	.0042 689	.0035 689	.0029 806	.0024 868	.0020 728
.80	.0078 889	.0066 835+	.0056 564	.0047 822	.0040 391	.0034 083
.81	.0120 461	.0103 290	.0088 475+	.0075 709	.0064 722	.0055 277
.82	.0181 542	.0157 517	.0136 532	.0118 226	.0102 277	.0088 398
.83	.0269 915-	.0236 934	.0207 775+	.0182 029	.0159 323	.0139 324
.84	.0395 707	.0351 346	.0311 654	.0276 184	.0244 527	.0216 307
.85	.0571 679	.0513 313	.0460 466	.0412 679	.0369 521	.0330 591
.86	.0813 289	.0738 319	.0669 640	.0606 806	.0549 391	.0496 991
.87	.1138 336	.1044 561	.0957 657	.0877 224	.0802 873	.0734 228
.88	.1565 937	.1452 072	.1345 336	.1245 412	.1151 984	.1064 734
.89	.2114 532	.1980 852	.1854 117	.1734 124	.1620 655+	.1513 485+
.90	.2798 622	.2647 663	.2502 939	.2364 374	.2231 871	.2105 317
.91	.3624 072	.3461 226	.3303 371	.3150 554	.3002 799	.2860 110
.92	.4582 129	.4415 880	.4252 957	.4093 500-	.3937 630	.3785 451
.93	.5642 901	.5484 386	.5327 353	.5171 990	.5018 473	.4866 960
.94	.6750 121	.6611 668	.6473 034	.6334 400	.6195 939	.6057 817
.95	.7820 355-	.7712 860	.7604 080	.7494 142	.7383 174	.7271 300
.96	.8751 352	.8680 820	.8608 692	.8535 030	.8459 895-	.8383 349
.97	.9444 736	.9409 160	.9372 401	.9334 468	.9295 374	.9255 131
.98	.9845 207	.9834 058	.9822 419	.9810 285+	.9797 651	.9784 511
.99	.9986 299	.9985 199	.9984 038	.9982 816	.9981 531	.9980 180
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 70

$q = 4.5$

$p = 7.5$  to 10

$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$= .5452\ 8223 \times \frac{1}{10^8}$	$.4284\ 0156 \times \frac{1}{10^8}$	$.3408\ 0140 \times \frac{1}{10^8}$	$.2741\ 7700 \times \frac{1}{10^8}$	$.2228\ 3168 \times \frac{1}{10^8}$	$.1827\ 8467 \times \frac{1}{10^8}$
.0000 001					
.0000 004	.0000 001				
.0000 011	.0000 004	.0000 001			
.0000 026	.0000 009	.0000 003	.0000 001		
.0000 056	.0000 021	.0000 008	.0000 003	.0000 001	
.0000 111	.0000 044	.0000 017	.0000 007	.0000 003	.0000 001
.0000 205 <sup>+</sup>	.0000 085 <sup>-</sup>	.0000 035 <sup>-</sup>	.0000 014	.0000 006	.0000 002
.0000 362	.0000 155 <sup>+</sup>	.0000 066	.0000 028	.0000 012	.0000 005 <sup>-</sup>
.0000 609	.0000 271	.0000 119	.0000 052	.0000 023	.0000 010
.0000 986	.0000 454	.0000 207	.0000 094	.0000 042	.0000 019
.0001 544	.0000 734	.0000 346	.0000 162	.0000 075 <sup>+</sup>	.0000 035 <sup>-</sup>
.0002 347	.0001 149	.0000 558	.0000 269	.0000 129	.0000 061
.0003 475 <sup>-</sup>	.0001 750 <sup>+</sup>	.0000 875 <sup>-</sup>	.0000 434	.0000 214	.0000 105 <sup>-</sup>
.0005 025 <sup>+</sup>	.0002 599	.0001 334	.0000 680	.0000 344	.0000 173
.0007 115 <sup>-</sup>	.0003 775 <sup>-</sup>	.0001 987	.0001 038	.0000 539	.0000 278
.0009 883	.0005 371	.0002 896	.0001 550 <sup>+</sup>	.0000 824	.0000 436
.0013 490	.0007 501	.0004 138	.0002 266	.0001 233	.0000 667
.0018 124	.0010 300	.0005 808	.0003 251	.0001 808	.0001 000
.0023 997	.0013 925 <sup>+</sup>	.0008 018	.0004 584	.0002 604	.0001 470
.0031 350 <sup>-</sup>	.0018 559	.0010 902	.0006 359	.0003 685 <sup>+</sup>	.0002 123
.0040 449	.0024 410	.0014 617	.0008 692	.0005 135 <sup>+</sup>	.0003 016
.0051 593	.0031 713	.0019 345 <sup>-</sup>	.0011 718	.0007 053	.0004 220
.0065 104	.0040 734	.0025 293	.0015 597	.0009 556	.0005 821
.0081 336	.0051 766	.0032 699	.0020 512	.0012 786	.0007 924
.0100 667	.0065 134	.0041 828	.0026 677	.0016 907	.0010 654
.0123 503	.0081 189	.0052 976	.0034 332	.0022 111	.0014 158
.0150 271	.0100 315 <sup>+</sup>	.0066 473	.0043 750 <sup>+</sup>	.0028 616	.0018 610
.0181 421	.0122 923	.0082 677	.0055 235 <sup>+</sup>	.0036 674	.0024 212
.0217 419	.0149 447	.0101 979	.0069 125 <sup>-</sup>	.0046 568	.0031 195 <sup>-</sup>
.0258 749	.0180 351	.0124 801	.0085 790	.0058 614	.0039 822
.0305 903	.0216 116	.0151 592	.0105 635 <sup>-</sup>	.0073 166	.0050 394
.0359 379	.0257 244	.0182 831	.0129 098	.0090 610	.0063 245 <sup>+</sup>
.0419 678	.0304 250 <sup>-</sup>	.0219 020	.0156 648	.0111 372	.0078 748
.0487 297	.0357 659	.0260 683	.0188 786	.0135 912	.0097 313
.0562 722	.0418 003	.0308 362	.0226 039	.0164 724	.0119 392
.0646 425 <sup>+</sup>	.0485 811	.0362 614	.0268 959	.0198 337	.0145 473
.0738 857	.0561 608	.0424 000	.0318 119	.0237 308	.0176 084
.0840 438	.0645 906	.0493 088	.0374 109	.0282 226	.0211 787
.0951 556	.0739 195 <sup>+</sup>	.0570 439	.0437 529	.0333 698	.0253 178
.1072 557	.0841 943	.0656 605 <sup>-</sup>	.0508 983	.0392 353	.0300 884
.1203 739	.0954 579	.0752 119	.0589 074	.0458 832	.0355 557
.1345 346	.1077 495 <sup>+</sup>	.0857 489	.0678 394	.0533 781	.0417 869
.1497 561	.1211 032	.0973 187	.0777 517	.0617 846	.0488 507
.1660 502	.1355 474	.1099 643	.0886 992	.0711 661	.0568 163
.1834 214	.1511 043	.1237 237	.1007 329	.0815 842	.0657 528
.2018 664	.1677 889	.1386 287	.1138 995 <sup>+</sup>	.0930 976	.0757 279
.2213 740	.1856 083	.1547 044	.1282 401	.1057 610	.0868 075 <sup>+</sup>
.2419 242	.2045 615 <sup>+</sup>	.1719 680	.1437 891	.1196 242	.0990 539
.2634 884	.2246 382	.1904 282	.1605 735 <sup>-</sup>	.1347 305 <sup>+</sup>	.1125 249
.2860 286	.2458 189	.2100 845 <sup>+</sup>	.1786 115 <sup>+</sup>	.1511 162	.1272 726
.3094 979	.2680 739	.2309 264	.1979 122	.1688 088	.1433 419
.3338 399	.2913 635 <sup>-</sup>	.2529 328	.2184 738	.1878 263	.1607 695 <sup>-</sup>
.3589 893	.3156 374	.2760 712	.2402 836	.2081 760	.1795 820
.3848 715 <sup>+</sup>	.3408 348	.3002 979	.2633 169	.2298 531	.1997 952
.4114 035 <sup>-</sup>	.3668 846	.3255 572	.2875 361	.2528 403	.2214 123
.4384 937	.3937 050 <sup>-</sup>	.3517 811	.3128 906	.2771 064	.2444 229
.4660 429	.4212 042	.3788 901	.3393 165 <sup>-</sup>	.3026 058	.2688 021
.4939 449	.4492 810	.4067 925 <sup>+</sup>	.3607 359	.3292 782	.2945 090
.5220 870	.4778 250 <sup>-</sup>	.4353 852	.3950 574	.3570 475 <sup>-</sup>	.3214 863
.5503 513	.5067 176	.4645 540	.4241 762	.3858 223	.3496 596
.5786 153	.5358 330	.4941 745 <sup>+</sup>	.4539 742	.4154 956	.3789 369
.6067 533	.5650 390	.5241 132	.4843 211	.4459 453	.4092 085 <sup>+</sup>
.6346 377	.5941 985 <sup>-</sup>	.5542 279	.5150 750 <sup>+</sup>	.4770 345 <sup>-</sup>	.4403 474
.6621 401	.6231 706	.5843 699	.5460 840	.5086 126	.4722 092
.6891 327	.6518 125 <sup>-</sup>	.6143 849	.5771 867	.5405 165 <sup>+</sup>	.5046 339

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION $x = .71 \text{ to } 1.00$ 
$$q = 4.5$$
 $p = 7.51$ 

	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$B(p, q) = .5452\ 8223 \times \frac{1}{10^8}$	$.4284\ 0156 \times \frac{1}{10^8}$	$.3408\ 0140 \times \frac{1}{10^8}$	$.2741\ 7700 \times \frac{1}{10^8}$	$.2228\ 3168 \times \frac{1}{10^8}$	$.1827\ 8467 \times \frac{1}{10^8}$	
$\cdot 71$	$\cdot 7154\ 901$	$\cdot 6799\ 806$	$\cdot 6441\ 151$	$\cdot 6082\ 150$	$\cdot 5725\ 722$	$\cdot 5374\ 460$
$\cdot 72$	$\cdot 7410\ 902$	$\cdot 7075\ 329$	$\cdot 6734\ 005^+$	$\cdot 6389\ 951$	$\cdot 6045\ 960$	$\cdot 5704\ 573$
$\cdot 73$	$\cdot 7658\ 165^+$	$\cdot 7343\ 309$	$\cdot 7020\ 818$	$\cdot 6693\ 502$	$\cdot 6363\ 976$	$\cdot 6034\ 678$
$\cdot 74$	$\cdot 7895\ 592$	$\cdot 7602\ 379$	$\cdot 7300\ 018$	$\cdot 6991\ 025$	$\cdot 6677\ 819$	$\cdot 6362\ 689$
$\cdot 75$	$\cdot 8122\ 167$	$\cdot 7851\ 292$	$\cdot 7570\ 080$	$\cdot 7280\ 761$	$\cdot 6985\ 519$	$\cdot 6686\ 458$
$\cdot 76$	$\cdot 8336\ 973$	$\cdot 8088\ 854$	$\cdot 7829\ 550^+$	$\cdot 7560\ 994$	$\cdot 7285\ 116$	$\cdot 7003\ 809$
$\cdot 77$	$\cdot 8539\ 207$	$\cdot 8313\ 987$	$\cdot 8077\ 068$	$\cdot 7830\ 082$	$\cdot 7574\ 695^+$	$\cdot 7312\ 573$
$\cdot 78$	$\cdot 8728\ 192$	$\cdot 8525\ 740$	$\cdot 8311\ 391$	$\cdot 8086\ 484$	$\cdot 7852\ 419$	$\cdot 7610\ 623$
$\cdot 79$	$\cdot 8903\ 389$	$\cdot 8723\ 306$	$\cdot 8531\ 419$	$\cdot 8328\ 792$	$\cdot 8116\ 564$	$\cdot 7895\ 923$
$\cdot 80$	$\cdot 9064\ 408$	$\cdot 8906\ 036$	$\cdot 8736\ 216$	$\cdot 8555\ 758$	$\cdot 8365\ 553$	$\cdot 8166\ 559$
$\cdot 81$	$\cdot 9211\ 018$	$\cdot 9073\ 458$	$\cdot 8925\ 031$	$\cdot 8766\ 322$	$\cdot 8597\ 997$	$\cdot 8420\ 794$
$\cdot 82$	$\cdot 9343\ 153$	$\cdot 9225\ 282$	$\cdot 9097\ 316$	$\cdot 8959\ 641$	$\cdot 8812\ 724$	$\cdot 8657\ 104$
$\cdot 83$	$\cdot 9460\ 915^+$	$\cdot 9361\ 419$	$\cdot 9252\ 743$	$\cdot 9135\ 108$	$\cdot 9008\ 814$	$\cdot 8874\ 223$
$\cdot 84$	$\cdot 9564\ 575^+$	$\cdot 9481\ 976$	$\cdot 9391\ 214$	$\cdot 9292\ 379$	$\cdot 9185\ 628$	$\cdot 9071\ 181$
$\cdot 85$	$\cdot 9654\ 572$	$\cdot 9587\ 268$	$\cdot 9512\ 872$	$\cdot 9431\ 377$	$\cdot 9342\ 832$	$\cdot 9247\ 339$
$\cdot 86$	$\cdot 9731\ 507$	$\cdot 9677\ 809$	$\cdot 9618\ 102$	$\cdot 9552\ 315$	$\cdot 9480\ 416$	$\cdot 9402\ 419$
$\cdot 87$	$\cdot 9796\ 133$	$\cdot 9754\ 305$	$\cdot 9707\ 528$	$\cdot 9655\ 686$	$\cdot 9598\ 700$	$\cdot 9536\ 523$
$\cdot 88$	$\cdot 9849\ 339$	$\cdot 9817\ 647$	$\cdot 9782\ 002$	$\cdot 9742\ 270$	$\cdot 9698\ 345^+$	$\cdot 9650\ 144$
$\cdot 89$	$\cdot 9892\ 136$	$\cdot 9868\ 888$	$\cdot 9842\ 590$	$\cdot 9813\ 112$	$\cdot 9780\ 337$	$\cdot 9744\ 167$
$\cdot 90$	$\cdot 9925\ 635^+$	$\cdot 9909\ 221$	$\cdot 9890\ 549$	$\cdot 9869\ 502$	$\cdot 9845\ 969$	$\cdot 9819\ 853$
$\cdot 91$	$\cdot 9951\ 016$	$\cdot 9939\ 949$	$\cdot 9927\ 290$	$\cdot 9912\ 942$	$\cdot 9896\ 810$	$\cdot 9878\ 807$
$\cdot 92$	$\cdot 9969\ 500$	$\cdot 9962\ 450^+$	$\cdot 9954\ 342$	$\cdot 9945\ 101$	$\cdot 9934\ 655^+$	$\cdot 9922\ 934$
$\cdot 93$	$\cdot 9982\ 315$	$\cdot 9978\ 135$	$\cdot 9973\ 301$	$\cdot 9967\ 763$	$\cdot 9961\ 468$	$\cdot 9954\ 366$
$\cdot 94$	$\cdot 9990\ 657$	$\cdot 9988\ 401$	$\cdot 9985\ 777$	$\cdot 9982\ 754$	$\cdot 9979\ 300$	$\cdot 9975\ 383$
$\cdot 95$	$\cdot 9995\ 654$	$\cdot 9994\ 581$	$\cdot 9993\ 328$	$\cdot 9991\ 876$	$\cdot 9990\ 208$	$\cdot 9988\ 307$
$\cdot 96$	$\cdot 9998\ 318$	$\cdot 9997\ 894$	$\cdot 9997\ 366$	$\cdot 9996\ 817$	$\cdot 9996\ 147$	$\cdot 9995\ 380$
$\cdot 97$	$\cdot 9999\ 513$	$\cdot 9999\ 388$	$\cdot 9999\ 240$	$\cdot 9999\ 067$	$\cdot 9998\ 867$	$\cdot 9998\ 635^+$
$\cdot 98$	$\cdot 9999\ 917$	$\cdot 9999\ 895^+$	$\cdot 9999\ 870$	$\cdot 9999\ 839$	$\cdot 9999\ 804$	$\cdot 9999\ 763$
$\cdot 99$	$\cdot 9999\ 996$	$\cdot 9999\ 995^+$	$\cdot 9999\ 994$	$\cdot 9999\ 992$	$\cdot 9999\ 991$	$\cdot 9999\ 989$
$\cdot 1.00$	$\cdot 1.0000\ 000$	$\cdot 1.0000\ 000$	$\cdot 1.0000\ 000$	$\cdot 1.0000\ 000$	$\cdot 1.0000\ 000$	$\cdot 1.0000\ 000$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q = 4.5$

$p = 10.5 \text{ to } 15$

$0.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$0.721 \times \frac{1}{10^3}$	$1.260 \ 5839 \times \frac{1}{10^3}$	$8.946 \ 0793 \times \frac{1}{10^4}$	$6.506 \ 2395 \times \frac{1}{10^4}$	$4.833 \ 2065 \times \frac{1}{10^4}$	$3.657 \ 5617 \times \frac{1}{10^4}$
001					
002	0000 001				
004	0000 002				
008	0000 004	0000 001			
016	0000 007	0000 002			
029	0000 014	0000 003	0000 001		
051	0000 025	0000 006	0000 001		
087	0000 043	0000 011	0000 003		
143	0000 073	0000 019	0000 005	0000 001	
229	0000 120	0000 032	0000 009	0000 002	0000 001
359	0000 192	0000 054	0000 015 <sup>+</sup>	0000 004	0000 001
550	0000 301	0000 089	0000 026	0000 007	0000 002
825 <sup>+</sup>	0000 461	0000 142	0000 043	0000 013	0000 004
216	0000 693	0000 222	0000 070	0000 022	0000 007
762	0001 024	0000 341	0000 112	0000 036	0000 012
511	0001 487	0000 514	0000 175	0000 059	0000 019
527	0002 126	0000 762	0000 269	0000 093	0000 032
884	0002 996	0001 112	0000 406	0000 146	0000 052
677	0004 164	0001 598	0000 603	0000 225	0000 083
017	0005 715	0002 265	0000 883	0000 340	0000 129
039	0007 750	0003 168	0001 275	0000 506	0000 198
900	0010 391	0004 379	0001 816	0000 743	0000 300
787	0013 785	0005 982	0002 555 <sup>+</sup>	0001 076	0000 448
915	0018 103	0008 082	0003 552	0001 540	0000 659
531	0023 547	0010 807	0004 883	0002 176	0000 958
918	0030 352	0014 308	0006 641	0003 041	0001 375 <sup>+</sup>
396	0038 784	0018 767	0008 941	0004 202	0001 951
324	0049 153	0024 394	0011 922	0005 748	0002 738
101	0061 803	0031 438	0015 749	0007 784	0003 802
169	0077 124	0040 185	0020 622	0010 442	0005 225
010	0095 550 <sup>+</sup>	0050 964	0026 775 <sup>+</sup>	0013 881	0007 112
150	0117 558	0064 148	0034 483	0018 293	0009 591
155	0143 673	0080 160	0044 064	0023 905 <sup>+</sup>	0012 818
629	0174 403	0099 473	0055 885	0030 989	0016 986
215 <sup>+</sup>	0210 543	0122 611	0070 366	0039 862	0022 323
586	0252 567	0150 154	0087 981	0050 892	0029 103
443	0301 229	0182 733	0109 265 <sup>+</sup>	0064 506	0037 651
508	0357 258	0221 033	0134 813	0081 192	0048 348
519	0421 412	0265 790	0165 284	0101 502	0061 637
217	0494 468	0317 787	0201 398	0126 059	0078 030
340	0577 218	0377 849	0243 940	0155 559	0098 112
612	0670 458	0446 838	0293 752	0190 772	0122 548
729	0774 975	0525 643	0351 735	0232 540	0152 084
347	0891 534	0615 170	0418 837	0281 781	0187 555
071	1020 868	0716 331	0496 049	0339 479	0220 880
433	1163 660	0830 029	0584 393	0406 684	0280 067
886	1320 527	0957 143	0684 908	0484 501	0339 207
9782	1492 002	1098 511	0798 638	0574 079	0408 467
359	1678 520	1254 907	0926 607	0676 594	0489 084
726	1880 398	1427 026	1069 807	0793 237	0582 352
846	2097 815 <sup>+</sup>	1615 456	1229 167	0925 187	0689 603
526	2330 800	1820 660	1405 533	1073 592	0812 188
402	2579 208	2042 947	1599 637	1239 534	0951 451
926	2842 713	2282 454	1812 070	1424 004	1108 699
4362	3120 787	2539 121	2043 250	1627 863	1285 169
2777	3412 692	2812 665	2293 391	1851 809	1481 984
2040	3717 473	3102 568	2562 475 <sup>+</sup>	2096 334	1700 114
0817	4033 949	3408 052	2850 221	2361 689	1940 331
7584	4360 714	3728 070	3156 054	2647 840	2203 152

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $1.00$  $q = 4.5$  $p =$ 

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .1512\ 0721 \times \frac{1}{10^8}$	$.1260\ 5839 \times \frac{1}{10^8}$	$.8946\ 0793 \times \frac{1}{10^4}$	$.6506\ 2395 \times \frac{1}{10^4}$	$.4833\ 2065 \times \frac{1}{10^4}$	$.3657\ 1000 \times \frac{1}{10^4}$	
.71	.5030 626	.4696 140	.4061 294	.3479 088	.2954 430	.2488 910
.72	.5368 054	.5038 386	.4406 113	.3818 100	.3280 746	.2797 100
.73	.5707 826	.5385 407	.4760 632	.4171 521	.3625 687	.3127 100
.74	.6047 761	.5734 981	.5122 681	.4537 424	.3987 732	.3479 100
.75	.6385 574	.6084 726	.5489 832	.4913 529	.4364 932	.3850 100
.76	.6718 904	.6432 137	.5859 426	.5297 215	.4754 897	.4239 100
.77	.7045 353	.6774 622	.6228 601	.5685 539	.5154 799	.4643 100
.78	.7362 527	.7109 542	.6594 339	.6075 272	.5561 396	.5060 100
.79	.7668 083	.7434 267	.6953 516	.6462 949	.5971 062	.5485 100
.80	.7959 774	.7746 224	.7302 963	.6844 925 <sup>+</sup>	.6379 841	.5914 100
.81	.8235 505 <sup>-</sup>	.8042 960	.7639 535 <sup>+</sup>	.7217 453	.6783 519	.6344 100
.82	.8493 381	.8322 197	.7960 192	.7576 769	.7177 713	.6768 100
.83	.8731 761	.8581 901	.8262 075 <sup>-</sup>	.7919 194	.7557 982	.7183 100
.84	.8949 312	.8820 343	.8542 597	.8241 243	.7919 953	.7582 100
.85	.9145 049	.9036 156	.8799 534	.8539 743	.8259 469	.7961 100
.86	.9318 380	.9228 390	.9031 105 <sup>+</sup>	.8811 954	.8572 742	.8315 100
.87	.9469 138	.9396 564	.9236 058	.9055 689	.8856 520	.8639 100
.88	.9597 606	.9540 697	.9413 739	.9269 430	.9108 248	.8930 100
.89	.9704 519	.9661 329	.9564 146	.9452 425 <sup>+</sup>	.9326 224	.9185 100
.90	.9791 065 <sup>+</sup>	.9759 530	.9687 964	.9604 769	.9509 735 <sup>+</sup>	.9402 100
.91	.9858 853	.9836 873	.9786 572	.9727 446	.9659 157	.9581 100
.92	.9909 870	.9895 401	.9862 012	.9822 334	.9776 003	.9722 100
.93	.9946 407	.9937 545 <sup>+</sup>	.9916 925 <sup>+</sup>	.9892 156	.9862 918	.9828 100
.94	.9970 969	.9966 028	.9954 437	.9940 364	.9923 573	.9903 100
.95	.9986 153	.9983 729	.9977 996	.9970 961	.9962 479	.9952 100
.96	.9994 506	.9993 517	.9991 161	.9988 239	.9984 678	.9980 100
.97	.9998 370	.9998 069	.9997 345 <sup>+</sup>	.9996 439	.9995 322	.9993 100
.98	.9999 716	.9999 662	.9999 531	.9999 366	.9999 160	.9998 100
.99	.9999 986	.9999 984	.9999 977	.9999 969	.9999 959	.9999 100
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 100

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .12$  to  $.70$  $q = 4.5$  $p = 10.5$  to  $15$ 

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .1512\ 0721 \times \frac{1}{10^3}$	$.1260\ 5839 \times \frac{1}{10^3}$	$.8946\ 0793 \times \frac{1}{10^4}$	$.6506\ 2395 \times \frac{1}{10^4}$	$.4833\ 2065 \times \frac{1}{10^4}$	$.3657\ 5617 \times \frac{1}{10^4}$	
.12	.0000 001					
.13	.0000 002	.0000 001				
.14	.0000 004	.0000 002				
.15	.0000 008	.0000 004	.0000 001			
.16	.0000 016	.0000 007	.0000 002			
.17	.0000 029	.0000 014	.0000 003	.0000 001		
.18	.0000 051	.0000 025 <sup>-</sup>	.0000 006	.0000 001		
.19	.0000 087	.0000 043	.0000 011	.0000 003		
.20	.0000 143	.0000 073	.0000 019	.0000 005 <sup>-</sup>	.0000 001	
.21	.0000 229	.0000 120	.0000 032	.0000 009	.0000 002	.0000 001
.22	.0000 359	.0000 192	.0000 054	.0000 015 <sup>+</sup>	.0000 004	.0000 001
.23	.0000 550 <sup>-</sup>	.0000 301	.0000 089	.0000 026	.0000 007	.0000 002
.24	.0000 825 <sup>+</sup>	.0000 461	.0000 142	.0000 043	.0000 013	.0000 004
.25	.0001 216	.0000 693	.0000 222	.0000 070	.0000 022	.0000 007
.26	.0001 762	.0001 024	.0000 341	.0000 112	.0000 036	.0000 012
.27	.0002 511	.0001 487	.0000 514	.0000 175 <sup>-</sup>	.0000 059	.0000 019
.28	.0003 527	.0002 126	.0000 762	.0000 269	.0000 093	.0000 032
.29	.0004 884	.0002 996	.0001 112	.0000 406	.0000 146	.0000 052
.30	.0006 677	.0004 164	.0001 598	.0000 603	.0000 225 <sup>-</sup>	.0000 083
.31	.0009 017	.0005 715 <sup>-</sup>	.0002 265 <sup>-</sup>	.0000 883	.0000 340	.0000 129
.32	.0012 039	.0007 750 <sup>-</sup>	.0003 168	.0001 275 <sup>-</sup>	.0000 506	.0000 198
.33	.0015 900	.0010 391	.0004 379	.0001 816	.0000 743	.0000 300
.34	.0020 787	.0013 785 <sup>-</sup>	.0005 982	.0002 555 <sup>+</sup>	.0001 076	.0000 448
.35	.0026 915 <sup>-</sup>	.0018 103	.0008 082	.0003 552	.0001 540	.0000 659
.36	.0034 531	.0023 547	.0010 807	.0004 883	.0002 176	.0000 958
.37	.0043 918	.0030 352	.0014 308	.0006 641	.0003 041	.0001 375 <sup>+</sup>
.38	.0055 396	.0038 784	.0018 767	.0008 941	.0004 202	.0001 951
.39	.0069 324	.0049 153	.0024 394	.0011 922	.0005 748	.0002 738
.40	.0086 101	.0061 803	.0031 438	.0015 749	.0007 784	.0003 802
.41	.0106 169	.0077 124	.0040 185 <sup>-</sup>	.0020 622	.0010 442	.0005 225 <sup>-</sup>
.42	.0130 010	.0095 550 <sup>+</sup>	.0050 964	.0026 775 <sup>+</sup>	.0013 881	.0007 112
.43	.0158 150 <sup>-</sup>	.0117 558	.0064 148	.0034 483	.0018 293	.0009 591
.44	.0191 155 <sup>-</sup>	.0143 673	.0080 160	.0044 064	.0023 905 <sup>+</sup>	.0012 818
.45	.0229 629	.0174 463	.0099 473	.0055 885 <sup>-</sup>	.0030 989	.0016 986
.46	.0274 215 <sup>+</sup>	.0210 543	.0122 611	.0070 366	.0039 862	.0022 323
.47	.0325 586	.0252 567	.0150 154	.0087 981	.0050 892	.0029 103
.48	.0384 443	.0301 229	.0182 733	.0109 265 <sup>+</sup>	.0064 506	.0037 651
.49	.0451 508	.0357 258	.0221 033	.0134 813	.0081 192	.0048 348
.50	.0527 519	.0421 412	.0265 790	.0165 284	.0101 502	.0061 637
.51	.0613 217	.0494 468	.0317 787	.0201 398	.0126 059	.0078 030
.52	.0709 340	.0577 218	.0377 849	.0243 940	.0155 559	.0098 112
.53	.0816 612	.0670 458	.0446 838	.0293 752	.0190 772	.0122 548
.54	.0935 729	.0774 975 <sup>-</sup>	.0525 643	.0351 735 <sup>-</sup>	.0232 540	.0152 084
.55	.1067 347	.0891 534	.0615 170	.0418 837	.0281 781	.0187 555 <sup>-</sup>
.56	.1212 071	.1020 868	.0716 331	.0496 049	.0339 479	.0229 880
.57	.1370 433	.1163 660	.0830 029	.0584 393	.0406 684	.0280 067
.58	.1542 886	.1320 527	.0957 143	.0684 908	.0484 501	.0339 207
.59	.1729 782	.1492 002	.1098 511	.0798 638	.0574 079	.0408 467
.60	.1931 359	.1678 520	.1254 907	.0926 607	.0676 594	.0489 084
.61	.2147 726	.1880 398	.1427 026	.1069 807	.0793 237	.0582 352
.62	.2378 846	.2097 815 <sup>+</sup>	.1615 456	.1229 167	.0925 187	.0689 603
.63	.2624 526	.2330 800	.1820 660	.1405 533	.1073 592	.0812 188
.64	.2884 402	.2579 208	.2042 947	.1599 637	.1239 534	.0951 451
.65	.3157 926	.2842 713	.2282 454	.1812 070	.1424 004	.1108 699
.66	.3444 362	.3120 787	.2539 121	.2043 250 <sup>-</sup>	.1627 863	.1285 169
.67	.3742 777	.3412 692	.2812 665 <sup>-</sup>	.2293 391	.1851 809	.1481 984
.68	.4052 040	.3717 473	.3102 568	.2562 475 <sup>+</sup>	.2096 334	.1700 114
.69	.4370 817	.4033 949	.3408 052	.2850 221	.2361 689	.1940 331
.70	.4697 584	.4360 714	.3728 070	.3156 054	.2647 840	.2203 152

TABLE I. THE  $I_x(p, q)$  FUNCTION

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 $x = .71$  to  $1.00$  $q = 4.5$  $p = 10.5$  to  $15$ 

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .1512\ 0721 \times \frac{1}{10^8}$	$.1260\ 5839 \times \frac{1}{10^8}$	$.8946\ 0793 \times \frac{1}{10^4}$	$.6506\ 2395 \times \frac{1}{10^4}$	$.4833\ 2065 \times \frac{1}{10^4}$	$.3657\ 5617 \times \frac{1}{10^4}$	
.71	.5030 626	.4696 140	.4061 294	.3479 088	.2954 430	.2488 796
.72	.5368 054	.5038 386	.4406 113	.3818 100	.3280 746	.2797 128
.73	.5707 826	.5385 407	.4760 632	.4171 521	.3625 687	.3127 613
.74	.6047 761	.5734 981	.5122 681	.4537 424	.3987 732	.3479 267
.75	.6385 574	.6084 726	.5489 832	.4913 529	.4364 932	.3850 623
.76	.6718 904	.6432 137	.5859 426	.5297 215	.4754 897	.4239 695
.77	.7045 353	.6774 622	.6228 601	.5685 539	.5154 799	.4643 963
.78	.7362 527	.7109 542	.6594 339	.6075 272	.5561 396	.5060 367
.79	.7668 083	.7434 267	.6953 516	.6462 949	.5971 062	.5485 323
.80	.7959 774	.7746 224	.7302 963	.6844 925 <sup>+</sup>	.6379 841	.5914 757
.81	.8235 505 <sup>-</sup>	.8042 960	.7639 535 <sup>+</sup>	.7217 453	.6783 519	.6344 162
.82	.8493 381	.8322 197	.7960 192	.7576 769	.7177 713	.6768 682
.83	.8731 761	.8581 901	.8262 075 <sup>-</sup>	.7919 194	.7557 982	.7183 225 <sup>-</sup>
.84	.8949 312	.8820 343	.8542 597	.8241 243	.7919 953	.7582 599
.85	.9145 049	.9036 156	.8799 534	.8539 743	.8259 469	.7961 678
.86	.9318 380	.9228 390	.9031 105 <sup>+</sup>	.8811 954	.8572 742	.8315 589
.87	.9469 138	.9396 564	.9236 058	.9055 689	.8856 520	.8639 923
.88	.9597 606	.9540 697	.9413 739	.9269 430	.9108 248	.8930 947
.89	.9704 519	.9661 329	.9564 146	.9452 425 <sup>+</sup>	.9326 224	.9185 825 <sup>-</sup>
.90	.9791 065 <sup>+</sup>	.9759 530	.9687 964	.9604 769	.9509 735 <sup>+</sup>	.9402 822
.91	.9858 853	.9836 873	.9786 572	.9727 446	.9659 157	.9581 477
.92	.9909 870	.9895 401	.9862 012	.9822 334	.9776 003	.9722 722
.93	.9946 407	.9937 545 <sup>+</sup>	.9916 925 <sup>+</sup>	.9892 156	.9862 918	.9828 929
.94	.9970 969	.9966 028	.9954 437	.9940 364	.9923 573	.9903 844
.95	.9986 153	.9983 729	.9977 996	.9970 961	.9962 479	.9952 407
.96	.9994 506	.9993 517	.9991 161	.9988 239	.9984 678	.9980 405 <sup>+</sup>
.97	.9998 370	.9998 069	.9997 345 <sup>+</sup>	.9996 439	.9995 322	.9993 969
.98	.9999 716	.9999 662	.9999 531	.9999 366	.9999 160	.9998 908
.99	.9999 986	.9999 984	.9999 977	.9999 969	.9999 959	.9999 946
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .23$  to  $.80$  $q = 4.5$  $p = 16$  to  $21$ 

	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$B(p, q) = .2813\ 5090 \times \frac{1}{10^4}$	$.2195\ 9094 \times \frac{1}{10^4}$	$.1736\ 3005 \times \frac{1}{10^4}$	$.1389\ 0404 \times \frac{1}{10^4}$	$.1123\ 0539 \times \frac{1}{10^4}$	$.9167\ 7872 \times \frac{1}{10^4}$	
$x$						
.23	.0000 001					
.24	.0000 001					
.25	.0000 002	.0000 001				
.26	.0000 004	.0000 001				
.27	.0000 006	.0000 002	.0000 001			
.28	.0000 011	.0000 004	.0000 001			
.29	.0000 018	.0000 006	.0000 002	.0000 001		
.30	.0000 030	.0000 011	.0000 004	.0000 001		
.31	.0000 048	.0000 018	.0000 007	.0000 002	.0000 001	
.32	.0000 077	.0000 029	.0000 011	.0000 004	.0000 002	.0000 001
.33	.0000 120	.0000 047	.0000 019	.0000 007	.0000 003	.0000 001
.34	.0000 184	.0000 075 <sup>+</sup>	.0000 030	.0000 012	.0000 005 <sup>-</sup>	.0000 002
.35	.0000 279	.0000 117	.0000 049	.0000 020	.0000 008	.0000 003
.36	.0000 417	.0000 180	.0000 077	.0000 033	.0000 014	.0000 006
.37	.0000 615 <sup>+</sup>	.0000 273	.0000 120	.0000 052	.0000 023	.0000 010
.38	.0000 806	.0000 408	.0000 184	.0000 082	.0000 037	.0000 016
.39	.0001 290	.0000 602	.0000 279	.0000 128	.0000 058	.0000 026
.40	.0001 837	.0000 879	.0000 417	.0000 196	.0000 092	.0000 043
.41	.0002 587	.0001 268	.0000 617	.0000 297	.0000 142	.0000 068
.42	.0003 605 <sup>+</sup>	.0001 810	.0000 901	.0000 445 <sup>+</sup>	.0000 218	.0000 106
.43	.0004 975 <sup>+</sup>	.0002 557	.0001 303	.0000 659	.0000 331	.0000 165 <sup>-</sup>
.44	.0006 802	.0003 575 <sup>+</sup>	.0001 863	.0000 964	.0000 495 <sup>-</sup>	.0000 252
.45	.0009 214	.0004 951	.0002 638	.0001 395 <sup>-</sup>	.0000 732	.0000 382
.46	.0012 372	.0006 793	.0003 699	.0001 998	.0001 072	.0000 572
.47	.0016 472	.0009 237	.0005 137	.0002 835 <sup>-</sup>	.0001 553	.0000 846
.48	.0021 753	.0012 452	.0007 069	.0003 983	.0002 228	.0001 230
.49	.0028 500 <sup>+</sup>	.0016 647	.0009 643	.0005 544	.0003 165 <sup>+</sup>	.0001 796
.50	.0037 055 <sup>-</sup>	.0022 074	.0013 042	.0007 648	.0004 454	.0002 578
.51	.0047 820	.0029 043	.0017 495 <sup>-</sup>	.0010 460	.0006 211	.0003 665 <sup>+</sup>
.52	.0061 269	.0037 920	.0023 279	.0014 185 <sup>+</sup>	.0008 585 <sup>+</sup>	.0005 164
.53	.0077 952	.0049 145 <sup>+</sup>	.0030 735 <sup>-</sup>	.0019 080	.0011 765 <sup>-</sup>	.0007 210
.54	.0098 500 <sup>+</sup>	.0063 235 <sup>+</sup>	.0040 272	.0025 460	.0015 988	.0009 979
.55	.0123 638	.0080 794	.0052 378	.0033 711	.0021 552	.0013 695 <sup>-</sup>
.56	.0154 184	.0102 521	.0067 633	.0044 298	.0028 822	.0018 630
.57	.0191 055 <sup>+</sup>	.0129 220	.0086 717	.0057 780	.0038 247	.0025 165 <sup>+</sup>
.58	.0235 272	.0161 804	.0110 419	.0074 821	.0050 370	.0033 708
.59	.0287 956	.0201 302	.0139 650 <sup>-</sup>	.0096 202	.0065 846	.0044 801
.60	.0350 327	.0248 861	.0175 447	.0122 833	.0085 450 <sup>+</sup>	.0059 095 <sup>+</sup>
.61	.0423 696	.0305 746	.0218 983	.0155 766	.0110 101	.0077 370
.62	.0509 459	.0373 339	.0271 568	.0196 201	.0140 866	.0100 555 <sup>-</sup>
.63	.0609 077	.0453 126	.0334 649	.0245 495 <sup>+</sup>	.0178 982	.0129 745 <sup>+</sup>
.64	.0724 057	.0546 691	.0409 805 <sup>+</sup>	.0305 164	.0225 857	.0166 218
.65	.0855 930	.0655 690	.0498 737	.0376 880	.0283 083	.0211 445 <sup>-</sup>
.66	.1006 216	.0781 833	.0603 250 <sup>+</sup>	.0462 468	.0352 435 <sup>+</sup>	.0267 105 <sup>-</sup>
.67	.1176 385 <sup>-</sup>	.0926 844	.0725 230	.0563 882	.0435 866	.0335 085 <sup>+</sup>
.68	.1367 818	.1092 427	.0866 607	.0683 191	.0535 492	.0417 481
.69	.1581 755 <sup>+</sup>	.1280 215 <sup>+</sup>	.1029 316	.0822 534	.0653 571	.0516 584
.70	.1819 239	.1491 713	.1215 242	.0984 082	.0792 462	.0634 855 <sup>+</sup>
.71	.2081 056	.1728 234	.1426 155 <sup>+</sup>	.1169 976	.0954 583	.0774 892
.72	.2367 675 <sup>+</sup>	.1990 830	.1663 640	.1382 256	.1142 340	.0939 371
.73	.2679 181	.2280 216	.1929 010	.1622 777	.1358 048	.1130 976
.74	.3015 209	.2596 686	.2223 216	.1893 109	.1603 832	.1352 306
.75	.3374 887	.2940 035 <sup>-</sup>	.2546 749	.2194 430	.1881 516	.1605 760
.76	.3756 779	.3309 478	.2899 540	.2527 407	.2192 488	.1893 405 <sup>-</sup>
.77	.4158 838	.3703 579	.3280 858	.2892 071	.2537 560	.2216 815 <sup>+</sup>
.78	.4578 377	.4120 185 <sup>-</sup>	.3689 215 <sup>-</sup>	.3287 694	.2916 817	.2576 907
.79	.5012 052	.4556 380	.4122 286	.3712 670	.3329 463	.2973 751
.80	.5455 874	.5008 463	.4576 844	.4164 407	.3773 677	.3406 392

TABLE I. THE  $I_0(p, q)$  FUNCTION

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 $x = .81$  to  $1.00$  $q = 4.5$  $p = 16$  to  $21$ 

	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$I(p, q) = .281150991 \times 10^{-1}$		$.210590994 \times 10^{-1}$	$.12361005 \times 10^{-1}$	$.13890404 \times 10^{-1}$	$.112305491 \times 10^{-1}$	$.91677872 \times 10^{-1}$
$\cdot 81$	.5905 244	.5471 949	.5048 724	.4639 250 <sup>*</sup>	.4246 481	.3872 662
$\cdot 82$	.6155 015	.5641 606	.5242 818	.4842 432	.4443 637	.4069 033
$\cdot 83$	.6409 598	.5811 516	.5423 131	.5018 071	.4620 908	.4250 492
$\cdot 84$	.7213 099	.6675 299	.6212 869	.5819 231	.5428 506	.5040 484
$\cdot 85$	.7649 493	.7126 059	.6694 600	.6298 047	.5919 279	.5548 035 <sup>*</sup>
$\cdot 86$	.8042 846	.7536 957	.7100 481	.6713 938	.6345 781	.5983 272
$\cdot 87$	.8410 516	.7916 090	.7482 562	.7093 907	.6737 122	.6384 179
$\cdot 88$	.8748 517	.8262 116	.7833 120	.7442 929	.7093 035 <sup>*</sup>	.6750 986
$\cdot 89$	.9041 714	.8564 551	.8135 147	.7749 420	.7403 423	.7063 222
$\cdot 90$	.9284 148	.8813 978	.8382 705	.7980 836	.7638 076	.7307 810
$\cdot 91$	.9484 294	.9047 604	.8620 500	.8196 171	.7851 888	.7518 929
$\cdot 92$	.9642 266	.9214 486	.8791 527	.8370 648	.8026 604	.7690 277
$\cdot 93$	.9780 913	.9355 756	.8936 200	.8511 153	.8160 529	.7821 281
$\cdot 94$	.9890 071	.9474 768	.9052 506	.8631 716	.8274 594	.7931 501
$\cdot 95$	.9979 606	.9560 942	.9131 289	.8703 527	.8343 544	.8001 239
$\cdot 96$	.9997 546	.9590 427	.9162 575 <sup>*</sup>	.8734 718	.8375 786	.8035 711
$\cdot 97$	.9999 349	.9599 415	.9168 196	.8735 601	.8376 621	.8036 224
$\cdot 98$	.9999 603	.9599 239	.9167 809	.8735 306	.8376 722	.8036 050
$\cdot 99$	.9999 930	.9599 911	.9169 889	.8739 862	.8380 831	.8039 794
$1.00$	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .34$  to  $.90$  $q = 4.5$ 

	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$B(p, q) = .7549\ 9424 \times 10^5$	$.6267\ 8767 \times 10^5$	$.5242\ 2241 \times 10^5$	$.4414\ 5945 \times 10^5$	$.3741\ 1055 \times 10^5$	
$x$					
.34	.0000 001				
.35	.0000 001	.0000 001			
.36	.0000 002	.0000 001			
.37	.0000 004	.0000 002	.0000 001		
.38	.0000 007	.0000 003	.0000 001	.0000 001	
.39	.0000 012	.0000 005 <sup>†</sup>	.0000 002	.0000 001	
.40	.0000 020	.0000 009	.0000 004	.0000 002	.0000 001
.41	.0000 032	.0000 015 <sup>†</sup>	.0000 007	.0000 004	.0000 002
.42	.0000 052	.0000 025 <sup>†</sup>	.0000 012	.0000 007	.0000 004
.43	.0000 082	.0000 040	.0000 020	.0000 010	.0000 005
.44	.0000 128	.0000 065 <sup>†</sup>	.0000 032	.0000 016	.0000 008
.45	.0000 198	.0000 102	.0000 052	.0000 027	.0000 014
.46	.0000 303	.0000 160	.0000 084	.0000 044	.0000 024
.47	.0000 458	.0000 246	.0000 132	.0000 070	.0000 047
.48	.0000 685 <sup>†</sup>	.0000 376	.0000 206	.0000 112	.0000 061
.49	.0001 013	.0000 568	.0000 317	.0000 176	.0000 097
.50	.0001 483	.0000 849	.0000 484	.0000 274	.0000 155
.51	.0002 150 <sup>†</sup>	.0001 255 <sup>†</sup>	.0000 729	.0000 421	.0000 242
.52	.0003 088	.0001 837	.0001 087	.0000 641	.0000 406
.53	.0004 393	.0002 662	.0001 606	.0000 964	.0000 676
.54	.0006 193	.0003 823	.0002 348	.0001 416	.0000 875
.55	.0008 653	.0005 439	.0003 402	.0002 118	.0001 314
.56	.0011 987	.0007 668	.0004 882	.0003 094	.0001 954
.57	.0016 466	.0010 718	.0006 934	.0004 478	.0002 876
.58	.0022 432	.0014 892	.0009 787	.0006 420	.0004 195
.59	.0030 316	.0020 410	.0013 676	.0009 124	.0006 061
.60	.0040 647	.0027 817	.0018 947	.0012 849	.0008 678
.61	.0054 077	.0037 667	.0026 012	.0017 944	.0012 415 <sup>†</sup>
.62	.0071 396	.0050 442	.0035 473	.0024 819	.0017 124
.63	.0093 556	.0067 130	.0047 948	.0034 101	.0024 156
.64	.0121 687	.0088 654	.0064 295 <sup>†</sup>	.0046 412	.0033 499
.65	.0157 110	.0116 190	.0085 538	.0062 708	.0045 791
.66	.0201 490	.0151 137	.0112 914	.0084 409	.0062 260
.67	.0256 308	.0195 131	.0147 904	.0111 694	.0084 956
.68	.0323 859	.0250 068	.0192 254	.0147 268	.0112 287
.69	.0406 309	.0318 115 <sup>†</sup>	.0248 960	.0192 576	.0148 961
.70	.0506 143	.0401 210	.0317 484	.0249 926	.0196 015
.71	.0626 047	.0503 555 <sup>†</sup>	.0403 451	.0311 811	.0255 819
.72	.0768 876	.0626 591	.0508 556	.0391 178	.0341 249
.73	.0937 586	.0773 222	.0643 727	.0500 000	.0444 000

TABLE 1. THE  $I_x(p, q)$  FUNCTION. $x = .91$  to  $1.00$  $q = 4.5$ 

	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$I(p, q) = 7549.9424 \times 10^5$	$9267.8767 \times 10^5$	$95242.2241 \times 10^5$	$94414.5045 \times 10^5$	$93$	
.91	8777.916	8629.105	8473.079	8310.390	8147.861
.92	9144.814	9044.417	8915.337	8790.861	8666.861
.93	9442.402	9364.921	9281.897	9193.425 <sup>†</sup>	9103.425 <sup>†</sup>
.94	9668.626	9619.633	9566.573	9509.422	9419.422
.95	9826.517	9799.296	9769.499	9737.065 <sup>†</sup>	9666.861
.96	9924.426	9911.870	9897.981	9882.704	9810.342
.97	9975.481	9971.059	9966.229	9960.861	9937.065 <sup>†</sup>
.98	9995.281	9994.497	9993.421	9992.313	9991.313
.99	9999.752	9999.704	9999.649	9999.586	9999.523
1.00	1.0000.000	1.0000.000	1.0000.000	1.0000.000	1.0000.000

† Values of  $I_x(p, q)$  for  $x = .91$  to  $1.00$  are given in Table 1.

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .42$  to  $1.00$  $q = 4.5$ 

	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$
$B(p, q) = .2733\ 5478 \times \frac{x}{10^8}$	$.2355\ 0566 \times \frac{x}{10^8}$	$.2038\ 7057 \times \frac{x}{10^8}$	$.1772\ 7876 \times \frac{x}{10^8}$	$.1548\ 0$	
$x$					
.42	.0000 001				
.43	.0000 001	.0000 001			
.44	.0000 002	.0000 001			
.45	.0000 003	.0000 002	.0000 001		
.46	.0000 006	.0000 003	.0000 002	.0000 001	
.47	.0000 010	.0000 005 <sup>+</sup>	.0000 003	.0000 001	.0000 000
.48	.0000 018	.0000 009	.0000 005 <sup>+</sup>	.0000 003	.0000 000
.49	.0000 029	.0000 016	.0000 009	.0000 005 <sup>+</sup>	.0000 000
.50	.0000 049	.0000 027	.0000 015 <sup>+</sup>	.0000 008	.0000 000
.51	.0000 079	.0000 045 <sup>+</sup>	.0000 026	.0000 014	.0000 000
.52	.0000 128	.0000 074	.0000 043	.0000 025 <sup>+</sup>	.0000 000
.53	.0000 204	.0000 120	.0000 071	.0000 042	.0000 000
.54	.0000 321	.0000 193	.0000 116	.0000 069	.0000 000
.55	.0000 499	.0000 306	.0000 187	.0000 114	.0000 000
.56	.0000 769	.0000 480	.0000 299	.0000 185 <sup>+</sup>	.0000 000
.57	.0001 173	.0000 745 <sup>+</sup>	.0000 472	.0000 298	.0000 000
.58	.0001 770	.0001 144	.0000 737	.0000 473	.0000 000
.59	.0002 645 <sup>+</sup>	.0001 738	.0001 139	.0000 744	.0000 000
.60	.0003 914	.0002 615 <sup>+</sup>	.0001 742	.0001 157	.0000 000
.61	.0005 738	.0003 897	.0002 638	.0001 780	.0001 000
.62	.0008 332	.0005 749	.0003 955 <sup>+</sup>	.0002 712	.0001 000
.63	.0011 989	.0008 403	.0005 872	.0004 091	.0002 000
.64	.0017 093	.0012 167	.0008 634	.0006 109	.0004 000
.65	.0024 154	.0017 456	.0012 576	.0009 034	.0006 000
.66	.0033 831	.0024 816	.0018 147	.0013 232	.0009 000
.67	.0046 971	.0034 962	.0025 944	.0019 198	.0014 000
.68	.0064 650 <sup>+</sup>	.0048 818	.0036 752	.0027 591	.0020 000
.69	.0088 216	.0067 561	.0051 589	.0039 283	.0029 000
.70	.0119 340	.0092 677	.0071 760	.0055 411	.0042 000
.71	.0160 064	.0126 012	.0098 917	.0077 436	.0060 000
.72	.0212 847	.0169 830	.0135 120	.0107 213	.0084 000
.73	.0280 607	.0226 867	.0182 903	.0147 065 <sup>+</sup>	.0117 000
.74	.0366 748	.0300 376	.0245 334	.0199 850 <sup>+</sup>	.0162 000
.75	.0475 169	.0394 155 <sup>+</sup>	.0326 062	.0269 033	.0221 000
.76	.0610 241	.0512 552	.0429 348	.0358 735 <sup>+</sup>	.0299 000
.77	.0776 745 <sup>+</sup>	.0660 431	.0560 061	.0473 759	.0399 000
.78	.0979 758	.0843 090	.0723 624	.0619 568	.0529 000
.79	.1224 476	.1066 111	.0925 902	.0802 215 <sup>+</sup>	.0695 000
.80	.1515 958	.1335 128	.1173 005 <sup>+</sup>	.1028 175 <sup>+</sup>	.0899 000
.81	.1858 790	.1655 513	.1470 987	.1304 084	.1155 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .49$  to  $1.00$  $q = 4.5$ 

	$p = .34$	$p = .35$	$p = .36$	$p = .37$	$p = .38$
$I(p, q) = .11043451 \times 10^4$	$.10517403 \times 10^4$	$.93458531 \times 10^3$	$.83974252 \times 10^3$	$.77000000$	$.72000000$
$.49$	.000000001				
$.50$	.000000001	.000000001			
$.51$	.000000001	.000000001	.000000001		
$.52$	.000000001	.000000001	.000000002	.000000001	
$.53$	.000000008	.000000001	.000000001	.000000002	.000000001
$.54$	.000000001	.000000001	.000000001	.000000001	.000000001
$.55$	.000000021	.000000011	.000000001	.000000001	.000000001
$.56$	.000000001	.000000027	.000000015	.000000010	.000000001
$.57$	.000000001	.000000015	.000000020	.000000018	.000000001
$.58$	.000000121	.000000018	.000000030	.000000041	.000000001
$.59$	.000000201	.000000117	.000000051	.000000055	.000000001
$.60$	.000000411	.000000219	.000000111	.000000091	.000000001
$.61$	.000000638	.000000404	.000000240	.000000150	.000000001
$.62$	.000000861	.000000551	.000000320	.000000207	.000000001
$.63$	.000001161	.000000701	.000000440	.000000314	.000000001
$.64$	.00000129	.000000821	.000000542	.000000427	.000000001
$.65$	.000001201	.000000911	.000000616	.000000527	.000000001
$.66$	.000000950	.000000950	.000000676	.000000682	.000000001
$.67$	.000000652	.000000610	.000000701	.000000701	.000000001
$.68$	.000001101	.000000541	.000000646	.000000600	.000000001
$.69$	.000001001	.000000401	.000000500	.000000454	.000000001
$.70$	.000000711	.000000217	.000000300	.000000254	.000000001
$.71$	.000000401	.000000101	.000000150	.000000103	.000000001
$.72$	.000000101	.000000011	.000000025	.000000018	.000000001
$.73$	.000000012	.000000001	.000000004	.000000001	.000000001
$.74$	.000000002	.000000000	.000000000	.000000000	.000000001
$.75$	.000000001	.000000000	.000000000	.000000000	.000000001
$.76$	.000000000	.000000000	.000000000	.000000000	.000000001
$.77$	.000000000	.000000000	.000000000	.000000000	.000000001
$.78$	.000000000	.000000000	.000000000	.000000000	.000000001
$.79$	.000000000	.000000000	.000000000	.000000000	.000000001
$.80$	.000000000	.000000000	.000000000	.000000000	.000000001
$.81$	.000000000	.000000000	.000000000	.000000000	.000000001
$.82$	.000000000	.000000000	.000000000	.000000000	.000000001
$.83$	.000000000	.000000000	.000000000	.000000000	.000000001
$.84$	.000000000	.000000000	.000000000	.000000000	.000000001
$.85$	.000000000	.000000000	.000000000	.000000000	.000000001
$.86$	.000000000	.000000000	.000000000	.000000000	.000000001
$.87$	.000000000	.000000000	.000000000	.000000000	.000000001
$.88$	.000000000	.000000000	.000000000	.000000000	.000000001
$.89$	.000000000	.000000000	.000000000	.000000000	.000000001
$.90$	.000000000	.000000000	.000000000	.000000000	.000000001
$.91$	.000000000	.000000000	.000000000	.000000000	.000000001
$.92$	.000000000	.000000000	.000000000	.000000000	.000000001
$.93$	.000000000	.000000000	.000000000	.000000000	.000000001
$.94$	.000000000	.000000000	.000000000	.000000000	.000000001
$.95$	.000000000	.000000000	.000000000	.000000000	.000000001
$.96$	.000000000	.000000000	.000000000	.000000000	.000000001
$.97$	.000000000	.000000000	.000000000	.000000000	.000000001
$.98$	.000000000	.000000000	.000000000	.000000000	.000000001
$.99$	.000000000	.000000000	.000000000	.000000000	.000000001
$1.00$	.000000000	.000000000	.000000000	.000000000	.000000001

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

54 to 100

$q = 4.5$

$p =$

	$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$p, q) =$	$.5937\ 3149 \times \frac{1}{10^8}$	$.5336\ 9123 \times \frac{1}{10^8}$	$.4809\ 0858 \times \frac{1}{10^8}$	$.4343\ 6904 \times \frac{1}{10^8}$	$.3932\ 1829 \times \frac{1}{10^8}$	$.3567\ 34$
$\cdot 54$	.0000 001					
$\cdot 55$	.0000 001	.0000 001				
$\cdot 56$	.0000 002	.0000 001	.0000 001			
$\cdot 57$	.0000 004	.0000 003	.0000 002	.0000 001	.0000 001	
$\cdot 58$	.0000 008	.0000 005	.0000 003	.0000 002	.0000 001	.0000 00
$\cdot 59$	.0000 014	.0000 009	.0000 006	.0000 004	.0000 002	.0000 00
$\cdot 60$	.0000 026	.0000 017	.0000 011	.0000 007	.0000 005	.0000 00
$\cdot 61$	.0000 046	.0000 031	.0000 020	.0000 013	.0000 009	.0000 00
$\cdot 62$	.0000 082	.0000 055	.0000 037	.0000 024	.0000 016	.0000 01
$\cdot 63$	.0000 142	.0000 097	.0000 066	.0000 045	.0000 030	.0000 02
$\cdot 64$	.0000 243	.0000 168	.0000 116	.0000 080	.0000 055 <sup>+</sup>	.0000 03
$\cdot 65$	.0000 413	.0000 290	.0000 204	.0000 143	.0000 100	.0000 07
$\cdot 66$	.0000 693	.0000 494	.0000 352	.0000 250 <sup>+</sup>	.0000 178	.0000 12
$\cdot 67$	.0001 148	.0000 831	.0000 601	.0000 433	.0000 312	.0000 22
$\cdot 68$	.0001 880	.0001 381	.0001 013	.0000 742	.0000 542	.0000 39
$\cdot 69$	.0003 044	.0002 269	.0001 688	.0001 254	.0000 930	.0000 68
$\cdot 70$	.0004 873	.0003 684	.0002 780	.0002 095	.0001 576	.0001 18
$\cdot 71$	.0007 713	.0005 913	.0004 525	.0003 457	.0002 638	.0002 01
$\cdot 72$	.0012 072	.0009 381	.0007 278	.0005 638	.0004 361	.0003 36
$\cdot 73$	.0018 680	.0014 713	.0011 571	.0009 085 <sup>+</sup>	.0007 123	.0005 57
$\cdot 74$	.0028 580	.0022 812	.0018 180	.0014 466	.0011 494	.0009 12
$\cdot 75$	.0043 231	.0034 960	.0028 228	.0022 758	.0018 321	.0014 74
$\cdot 76$	.0064 645 <sup>+</sup>	.0052 955	.0043 311	.0035 371	.0028 846	.0023 44
$\cdot 77$	.0095 553	.0079 269	.0065 660	.0054 308	.0044 856	.0036 99
$\cdot 78$	.0139 584	.0117 246	.0098 335 <sup>+</sup>	.0082 356	.0068 877	.0057 51
$\cdot 79$	.0201 474	.0171 315 <sup>+</sup>	.0145 455 <sup>+</sup>	.0123 324	.0104 417	.0088 29
$\cdot 80$	.0287 267	.0247 219	.0212 445 <sup>+</sup>	.0182 308	.0156 236	.0133 74
$\cdot 81$	.0404 480	.0352 224	.0306 282	.0265 969	.0230 657	.0199 77
$\cdot 82$	.0562 198	.0495 266	.0435 697	.0382 780	.0335 856	.0294 31
$\cdot 83$	.0771 022	.0686 980	.0611 271	.0543 196	.0482 095	.0427 37
$\cdot 84$	.1042 801	.0939 511	.0845 340	.0759 644	.0681 801	.0611 21
$\cdot 85$	.1390 037	.1266 010	.1151 587	.1046 222	.0949 373	.0860 56
$\cdot 86$	.1824 859	.1679 694	.1544 194	.1417 953	.1300 549	.1191 51
$\cdot 87$	.2357 492	.2192 347	.2036 406	.1889 431	.1751 156	.1621 21
$\cdot 88$	.2994 152	.2812 193	.2638 400	.2472 718	.2315 050 <sup>+</sup>	.2165 29
$\cdot 89$	.3734 449	.3541 156	.3354 439	.3174 413	.3001 148	.2834 61
$\cdot 90$	.4568 509	.4371 698	.4179 447	.3992 002	.3809 570	.3632 31
$\cdot 91$	.5474 282	.5283 693	.5095 452	.4909 878	.4727 260	.4547 81
$\cdot 92$	.6415 828	.6242 141	.6068 708	.5895 853	.5723 882	.5553 08
$\cdot 93$	.7343 684	.7196 242	.7048 785 <sup>+</sup>	.6899 535 <sup>+</sup>	.6749 435 <sup>+</sup>	.6598 73
$\cdot 94$	.8198 648	.8086 150 <sup>+</sup>	.7971 375	.7854 495 <sup>+</sup>	.7735 686	.7615 11
$\cdot 95$	.8920 181	.8844 557	.8766 581	.8686 331	.8603 889	.8519 31
$\cdot 96$	.9459 760	.9417 642	.9373 757	.9328 117	.9280 736	.9231 61
$\cdot 97$	.9797 353	.9779 884	.9761 493	.9742 166	.9721 894	.9700 66
$\cdot 98$	.9955 171	.9950 925	.9946 408	.9941 613	.9936 531	.9931 11
$\cdot 99$	.9997 278	.9996 997	.9996 694	.9996 370	.9996 022	.9995 61
100	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 00

TABLE I. THE  $I_{\pi}(p, q)$  FUNCTION $x = .50$  to  $1.00$  $q = 4.5$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$H(p, q) = 3243.0374 \times 10^6$		$2954.0539 \times 10^6$	$2605.9327 \times 10^6$	$22464$
$y$				
.50	.0000.001	.0000.001		
.60	.0000.002	.0000.001	.0000.001	
.61	.0000.001	.0000.002	.0000.002	.0000
.62	.0000.002	.0000.003	.0000.003	.0000
.63	.0000.014	.0000.009	.0000.006	.0000
.64	.0000.026	.0000.018	.0000.012	.0000
.65	.0000.049	.0000.034	.0000.024	.0000
.66	.0000.089	.0000.063	.0000.044	.0000
.67	.0000.146	.0000.116	.0000.083	.0000
.68	.0000.289	.0000.210	.0000.153	.0000
.69	.0000.519	.0000.377	.0000.278	.0000
.70	.0000.888	.0000.666	.0000.498	.0000
.71	.0001.529	.0001.462	.0000.882	.0000
.72	.0002.598	.0002.002	.0001.530	.0001
.73	.0004.461	.0003.405	.0002.656	.0002
.74	.0007.227	.0005.719	.0004.520	.0003
.75	.0011.826	.0009.482	.0007.594	.0006
.76	.0019.107	.0015.520	.0012.592	.0010
.77	.0031.427	.0025.074	.0020.604	.0016
.78	.0047.987	.0039.928	.0034.266	.0027
.79	.0074.564	.0062.861	.0052.084	.0044
.80	.0114.405	.0097.591	.0083.225	.0070
.81	.0172.523	.0139.427	.0128.828	.0111
.82	.0257.607	.0225.244	.0196.673	.0171
.83	.0378.474	.0344.629	.0295.619	.0260
.84	.0547.104	.0496.547	.0447.400	.0390
.85	.0779.087	.0704.619	.0646.607	.0574
.86	.1099.521	.0997.024	.0910.630	.0830
.87	.1519.514	.1385.509	.1278.040	.1179
.88	.2074.404	.1888.607	.1764.477	.1641
.89	.2804.627	.2522.661	.2345.546	.2206
.90	.3769.172	.3424.877	.3142.856	.2977
.91	.4994.605	.4599.605	.4301.446	.4066
.92	.6518.228	.6026.066	.5630.430	.5386
.93	.8412.609	.7826.826	.7445.425	.7094
.94	.10792.926	.10042.424	.9244.645	.8718
.95	.13812.767	.12844.268	.12054.912	.1161
.96	.1780.525	.1648.440	.1537.202	.1418
.97	.2308.474	.2155.408	.2031.164	.1906
.98	.2975.476	.2794.475	.2644.481	.2506
.99	.3825.254	.3604.542	.3434.484	.3294
1.00	1.0000.000	1.0000.000	1.0000.000	1.0000.000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

2 to 60

$q = 5$

$p = 5$

	$p = 5$	$p = 5.5$	$p = 6$	$p = 6.5$	$p = 7$	$p = 7.5$
$q) = .1587\ 3016 \times \frac{1}{10^4}$		$.1108\ 4890 \times \frac{1}{10^4}$	$.7936\ 5079 \times \frac{1}{10^4}$	$.5806\ 3711 \times \frac{1}{10^4}$	$.4329\ 0043 \times \frac{1}{10^4}$	$.3281\ 8619$
2	.0000 004	.0000 001				
3	.0000 028	.0000 006	.0000 001			
4	.0000 113	.0000 029	.0000 007	.0000 002		
5	.0000 332	.0000 096	.0000 028	.0000 008	.0000 002	.0000 001
6	.0000 798	.0000 254	.0000 079	.0000 024	.0000 007	.0000 002
7	.0001 666	.0000 572	.0000 193	.0000 064	.0000 021	.0000 007
8	.0003 136	.0001 149	.0000 415	.0000 147	.0000 052	.0000 018
9	.0005 453	.0002 119	.0000 810	.0000 305 <sup>+</sup>	.0000 114	.0000 042
0	.0008 909	.0003 646	.0001 469	.0000 584	.0000 229	.0000 089
1	.0013 838	.0005 936	.0002 507	.0001 044	.0000 429	.0000 175
2	.0020 615	.0009 230	.0004 069	.0001 769	.0000 760	.0000 323
3	.0029 649	.0013 808	.0006 332	.0002 864	.0001 279	.0000 565
4	.0041 384	.0019 986	.0009 505	.0004 459	.0002 066	.0000 947
5	.0056 287	.0028 117	.0013 832	.0006 713	.0003 219	.0001 527
6	.0074 847	.0038 587	.0019 593	.0009 815 <sup>+</sup>	.0004 858	.0002 379
7	.0097 568	.0051 808	.0027 098	.0013 985	.0007 131	.0003 598
8	.0124 962	.0068 224	.0036 694	.0019 475	.0010 214	.0005 300
9	.0157 541	.0088 297	.0048 757	.0026 570	.0014 309	.0007 625
0	.0195 814	.0112 506	.0063 694	.0035 589	.0019 654	.0010 739
1	.0240 280	.0141 343	.0081 935 <sup>+</sup>	.0046 883	.0026 515	.0014 839
2	.0291 417	.0175 304	.0103 936	.0060 831	.0035 193	.0020 149
3	.0349 682	.0214 888	.0130 167	.0077 843	.0046 020	.0026 926
4	.0415 503	.0260 588	.0161 116	.0098 356	.0059 361	.0035 460
5	.0489 273	.0312 883	.0197 277	.0122 827	.0075 612	.0046 073
6	.0571 345	.0372 238	.0239 148	.0151 734	.0095 196	.0059 122
7	.0662 028	.0439 094	.0287 224	.0185 509	.0118 563	.0074 993
8	.0761 583	.0513 861	.0341 994	.0224 834	.0146 187	.0094 105
9	.0870 218	.0596 916	.0403 932	.0270 037	.0178 560	.0116 907
0	.0988 087	.0688 598	.0473 490	.0321 685	.0216 192	.0143 873
1	.1115 286	.0789 198	.0551 097	.0380 276	.0259 599	.0175 500
2	.1251 852	.0898 962	.0637 149	.0446 299	.0309 308	.0212 307
3	.1397 759	.1018 081	.0732 005	.0520 222	.0365 839	.0254 824
4	.1552 923	.1146 689	.0835 979	.0602 487	.0429 711	.0303 595
5	.1717 193	.1284 861	.0949 341	.0693 508	.0501 427	.0359 166
6	.1890 360	.1432 612	.1072 304	.0793 658	.0581 470	.0422 081
7	.2072 151	.1589 890	.1205 026	.0903 267	.0670 298	.0492 878
8	.2262 237	.1756 580	.1347 603	.1022 617	.0768 336	.0572 076
9	.2460 227	.1932 500 <sup>+</sup>	.1500 068	.1151 933	.0875 966	.0660 176
0	.2665 677	.2117 404	.1662 386	.1291 382	.0993 526	.0757 166
1	.2878 090	.2310 979	.1834 452	.1441 064	.1121 300	.0856 266
2	.3096 920	.2512 848	.2016 092	.1601 012	.1259 511	.0955 366
3	.3321 576	.2722 571	.2207 058	.1771 186	.1408 320	.1054 466
4	.3551 423	.2939 650 <sup>+</sup>	.2407 033	.1951 471	.1567 813	.1153 566
5	.3785 703	.3162 238	.2617 018			
6						
7						
8						
9						
0						





# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.60

$q = 5$

$p = 8$  to 10

$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$
$= .2525 \ 2525 \times \frac{1}{10^8}$	$\cdot 1969 \ 1172 \times \frac{1}{10^8}$	$\cdot 1554 \ 0016 \times \frac{1}{10^8}$	$\cdot 1239 \ 8145 \times \frac{1}{10^8}$	$\cdot 9990 \ 0100 \times \frac{1}{10^8}$	$\cdot 8122 \ 9227 \times \frac{1}{10^8}$
$\cdot 0000 \ 001$	$\cdot 0000 \ 001$	$\cdot 0000 \ 001$	$\cdot 0000 \ 001$	$\cdot 0000 \ 001$	
$\cdot 0000 \ 002$	$\cdot 0000 \ 002$	$\cdot 0000 \ 002$	$\cdot 0000 \ 002$		
$\cdot 0000 \ 006$	$\cdot 0000 \ 006$	$\cdot 0000 \ 006$			
$\cdot 0000 \ 015^+$	$\cdot 0000 \ 013$	$\cdot 0000 \ 005^-$			
$\cdot 0000 \ 034$					
$\cdot 0000 \ 070$	$\cdot 0000 \ 028$	$\cdot 0000 \ 011$	$\cdot 0000 \ 004$	$\cdot 0000 \ 002$	$\cdot 0000 \ 001$
$\cdot 0000 \ 136$	$\cdot 0000 \ 057$	$\cdot 0000 \ 023$	$\cdot 0000 \ 010$	$\cdot 0000 \ 004$	$\cdot 0000 \ 002$
$\cdot 0000 \ 247$	$\cdot 0000 \ 107$	$\cdot 0000 \ 046$	$\cdot 0000 \ 020$	$\cdot 0000 \ 008$	$\cdot 0000 \ 004$
$\cdot 0000 \ 430$	$\cdot 0000 \ 193$	$\cdot 0000 \ 086$	$\cdot 0000 \ 038$	$\cdot 0000 \ 017$	$\cdot 0000 \ 007^-$
$\cdot 0000 \ 717$	$\cdot 0000 \ 334$	$\cdot 0000 \ 154$	$\cdot 0000 \ 071$	$\cdot 0000 \ 032$	$\cdot 0000 \ 015^-$
$\cdot 0001 \ 153$	$\cdot 0000 \ 554$	$\cdot 0000 \ 264$	$\cdot 0000 \ 125^+$	$\cdot 0000 \ 059$	$\cdot 0000 \ 027^-$
$\cdot 0001 \ 797$	$\cdot 0000 \ 890$	$\cdot 0000 \ 437$	$\cdot 0000 \ 213$	$\cdot 0000 \ 103$	$\cdot 0000 \ 050^-$
$\cdot 0002 \ 723$	$\cdot 0001 \ 387$	$\cdot 0000 \ 701$	$\cdot 0000 \ 352$	$\cdot 0000 \ 175^+$	$\cdot 0000 \ 087$
$\cdot 0004 \ 024$	$\cdot 0002 \ 105^+$	$\cdot 0001 \ 093$	$\cdot 0000 \ 503$	$\cdot 0000 \ 288$	$\cdot 0000 \ 147$
$\cdot 0005 \ 812$	$\cdot 0003 \ 119$	$\cdot 0001 \ 660$	$\cdot 0000 \ 877$	$\cdot 0000 \ 460$	$\cdot 0000 \ 240$
$\cdot 0008 \ 226$	$\cdot 0004 \ 521$	$\cdot 0002 \ 465^+$	$\cdot 0001 \ 334$	$\cdot 0000 \ 718$	$\cdot 0000 \ 383$
$\cdot 0011 \ 427$	$\cdot 0006 \ 425^+$	$\cdot 0003 \ 585^-$	$\cdot 0001 \ 985^+$	$\cdot 0001 \ 092$	$\cdot 0000 \ 597$
$\cdot 0015 \ 607$	$\cdot 0008 \ 969$	$\cdot 0005 \ 114$	$\cdot 0002 \ 895^+$	$\cdot 0001 \ 628$	$\cdot 0000 \ 910$
$\cdot 0020 \ 985^+$	$\cdot 0012 \ 314$	$\cdot 0007 \ 170$	$\cdot 0004 \ 145^+$	$\cdot 0002 \ 381$	$\cdot 0001 \ 359$
$\cdot 0027 \ 815^+$	$\cdot 0016 \ 651$	$\cdot 0009 \ 891$	$\cdot 0005 \ 834$	$\cdot 0003 \ 419$	$\cdot 0001 \ 991$
$\cdot 0036 \ 381$	$\cdot 0022 \ 200$	$\cdot 0013 \ 443$	$\cdot 0008 \ 083$	$\cdot 0004 \ 829$	$\cdot 0002 \ 867$
$\cdot 0047 \ 002$	$\cdot 0029 \ 214$	$\cdot 0018 \ 020$	$\cdot 0011 \ 037$	$\cdot 0006 \ 717$	$\cdot 0004 \ 063$
$\cdot 0060 \ 031$	$\cdot 0037 \ 979$	$\cdot 0023 \ 846$	$\cdot 0014 \ 868$	$\cdot 0009 \ 211$	$\cdot 0005 \ 672$
$\cdot 0075 \ 855^+$	$\cdot 0048 \ 815^+$	$\cdot 0031 \ 178$	$\cdot 0019 \ 776$	$\cdot 0012 \ 464$	$\cdot 0007 \ 809$
$\cdot 0094 \ 894$	$\cdot 0062 \ 080$	$\cdot 0040 \ 310$	$\cdot 0025 \ 994$	$\cdot 0016 \ 657$	$\cdot 0010 \ 611$
$\cdot 0117 \ 598$	$\cdot 0078 \ 164$	$\cdot 0051 \ 568$	$\cdot 0033 \ 790$	$\cdot 0022 \ 001$	$\cdot 0014 \ 242$
$\cdot 0144 \ 450^+$	$\cdot 0097 \ 495^-$	$\cdot 0065 \ 319$	$\cdot 0043 \ 465^+$	$\cdot 0028 \ 742$	$\cdot 0018 \ 896$
$\cdot 0175 \ 957$	$\cdot 0120 \ 533$	$\cdot 0081 \ 965^-$	$\cdot 0055 \ 363$	$\cdot 0037 \ 162$	$\cdot 0024 \ 801$
$\cdot 0212 \ 648$	$\cdot 0147 \ 773$	$\cdot 0101 \ 946$	$\cdot 0069 \ 862$	$\cdot 0047 \ 579$	$\cdot 0032 \ 218$
$\cdot 0255 \ 075^-$	$\cdot 0179 \ 736$	$\cdot 0125 \ 739$	$\cdot 0087 \ 382$	$\cdot 0060 \ 353$	$\cdot 0041 \ 448$
$\cdot 0303 \ 799$	$\cdot 0216 \ 971$	$\cdot 0153 \ 856$	$\cdot 0108 \ 384$	$\cdot 0075 \ 886$	$\cdot 0052 \ 832$
$\cdot 0359 \ 393$	$\cdot 0260 \ 050^+$	$\cdot 0186 \ 840$	$\cdot 0133 \ 365^-$	$\cdot 0094 \ 620$	$\cdot 0066 \ 755^+$
$\cdot 0422 \ 430$	$\cdot 0309 \ 562$	$\cdot 0225 \ 264$	$\cdot 0162 \ 862$	$\cdot 0117 \ 042$	$\cdot 0083 \ 644$
$\cdot 0493 \ 480$	$\cdot 0366 \ 105^-$	$\cdot 0269 \ 726$	$\cdot 0197 \ 446$	$\cdot 0143 \ 677$	$\cdot 0103 \ 974$
$\cdot 0573 \ 099$	$\cdot 0430 \ 284$	$\cdot 0320 \ 843$	$\cdot 0237 \ 720$	$\cdot 0175 \ 095^+$	$\cdot 0128 \ 263$
$\cdot 0661 \ 826$	$\cdot 0502 \ 704$	$\cdot 0379 \ 249$	$\cdot 0284 \ 314$	$\cdot 0211 \ 901$	$\cdot 0157 \ 074$
$\cdot 0760 \ 168$	$\cdot 0583 \ 957$	$\cdot 0445 \ 582$	$\cdot 0337 \ 882$	$\cdot 0254 \ 733$	$\cdot 0191 \ 014$
$\cdot 0868 \ 601$	$\cdot 0674 \ 620$	$\cdot 0520 \ 483$	$\cdot 0399 \ 092$	$\cdot 0304 \ 262$	$\cdot 0230 \ 730$
$\cdot 0987 \ 553$	$\cdot 0775 \ 240$	$\cdot 0604 \ 581$	$\cdot 0468 \ 622$	$\cdot 0361 \ 181$	$\cdot 0276 \ 905^+$
$\cdot 1117 \ 400$	$\cdot 0886 \ 330$	$\cdot 0698 \ 492$	$\cdot 0547 \ 151$	$\cdot 0426 \ 201$	$\cdot 0330 \ 255^-$
$\cdot 1258 \ 456$	$\cdot 1008 \ 359$	$\cdot 0802 \ 801$	$\cdot 0635 \ 349$	$\cdot 0500 \ 043$	$\cdot 0391 \ 519$
$\cdot 1410 \ 967$	$\cdot 1141 \ 738$	$\cdot 0918 \ 057$	$\cdot 0733 \ 869$	$\cdot 0583 \ 426$	$\cdot 0461 \ 455^+$
$\cdot 1575 \ 100$	$\cdot 1286 \ 816$	$\cdot 1044 \ 704$	$\cdot 0843 \ 333$	$\cdot 0677 \ 063$	$\cdot 0540 \ 831$
$\cdot 1750 \ 940$	$\cdot 1443 \ 868$	$\cdot 1183 \ 363$	$\cdot 0964 \ 325^+$	$\cdot 0781 \ 644$	$\cdot 0630 \ 412$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .61$  to  $1.00$  $q = 5$ 

	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p =$
$I(p, q) = .2525 \pm .0001$		$.1909 \pm .0002$	$.1551 \pm .0016$	$.1239 \pm .0145$	$.0000$
.61	.49667 745	.4222 042	.3811 551	.3420 826	.3057
.62	.49457 101	.4190 052	.3780 831	.3389 128	.3033
.63	.49248 865	.4157 065	.3750 064	.3358 722	.3009
.64	.49041 444	.4124 077	.3719 047	.3328 238	.2986
.65	.48834 450	.4091 088	.3688 027	.3297 862	.2962
.66	.48627 567	.4058 096	.3657 181	.3267 603	.2938
.67	.48420 448	.4025 105	.3626 264	.3237 447	.2914
.68	.48212 117	.4001 2 644	.3595 126	.3207 381	.2890
.69	.48003 205	.3977 442	.3564 020	.3177 311	.2866
.70	.47793 000	.3953 2 44	.3533 146	.3147 280	.2842
.71	.47582 105	.3928 705	.3502 411	.3117 296	.2818
.72	.47370 644	.3904 185	.3471 818	.3087 262	.2794
.73	.47158 648	.3879 201	.3441 244	.3057 206	.2770
.74	.46946 144	.3854 255	.3410 422	.3027 172	.2746
.75	.46733 564	.3829 254	.3379 649	.3001 044	.2722
.76	.46520 100	.3804 295	.3348 487	.2974 870	.2698
.77	.46306 224	.3779 278	.3317 459	.2948 491	.2674
.78	.46091 860	.3754 285	.3286 267	.2921 278	.2650
.79	.45877 068	.3729 268	.3255 205	.2894 427	.2626
.80	.45662 145	.3704 262	.3224 064	.2867 067	.2602
.81	.45447 088	.3679 294	.3193 259	.2840 154	.2578
.82	.45231 604	.3654 265	.3162 428	.2813 204	.2554
.83	.45015 744	.3629 274	.3131 244	.2786 226	.2530
.84	.44799 442	.3604 269	.3100 244	.2759 244	.2506
.85	.44582 744	.3579 262	.3069 244	.2732 262	.2482
.86	.44365 604	.3554 254	.3038 244	.2705 262	.2458
.87	.44148 000	.3529 245	.3007 244	.2678 262	.2434
.88	.43930 444	.3504 237	.2976 244	.2651 262	.2410
.89	.43712 444	.3479 228	.2945 244	.2624 262	.2386
.90	.43494 000	.3454 219	.2914 244	.2597 262	.2362
.91	.43275 444	.3429 210	.2883 244	.2570 262	.2338
.92	.43056 444	.3404 201	.2852 244	.2543 262	.2314
.93	.42837 000	.3379 192	.2821 244	.2516 262	.2290
.94	.42617 444	.3354 183	.2790 244	.2489 262	.2266
.95	.42397 444	.3329 174	.2759 244	.2462 262	.2242
.96	.42177 444	.3304 165	.2728 244	.2435 262	.2218
.97	.41957 444	.3279 156	.2697 244	.2408 262	.2194
.98	.41737 444	.3254 147	.2666 244	.2381 262	.2170
.99	.41517 444	.3229 138	.2635 244	.2354 262	.2146
1.00	.41297 444	.3204 129	.2604 244	.2327 262	.2122

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

2 to 70

$q = 5$

$p = 11$

	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$
$q) = .6660\ 0067 \times \frac{1}{10^4}$	$.4578\ 7546 \times \frac{1}{10^4}$	$.3232\ 0621 \times \frac{1}{10^4}$	$.2334\ 2670 \times \frac{1}{10^4}$	$.1719\ 9862 \times \frac{1}{10^4}$	$.1289\ 9897$	
2	.0000 001					
3	.0000 001					
4	.0000 003	.0000 001				
5	.0000 007	.0000 001				
6	.0000 013	.0000 003	.0000 001			
7	.0000 024	.0000 005 <sup>+</sup>	.0000 001			
8	.0000 043	.0000 010	.0000 002	.0000 001		
9	.0000 074	.0000 019	.0000 005 <sup>-</sup>	.0000 001		
0	.0000 125 <sup>-</sup>	.0000 033	.0000 009	.0000 002	.0000 001	
1	.0000 204	.0000 057	.0000 015 <sup>+</sup>	.0000 004	.0000 001	
2	.0000 325 <sup>-</sup>	.0000 095 <sup>-</sup>	.0000 027	.0000 008	.0000 002	.0000 001
3	.0000 506	.0000 154	.0000 046	.0000 014	.0000 004	.0000 001
4	.0000 771	.0000 245 <sup>-</sup>	.0000 076	.0000 023	.0000 007	.0000 002
5	.0001 153	.0000 381	.0000 124	.0000 039	.0000 012	.0000 004
6	.0001 693	.0000 582	.0000 196	.0000 065 <sup>+</sup>	.0000 021	.0000 007
7	.0002 444	.0000 871	.0000 305 <sup>+</sup>	.0000 105 <sup>+</sup>	.0000 036	.0000 012
8	.0003 474	.0001 284	.0000 466	.0000 166	.0000 059	.0000 020
9	.0004 866	.0001 861	.0000 699	.0000 259	.0000 094	.0000 034
0	.0006 722	.0002 658	.0001 033	.0000 395 <sup>-</sup>	.0000 149	.0000 056
1	.0009 169	.0003 745 <sup>-</sup>	.0001 502	.0000 594	.0000 231	.0000 089
2	.0012 356	.0005 206	.0002 155 <sup>+</sup>	.0000 878	.0000 353	.0000 140
3	.0016 463	.0007 148	.0003 050 <sup>+</sup>	.0001 281	.0000 531	.0000 217
4	.0021 700	.0009 702	.0004 263	.0001 844	.0000 787	.0000 332
5	.0028 314	.0013 023	.0005 887	.0002 621	.0001 151	.0000 499
6	.0036 589	.0017 298	.0008 038	.0003 679	.0001 661	.0000 741
7	.0046 850 <sup>+</sup>	.0022 748	.0010 858	.0005 105 <sup>-</sup>	.0002 368	.0001 085
8	.0059 467	.0029 633	.0014 517	.0007 006	.0003 336	.0001 570
9	.0074 855 <sup>+</sup>	.0038 254	.0019 222	.0009 515 <sup>+</sup>	.0004 648	.0002 244
0	.0093 477	.0048 957	.0025 214	.0012 794	.0006 407	.0003 170
1	.0115 843	.0062 138	.0032 779	.0017 038	.0008 741	.0004 432
2	.0142 514	.0078 243	.0042 252	.0022 484	.0011 809	.0006 130
3	.0174 098	.0097 774	.0054 015 <sup>-</sup>	.0029 409	.0015 805 <sup>+</sup>	.0008 396
4	.0211 247	.0121 287	.0068 510	.0038 143	.0020 964	.0011 389
5	.0254 659	.0149 394	.0086 235 <sup>-</sup>	.0049 068	.0027 564	.0015 307
6	.0305 067	.0182 764	.0107 751	.0062 628	.0035 941	.0020 391
7	.0363 239	.0222 119	.0133 684	.0079 330	.0046 485 <sup>+</sup>	.0026 931
8	.0429 969	.0268 234	.0164 724	.0099 751	.0059 654	.0035 275
9	.0506 066	.0321 929	.0201 626	.0124 540	.0075 976	.0045 833
0	.0592 346	.0384 064	.0245 209	.0154 419	.0096 054	.0059 090
1	.0689 623	.0455 532	.0296 350 <sup>-</sup>	.0190 187	.0120 575 <sup>+</sup>	.0075 606
2	.0798 689	.0537 247	.0355 980	.0232 719	.0150 310	.0096 030
3	.0920 308	.0630 135 <sup>-</sup>	.0425 080	.0282 960	.0186 116	.0121 102
4	.1055 192	.0735 116	.0504 661	.0341 925 <sup>-</sup>	.0228 939	.0151 657
5	.1203 993	.0853 092	.0595 764	.0410 686	.0279 810	.0188 633
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TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $1.00$  $q = 5$ 

	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p$
$H(p, q) = .66666667 \times 10^6$		$.47587546 \times 10^6$	$.32320621 \times 10^6$	$.23342070 \times 10^6$	.17
.71	.34997251	.4852660	.3240087	.23671302	.131
.72	.3845548	.5212280	.36004353	.2631966	.135
.73	.4180064	.5575199	.39772412	.2909469	.138
.74	.45105577	.5949600	.43656562	.3202333	.142
.75	.48364859	.63291892	.47488864	.35186693	.146
.76	.51590449	.6709286	.51211451	.38586329	.150
.77	.54794594	.70868947	.5500086	.4202702	.154
.78	.57989297	.74647097	.58722221	.45687022	.159
.79	.61169492	.78424410	.62334079	.49580320	.163
.80	.64357664	.82222454	.65822332	.53693538	.167
.81	.67544904	.86022864	.69143466	.57932631	.171
.82	.70741060	.89824458	.72243525	.62384509	.175
.83	.73946126	.93626141	.75142612	.67044509	.178
.84	.77122087	.97428694	.78776286	.71917586	.182
.85	.80269946	1.01232487	.82192990	.76994486	.185
.86	.83391541	.10481822	.85221610	.82271434	.188
.87	.86485542	.09529005 <sup>+</sup>	.87901942	.87757383	.190
.88	.89553941	.06517908	.90554126	.9344729	.193
.89	.925912419	.0751511	.93079115 <sup>+</sup>	.99394857	.194
.90	.95622795 <sup>+</sup>	.08290960	.9528558	.10718061	.196
.91	.98647594	.08869390	.9714714	.12814594	.197
.92	1.01669494	.09242428	.98700901	.15583122	.198
.93	1.04672193	.09491572	.9999098	.18944486	.199
.94	1.07656967	.09660564	1.0000000	.22905599	.200
.95	1.10621754	.09761477	.0000000	.27584546	.201
.96	1.13569286	.09806046	.0000000	.32994448	.202
.97	1.16500444	.09809106	.0000000	.39148599	.203
.98	1.19419949	.09809884	.0000000	.46069709	.204
.99	1.22329999	.09809996	.0000000	.53780012	.205
1.00	1.25239999	1.00000000	1.00000000	1.00000000	1.000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

24 to .80

$q = 5$

$p = 1$

	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$
$\gamma(q) = .9828\ 4928 \times \frac{1}{10^8}$	$.7594\ 7444 \times \frac{1}{10^8}$	$.5943\ 7130 \times \frac{1}{10^8}$	$.4705\ 4395 \times \frac{1}{10^8}$	$.3764\ 3516 \times \frac{1}{10^8}$	$.3040\ 437$	
24	.0000 001					
25	.0000 001					
26	.0000 002	.0000 001				
27	.0000 004	.0000 001				
28	.0000 007	.0000 002	.0000 001			
29	.0000 012	.0000 004	.0000 001	.0000 001		
30	.0000 020	.0000 007	.0000 003	.0000 001		
31	.0000 034	.0000 013	.0000 005 <sup>-</sup>	.0000 002	.0000 001	
32	.0000 055 <sup>+</sup>	.0000 021	.0000 008	.0000 003	.0000 001	
33	.0000 088	.0000 035 <sup>+</sup>	.0000 014	.0000 006	.0000 002	.0000 001
34	.0000 138	.0000 057	.0000 023	.0000 010	.0000 004	.0000 002
35	.0000 214	.0000 091	.0000 038	.0000 016	.0000 007	.0000 003
36	.0000 327	.0000 143	.0000 062	.0000 027	.0000 011	.0000 005
37	.0000 492	.0000 221	.0000 098	.0000 043	.0000 019	.0000 008
38	.0000 731	.0000 337	.0000 154	.0000 070	.0000 031	.0000 014
39	.0001 071	.0000 507	.0000 238	.0000 110	.0000 051	.0000 023
40	.0001 552	.0000 753	.0000 362	.0000 172	.0000 082	.0000 038
41	.0002 223	.0001 104	.0000 544	.0000 266	.0000 129	.0000 062
42	.0003 149	.0001 602	.0000 808	.0000 404	.0000 201	.0000 099
43	.0004 413	.0002 298	.0001 186	.0000 607	.0000 309	.0000 156
44	.0006 123	.0003 261	.0001 722	.0000 902	.0000 469	.0000 242
45	.0008 413	.0004 580	.0002 472	.0001 324	.0000 704	.0000 372
46	.0011 450 <sup>+</sup>	.0006 370	.0003 513	.0001 923	.0001 045 <sup>-</sup>	.0000 564
47	.0015 444	.0008 774	.0004 943	.0002 763	.0001 533	.0000 845
48	.0020 047	.0011 974	.0006 886	.0003 929	.0002 226	.0001 253
49	.0027 371	.0016 196	.0009 504	.0005 534	.0003 200	.0001 838
50	.0035 987	.0021 718	.0012 997	.0007 719	.0004 553	.0002 668
51	.0046 938	.0028 877	.0017 619	.0010 669	.0006 415 <sup>+</sup>	.0003 833
52	.0060 748	.0038 085 <sup>-</sup>	.0023 681	.0014 614	.0008 956	.0005 454
53	.0078 030	.0049 830	.0031 563	.0019 843	.0012 389	.0007 686
54	.0099 492	.0064 694	.0041 728	.0026 715 <sup>+</sup>	.0016 987	.0010 733
55	.0125 948	.0083 360	.0054 731	.0035 670	.0023 090	.0014 853
56	.0158 324	.0106 620	.0071 232	.0047 242	.0031 121	.0020 374
57	.0197 059	.0135 389	.0092 008	.0062 074	.0041 600	.0027 707
58	.0245 110	.0170 707	.0117 964	.0080 933	.0055 159	.0037 363
59	.0301 948	.0213 748	.0150 146	.0104 721	.0072 560	.0049 971
60	.0369 556	.0265 819	.0189 745 <sup>-</sup>	.0134 491	.0094 708	.0066 292

61	.0449 419	.0328 361	.0238 105 <sup>+</sup>	.0171 458	.0122 673	.0087 246
62	.0543 105 <sup>-</sup>	.0402 940	.0296 725 <sup>+</sup>	.0217 009	.0157 700	.0113 924
63	.0652 248	.0491 235 <sup>-</sup>	.0367 254	.0272 703	.0201 222	.0147 611
64	.0778 519	.0595 019	.0451 481	.0340 277	.0254 873	.0189 802
65	.0923 592	.0716 134	.0551 319	.0421 636	.0320 484	.0242 211
66	.1089 101	.0856 449	.0668 777	.0518 837	.0400 084	.0306 779
67	.1276 580	.1017 824	.0805 021	.0621 062	.0495 885 <sup>+</sup>	.0385 572

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $1.00$  $q = 5$ 

	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$B(p, q) = .9828\ 4928 \times \frac{x}{10^8}$		$.7594\ 7444 \times \frac{x}{10^8}$	$.5943\ 7130 \times \frac{x}{10^8}$	$.4705\ 4395 \times \frac{x}{10^8}$	$.3764\ 35$
$x$					
.81	.6317 331	.5905 160	.5497 111	.5097 008	.4708 108
.82	.6768 640	.6381 149	.5992 796	.5607 305 <sup>+</sup>	.5227 988
.83	.7207 849	.6850 053	.6487 088	.6122 403	.5759 178
.84	.7628 742	.7304 842	.6972 304	.6634 165 <sup>-</sup>	.6293 321
.85	.8025 286	.7738 444	.7440 448	.7133 825 <sup>-</sup>	.6821 070
.86	.8391 894	.8144 050 <sup>-</sup>	.7883 539	.7612 333	.7332 448
.87	.8723 707	.8515 446	.8293 995 <sup>+</sup>	.8060 772	.7817 288
.88	.9016 879	.8847 376	.8665 067	.8470 860	.8265 777
.89	.9268 831	.9135 890	.8991 280	.8835 482	.8669 090
.90	.9478 476	.9378 663	.9268 869	.9149 251	.9020 064
.91	.9646 367	.9575 249	.9496 150 <sup>-</sup>	.9409 016	.9313 865
.92	.9774 739	.9727 216	.9673 779	.9614 266	.9548 565
.93	.9867 434	.9838 141	.9804 844	.9767 359	.9725 526
.94	.9929 660	.9913 404	.9894 729	.9873 478	.9849 507
.95	.9967 597	.9959 777	.9950 697	.9940 255 <sup>-</sup>	.9928 351
.96	.9987 835 <sup>+</sup>	.9984 774	.9981 181	.9977 006	.9972 197
.97	.9996 695 <sup>-</sup>	.9995 828	.9994 801	.9993 594	.9992 190
.98	.9999 502	.9999 366	.9999 203	.9999 010	.9998 783
.99	.9999 982	.9999 977	.9999 971	.9999 964	.9999 955
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to go

$q = 5$

$p = 2$

$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$
$= .2477\ 3938 \times 10^1$	$.2035\ 0020 \times 10^1$	$.1684\ 1306 \times 10^1$	$.1403\ 4497 \times 10^1$	$.1177\ 0868 \times 10^1$	$.9941\ 6000$
.0000 001					
.0000 001					
.0000 002	.0000 001				
.0000 004	.0000 002	.0000 001			
.0000 006	.0000 003	.0000 001	.0000 001		
.0000 011	.0000 005	.0000 002	.0000 001		
.0000 018	.0000 008	.0000 004	.0000 002	.0000 001	
.0000 030	.0000 014	.0000 007	.0000 003	.0000 001	.0000 001
.0000 049	.0000 024	.0000 011	.0000 006	.0000 003	.0000 001
.0000 078	.0000 039	.0000 019	.0000 010	.0000 005	.0000 002
.0000 124	.0000 064	.0000 032	.0000 016	.0000 008	.0000 003
.0000 195 <sup>+</sup>	.0000 102	.0000 053	.0000 027	.0000 014	.0000 007
.0000 303	.0000 161	.0000 086	.0000 045 <sup>+</sup>	.0000 023	.0000 012
.0000 463	.0000 252	.0000 137	.0000 074	.0000 039	.0000 021
.0000 701	.0000 390	.0000 216	.0000 119	.0000 065 <sup>+</sup>	.0000 039
.0001 049	.0000 596	.0000 337	.0000 189	.0000 106	.0000 066
.0001 554	.0000 900	.0000 519	.0000 297	.0000 170	.0000 107
.0002 276	.0001 344	.0000 790	.0000 462	.0000 269	.0000 156
.0003 301	.0001 987	.0001 190	.0000 710	.0000 421	.0000 249
.0004 741	.0002 908	.0001 775	.0001 078	.0000 652	.0000 392
.0006 742	.0004 212	.0002 618	.0001 620	.0000 928	.0000 642
.0009 499	.0006 042	.0003 824	.0002 409	.0001 511	.0000 914
.0013 261	.0008 585 <sup>+</sup>	.0005 530	.0003 546	.0002 263	.0001 494
.0018 348	.0012 086	.0007 921	.0005 167	.0003 456	.0002 171
.0025 165 <sup>+</sup>	.0016 859	.0011 239	.0007 458	.0004 928	.0003 214
.0034 220	.0023 311	.0015 801	.0010 662	.0007 164	.0004 794
.0046 142	.0031 950 <sup>+</sup>	.0022 015 <sup>+</sup>	.0015 191	.0010 414	.0006 476
.0061 706	.0043 417	.0030 401	.0021 191	.0014 709	.0010 169
.0081 848	.0058 503	.0041 616	.0029 171	.0020 784	.0014 595
.0107 696	.0078 176	.0056 479	.0040 622	.0029 096	.0020 758
.0140 586	.0103 609	.0076 000	.0055 502	.0040 496	.0029 244
.0182 083	.0136 203	.0101 411	.0075 177	.0055 592	.0040 899
.0234 000	.0177 613	.0134 195 <sup>+</sup>	.0100 954	.0075 649	.0056 457
.0298 405 <sup>+</sup>	.0229 766	.0176 114	.0134 415 <sup>+</sup>	.0102 179	.0076 182
.0377 620	.0294 877	.0229 234	.0177 455	.0136 828	.0102 189
.0474 217	.0375 449	.0295 041	.0242 494	.0184 647	.0134 692
.0590 987	.0474 269	.0375 939	.0320 539	.0242 014	.0177 091
.0730 899	.0594 375 <sup>+</sup>	.0481 287	.0405 148	.0311 136	.0249 097
.0897 032	.0739 011	.0606 273	.0505 411	.0404 410	.0324 173
.1092 489	.0911 555 <sup>+</sup>	.0757 459	.0626 930	.0517 059	.0424 936
.1320 277	.1115 419	.0938 541	.0786 716	.0657 084	.0546 930
.1583 163	.1353 873	.1153 243	.0978 699	.0827 694 <sup>+</sup>	.0676 574
.1883 500 <sup>+</sup>	.1620 061	.1408 155 <sup>+</sup>	.1206 662	.1044 894	.0846 504

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .91$  to  $1.00$  $q = 5$ 

	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$
$B(p, q) = .2477\ 3938 \times \frac{1}{10^8}$	$.2035\ 0020 \times \frac{1}{10^8}$	$.1684\ 1396 \times \frac{1}{10^8}$	$.1403\ 4497 \times \frac{1}{10^8}$	$.1177\ 0$	
$.91$	$.9099\ 929$	$.8981\ 504$	$.8855\ 777$	$.8723\ 059$	$.8583\ 70$
$.92$	$.9398\ 367$	$.9313\ 869$	$.9223\ 174$	$.9126\ 385^-$	$.9023\ 6$
$.93$	$.9628\ 307$	$.9572\ 732$	$.9512\ 434$	$.9447\ 384$	$.9377\ 5$
$.94$	$.9792\ 883$	$.9760\ 000$	$.9723\ 938$	$.9684\ 616$	$.9641\ 90$
$.95$	$.9899\ 773$	$.9882\ 916$	$.9864\ 233$	$.9843\ 645^-$	$.9821\ 0$
$.96$	$.9960\ 464$	$.9953\ 437$	$.9945\ 565^+$	$.9936\ 800$	$.9927\ 0$
$.97$	$.9988\ 710$	$.9986\ 593$	$.9984\ 198$	$.9981\ 503$	$.9978\ 4$
$.98$	$.9998\ 212$	$.9997\ 859$	$.9997\ 455^+$	$.9996\ 997$	$.9996\ 4$
$.99$	$.9999\ 933$	$.9999\ 919$	$.9999\ 903$	$.9999\ 884$	$.9999\ 8$
$1.00$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 00$



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 1.00

$q = 5$

$p$

$p = 29$	$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$
$\cdot 8426\ 8716 \times \frac{1}{10^6}$	$\cdot 7187\ 6258 \times \frac{1}{10^6}$	$\cdot 6160\ 8221 \times \frac{1}{10^6}$	$\cdot 5395\ 1524 \times \frac{1}{10^6}$	$\cdot 4588\ 2392 \times \frac{1}{10^6}$	$\cdot 3794\ 1322$
$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 0000\ 001$	$\cdot 0000\ 001$	$\cdot 0000\ 001$			
$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$			
$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 0000\ 011$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 019$	$\cdot 0000\ 010$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 033$	$\cdot 0000\ 018$	$\cdot 0000\ 010$	$\cdot 0000\ 005$	$\cdot 0000\ 003$	$\cdot 0000\ 002$
$\cdot 0000\ 055$	$\cdot 0000\ 031$	$\cdot 0000\ 017$	$\cdot 0000\ 010$	$\cdot 0000\ 005$	$\cdot 0000\ 003$
$\cdot 0000\ 090$	$\cdot 0000\ 052$	$\cdot 0000\ 030$	$\cdot 0000\ 017$	$\cdot 0000\ 010$	$\cdot 0000\ 005$
$\cdot 0000\ 146$	$\cdot 0000\ 086$	$\cdot 0000\ 050$	$\cdot 0000\ 029$	$\cdot 0000\ 017$	$\cdot 0000\ 009$
$\cdot 0000\ 235$	$\cdot 0000\ 141$	$\cdot 0000\ 084$	$\cdot 0000\ 050$	$\cdot 0000\ 029$	$\cdot 0000\ 016$
$\cdot 0000\ 374$	$\cdot 0000\ 228$	$\cdot 0000\ 138$	$\cdot 0000\ 083$	$\cdot 0000\ 050$	$\cdot 0000\ 029$
$\cdot 0000\ 587$	$\cdot 0000\ 364$	$\cdot 0000\ 225$	$\cdot 0000\ 148$	$\cdot 0000\ 083$	$\cdot 0000\ 052$
$\cdot 0000\ 911$	$\cdot 0000\ 575$	$\cdot 0000\ 362$	$\cdot 0000\ 222$	$\cdot 0000\ 143$	$\cdot 0000\ 085$
$\cdot 0001\ 390$	$\cdot 0000\ 868$	$\cdot 0000\ 575$	$\cdot 0000\ 362$	$\cdot 0000\ 244$	$\cdot 0000\ 143$
$\cdot 0002\ 126$	$\cdot 0001\ 388$	$\cdot 0000\ 904$	$\cdot 0000\ 586$	$\cdot 0000\ 362$	$\cdot 0000\ 243$
$\cdot 0003\ 195$	$\cdot 0002\ 123$	$\cdot 0001\ 395$	$\cdot 0000\ 912$	$\cdot 0000\ 586$	$\cdot 0000\ 362$
$\cdot 0004\ 755$	$\cdot 0003\ 211$	$\cdot 0002\ 161$	$\cdot 0001\ 450$	$\cdot 0000\ 912$	$\cdot 0000\ 586$
$\cdot 0007\ 004$	$\cdot 0004\ 807$	$\cdot 0003\ 288$	$\cdot 0002\ 242$	$\cdot 0001\ 524$	$\cdot 0001\ 044$
$\cdot 0010\ 216$	$\cdot 0007\ 124$	$\cdot 0004\ 951$	$\cdot 0003\ 431$	$\cdot 0002\ 420$	$\cdot 0001\ 644$
$\cdot 0014\ 755$	$\cdot 0010\ 452$	$\cdot 0007\ 379$	$\cdot 0005\ 194$	$\cdot 0004\ 645$	$\cdot 0002\ 550$
$\cdot 0021\ 108$	$\cdot 0015\ 184$	$\cdot 0010\ 886$	$\cdot 0007\ 781$	$\cdot 0007\ 545$	$\cdot 0004\ 941$
$\cdot 0029\ 911$	$\cdot 0021\ 843$	$\cdot 0015\ 890$	$\cdot 0011\ 548$	$\cdot 0008\ 448$	$\cdot 0006\ 024$
$\cdot 0041\ 987$	$\cdot 0031\ 120$	$\cdot 0022\ 902$	$\cdot 0016\ 945$	$\cdot 0012\ 448$	$\cdot 0009\ 140$
$\cdot 0058\ 393$	$\cdot 0043\ 016$	$\cdot 0032\ 923$	$\cdot 0024\ 608$	$\cdot 0018\ 440$	$\cdot 0014\ 641$
$\cdot 0080\ 459$	$\cdot 0061\ 385$	$\cdot 0046\ 685$	$\cdot 0035\ 300$	$\cdot 0026\ 666$	$\cdot 0020\ 153$
$\cdot 0100\ 847$	$\cdot 0084\ 904$	$\cdot 0065\ 560$	$\cdot 0050\ 420$	$\cdot 0035\ 667$	$\cdot 0026\ 575$
$\cdot 0148\ 596$	$\cdot 0116\ 578$	$\cdot 0091\ 178$	$\cdot 0071\ 104$	$\cdot 0055\ 225$	$\cdot 0039\ 175$
$\cdot 0199\ 175$	$\cdot 0158\ 397$	$\cdot 0125\ 585$	$\cdot 0099\ 284$	$\cdot 0076\ 274$	$\cdot 0054\ 549$
$\cdot 0264\ 523$	$\cdot 0213\ 193$	$\cdot 0171\ 308$	$\cdot 0137\ 260$	$\cdot 0109\ 681$	$\cdot 0076\ 117$
$\cdot 0348\ 080$	$\cdot 0284\ 237$	$\cdot 0231\ 417$	$\cdot 0187\ 883$	$\cdot 0132\ 141$	$\cdot 0122\ 968$
$\cdot 0453\ 792$	$\cdot 0375\ 355$	$\cdot 0300\ 572$	$\cdot 0254\ 612$	$\cdot 0208\ 865$	$\cdot 0170\ 895$
$\cdot 0586\ 084$	$\cdot 0490\ 933$	$\cdot 0410\ 055$	$\cdot 0341\ 520$	$\cdot 0281\ 685$	$\cdot 0245\ 105$
$\cdot 0740\ 797$	$\cdot 0635\ 880$	$\cdot 0537\ 761$	$\cdot 0454\ 567$	$\cdot 0381\ 562$	$\cdot 0340\ 242$
$\cdot 0950\ 060$	$\cdot 0815\ 534$	$\cdot 0698\ 148$	$\cdot 0596\ 009$	$\cdot 0509\ 660$	$\cdot 0451\ 344$
$\cdot 1192\ 103$	$\cdot 1035\ 503$	$\cdot 0897\ 068$	$\cdot 0775\ 176$	$\cdot 0689\ 175$	$\cdot 0613\ 371$
$\cdot 1480\ 988$	$\cdot 1301\ 423$	$\cdot 1130\ 666$	$\cdot 0997\ 260$	$\cdot 0919\ 216$	$\cdot 0833\ 373$
$\cdot 1821\ 257$	$\cdot 1618\ 631$	$\cdot 1444\ 917$	$\cdot 1266\ 982$	$\cdot 1149\ 645$	$\cdot 1056\ 675$
$\cdot 2216\ 493$	$\cdot 1991\ 797$	$\cdot 1785\ 354$	$\cdot 1596\ 640$	$\cdot 1424\ 675$	$\cdot 1296\ 465$
$\cdot 2668\ 806$	$\cdot 2423\ 956$	$\cdot 2196\ 400$	$\cdot 1985\ 244$	$\cdot 1791\ 406$	$\cdot 1642\ 253$
$\cdot 3178\ 263$	$\cdot 2916\ 779$	$\cdot 2670\ 810$	$\cdot 2440\ 344$	$\cdot 2254\ 087$	$\cdot 2042\ 087$
$\cdot 3742\ 301$	$\cdot 3469\ 004$	$\cdot 3208\ 825$	$\cdot 2962\ 075$	$\cdot 2728\ 806$	$\cdot 2509\ 265$
$\cdot 4355\ 177$	$\cdot 4076\ 218$	$\cdot 3807\ 485$			

# TABLE I. THE $I_z(p, q)$ FUNCTION

$\lambda = .49$  to  $1.00$

$q = 5$

$p$	$p = 35$	$p = 36$	$p = 37$	$p = 38$	$p = 39$
$I_z(p, q) = 34.36877 \times 10^{-6}$	$30.304707 \times 10^{-6}$	$26.688088 \times 10^{-6}$	$23.510935 \times 10^{-6}$	$20.777$	
.40	000000001				
.45	000000002	000000001	000000001		
.50	000000003	000000002	000000001	000000001	
.55	000000004	000000003	000000002	000000001	000000000
.60	000000005	000000004	000000003	000000002	000000001
.65	000000006	000000005	000000004	000000003	000000002
.70	000000007	000000006	000000005	000000004	000000003
.75	000000008	000000007	000000006	000000005	000000004
.80	000000009	000000008	000000007	000000006	000000005
.85	000000010	000000009	000000008	000000007	000000006
.90	000000011	000000010	000000009	000000008	000000007
.95	000000012	000000011	000000010	000000009	000000008
1.00	000000013	000000012	000000011	000000010	000000009
.41	000000014	000000013	000000012	000000011	000000010
.42	000000015	000000014	000000013	000000012	000000011
.43	000000016	000000015	000000014	000000013	000000012
.44	000000017	000000016	000000015	000000014	000000013
.45	000000018	000000017	000000016	000000015	000000014
.46	000000019	000000018	000000017	000000016	000000015
.47	000000020	000000019	000000018	000000017	000000016
.48	000000021	000000020	000000019	000000018	000000017
.49	000000022	000000021	000000020	000000019	000000018
.50	000000023	000000022	000000021	000000020	000000019
.51	000000024	000000023	000000022	000000021	000000020
.52	000000025	000000024	000000023	000000022	000000021
.53	000000026	000000025	000000024	000000023	000000022
.54	000000027	000000026	000000025	000000024	000000023
.55	000000028	000000027	000000026	000000025	000000024
.56	000000029	000000028	000000027	000000026	000000025
.57	000000030	000000029	000000028	000000027	000000026
.58	000000031	000000030	000000029	000000028	000000027
.59	000000032	000000031	000000030	000000029	000000028
.60	000000033	000000032	000000031	000000030	000000029
.61	000000034	000000033	000000032	000000031	000000030
.62	000000035	000000034	000000033	000000032	000000031
.63	000000036	000000035	000000034	000000033	000000032
.64	000000037	000000036	000000035	000000034	000000033
.65	000000038	000000037	000000036	000000035	000000034
.66	000000039	000000038	000000037	000000036	000000035
.67	000000040	000000039	000000038	000000037	000000036
.68	000000041	000000040	000000039	000000038	000000037
.69	000000042	000000041	000000040	000000039	000000038
.70	000000043	000000042	000000041	000000040	000000039
.71	000000044	000000043	000000042	000000041	000000040
.72	000000045	000000044	000000043	000000042	000000041
.73	000000046	000000045	000000044	000000043	000000042
.74	000000047	000000046	000000045	000000044	000000043
.75	000000048	000000047	000000046	000000045	000000044
.76	000000049	000000048	000000047	000000046	000000045
.77	000000050	000000049	000000048	000000047	000000046
.78	000000051	000000050	000000049	000000048	000000047
.79	000000052	000000051	000000050	000000049	000000048
.80	000000053	000000052	000000051	000000050	000000049
.81	000000054	000000053	000000052	000000051	000000050
.82	000000055	000000054	000000053	000000052	000000051
.83	000000056	000000055	000000054	000000053	000000052
.84	000000057	000000056	000000055	000000054	000000053
.85	000000058	000000057	000000056	000000055	000000054
.86	000000059	000000058	000000057	000000056	000000055
.87	000000060	000000059	000000058	000000057	000000056
.88	000000061	000000060	000000059	000000058	000000057
.89	000000062	000000061	000000060	000000059	000000058
.90	000000063	000000062	000000061	000000060	000000059
.91	000000064	000000063	000000062	000000061	000000060
.92	000000065	000000064	000000063	000000062	000000061
.93	000000066	000000065	000000064	000000063	000000062
.94	000000067	000000066	000000065	000000064	000000063
.95	000000068	000000067	000000066	000000065	000000064
.96	000000069	000000068	000000067	000000066	000000065
.97	000000070	000000069	000000068	000000067	000000066
.98	000000071	000000070	000000069	000000068	000000067
.99	000000072	000000071	000000070	000000069	000000068
1.00	000000073	000000072	000000071	000000070	000000069

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

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$q = 6.5$

$p = 6$

$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$
$\cdot 1730\ 2225 \times \frac{1}{10^5}$	$\cdot 1211\ 7644 \times \frac{1}{10^5}$	$\cdot 8651\ 1124 \times \frac{1}{10^4}$	$\cdot 6283\ 2229 \times \frac{1}{10^4}$	$\cdot 4634\ 5245 \times \frac{1}{10^4}$	$\cdot 3466\ 6057$
$\cdot 0000\ 001$	$\cdot 0000\ 002$				
$\cdot 0000\ 006$	$\cdot 0000\ 007$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 0000\ 024$	$\cdot 0000\ 007$	$\cdot 0000\ 008$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 076$	$\cdot 0000\ 025$	$\cdot 0000\ 008$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0000\ 196$	$\cdot 0000\ 069$	$\cdot 0000\ 024$	$\cdot 0000\ 008$	$\cdot 0000\ 008$	$\cdot 0000\ 003$
$\cdot 0000\ 445$	$\cdot 0000\ 166$	$\cdot 0000\ 061$	$\cdot 0000\ 022$	$\cdot 0000\ 021$	$\cdot 0000\ 008$
$\cdot 0000\ 908$	$\cdot 0000\ 360$	$\cdot 0000\ 140$	$\cdot 0000\ 054$	$\cdot 0000\ 048$	$\cdot 0000\ 019$
$\cdot 0001\ 711$	$\cdot 0000\ 714$	$\cdot 0000\ 294$	$\cdot 0000\ 119$		
$\cdot 0003\ 019$	$\cdot 0001\ 320$	$\cdot 0000\ 569$	$\cdot 0000\ 243$	$\cdot 0000\ 102$	$\cdot 0000\ 043$
$\cdot 0005\ 043$	$\cdot 0002\ 301$	$\cdot 0001\ 036$	$\cdot 0000\ 461$	$\cdot 0000\ 203$	$\cdot 0000\ 088$
$\cdot 0008\ 048$	$\cdot 0003\ 820$	$\cdot 0001\ 789$	$\cdot 0000\ 828$	$\cdot 0000\ 379$	$\cdot 0000\ 172$
$\cdot 0012\ 353$	$\cdot 0006\ 081$	$\cdot 0002\ 954$	$\cdot 0001\ 418$	$\cdot 0000\ 673$	$\cdot 0000\ 317$
$\cdot 0018\ 330$	$\cdot 0009\ 334$	$\cdot 0004\ 691$	$\cdot 0002\ 330$	$\cdot 0001\ 144$	$\cdot 0000\ 557$
$\cdot 0026\ 412$	$\cdot 0013\ 880$	$\cdot 0007\ 200$	$\cdot 0003\ 691$	$\cdot 0001\ 872$	$\cdot 0000\ 940$
$\cdot 0037\ 085$	$\cdot 0020\ 074$	$\cdot 0010\ 727$	$\cdot 0005\ 665$	$\cdot 0002\ 960$	$\cdot 0001\ 531$
$\cdot 0050\ 884$	$\cdot 0028\ 321$	$\cdot 0015\ 562$	$\cdot 0008\ 452$	$\cdot 0004\ 542$	$\cdot 0002\ 417$
$\cdot 0068\ 397$	$\cdot 0039\ 082$	$\cdot 0022\ 049$	$\cdot 0012\ 296$	$\cdot 0006\ 785$	$\cdot 0003\ 707$
$\cdot 0090\ 250$	$\cdot 0052\ 867$	$\cdot 0030\ 579$	$\cdot 0017\ 485$	$\cdot 0009\ 893$	$\cdot 0005\ 544$
$\cdot 0117\ 106$	$\cdot 0070\ 235$	$\cdot 0041\ 599$	$\cdot 0024\ 358$	$\cdot 0014\ 114$	$\cdot 0008\ 100$
$\cdot 0149\ 651$	$\cdot 0091\ 789$	$\cdot 0055\ 603$	$\cdot 0033\ 303$	$\cdot 0019\ 740$	$\cdot 0011\ 589$
$\cdot 0188\ 590$	$\cdot 0118\ 171$	$\cdot 0073\ 137$	$\cdot 0044\ 759$	$\cdot 0027\ 110$	$\cdot 0016\ 265$
$\cdot 0234\ 636$	$\cdot 0150\ 051$	$\cdot 0094\ 791$	$\cdot 0059\ 217$	$\cdot 0036\ 616$	$\cdot 0022\ 429$
$\cdot 0288\ 494$	$\cdot 0188\ 125$	$\cdot 0121\ 195$	$\cdot 0077\ 217$	$\cdot 0048\ 699$	$\cdot 0030\ 428$
$\cdot 0350\ 856$	$\cdot 0233\ 099$	$\cdot 0153\ 015$	$\cdot 0099\ 346$	$\cdot 0063\ 854$	$\cdot 0040\ 662$
$\cdot 0422\ 387$	$\cdot 0285\ 687$	$\cdot 0190\ 942$	$\cdot 0126\ 235$	$\cdot 0082\ 625$	$\cdot 0053\ 585$
$\cdot 0503\ 710$	$\cdot 0346\ 590$	$\cdot 0235\ 686$	$\cdot 0158\ 549$	$\cdot 0105\ 605$	$\cdot 0069\ 700$
$\cdot 0595\ 402$	$\cdot 0416\ 494$	$\cdot 0287\ 968$	$\cdot 0196\ 987$	$\cdot 0133\ 432$	$\cdot 0089\ 565$
$\cdot 0697\ 976$	$\cdot 0496\ 054$	$\cdot 0348\ 505$	$\cdot 0242\ 267$	$\cdot 0166\ 781$	$\cdot 0113\ 787$
$\cdot 0811\ 875$	$\cdot 0585\ 882$	$\cdot 0418\ 003$	$\cdot 0295\ 122$	$\cdot 0206\ 363$	$\cdot 0143\ 018$
$\cdot 0937\ 462$	$\cdot 0686\ 536$	$\cdot 0497\ 143$	$\cdot 0356\ 287$	$\cdot 0252\ 912$	$\cdot 0177\ 952$
$\cdot 1075\ 009$	$\cdot 0798\ 511$	$\cdot 0586\ 568$	$\cdot 0426\ 490$	$\cdot 0307\ 180$	$\cdot 0219\ 321$
$\cdot 1224\ 692$	$\cdot 0922\ 225$	$\cdot 0686\ 876$	$\cdot 0506\ 437$	$\cdot 0369\ 924$	$\cdot 0267\ 879$
$\cdot 1386\ 584$	$\cdot 1058\ 011$	$\cdot 0798\ 602$	$\cdot 0596\ 803$	$\cdot 0441\ 893$	$\cdot 0324\ 402$
$\cdot 1560\ 651$	$\cdot 1206\ 108$	$\cdot 0922\ 210$	$\cdot 0698\ 217$	$\cdot 0523\ 823$	$\cdot 0389\ 672$
$\cdot 1746\ 748$	$\cdot 1366\ 654$	$\cdot 1058\ 081$	$\cdot 0811\ 249$	$\cdot 0616\ 416$	$\cdot 0464\ 468$
$\cdot 1944\ 616$	$\cdot 1539\ 675$	$\cdot 1206\ 501$	$\cdot 0936\ 401$	$\cdot 0720\ 330$	$\cdot 0549\ 551$
$\cdot 2153\ 885$	$\cdot 1725\ 088$	$\cdot 1367\ 657$	$\cdot 1074\ 090$	$\cdot 0836\ 166$	$\cdot 0645\ 651$
$\cdot 2374\ 070$	$\cdot 1922\ 689$	$\cdot 1541\ 621$	$\cdot 1224\ 639$	$\cdot 0964\ 456$	$\cdot 0753\ 455$
$\cdot 2604\ 580$	$\cdot 2132\ 157$	$\cdot 1728\ 350$	$\cdot 1388\ 267$	$\cdot 1105\ 643$	$\cdot 0873\ 588$
$\cdot 2844\ 717$	$\cdot 2353\ 050$	$\cdot 1927\ 680$	$\cdot 1565\ 078$	$\cdot 1260\ 076$	$\cdot 1006\ 603$
$\cdot 3093\ 686$	$\cdot 2584\ 808$	$\cdot 2139\ 317$	$\cdot 1755\ 054$	$\cdot 1427\ 995$	$\cdot 1152\ 964$
$\cdot 3350\ 600$	$\cdot 2826\ 758$	$\cdot 2362\ 845$	$\cdot 1958\ 048$	$\cdot 1609\ 517$	$\cdot 1313\ 031$
$\cdot 3614\ 487$	$\cdot 3078\ 114$	$\cdot 2597\ 715$	$\cdot 2173\ 780$	$\cdot 1804\ 633$	$\cdot 1487\ 052$
$\cdot 3884\ 302$	$\cdot 3337\ 988$	$\cdot 2843\ 256$	$\cdot 2401\ 832$	$\cdot 2013\ 195$	$\cdot 1675\ 141$
$\cdot 4158\ 930$	$\cdot 3605\ 305$	$\cdot 3068\ 675$	$\cdot 2641\ 650$	$\cdot 2223\ 639$	$\cdot 1873\ 281$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .61$  to  $.98$  $q = 6.5$ 

	$p = 6.5$	$p = 7$	$p = 7.5$	$p = 8$	$p =$
$B(p, q) = .1730\ 2225 \times \frac{1}{10^3}$		$.1211\ 7644 \times \frac{1}{10^3}$	$.8651\ 1124 \times \frac{1}{10^1}$	$.6283\ 2229 \times \frac{1}{10^4}$	$.4634$
$x$					
.61	.7846 115 <sup>+</sup>	.7461 352	.7059 887	.6647 187	.6228
.62	.8055 384	.7695 671	.7317 269	.6925 089	.6524
.63	.8253 252	.7919 019	.7564 585 <sup>-</sup>	.7194 285 <sup>-</sup>	.6812
.64	.8439 349	.8130 753	.7800 908	.7453 564	.7092
.65	.8613 416	.8330 353	.8025 434	.7701 832	.7362
.66	.8775 308	.8517 425 <sup>-</sup>	.8237 493	.7938 124	.7622
.67	.8924 991	.8691 703	.8436 551	.8161 615 <sup>+</sup>	.7869
.68	.9062 538	.8853 051	.8622 219	.8371 634	.8103
.69	.9188 125 <sup>-</sup>	.9001 455 <sup>+</sup>	.8794 253	.8567 666	.8323
.70	.9302 024	.9137 027	.8952 554	.8749 361	.8528
.71	.9404 598	.9259 994	.9097 165 <sup>+</sup>	.8916 531	.8718
.72	.9496 290	.9370 691	.9228 266	.9069 155 <sup>+</sup>	.8893
.73	.9577 613	.9469 554	.9346 168	.9207 369	.9053
.74	.9649 144	.9557 110	.9451 302	.9331 461	.9197
.75	.9711 506	.9633 961	.9544 207	.9441 863	.9326
.76	.9765 364	.9700 775 <sup>+</sup>	.9625 519	.9539 134	.9441
.77	.9811 410	.9758 275 <sup>-</sup>	.9695 957	.9623 952	.9541
.78	.9850 349	.9807 217	.9756 302	.9697 090	.9629
.79	.9882 894	.9848 385 <sup>+</sup>	.9807 388	.9759 404	.9703
.80	.9909 750 <sup>-</sup>	.9882 571	.9850 078	.9811 808	.9767
.81	.9931 603	.9910 564	.9885 254	.9855 258	.9820
.82	.9949 116	.9933 136	.9913 794	.9890 728	.9863
.83	.9962 915 <sup>+</sup>	.9951 031	.9936 558	.9919 193	.9898
.84	.9973 588	.9964 954	.9954 375 <sup>+</sup>	.9941 607	.9926
.85	.9981 670	.9975 500	.9968 030	.9958 887	.9947
.86	.9987 647	.9983 451	.9978 249	.9971 895 <sup>+</sup>	.9964
.87	.9991 952	.9989 166	.9985 693	.9981 426	.9976
.88	.9994 957	.9993 179	.9990 950 <sup>+</sup>	.9988 196	.9984
.89	.9996 981	.9995 898	.9994 532	.9992 834	.9990
.90	.9998 289	.9997 664	.9996 871	.9995 881	.9994
.91	.9999 092	.9998 754	.9998 324	.9997 783	.9997
.92	.9999 555 <sup>+</sup>	.9999 387	.9999 172	.9998 900	.9998
.93	.9999 804	.9999 728	.9999 631	.9999 507	.9999
.94	.9999 924	.9999 895 <sup>-</sup>	.9999 856	.9999 807	.9999
.95	.9999 976	.9999 966	.9999 953	.9999 937	.9999
.96	.9999 994	.9999 992	.9999 988	.9999 984	.9999
.97	.9999 999	.9999 999	.9999 998	.9999 997	.9999
.98	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\cdot 08$  to  $\cdot 70$

$q = 6.5$

$p =$

	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$p, q) = \cdot 2626\ 2305 \times \frac{1}{10^4}$	$\cdot 2012\ 8678 \times \frac{1}{10^4}$	$\cdot 1559\ 3244 \times \frac{1}{10^4}$	$\cdot 1219\ 9199 \times \frac{1}{10^4}$	$\cdot 7668\ 0679 \times \frac{1}{10^3}$	$\cdot 4973\ 88$	
$\cdot 08$	$\cdot 0000\ 001$					
$\cdot 09$	$\cdot 0000\ 003$	$\cdot 0000\ 001$				
$\cdot 10$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$			
$\cdot 11$	$\cdot 0000\ 018$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 12$	$\cdot 0000\ 038$	$\cdot 0000\ 016$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 13$	$\cdot 0000\ 077$	$\cdot 0000\ 034$	$\cdot 0000\ 015^+$	$\cdot 0000\ 007$	$\cdot 0000\ 001$	
$\cdot 14$	$\cdot 0000\ 147$	$\cdot 0000\ 068$	$\cdot 0000\ 031$	$\cdot 0000\ 014$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 15$	$\cdot 0000\ 268$	$\cdot 0000\ 128$	$\cdot 0000\ 061$	$\cdot 0000\ 029$	$\cdot 0000\ 006$	$\cdot 0000\ 001$
$\cdot 16$	$\cdot 0000\ 468$	$\cdot 0000\ 231$	$\cdot 0000\ 113$	$\cdot 0000\ 055^-$	$\cdot 0000\ 013$	$\cdot 0000\ 001$
$\cdot 17$	$\cdot 0000\ 785^+$	$\cdot 0000\ 399$	$\cdot 0000\ 201$	$\cdot 0000\ 101$	$\cdot 0000\ 025^-$	$\cdot 0000\ 001$
$\cdot 18$	$\cdot 0001\ 275^-$	$\cdot 0000\ 667$	$\cdot 0000\ 346$	$\cdot 0000\ 178$	$\cdot 0000\ 046$	$\cdot 0000\ 001$
$\cdot 19$	$\cdot 0002\ 008$	$\cdot 0001\ 078$	$\cdot 0000\ 575^-$	$\cdot 0000\ 304$	$\cdot 0000\ 084$	$\cdot 0000\ 001$
$\cdot 20$	$\cdot 0003\ 079$	$\cdot 0001\ 696$	$\cdot 0000\ 927$	$\cdot 0000\ 503$	$\cdot 0000\ 145^+$	$\cdot 0000\ 001$
$\cdot 21$	$\cdot 0004\ 608$	$\cdot 0002\ 600$	$\cdot 0001\ 456$	$\cdot 0000\ 809$	$\cdot 0000\ 245^+$	$\cdot 0000\ 001$
$\cdot 22$	$\cdot 0006\ 745^-$	$\cdot 0003\ 893$	$\cdot 0002\ 230$	$\cdot 0001\ 269$	$\cdot 0000\ 403$	$\cdot 0000\ 001$
$\cdot 23$	$\cdot 0009\ 674$	$\cdot 0005\ 707$	$\cdot 0003\ 342$	$\cdot 0001\ 943$	$\cdot 0000\ 645^-$	$\cdot 0000\ 001$
$\cdot 24$	$\cdot 0013\ 619$	$\cdot 0008\ 204$	$\cdot 0004\ 905^-$	$\cdot 0002\ 912$	$\cdot 0001\ 007$	$\cdot 0000\ 001$
$\cdot 25$	$\cdot 0018\ 848$	$\cdot 0011\ 582$	$\cdot 0007\ 064$	$\cdot 0004\ 279$	$\cdot 0001\ 541$	$\cdot 0000\ 001$
$\cdot 26$	$\cdot 0025\ 672$	$\cdot 0016\ 080$	$\cdot 0009\ 998$	$\cdot 0006\ 174$	$\cdot 0002\ 310$	$\cdot 0000\ 001$
$\cdot 27$	$\cdot 0034\ 457$	$\cdot 0021\ 982$	$\cdot 0013\ 921$	$\cdot 0008\ 756$	$\cdot 0003\ 400$	$\cdot 0001\ 001$
$\cdot 28$	$\cdot 0045\ 615^+$	$\cdot 0029\ 620$	$\cdot 0019\ 093$	$\cdot 0012\ 224$	$\cdot 0004\ 919$	$\cdot 0001\ 001$
$\cdot 29$	$\cdot 0059\ 618$	$\cdot 0039\ 376$	$\cdot 0025\ 819$	$\cdot 0016\ 816$	$\cdot 0007\ 002$	$\cdot 0002\ 001$
$\cdot 30$	$\cdot 0076\ 988$	$\cdot 0051\ 688$	$\cdot 0034\ 454$	$\cdot 0022\ 813$	$\cdot 0009\ 818$	$\cdot 0004\ 001$
$\cdot 31$	$\cdot 0098\ 302$	$\cdot 0067\ 050^+$	$\cdot 0045\ 409$	$\cdot 0030\ 548$	$\cdot 0013\ 574$	$\cdot 0005\ 001$
$\cdot 32$	$\cdot 0124\ 189$	$\cdot 0086\ 011$	$\cdot 0059\ 149$	$\cdot 0040\ 408$	$\cdot 0018\ 519$	$\cdot 0008\ 001$
$\cdot 33$	$\cdot 0155\ 325^-$	$\cdot 0109\ 176$	$\cdot 0076\ 200$	$\cdot 0052\ 836$	$\cdot 0024\ 948$	$\cdot 0011\ 001$
$\cdot 34$	$\cdot 0192\ 430$	$\cdot 0137\ 202$	$\cdot 0097\ 144$	$\cdot 0068\ 335^+$	$\cdot 0033\ 213$	$\cdot 0015\ 001$
$\cdot 35$	$\cdot 0236\ 261$	$\cdot 0170\ 798$	$\cdot 0122\ 622$	$\cdot 0087\ 468$	$\cdot 0043\ 719$	$\cdot 0021\ 001$
$\cdot 36$	$\cdot 0287\ 603$	$\cdot 0210\ 717$	$\cdot 0153\ 330$	$\cdot 0110\ 859$	$\cdot 0056\ 937$	$\cdot 0028\ 001$
$\cdot 37$	$\cdot 0347\ 260$	$\cdot 0257\ 749$	$\cdot 0190\ 016$	$\cdot 0139\ 195^-$	$\cdot 0073\ 397$	$\cdot 0037\ 001$
$\cdot 38$	$\cdot 0416\ 043$	$\cdot 0312\ 713$	$\cdot 0233\ 471$	$\cdot 0173\ 216$	$\cdot 0093\ 702$	$\cdot 0049\ 001$
$\cdot 39$	$\cdot 0494\ 762$	$\cdot 0376\ 449$	$\cdot 0284\ 529$	$\cdot 0213\ 716$	$\cdot 0118\ 518$	$\cdot 0064\ 001$
$\cdot 40$	$\cdot 0584\ 206$	$\cdot 0449\ 803$	$\cdot 0344\ 048$	$\cdot 0261\ 538$	$\cdot 0148\ 580$	$\cdot 0082\ 001$
$\cdot 41$	$\cdot 0685\ 133$	$\cdot 0533\ 614$	$\cdot 0412\ 907$	$\cdot 0317\ 560$	$\cdot 0184\ 687$	$\cdot 0105\ 001$
$\cdot 42$	$\cdot 0798\ 252$	$\cdot 0628\ 701$	$\cdot 0491\ 990$	$\cdot 0382\ 687$	$\cdot 0227\ 699$	$\cdot 0132\ 001$
$\cdot 43$	$\cdot 0924\ 211$	$\cdot 0735\ 849$	$\cdot 0582\ 168$	$\cdot 0457\ 842$	$\cdot 0278\ 527$	$\cdot 0166\ 001$
$\cdot 44$	$\cdot 1063\ 576$	$\cdot 0855\ 786$	$\cdot 0684\ 290$	$\cdot 0543\ 945^-$	$\cdot 0338\ 129$	$\cdot 0206\ 001$
$\cdot 45$	$\cdot 1216\ 819$	$\cdot 0989\ 173$	$\cdot 0799\ 162$	$\cdot 0641\ 903$	$\cdot 0407\ 496$	$\cdot 0253\ 001$
$\cdot 46$	$\cdot 1384\ 302$	$\cdot 1136\ 583$	$\cdot 0927\ 527$	$\cdot 0752\ 591$	$\cdot 0487\ 636$	$\cdot 0309\ 001$
$\cdot 47$	$\cdot 1566\ 261$	$\cdot 1298\ 483$	$\cdot 1070\ 050^-$	$\cdot 0876\ 828$	$\cdot 0579\ 562$	$\cdot 0375\ 001$
$\cdot 48$	$\cdot 1762\ 793$	$\cdot 1475\ 219$	$\cdot 1227\ 296$	$\cdot 1015\ 366$	$\cdot 0684\ 272$	$\cdot 0452\ 001$
$\cdot 49$	$\cdot 1973\ 847$	$\cdot 1667\ 000$	$\cdot 1399\ 714$	$\cdot 1168\ 859$	$\cdot 0802\ 726$	$\cdot 0541\ 001$
$\cdot 50$	$\cdot 2199\ 211$	$\cdot 1873\ 883$	$\cdot 1587\ 616$	$\cdot 1337\ 852$	$\cdot 0935\ 828$	$\cdot 0642\ 001$
$\cdot 51$	$\cdot 2438\ 504$	$\cdot 2095\ 760$	$\cdot 1791\ 161$	$\cdot 1522\ 751$	$\cdot 1084\ 400$	$\cdot 0758\ 001$

TABLE I. THE  $I_{\omega}(p, q)$  FUNCTION[illegible]

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

16 to 80		$q = 6.5$		$p = 1$	
$p = 14$		$p = 15$		$p = 16$	
$p = 17$		$p = 18$		$p = 19$	
$p, q) = .3315\ 9213 \times \frac{1}{10^3}$					
$p, q) = .2264\ 5316 \times \frac{1}{10^3}$					
$p, q) = .1579\ 9058 \times \frac{1}{10^3}$					
$p, q) = .1123\ 4885 \times \frac{1}{10^3}$					
$p, q) = .8127\ 3640 \times \frac{1}{10^3}$					
$p, q) = .5971\ 124$					
$p, q) = .0000\ 001$					
$p, q) = .0000\ 001$					
$p, q) = .0000\ 003$					
$p, q) = .0000\ 006$					
$p, q) = .0000\ 011$					
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$p, q) = .0000\ 006$					
$p, q) = .0000\ 012$					
$p, q) = .0000\ 004$					
$p, q) = .0000\ 007$					
$p, q) = .0000\ 013$					
$p, q) = .0000\ 022$					
$p, q) = .0000\ 039$					
$p, q) = .0000\ 066$					
$p, q) = .0000\ 109$					
$p, q) = .0000\ 177$					
$p, q) = .0000\ 277$					
$p, q) = .0000\ 435$					
$p, q) = .0000\ 672$					
$p, q) = .0001\ 021$					
$p, q) = .0001\ 528$					
$p, q) = .0002\ 252$					
$p, q) = .0003\ 274$					
$p, q) = .0004\ 695$					
$p, q) = .0006\ 648$					
$p, q) = .0009\ 300$					
$p, q) = .0012\ 858$					
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$p, q) = .0118\ 914$					
$p, q) = .015$					

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.99$  $q = 6.5$ 

	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$B(p, q) = .3315\ 9213 \times \frac{1}{10^8}$		$.2264\ 5316 \times \frac{1}{10^8}$	$.1579\ 9058 \times \frac{1}{10^8}$	$.1123\ 4885 \times \frac{1}{10^8}$	$.8127\ 3011 \times \frac{1}{10^8}$
$x$					
.81	.9023 977	.8792 793	.8536 873	.8258 321	.7959 601
.82	.9220 723	.9027 556	.8811 080	.8572 551	.8313 601
.83	.9390 583	.9232 720	.9053 652	.8853 934	.8634 501
.84	.9534 348	.9408 468	.9263 957	.9100 841	.8919 401
.85	.9653 374	.9555 712	.9442 261	.9312 679	.9166 801
.86	.9749 514	.9676 050	.9589 704	.9489 921	.9376 301
.87	.9825 034	.9771 680	.9708 241	.9634 078	.9548 601
.88	.9882 495 <sup>-</sup>	.9845 281	.9800 526	.9747 602	.9685 901
.89	.9924 635 <sup>+</sup>	.9899 874	.9869 755 <sup>+</sup>	.9833 735 <sup>+</sup>	.9791 301
.90	.9954 235 <sup>+</sup>	.9938 652	.9919 484	.9896 304	.9868 601
.91	.9973 987	.9964 816	.9953 410	.9939 463	.9922 601
.92	.9986 371	.9981 401	.9975 151	.9967 426	.9958 001
.93	.9993 559	.9991 132	.9988 046	.9984 190	.9979 401
.94	.9997 340	.9996 305 <sup>-</sup>	.9994 975 <sup>-</sup>	.9993 295 <sup>-</sup>	.9991 201
.95	.9999 086	.9998 719	.9998 242	.9997 634	.9996 801
.96	.9999 759	.9999 660	.9999 529	.9999 360	.9999 101
.97	.9999 958	.9999 941	.9999 917	.9999 886	.9999 841
.98	.9999 997	.9999 995 <sup>+</sup>	.9999 993	.9999 991	.9999 989
.99	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

7 to 80

$q = 6.5$

$p = 2$

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$q) = .4449\ 0732 \times \frac{1}{10^6}$	$.3357\ 7911 \times \frac{1}{10^6}$	$.2564\ 1314 \times \frac{1}{10^6}$	$.1979\ 3295 \times \frac{1}{10^6}$	$.1543\ 2060 \times \frac{1}{10^6}$	$.1214\ 326$	
7	.0000 001					
8	.0000 002	.0000 001				
9	.0000 003	.0000 001				
0	.0000 006	.0000 002	.0000 001			
1	.0000 011	.0000 004	.0000 002	.0000 001		
2	.0000 019	.0000 008	.0000 003	.0000 001		
3	.0000 033	.0000 014	.0000 006	.0000 002	.0000 001	
4	.0000 056	.0000 024	.0000 010	.0000 004	.0000 002	.0000 001
5	.0000 093	.0000 041	.0000 018	.0000 008	.0000 003	.0000 001
6	.0000 150 <sup>+</sup>	.0000 068	.0000 030	.0000 013	.0000 006	.0000 003
7	.0000 240	.0000 111	.0000 051	.0000 023	.0000 011	.0000 005
8	.0000 378	.0000 180	.0000 085 <sup>-</sup>	.0000 040	.0000 018	.0000 008
9	.0000 585 <sup>+</sup>	.0000 286	.0000 138	.0000 066	.0000 032	.0000 015
0	.0000 894	.0000 447	.0000 222	.0000 109	.0000 053	.0000 026
1	.0001 346	.0000 690	.0000 351	.0000 177	.0000 089	.0000 044
2	.0002 001	.0001 051	.0000 547	.0000 282	.0000 145 <sup>-</sup>	.0000 074
3	.0002 937	.0001 578	.0000 841	.0000 444	.0000 233	.0000 123
4	.0004 259	.0002 340	.0001 275 <sup>+</sup>	.0000 689	.0000 370	.0000 197
5	.0006 104	.0003 429	.0001 910	.0001 055 <sup>+</sup>	.0000 579	.0000 311
6	.0008 649	.0004 964	.0002 825 <sup>+</sup>	.0001 595 <sup>+</sup>	.0000 894	.0000 493
7	.0012 123	.0007 106	.0004 130	.0002 382	.0001 364	.0000 773
8	.0016 813	.0010 059	.0005 968	.0003 513	.0002 054	.0001 192
9	.0023 079	.0014 088	.0008 528	.0005 123	.0003 055 <sup>+</sup>	.0001 810
0	.0031 369	.0019 527	.0012 056	.0007 386	.0004 493	.0002 711
1	.0042 225 <sup>+</sup>	.0026 795 <sup>-</sup>	.0016 864	.0010 533	.0006 533	.0004 021
2	.0056 307	.0036 408	.0023 350 <sup>+</sup>	.0014 863	.0009 394	.0005 891
3	.0074 399	.0048 998	.0032 010	.0020 755 <sup>-</sup>	.0013 363	.0008 541
4	.0097 429	.0065 330	.0043 456	.0028 691	.0018 812	.0012 251
5	.0126 476	.0086 312	.0058 437	.0039 272	.0026 211	.0017 381
6	.0162 783	.0113 020	.0077 854	.0053 238	.0036 157	.0024 391
7	.0207 758	.0146 700	.0102 782	.0071 490	.0049 389	.0033 901
8	.0262 983	.0188 787	.0134 482	.0095 111	.0066 816	.0046 641
9	.0330 198	.0240 901	.0174 417	.0125 386	.0089 539	.0063 541
0	.0411 295 <sup>-</sup>	.0304 852	.0224 260	.0163 816	.0118 877	.0085 731
1	.0508 293	.0382 627	.0285 894	.0212 135 <sup>+</sup>	.0156 383	.0114 581
2	.0623 302	.0476 366	.0361 406	.0272 313	.0203 865 <sup>+</sup>	.0151 701
3	.0758 482	.0588 335 <sup>+</sup>	.0453 069	.0346 547	.0263 390	.0198 991
4	.0915 982	.0720 878	.0563 308	.0437 252	.0337 285 <sup>-</sup>	.0258 641
5	.1097 873	.0876 355 <sup>-</sup>	.0694 657	.0547 027	.0428 121	.0333 121
6	.1306 066	.1057 070	.0849 692	.0678 605 <sup>+</sup>	.0538 686	.0425 171
7	.1542 216	.1265 180	.1030 952	.0834 786	.0671 926	.0537 801
8	.1807 627	.1502 589	.1240 838	.1018 349	.0830 878	.0674 181
9	.2103 140	.1770 838	.1481 498	.1231 942	.1018 571	.0837 601
0	.2429 029	.2070 974	.1754 693	.1477 946	.1237 899	.1031 351

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $1.00$  $q = 6.5$ 

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$
$B(p, q) = .4449\ 0732 \times \frac{x}{10^8}$	$.3357\ 7911 \times \frac{x}{10^8}$	$.2564\ 1314 \times \frac{x}{10^8}$	$.1979\ 3295 \times \frac{x}{10^8}$	$.1543\ 2036$	
$x$					
.81	.7314 059	.6973 416	.6625 231	.6272 693	.5918 852
.82	.7743 498	.7437 126	.7120 105 <sup>-</sup>	.6795 157	.6464 982
.83	.8142 355 <sup>-</sup>	.7873 090	.7591 066	.7298 467	.6997 535
.84	.8505 201	.8274 492	.8029 939	.7773 159	.7505 885
.85	.8827 810	.8635 646	.8429 527	.8210 526	.7979 860
.86	.9107 410	.8952 349	.8784 071	.8603 173	.8410 398
.87	.9342 870	.9222 167	.9089 653	.8945 544	.8790 187
.88	.9534 783	.9444 620	.9344 495 <sup>+</sup>	.9234 358	.9114 260
.89	.9685 427	.9621 224	.9549 119	.9468 901	.9380 435
.90	.9798 594	.9755 387	.9706 317	.9651 112	.9589 548
.91	.9879 289	.9852 118	.9820 917	.9785 425 <sup>+</sup>	.9745 405
.92	.9933 315 <sup>+</sup>	.9917 592	.9899 338	.9878 346	.9854 415
.93	.9966 772	.9958 578	.9948 961	.9937 782	.9924 900
.94	.9985 531	.9981 805 <sup>+</sup>	.9977 386	.9972 192	.9966 143
.95	.9994 758	.9993 351	.9991 663	.9989 660	.9987 301
.96	.9998 545 <sup>-</sup>	.9998 138	.9997 645 <sup>+</sup>	.9997 054	.9996 350
.97	.9999 735 <sup>-</sup>	.9999 658	.9999 563	.9999 449	.9999 311
.98	.9999 978	.9999 971	.9999 962	.9999 952	.9999 940
.99	1.0000 000	1.0000 000	1.0000 000	.9999 999	.9999 999
1.00				1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

5 to 1.00

$q = 6.5$

$p = 2$

	$p = 26$	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$
$q) = .9637\ 5084 \times \frac{1}{10^7}$		$.7710\ 0067 \times \frac{1}{10^7}$	$.6214\ 0353 \times \frac{1}{10^7}$	$.5043\ 2750 \times \frac{1}{10^7}$	$.4119\ 8585 \times \frac{1}{10^7}$	$.3386\ 185$
5	.0000 001					
6	.0000 001					
7	.0000 002	.0000 001				
8	.0000 004	.0000 002	.0000 001			
9	.0000 007	.0000 003	.0000 002	.0000 001		
0	.0000 012	.0000 006	.0000 003	.0000 001	.0000 001	
1	.0000 022	.0000 011	.0000 005 <sup>+</sup>	.0000 003	.0000 001	.0000 001
2	.0000 037	.0000 019	.0000 009	.0000 005 <sup>-</sup>	.0000 002	.0000 001
3	.0000 063	.0000 032	.0000 017	.0000 008	.0000 004	.0000 002
4	.0000 104	.0000 055 <sup>-</sup>	.0000 029	.0000 015 <sup>-</sup>	.0000 008	.0000 004
5	.0000 171	.0000 092	.0000 049	.0000 026	.0000 014	.0000 007
6	.0000 275 <sup>+</sup>	.0000 152	.0000 083	.0000 045 <sup>+</sup>	.0000 024	.0000 013
7	.0000 438	.0000 246	.0000 138	.0000 076	.0000 042	.0000 023
8	.0000 688	.0000 394	.0000 225 <sup>-</sup>	.0000 128	.0000 072	.0000 041
9	.0001 066	.0000 624	.0000 363	.0000 210	.0000 121	.0000 069
0	.0001 630	.0000 973	.0000 578	.0000 341	.0000 201	.0000 117
1	.0002 464	.0001 500 <sup>+</sup>	.0000 908	.0000 547	.0000 328	.0000 196
2	.0003 681	.0002 284	.0001 409	.0000 865 <sup>+</sup>	.0000 529	.0000 321
3	.0005 434	.0003 435 <sup>+</sup>	.0002 160	.0001 351	.0000 841	.0000 521
4	.0007 934	.0005 108	.0003 271	.0002 084	.0001 321	.0000 834
5	.0011 457	.0007 509	.0004 895 <sup>+</sup>	.0003 175 <sup>+</sup>	.0002 050 <sup>-</sup>	.0001 317
6	.0016 367	.0010 917	.0007 243	.0004 782	.0003 142	.0002 055
7	.0023 136	.0015 700	.0010 598	.0007 118	.0004 759	.0003 167
8	.0032 369	.0022 339	.0015 337	.0010 477	.0007 124	.0004 823
9	.0044 831	.0031 456	.0021 957	.0015 251	.0010 545 <sup>-</sup>	.0007 259
0	.0061 474	.0043 839	.0031 102	.0021 959	.0015 433	.0010 799
1	.0083 472	.0060 481	.0043 599	.0031 279	.0022 338	.0015 885
2	.0112 247	.0082 610	.0060 491	.0044 084	.0031 983	.0023 104
3	.0149 502	.0111 725 <sup>-</sup>	.0083 077	.0061 484	.0045 300	.0033 235
4	.0197 240	.0149 629	.0112 951	.0084 866	.0063 483	.0047 289
5	.0257 785 <sup>-</sup>	.0198 458	.0152 040	.0115 942	.0088 028	.0066 558
6	.0333 778	.0260 695 <sup>+</sup>	.0202 635 <sup>-</sup>	.0156 788	.0120 790	.0092 676
7	.0428 172	.0339 181	.0267 411	.0209 880	.0164 023	.0127 667
8	.0544 190	.0437 095 <sup>+</sup>	.0349 436	.0278 119	.0220 426	.0174 003
9	.0685 270	.0557 923	.0452 154	.0364 838	.0293 164	.0234 642
0	.0854 972	.0705 383	.0579 341	.0473 780	.0385 874	.0313 058
1	.1056 851	.0883 324	.0735 023	.0609 047	.0502 640	.0413 241
2	.1294 297	.1095 581	.0923 360	.0775 004	.0647 929	.0539 661
3	.1570 340	.1345 786	.1148 470	.0976 135 <sup>+</sup>	.0826 471	.0697 187
4	.1887 416	.1637 144	.1414 216	.1216 845 <sup>-</sup>	.1043 090	.0890 939
5	.2247 115 <sup>-</sup>	.1972 158	.1723 933	.1501 195 <sup>-</sup>	.1302 459	.1126 081
6	.2649 909	.2352 339	.2080 118	.1832 591	.1608 793	.1407 524
7	.3094 890	.2777 896	.2484 089	.2213 420	.1965 477	.1739 560
8	.3579 531	.3247 431	.2935 626	.2644 645 <sup>+</sup>	.2374 635 <sup>+</sup>	.2125 416
9	.4099 402	.3757 657	.3429 655 <sup>+</sup>	.3149 655 <sup>+</sup>	.2864 653	.2566 653

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .42$  to  $1.00$  $q = 6.5$ 

	$p = 32$	$p = 33$	$p = 34$	$p = 35$	$p =$
$B(p, q) = .2799\ 2463 \times \frac{1}{10^7}$		$.2326\ 6463 \times \frac{1}{10^7}$	$.1943\ 7804 \times \frac{1}{10^7}$	$.1631\ 8157 \times \frac{1}{10^7}$	$.137$
$x$					
.42	.0000 001				
.43	.0000 001	.0000 001			
.44	.0000 002	.0000 001			
.45	.0000 004	.0000 002	.0000 001		
.46	.0000 007	.0000 004	.0000 002	.0000 001	
.47	.0000 013	.0000 007	.0000 004	.0000 002	.0000
.48	.0000 023	.0000 013	.0000 007	.0000 004	.0000
.49	.0000 040	.0000 023	.0000 013	.0000 007	.0000
.50	.0000 068	.0000 040	.0000 023	.0000 013	.0000
.51	.0000 116	.0000 069	.0000 041	.0000 024	.0000
.52	.0000 195-	.0000 117	.0000 071	.0000 042	.0000
.53	.0000 322	.0000 198	.0000 121	.0000 074	.0000
.54	.0000 524	.0000 328	.0000 205-	.0000 127	.0000
.55	.0000 843	.0000 537	.0000 341	.0000 216	.0000
.56	.0001 339	.0000 869	.0000 501	.0000 362	.0000
.57	.0002 099	.0001 386	.0000 911	.0000 597	.0000
.58	.0003 252	.0002 184	.0001 461	.0000 974	.0000
.59	.0004 976	.0003 398	.0002 312	.0001 567	.0001
.60	.0007 526	.0005 224	.0003 613	.0002 490	.0001
.61	.0011 249	.0007 936	.0005 578	.0003 907	.0002
.62	.0016 623	.0011 914	.0008 508	.0006 054	.0004
.63	.0024 286	.0017 679	.0012 823	.0009 268	.0006
.64	.0035 085+	.0025 933	.0019 099	.0014 018	.0010
.65	.0050 126	.0037 610	.0028 118	.0020 951	.0015
.66	.0070 828	.0053 930	.0040 919	.0030 943	.0023
.67	.0098 987	.0076 468	.0058 866	.0045 165-	.0034
.68	.0136 834	.0107 215+	.0083 717	.0065 154	.0050
.69	.0187 097	.0148 653	.0117 706	.0092 898	.0073
.70	.0253 044	.0203 813	.0163 608	.0130 912	.0104
.71	.0338 506	.0276 324	.0224 817	.0182 331	.0147
.72	.0447 878	.0370 436	.0305 385+	.0250 972	.0205
.73	.0586 065-	.0491 003	.0410 042	.0341 380	.0283
.74	.0758 372	.0643 411	.0544 161	.0458 835+	.0385
.75	.0970 328	.0833 435+	.0713 655-	.0609 287	.0518
.76	.1227 421	.1067 016	.0924 791	.0799 214	.0688
.77	.1534 740	.1349 923	.1183 895+	.1035 374	.0903
.78	.1896 536	.1687 325+	.1496 944	.1324 424	.1168
.79	.2315 689	.2083 254	.1869 027	.1672 410	.1492
.80	.2793 139	.2539 986	.2303 710	.2084 112	.1880
.81	.3327 292	.3057 376	.2802 306	.2562 278	.2337
.82	.3913 482	.3632 212	.3363 131	.3106 791	.2863
.83	.4543 571	.4257 663	.3980 809	.3713 848	.3457
.84	.5205 759	.4922 928	.4645 755-	.4375 266	.4112
.85	.5884 729	.5613 206	.5343 945-	.5078 049	.4811
.86	.6584 729+				

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$x$  .48 to 1.00

$q = 6.5$

$p =$

	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$(p, q) = .9915\ 5563 \times \frac{1}{10^8}$		$.8467\ 2166 \times \frac{1}{10^8}$	$.7257\ 6142 \times \frac{1}{10^8}$	$.6243\ 1090 \times \frac{1}{10^8}$	$.5388\ 7888 \times \frac{1}{10^8}$	$.4666\ 58$
$x$						
.48	.0000 001					
.49	.0000 001	.0000 001				
.50	.0000 002	.0000 001	.0000 001			
.51	.0000 005-	.0000 003	.0000 002	.0000 001	.0000 001	
.52	.0000 009	.0000 005+	.0000 003	.0000 002	.0000 001	.0000 00
.53	.0000 016	.0000 010	.0000 006	.0000 004	.0000 002	.0000 00
.54	.0000 030	.0000 018	.0000 011	.0000 007	.0000 004	.0000 00
.55	.0000 054	.0000 033	.0000 021	.0000 013	.0000 008	.0000 00
.56	.0000 095-	.0000 060	.0000 038	.0000 024	.0000 015+	.0000 01
.57	.0000 165-	.0000 107	.0000 069	.0000 044	.0000 028	.0000 01
.58	.0000 283	.0000 186	.0000 122	.0000 080	.0000 052	.0000 03
.59	.0000 479	.0000 320	.0000 214	.0000 142	.0000 095-	.0000 06
.60	.0000 799	.0000 544	.0000 369	.0000 250-	.0000 169	.0000 11
.61	.0001 316	.0000 911	.0000 628	.0000 432	.0000 297	.0000 20
.62	.0002 140	.0001 504	.0001 054	.0000 737	.0000 514	.0000 35
.63	.0003 433	.0002 451	.0001 745+	.0001 239	.0000 878	.0000 62
.64	.0005 438	.0003 943	.0002 851	.0002 056	.0001 479	.0001 06
.65	.0008 504	.0006 260	.0004 596	.0003 365-	.0002 458	.0001 79
.66	.0013 132	.0009 811	.0007 311	.0005 434	.0004 028	.0002 98
.67	.0020 024	.0015 181	.0011 479	.0008 658	.0006 514	.0004 88
.68	.0030 155-	.0023 193	.0017 791	.0013 613	.0010 391	.0007 91
.69	.0044 848	.0034 985-	.0027 219	.0021 124	.0016 355-	.0012 63
.70	.0065 874	.0052 105-	.0041 107	.0032 350-	.0025 398	.0019 89
.71	.0095 558	.0076 621	.0061 280	.0048 890	.0038 913	.0030 90
.72	.0136 891	.0111 243	.0090 172	.0072 914	.0058 822	.0047 34
.73	.0193 646	.0159 445+	.0130 958	.0107 303	.0087 718	.0071 54
.74	.0270 468	.0225 590	.0187 697	.0155 800	.0129 031	.0106 62
.75	.0372 936	.0315 015-	.0265 447	.0223 160	.0187 191	.0156 68
.76	.0507 561	.0434 077	.0370 354	.0315 265+	.0267 781	.0226 96
.77	.0681 685+	.0590 110	.0509 653	.0439 183	.0377 642	.0324 05
.78	.0903 261	.0791 255+	.0691 570	.0603 124	.0524 882	.0455 86
.79	.1180 458	.1046 134	.0925 052	.0816 246	.0718 758	.0631 65
.80	.1521 080	.1363 313	.1219 301	.1088 250+	.0969 345+	.0861 76
.81	.1931 780	.1750 551	.1583 053	.1428 725+	.1286 950+	.1157 07
.82	.2417 081	.2213 800	.2023 602	.1846 195-	.1681 206	.1528 19
.83	.2978 246	.2756 025+	.2545 571	.2346 875+	.2159 835+	.1984 26
.84	.3612 116	.3375 910	.3149 515-	.2933 194	.2727 110	.2531 32
.85	.4310 063	.4066 612	.3830 495+	.3602 200	.3382 118	.3170 55
.86	.5057 268	.4814 789	.4576 850-	.4344 086	.4117 055+	.3896 24
.87	.5832 598	.5600 183	.5369 468	.5141 145+	.4915 858	.4694 19
.88	.6609 322	.6396 058	.6181 918	.5967 564	.5753 629	.5540 71
.89	.7356 889	.7170 778	.6981 780	.6790 443	.6597 309	.6402 91
.90	.8043 817	.7890 668	.7733 396	.7572 388	.7408 043	.7240 76
.91	.8641 541	.8524 038	.8402 021	.8275 728	.8145 205-	.8011 55

$x = .53$  to  $1.00$  $q = 6.5$ 

	$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$
$B(p, q) = .4053\ 7968 \times \frac{1}{10^8}$	$.3532\ 0210 \times \frac{1}{10^8}$	$.3086\ 2319 \times \frac{1}{10^8}$	$.2704\ 1270 \times \frac{1}{10^8}$	$.2375\ 5882 \times \frac{1}{10^8}$	
$x$					
.53	.0000 001				
.54	.0000 002	.0000 001	.0000 001		
.55	.0000 003	.0000 002	.0000 001	.0000 001	
.56	.0000 006	.0000 004	.0000 002	.0000 001	.0000 001
.57	.0000 012	.0000 007	.0000 005	.0000 003	.0000 002
.58	.0000 022	.0000 014	.0000 009	.0000 006	.0000 004
.59	.0000 041	.0000 027	.0000 018	.0000 012	.0000 008
.60	.0000 076	.0000 051	.0000 034	.0000 023	.0000 015 <sup>+</sup>
.61	.0000 139	.0000 094	.0000 064	.0000 044	.0000 030
.62	.0000 248	.0000 172	.0000 119	.0000 082	.0000 056
.63	.0000 437	.0000 308	.0000 216	.0000 151	.0000 106
.64	.0000 760	.0000 543	.0000 387	.0000 275 <sup>+</sup>	.0000 196
.65	.0001 302	.0000 944	.0000 684	.0000 494	.0000 356
.66	.0002 199	.0001 619	.0001 190	.0000 872	.0000 638
.67	.0003 661	.0002 736	.0002 040	.0001 518	.0001 128
.68	.0006 012	.0004 558	.0003 449	.0002 604	.0001 963
.69	.0009 736	.0007 488	.0005 747	.0004 402	.0003 365 <sup>-</sup>
.70	.0015 548	.0012 126	.0009 438	.0007 331	.0005 684
.71	.0024 487	.0019 362	.0015 279	.0012 034	.0009 460
.72	.0038 029	.0030 481	.0024 382	.0019 465 <sup>+</sup>	.0015 511
.73	.0058 236	.0047 303	.0038 346	.0031 026	.0025 056
.74	.0087 928	.0072 361	.0059 433	.0048 723	.0039 870
.75	.0130 871	.0109 095 <sup>-</sup>	.0090 766	.0075 376	.0062 483
.76	.0191 982	.0162 069	.0136 557	.0114 850 <sup>-</sup>	.0096 421
.77	.0277 505 <sup>+</sup>	.0237 187	.0202 347	.0172 312	.0146 480
.78	.0395 139	.0341 855 <sup>-</sup>	.0295 214	.0254 484	.0218 997
.79	.0554 046	.0485 069	.0423 917	.0369 831	.0322 103
.80	.0764 692	.0677 326	.0598 892	.0528 641	.0465 865 <sup>-</sup>
.81	.1038 419	.0930 295 <sup>-</sup>	.0832 009	.0742 879	.0662 236
.82	.1386 684	.1256 138	.1136 006	.1025 719	.0924 702
.83	.1819 897	.1666 422	.1523 468	.1390 629	.1267 471
.84	.2345 838	.2170 550 <sup>-</sup>	.2005 311	.1849 915 <sup>-</sup>	.1704 107
.85	.2967 718	.2773 758	.2588 742	.2412 676	.2245 506
.86	.3682 051	.3474 823	.3274 825 <sup>-</sup>	.3082 261	.2897 277
.87	.4476 675 <sup>+</sup>	.4263 781	.4055 925 <sup>-</sup>	.3853 469	.3656 720
.88	.5329 380	.5120 161	.4913 545 <sup>+</sup>	.4709 984	.4509 888
.89	.6207 767	.6012 379	.5817 230	.5622 780	.5429 469
.90	.7070 953	.6899 022	.6725 371	.6550 398	.6374 496
.91	.7873 564	.7732 608	.7588 661	.7442 007	.7292 935 <sup>+</sup>
.92	.8572 035 <sup>-</sup>	.8466 003	.8356 532	.8243 776	.8127 902
.93	.9132 457	.9060 831	.8986 077	.8908 243	.8827 387
.94	.9538 260	.9496 159	.9451 747	.9405 008	.9355 933
.95	.9795 114	.9774 611	.9752 751	.9729 502	.9704 831
.96	.9930 219	.9922 598	.9914 388	.9905 564	.9896 101
.97	.9984 370	.9982 517	.9980 500 <sup>-</sup>	.9978 309	.9975 935 <sup>+</sup>
.98	.9998 372	.9998 163	.9997 934	.9997 682	.9997 407
.99	.9999 974	.9999 970	.9999 966	.9999 962	.9999 957
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$= .04 \text{ to } .60$

$q = 7$

$p =$

	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$
$(p, q) = .8325 \ 0083 \times \frac{1}{10^4}$	$.5834 \ 4212 \times \frac{1}{10^4}$	$.4162 \ 5042 \times \frac{1}{10^4}$	$.3017 \ 8041 \times \frac{1}{10^4}$	$.2220 \ 0022 \times \frac{1}{10^4}$	$.1654 \ 92$	
$x$						
.04	.0000 002	.0000 001				
.05	.0000 010	.0000 003	.0000 001			
.06	.0000 035 <sup>-</sup>	.0000 011	.0000 004	.0000 001		
.07	.0000 097	.0000 034	.0000 012	.0000 004	.0000 001	
.08	.0000 233	.0000 087	.0000 032	.0000 012	.0000 004	.0000 001
.09	.0000 503	.0000 200	.0000 078	.0000 030	.0000 012	.0000 001
.10	.0000 993	.0000 416	.0000 172	.0000 070	.0000 028	.0000 011
.11	.0001 827	.0000 802	.0000 348	.0000 149	.0000 063	.0000 021
.12	.0003 171	.0001 454	.0000 658	.0000 294	.0000 130	.0000 051
.13	.0005 239	.0002 498	.0001 176	.0000 548	.0000 252	.0000 111
.14	.0008 298	.0004 104	.0002 004	.0000 968	.0000 462	.0000 211
.15	.0012 675 <sup>+</sup>	.0006 485 <sup>-</sup>	.0003 276	.0001 637	.0000 809	.0000 391
.16	.0018 758	.0009 904	.0005 165 <sup>+</sup>	.0002 663	.0001 359	.0000 681
.17	.0026 995 <sup>-</sup>	.0014 682	.0007 887	.0004 190	.0002 203	.0001 141
.18	.0037 897	.0021 193	.0011 708	.0006 396	.0003 458	.0001 851
.19	.0052 035 <sup>+</sup>	.0029 875 <sup>-</sup>	.0016 945 <sup>-</sup>	.0009 505 <sup>-</sup>	.0005 277	.0002 901
.20	.0070 036	.0041 222	.0023 972	.0013 788	.0007 850 <sup>-</sup>	.0004 421
.21	.0092 574	.0055 789	.0033 221	.0019 567	.0011 409	.0006 591
.22	.0120 368	.0074 185 <sup>-</sup>	.0045 182	.0027 220	.0016 235 <sup>+</sup>	.0009 591
.23	.0154 169	.0097 070	.0060 403	.0037 183	.0022 662	.0013 681
.24	.0194 752	.0125 150 <sup>-</sup>	.0079 489	.0049 949	.0031 078	.0019 161
.25	.0242 901	.0159 165 <sup>-</sup>	.0103 095 <sup>+</sup>	.0066 071	.0041 930	.0026 361
.26	.0299 400	.0199 884	.0131 924	.0086 157	.0055 723	.0035 711
.27	.0365 018	.0248 092	.0166 717	.0110 869	.0073 020	.0047 661
.28	.0440 494	.0304 579	.0208 246	.0140 914	.0094 445 <sup>-</sup>	.0062 741
.29	.0526 525 <sup>-</sup>	.0370 124	.0257 302	.0177 045 <sup>+</sup>	.0120 671	.0081 521
.30	.0623 752	.0445 486	.0314 685 <sup>+</sup>	.0220 043	.0152 425 <sup>+</sup>	.0104 661
.31	.0732 747	.0531 388	.0381 192	.0270 714	.0190 474	.0132 861
.32	.0853 996	.0628 501	.0457 600	.0329 874	.0235 618	.0166 861
.33	.0987 895 <sup>+</sup>	.0737 433	.0544 656	.0398 339	.0288 685 <sup>+</sup>	.0207 451
.34	.1134 732	.0858 713	.0643 058	.0476 909	.0350 512	.0255 461
.35	.1294 682	.0992 779	.0753 446	.0566 352	.0421 938	.0311 751
.36	.1467 798	.1139 967	.0876 380	.0667 393	.0503 786	.0377 171
.37	.1654 004	.1300 498	.1012 332	.0780 096	.0596 849	.0452 611
.38	.1853 096	.1474 471	.1161 672	.0906 848	.0701 875 <sup>-</sup>	.0538 891
.39	.2064 732	.1661 855 <sup>-</sup>	.1324 652	.1046 346	.0819 548	.0636 851
.40	.2288 440	.1862 481	.1501 401	.1199 579	.0950 474	.0747 241
.41	.2523 614	.2076 043	.1691 911	.1366 818	.1095 164	.0870 771
.42	.2769 524	.2302 092	.1896 034	.1548 205 <sup>-</sup>	.1254 018	.1008 061
.43	.3025 317	.2540 040	.2113 473	.1743 737	.1427 310	.1159 621
.44	.3290 026	.2789 161	.2343 784	.1953 265 <sup>+</sup>	.1615 177	.1325 851
.45	.3562 582	.3048 598	.2586 371	.2176 483	.1817 605 <sup>-</sup>	.1507 001
.46	.3841 825 <sup>-</sup>	.3317 369	.2840 492	.2412 926	.2034 419	.1703 201
.47	.4126 515 <sup>+</sup>	.3594 375 <sup>+</sup>	.3105 262	.2661 967	.2265 281	.1914 391
.48	.4415 240	.3878 418	.3370 660	.2922 821	.2500 681	.2142 361

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .61$  to  $.98$  $q = 7$ 

	$p = 7$	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$
$B(p, q) = .8325\ 0083 \times \frac{1}{10^4}$	$.5834\ 4212 \times \frac{1}{10^4}$	$.4162\ 5042 \times \frac{1}{10^4}$	$.3017\ 8041 \times \frac{1}{10^4}$	$.2220\ 0000$	
$\cdot 61$	$\cdot 7935\ 268$	$\cdot 7573\ 837$	$\cdot 7195\ 189$	$\cdot 6804\ 056$	$\cdot 6405\ 181$
$\cdot 62$	$\cdot 8146\ 904$	$\cdot 7810\ 645^-$	$\cdot 7455\ 481$	$\cdot 7085\ 601$	$\cdot 6705\ 281$
$\cdot 63$	$\cdot 8345\ 996$	$\cdot 8035\ 218$	$\cdot 7704\ 324$	$\cdot 7356\ 941$	$\cdot 6996\ 881$
$\cdot 64$	$\cdot 8532\ 202$	$\cdot 8246\ 932$	$\cdot 7940\ 784$	$\cdot 7616\ 833$	$\cdot 7278\ 381$
$\cdot 65$	$\cdot 8705\ 318$	$\cdot 8445\ 303$	$\cdot 8164\ 081$	$\cdot 7864\ 181$	$\cdot 7548\ 481$
$\cdot 66$	$\cdot 8865\ 268$	$\cdot 8630\ 004$	$\cdot 8373\ 594$	$\cdot 8098\ 051$	$\cdot 7805\ 711$
$\cdot 67$	$\cdot 9012\ 105^-$	$\cdot 8800\ 851$	$\cdot 8568\ 865^+$	$\cdot 8317\ 682$	$\cdot 8049\ 181$
$\cdot 68$	$\cdot 9146\ 004$	$\cdot 8957\ 811$	$\cdot 8749\ 607$	$\cdot 8522\ 492$	$\cdot 8277\ 881$
$\cdot 69$	$\cdot 9267\ 253$	$\cdot 9100\ 992$	$\cdot 8915\ 699$	$\cdot 8712\ 089$	$\cdot 8491\ 181$
$\cdot 70$	$\cdot 9376\ 248$	$\cdot 9230\ 637$	$\cdot 9067\ 181$	$\cdot 8886\ 267$	$\cdot 8688\ 571$
$\cdot 71$	$\cdot 9473\ 475^+$	$\cdot 9347\ 113$	$\cdot 9204\ 253$	$\cdot 9045\ 004$	$\cdot 8869\ 741$
$\cdot 72$	$\cdot 9559\ 506$	$\cdot 9450\ 905^+$	$\cdot 9327\ 259$	$\cdot 9188\ 458$	$\cdot 9034\ 681$
$\cdot 73$	$\cdot 9634\ 982$	$\cdot 9542\ 596$	$\cdot 9436\ 682$	$\cdot 9316\ 960$	$\cdot 9183\ 381$
$\cdot 74$	$\cdot 9700\ 600$	$\cdot 9622\ 859$	$\cdot 9533\ 123$	$\cdot 9430\ 996$	$\cdot 9316\ 281$
$\cdot 75$	$\cdot 9757\ 099$	$\cdot 9692\ 436$	$\cdot 9617\ 292$	$\cdot 9531\ 193$	$\cdot 9433\ 781$
$\cdot 76$	$\cdot 9805\ 248$	$\cdot 9752\ 127$	$\cdot 9689\ 984$	$\cdot 9618\ 307$	$\cdot 9533\ 681$
$\cdot 77$	$\cdot 9845\ 831$	$\cdot 9802\ 770$	$\cdot 9752\ 064$	$\cdot 9693\ 194$	$\cdot 9625\ 781$
$\cdot 78$	$\cdot 9879\ 632$	$\cdot 9845\ 225^-$	$\cdot 9804\ 446$	$\cdot 9756\ 793$	$\cdot 9701\ 881$
$\cdot 79$	$\cdot 9907\ 426$	$\cdot 9880\ 359$	$\cdot 9848\ 073$	$\cdot 9810\ 104$	$\cdot 9766\ 081$
$\cdot 80$	$\cdot 9929\ 964$	$\cdot 9909\ 030$	$\cdot 9883\ 901$	$\cdot 9854\ 161$	$\cdot 9819\ 481$
$\cdot 81$	$\cdot 9947\ 965^-$	$\cdot 9932\ 071$	$\cdot 9912\ 874$	$\cdot 9890\ 013$	$\cdot 9863\ 181$
$\cdot 82$	$\cdot 9962\ 103$	$\cdot 9950\ 281$	$\cdot 9935\ 914$	$\cdot 9918\ 698$	$\cdot 9898\ 381$
$\cdot 83$	$\cdot 9973\ 005^+$	$\cdot 9964\ 409$	$\cdot 9953\ 898$	$\cdot 9941\ 226$	$\cdot 9926\ 181$
$\cdot 84$	$\cdot 9981\ 242$	$\cdot 9975\ 147$	$\cdot 9967\ 649$	$\cdot 9958\ 556$	$\cdot 9947\ 681$
$\cdot 85$	$\cdot 9987\ 325^-$	$\cdot 9983\ 124$	$\cdot 9977\ 925^+$	$\cdot 9971\ 583$	$\cdot 9963\ 981$
$\cdot 86$	$\cdot 9991\ 702$	$\cdot 9988\ 898$	$\cdot 9985\ 408$	$\cdot 9981\ 125^+$	$\cdot 9975\ 981$
$\cdot 87$	$\cdot 9994\ 761$	$\cdot 9992\ 958$	$\cdot 9990\ 699$	$\cdot 9987\ 912$	$\cdot 9984\ 581$
$\cdot 88$	$\cdot 9996\ 829$	$\cdot 9995\ 717$	$\cdot 9994\ 316$	$\cdot 9992\ 578$	$\cdot 9990\ 481$
$\cdot 89$	$\cdot 9998\ 173$	$\cdot 9997\ 520$	$\cdot 9996\ 694$	$\cdot 9995\ 662$	$\cdot 9994\ 381$
$\cdot 90$	$\cdot 9999\ 007$	$\cdot 9998\ 646$	$\cdot 9998\ 186$	$\cdot 9997\ 609$	$\cdot 9996\ 881$
$\cdot 91$	$\cdot 9999\ 497$	$\cdot 9999\ 311$	$\cdot 9999\ 073$	$\cdot 9998\ 773$	$\cdot 9998\ 381$
$\cdot 92$	$\cdot 9999\ 767$	$\cdot 9999\ 679$	$\cdot 9999\ 566$	$\cdot 9999\ 423$	$\cdot 9999\ 281$
$\cdot 93$	$\cdot 9999\ 903$	$\cdot 9999\ 866$	$\cdot 9999\ 818$	$\cdot 9999\ 757$	$\cdot 9999\ 681$
$\cdot 94$	$\cdot 9999\ 965^+$	$\cdot 9999\ 952$	$\cdot 9999\ 934$	$\cdot 9999\ 911$	$\cdot 9999\ 881$
$\cdot 95$	$\cdot 9999\ 990$	$\cdot 9999\ 986$	$\cdot 9999\ 980$	$\cdot 9999\ 974$	$\cdot 9999\ 981$
$\cdot 96$	$\cdot 9999\ 998$	$\cdot 9999\ 997$	$\cdot 9999\ 996$	$\cdot 9999\ 994$	$\cdot 9999\ 991$
$\cdot 97$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 991$
$\cdot 98$			$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

8 to 70

$q = 7$

$p = 10$

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$q) = .1248\ 7512 \times \frac{1}{10^4}$	$.9528\ 3550 \times \frac{1}{10^3}$	$.7345\ 5956 \times \frac{1}{10^3}$	$.4488\ 9751 \times \frac{1}{10^3}$	$.2835\ 1422 \times \frac{1}{10^3}$	$.1842\ 8424 \times \frac{1}{10^3}$	
8	.0000 001					
9	.0000 002	.0000 001				
0	.0000 005-	.0000 002	.0000 001			
1	.0000 011	.0000 005-	.0000 002			
2	.0000 025-	.0000 011	.0000 005-	.0000 001		
3	.0000 052	.0000 023	.0000 010	.0000 002		
4	.0000 103	.0000 048	.0000 022	.0000 005-	.0000 001	
5	.0000 192	.0000 093	.0000 044	.0000 010	.0000 002	
6	.0000 344	.0000 171	.0000 084	.0000 020	.0000 005-	.0000 001
7	.0000 592	.0000 303	.0000 154	.0000 039	.0000 010	.0000 002
8	.0000 983	.0000 518	.0000 271	.0000 072	.0000 019	.0000 005-
9	.0001 583	.0000 856	.0000 460	.0000 130	.0000 036	.0000 010
0	.0002 476	.0001 373	.0000 756	.0000 225-	.0000 065+	.0000 018
1	.0003 774	.0002 144	.0001 209	.0000 377	.0000 115-	.0000 034
2	.0005 621	.0003 267	.0001 885+	.0000 615-	.0000 196	.0000 061
3	.0008 195-	.0004 868	.0002 871	.0000 978	.0000 325+	.0000 106
4	.0011 714	.0007 105+	.0004 278	.0001 520	.0000 527	.0000 179
5	.0016 445-	.0010 175+	.0006 250+	.0002 312	.0000 835-	.0000 295+
6	.0022 702	.0014 318	.0008 965+	.0003 446	.0001 293	.0000 475+
7	.0030 857	.0019 822	.0012 642	.0005 042	.0001 963	.0000 749
8	.0041 338	.0027 028	.0017 546	.0007 250+	.0002 926	.0001 156
9	.0054 633	.0036 333	.0023 992	.0010 259	.0004 285-	.0001 753
0	.0071 295+	.0048 197	.0032 353	.0014 298	.0006 173	.0002 610
1	.0091 936	.0063 140	.0043 061	.0019 646	.0008 757	.0003 824
2	.0117 228	.0081 749	.0056 612	.0026 635-	.0012 246	.0005 516
3	.0147 901	.0104 671	.0073 567	.0035 657	.0016 891	.0007 840
4	.0184 736	.0132 618	.0094 554	.0047 167	.0023 000	.0010 991
5	.0228 559	.0166 359	.0120 267	.0061 690	.0030 938	.0015 207
6	.0280 230	.0206 714	.0151 462	.0079 820	.0041 133	.0020 778
7	.0340 635+	.0254 549	.0188 956	.0102 223	.0054 087	.0028 056
8	.0410 670	.0310 765-	.0233 618	.0129 640	.0070 372	.0037 456
9	.0491 230	.0376 284	.0286 360	.0162 879	.0090 643	.0049 468
0	.0583 189	.0452 038	.0348 127	.0202 816	.0115 629	.0064 659
1	.0687 387	.0538 953	.0419 885+	.0250 386	.0146 145-	.0083 680
2	.0804 607	.0637 929	.0502 603	.0306 575-	.0183 077	.0107 269
3	.0935 561	.0749 825-	.0597 237	.0372 405+	.0227 389	.0136 252
4	.1080 866	.0875 435-	.0704 711	.0448 925+	.0280 107	.0171 544
5	.1241 030	.1015 471	.0825 896	.0537 190	.0342 313	.0214 144
6	.1416 430	.1170 540	.0961 590	.0638 240	.0415 129	.0265 129
7	.1607 297	.1341 126	.1112 492	.0753 084	.0499 701	.0325 647
8	.1813 698	.1527 568	.1279 183	.0882 670	.0597 181	.0396 899
9	.2035 528	.1730 044	.1462 103	.1027 865-	.0708 699	.0480 128
0	.2272 491	.1948 550-	.1661 530	.1189 423	.0835 342	.0576 591
1	.2524 102	.2182 892	.1877 557	.1367 062	.0998 100	.0680 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.99$  $q = 7$ 

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$B(p, q) = .1248\ 7512 \times \frac{x}{100}$	$.9528\ 3550 \times \frac{x}{100}$	$.7345\ 5956 \times \frac{x}{100}$	$.4488\ 9751 \times \frac{x}{100}$	$.2835\ 1422$	
.71	.8473 907	.8255 220	.8024 238	.7530 828	.7005 346
.72	.8683 469	.8487 440	.8278 935 <sup>-</sup>	.7828 799	.7342 651
.73	.8875 136	.8701 311	.8515 139	.8108 997	.7664 271
.74	.9048 755 <sup>-</sup>	.8896 372	.8732 050 <sup>-</sup>	.8369 855 <sup>-</sup>	.7967 818
.75	.9204 427	.9072 456	.8929 184	.8610 152	.8251 241
.76	.9342 504	.9229 681	.9106 381	.8829 043	.8512 879
.77	.9463 564	.9368 439	.9263 796	.9026 072	.8751 501
.78	.9568 399	.9489 380	.9401 893	.9201 178	.8966 342
.79	.9657 978	.9593 388	.9521 416	.9354 686	.9157 112
.80	.9733 427	.9681 543	.9623 366	.9487 290	.9323 999
.81	.9795 984	.9755 094	.9708 958	.9600 017	.9467 654
.82	.9846 971	.9815 412	.9779 585 <sup>-</sup>	.9694 188	.9589 151
.83	.9887 751	.9863 950 <sup>+</sup>	.9836 765 <sup>+</sup>	.9771 365 <sup>-</sup>	.9689 941
.84	.9919 693	.9902 199	.9882 096	.9833 288	.9771 790
.85	.9944 137	.9931 643	.9917 200	.9881 815 <sup>-</sup>	.9836 698
.86	.9962 360	.9953 722	.9943 679	.9918 850 <sup>-</sup>	.9886 820
.87	.9975 547	.9969 793	.9963 064	.9946 280	.9924 377
.88	.9984 771	.9981 099	.9976 779	.9965 911	.9951 565 <sup>+</sup>
.89	.9990 973	.9988 744	.9986 107	.9979 414	.9970 479
.90	.9994 955 <sup>-</sup>	.9993 679	.9992 162	.9988 279	.9983 036
.91	.9997 373	.9996 694	.9995 882	.9993 784	.9990 921
.92	.9998 747	.9998 416	.9998 018	.9996 981	.9995 549
.93	.9999 465 <sup>+</sup>	.9999 321	.9999 146	.9998 687	.9998 047
.94	.9999 803	.9999 748	.9999 682	.9999 506	.9999 259
.95	.9999 940	.9999 923	.9999 903	.9999 848	.9999 769
.96	.9999 986	.9999 982	.9999 978	.9999 965 <sup>-</sup>	.9999 946
.97	.9999 998	.9999 997	.9999 997	.9999 995 <sup>-</sup>	.9999 992
.98	I.0000 000	I.0000 000	I.0000 000	I.0000 000	.9999 999
.99					I.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q$  to .80

$q = 7$

$p = 15$

$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$
$q = .1228\ 5616 \times \frac{1}{10^8}$	$.8376\ 5564 \times \frac{1}{10^8}$	$.5827\ 1696 \times \frac{1}{10^8}$	$.4127\ 5785 \times \frac{1}{10^8}$	$.2971\ 8565 \times \frac{1}{10^8}$	$.2171\ 7413 \times \frac{1}{10^8}$
.0000 001					
.0000 001					
.0000 003	.0000 001				
.0000 005 <sup>+</sup>	.0000 001				
.0000 010	.0000 003	.0000 001			
.0000 019	.0000 006	.0000 002			
.0000 034	.0000 011	.0000 003	.0000 001		
.0000 060	.0000 020	.0000 006	.0000 002	.0000 001	
.0000 102	.0000 035 <sup>-</sup>	.0000 012	.0000 004	.0000 001	
.0000 171	.0000 061	.0000 021	.0000 007	.0000 002	.0000 001
.0000 280	.0000 103	.0000 037	.0000 013	.0000 005 <sup>-</sup>	.0000 002
.0000 449	.0000 171	.0000 064	.0000 024	.0000 009	.0000 003
.0000 704	.0000 278	.0000 108	.0000 041	.0000 016	.0000 006
.0001 084	.0000 442	.0000 178	.0000 071	.0000 028	.0000 011
.0001 640	.0000 691	.0000 287	.0000 118	.0000 048	.0000 019
.0002 440	.0001 061	.0000 455 <sup>-</sup>	.0000 192	.0000 080	.0000 033
.0003 574	.0001 602	.0000 708	.0000 309	.0000 133	.0000 057
.0005 158	.0002 381	.0001 083	.0000 486	.0000 216	.0000 095 <sup>-</sup>
.0007 341	.0003 487	.0001 632	.0000 754	.0000 344	.0000 155 <sup>+</sup>
.0010 310	.0005 034	.0002 422	.0001 150 <sup>+</sup>	.0000 540	.0000 250 <sup>+</sup>
.0014 297	.0007 169	.0003 544	.0001 728	.0000 833	.0000 397
.0019 587	.0010 081	.0005 114	.0002 560	.0001 267	.0000 620
.0026 528	.0014 002	.0007 286	.0003 742	.0001 899	.0000 953
.0035 533	.0019 222	.0010 251	.0005 396	.0002 807	.0001 444
.0047 093	.0026 092	.0014 253	.0007 686	.0004 096	.0002 159
.0061 783	.0035 038	.0019 593	.0010 816	.0005 901	.0003 185 <sup>+</sup>
.0080 267	.0046 565 <sup>-</sup>	.0026 638	.0015 046	.0008 399	.0004 639
.0103 304	.0061 268	.0035 837	.0020 697	.0011 816	.0006 674
.0131 749	.0079 841	.0047 722	.0028 168	.0016 435 <sup>+</sup>	.0009 488
.0166 558	.0103 078	.0062 927	.0037 939	.0022 613	.0013 337
.0208 782	.0131 885 <sup>-</sup>	.0082 190	.0050 590	.0030 787	.0018 540
.0259 563	.0167 274	.0106 362	.0066 806	.0041 490	.0025 500 <sup>+</sup>
.0320 128	.0210 368	.0136 417	.0087 392	.0053 362	.0034 712
.0391 769	.0262 394	.0173 448	.0113 279	.0073 166	.0046 777
.0475 833	.0324 673	.0218 672	.0145 532	.0095 796	.0062 421
.0573 695 <sup>-</sup>	.0398 609	.0273 423	.0185 350 <sup>+</sup>	.0124 287	.0082 507
.0686 729	.0485 665 <sup>-</sup>	.0339 139	.0234 072	.0159 824	.0108 046
.0816 284	.0587 343	.0417 355 <sup>-</sup>	.0293 163	.0203 745 <sup>+</sup>	.0140 211
.0963 640	.0705 152	.0509 671	.0364 210	.0257 538	.0180 342
.1129 972	.0840 571	.0617 731	.0448 898	.0322 836	.0229 948
.1316 309	.0995 007	.0743 186	.0548 988	.0401 397	.0290 704
.1523 485 <sup>-</sup>	.1169 751	.0887 648	.0666 280	.0495 089	.0364 443
.1752 097	.1365 926	.1052 646	.0802 574	.0605 851	.0453 132
.2002 460	.1584 437	.1239 567	.0950 615 <sup>-</sup>	.0735 652	.0558 810

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.99$  $q = 7$ 

	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$
$B(p, q) = .1228\ 5616 \times \frac{1}{10^8}$	$.8376\ 5564 \times \frac{1}{10^8}$	$.5827\ 1696 \times \frac{1}{10^8}$	$.4127\ 5785 \times \frac{1}{10^8}$	$.2971\ 1111 \times \frac{1}{10^8}$	
$x$					
.81	.9129 635 <sup>-</sup>	.8924 016	.8695 009	.8444 043	.8173
.82	.9315 803	.9146 515 <sup>-</sup>	.8955 642	.8743 886	.8512
.83	.9474 051	.9337 961	.9182 649	.9008 242	.8815
.84	.9605 688	.9499 141	.9376 079	.9236 223	.9079
.85	.9712 590	.9631 599	.9536 941	.9428 085 <sup>-</sup>	.9304
.86	.9797 101	.9737 551	.9667 132	.9585 199	.9491
.87	.9861 912	.9819 751	.9769 317	.9709 953	.9641
.88	.9909 918	.9881 341	.9846 762	.9805 594	.9757
.89	.9944 079	.9925 667	.9903 135 <sup>-</sup>	.9876 003	.9843
.90	.9967 272	.9956 100	.9942 274	.9925 439	.9905
.91	.9982 162	.9975 855 <sup>-</sup>	.9967 963	.9958 247	.9946
.92	.9991 095 <sup>-</sup>	.9987 837	.9983 715 <sup>+</sup>	.9978 585 <sup>+</sup>	.9972
.93	.9996 022	.9994 517	.9992 593	.9990 172	.9987
.94	.9998 463	.9997 863	.9997 086	.9996 099	.9994
.95	.9999 513	.9999 316	.9999 060	.9998 730	.9998
.96	.9999 884	.9999 836	.9999 772	.9999 690	.9999
.97	.9999 982	.9999 975 <sup>-</sup>	.9999 965 <sup>-</sup>	.9999 952	.9999
.98	.9999 999	.9999 998	.9999 998	.9999 997	.9999
.99	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

	$q = 7$					$p = 2$
$\cdot 27$ to $\cdot 90$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$(p, q) = \cdot 1608\ 6973 \times \frac{1}{10^8}$	$\cdot 1206\ 5229 \times \frac{1}{10^8}$	$\cdot 9152\ 9327 \times \frac{1}{10^8}$	$\cdot 7017\ 2484 \times \frac{1}{10^8}$	$\cdot 5432\ 7084 \times \frac{1}{10^8}$	$\cdot 4244\ 3093 \times \frac{1}{10^8}$	
$x$						
$\cdot 27$	$\cdot 0000\ 001$					
$\cdot 28$	$\cdot 0000\ 001$					
$\cdot 29$	$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 30$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 31$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$			
$\cdot 32$	$\cdot 0000\ 014$	$\cdot 0000\ 005^+$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 33$	$\cdot 0000\ 024$	$\cdot 0000\ 010$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 34$	$\cdot 0000\ 041$	$\cdot 0000\ 018$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$
$\cdot 35$	$\cdot 0000\ 069$	$\cdot 0000\ 031$	$\cdot 0000\ 013$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 36$	$\cdot 0000\ 115^-$	$\cdot 0000\ 052$	$\cdot 0000\ 024$	$\cdot 0000\ 011$	$\cdot 0000\ 005^-$	$\cdot 0000\ 001$
$\cdot 37$	$\cdot 0000\ 187$	$\cdot 0000\ 088$	$\cdot 0000\ 041$	$\cdot 0000\ 019$	$\cdot 0000\ 009$	$\cdot 0000\ 001$
$\cdot 38$	$\cdot 0000\ 300$	$\cdot 0000\ 144$	$\cdot 0000\ 069$	$\cdot 0000\ 032$	$\cdot 0000\ 015^+$	$\cdot 0000\ 001$
$\cdot 39$	$\cdot 0000\ 474$	$\cdot 0000\ 233$	$\cdot 0000\ 114$	$\cdot 0000\ 055^+$	$\cdot 0000\ 026$	$\cdot 0000\ 011$
$\cdot 40$	$\cdot 0000\ 736$	$\cdot 0000\ 371$	$\cdot 0000\ 186$	$\cdot 0000\ 092$	$\cdot 0000\ 045^+$	$\cdot 0000\ 021$
$\cdot 41$	$\cdot 0001\ 127$	$\cdot 0000\ 583$	$\cdot 0000\ 299$	$\cdot 0000\ 152$	$\cdot 0000\ 077$	$\cdot 0000\ 031$
$\cdot 42$	$\cdot 0001\ 702$	$\cdot 0000\ 901$	$\cdot 0000\ 473$	$\cdot 0000\ 246$	$\cdot 0000\ 127$	$\cdot 0000\ 061$
$\cdot 43$	$\cdot 0002\ 537$	$\cdot 0001\ 374$	$\cdot 0000\ 738$	$\cdot 0000\ 394$	$\cdot 0000\ 208$	$\cdot 0000\ 101$
$\cdot 44$	$\cdot 0003\ 732$	$\cdot 0002\ 068$	$\cdot 0001\ 137$	$\cdot 0000\ 620$	$\cdot 0000\ 335^+$	$\cdot 0000\ 181$
$\cdot 45$	$\cdot 0005\ 424$	$\cdot 0003\ 073$	$\cdot 0001\ 726$	$\cdot 0000\ 962$	$\cdot 0000\ 532$	$\cdot 0000\ 291$
$\cdot 46$	$\cdot 0007\ 790$	$\cdot 0004\ 509$	$\cdot 0002\ 588$	$\cdot 0001\ 474$	$\cdot 0000\ 833$	$\cdot 0000\ 461$
$\cdot 47$	$\cdot 0011\ 058$	$\cdot 0006\ 536$	$\cdot 0003\ 832$	$\cdot 0002\ 229$	$\cdot 0001\ 287$	$\cdot 0000\ 731$
$\cdot 48$	$\cdot 0015\ 523$	$\cdot 0009\ 366$	$\cdot 0005\ 604$	$\cdot 0003\ 328$	$\cdot 0001\ 962$	$\cdot 0001\ 141$
$\cdot 49$	$\cdot 0021\ 557$	$\cdot 0013\ 270$	$\cdot 0008\ 101$	$\cdot 0004\ 908$	$\cdot 0002\ 953$	$\cdot 0001\ 761$
$\cdot 50$	$\cdot 0029\ 623$	$\cdot 0018\ 596$	$\cdot 0011\ 578$	$\cdot 0007\ 155^-$	$\cdot 0004\ 390$	$\cdot 0002\ 671$
$\cdot 51$	$\cdot 0040\ 293$	$\cdot 0025\ 784$	$\cdot 0016\ 366$	$\cdot 0010\ 309$	$\cdot 0006\ 449$	$\cdot 0004\ 001$
$\cdot 52$	$\cdot 0054\ 263$	$\cdot 0035\ 381$	$\cdot 0022\ 884$	$\cdot 0014\ 690$	$\cdot 0009\ 364$	$\cdot 0005\ 931$
$\cdot 53$	$\cdot 0072\ 371$	$\cdot 0048\ 061$	$\cdot 0031\ 663$	$\cdot 0020\ 705^-$	$\cdot 0013\ 445^-$	$\cdot 0008\ 671$
$\cdot 54$	$\cdot 0095\ 611$	$\cdot 0064\ 645^-$	$\cdot 0043\ 363$	$\cdot 0028\ 873$	$\cdot 0019\ 092$	$\cdot 0012\ 541$
$\cdot 55$	$\cdot 0125\ 146$	$\cdot 0086\ 115^+$	$\cdot 0058\ 794$	$\cdot 0039\ 847$	$\cdot 0026\ 820$	$\cdot 0017\ 931$
$\cdot 56$	$\cdot 0162\ 325^+$	$\cdot 0113\ 637$	$\cdot 0078\ 935^+$	$\cdot 0054\ 433$	$\cdot 0037\ 282$	$\cdot 0025\ 371$
$\cdot 57$	$\cdot 0208\ 681$	$\cdot 0148\ 569$	$\cdot 0104\ 961$	$\cdot 0073\ 620$	$\cdot 0051\ 290$	$\cdot 0035\ 501$
$\cdot 58$	$\cdot 0265\ 936$	$\cdot 0192\ 478$	$\cdot 0138\ 253$	$\cdot 0098\ 598$	$\cdot 0069\ 848$	$\cdot 0049\ 171$
$\cdot 59$	$\cdot 0335\ 996$	$\cdot 0247\ 140$	$\cdot 0180\ 418$	$\cdot 0130\ 782$	$\cdot 0094\ 175^+$	$\cdot 0067\ 391$
$\cdot 60$	$\cdot 0420\ 927$	$\cdot 0314\ 536$	$\cdot 0233\ 293$	$\cdot 0171\ 830$	$\cdot 0125\ 733$	$\cdot 0091\ 431$
$\cdot 61$	$\cdot 0522\ 933$	$\cdot 0396\ 841$	$\cdot 0298\ 949$	$\cdot 0223\ 656$	$\cdot 0166\ 246$	$\cdot 0122\ 821$
$\cdot 62$	$\cdot 0644\ 311$	$\cdot 0496\ 395^-$	$\cdot 0379\ 675^-$	$\cdot 0288\ 431$	$\cdot 0217\ 716$	$\cdot 0163\ 351$
$\cdot 63$	$\cdot 0787\ 400$	$\cdot 0615\ 661$	$\cdot 0477\ 957$	$\cdot 0368\ 572$	$\cdot 0282\ 432$	$\cdot 0215\ 131$
$\cdot 64$	$\cdot 0954\ 508$	$\cdot 0757\ 173$	$\cdot 0596\ 435^+$	$\cdot 0466\ 727$	$\cdot 0362\ 960$	$\cdot 0280\ 611$
$\cdot 65$	$\cdot 1147\ 828$	$\cdot 0923\ 459$	$\cdot 0737\ 844$	$\cdot 0585\ 721$	$\cdot 0462\ 121$	$\cdot 0362\ 491$
$\cdot 66$	$\cdot 1369\ 343$	$\cdot 1116\ 948$	$\cdot 0904\ 935^+$	$\cdot 0728\ 504$	$\cdot 0582\ 949$	$\cdot 0463\ 821$
$\cdot 67$	$\cdot 1620\ 715^-$	$\cdot 1339\ 865^-$	$\cdot 1100\ 376$	$\cdot 0898\ 060$	$\cdot 0728\ 620$	$\cdot 0587\ 851$
$\cdot 68$	$\cdot 1903\ 167$	$\cdot 1594\ 106$	$\cdot 1326\ 629$	$\cdot 1097\ 296$	$\cdot 0902\ 362$	$\cdot 0737\ 991$
$\cdot 69$	$\cdot 2217\ 362$	$\cdot 1881\ 107$	$\cdot 1585\ 814$	$\cdot 1328\ 909$	$\cdot 1107\ 328$	$\cdot 0917\ 741$
$\cdot 70$	$\cdot 2563\ 285^+$	$\cdot 2201\ 697$	$\cdot 1879\ 555^+$	$\cdot 1595\ 230$	$\cdot 1346\ 445^+$	$\cdot 1130\ 501$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .91$  to  $.99$  $q = 7$ 

	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p$
$B(p, q) = .1608\ 6973 \times \frac{x}{10^4}$	$.1206\ 5229 \times \frac{x}{10^4}$	$.9152\ 9327 \times \frac{x}{10^7}$	$.7017\ 2484 \times \frac{x}{10^7}$	$.543$	
$x$					
.91	.9915 644	.9896 106	.9873 478	.9847 514	.98
.92	.9955 564	.9944 788	.9932 170	.9917 532	.99
.93	.9979 055 <sup>+</sup>	.9973 745 <sup>-</sup>	.9967 459	.9960 088	.99
.94	.9991 463	.9989 203	.9986 499	.9983 295 <sup>+</sup>	.99
.95	.9997 146	.9996 358	.9995 406	.9994 265 <sup>+</sup>	.99
.96	.9999 284	.9999 078	.9998 826	.9998 522	.99
.97	.9999 886	.9999 852	.9999 809	.9999 758	.99
.98	.9999 992	.9999 990	.9999 986	.9999 983	.99
.99	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.00

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\gamma$  36 to 1000

$q = 7$

$p = 2$

	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$
$p, q) =$	$\cdot 3343\ 9967 \times \frac{1}{10^7}$	$\cdot 2655\ 5268 \times \frac{1}{10^7}$	$\cdot 2124\ 4214 \times \frac{1}{10^7}$	$\cdot 1711\ 3395 \times \frac{1}{10^7}$	$\cdot 1387\ 5725 \times \frac{1}{10^7}$	$\cdot 1131\ 9671 \times \frac{1}{10^7}$
$\gamma$						
36	·0000 001					
37	·0000 002	·0000 001				
38	·0000 003	·0000 001	·0000 001			
39	·0000 006	·0000 003	·0000 001	·0000 001		
40	·0000 011	·0000 005 <sup>+</sup>	·0000 003	·0000 001	·0000 001	
41	·0000 019	·0000 009	·0000 005 <sup>-</sup>	·0000 002	·0000 001	·0000 001
42	·0000 033	·0000 017	·0000 009	·0000 004	·0000 002	·0000 001
43	·0000 057	·0000 030	·0000 015 <sup>+</sup>	·0000 008	·0000 004	·0000 002
44	·0000 096	·0000 051	·0000 027	·0000 014	·0000 007	·0000 004
45	·0000 160	·0000 087	·0000 047	·0000 025 <sup>+</sup>	·0000 013	·0000 007
46	·0000 261	·0000 145 <sup>-</sup>	·0000 080	·0000 044	·0000 024	·0000 013
47	·0000 421	·0000 238	·0000 134	·0000 075 <sup>+</sup>	·0000 042	·0000 023
48	·0000 668	·0000 387	·0000 222	·0000 127	·0000 072	·0000 041
49	·0001 048	·0000 618	·0000 363	·0000 212	·0000 123	·0000 071
50	·0001 620	·0000 976	·0000 584	·0000 348	·0000 206	·0000 122
51	·0002 474	·0001 519	·0000 928	·0000 563	·0000 341	·0000 205 <sup>-</sup>
52	·0003 732	·0002 335 <sup>+</sup>	·0001 453	·0000 900	·0000 554	·0000 340
53	·0005 561	·0003 545 <sup>+</sup>	·0002 248	·0001 418	·0000 890	·0000 556
54	·0008 190	·0005 317	·0003 434	·0002 206	·0001 410	·0000 898
55	·0011 923	·0007 881	·0005 181	·0003 389	·0002 206	·0001 429
56	·0017 163	·0011 544	·0007 724	·0005 142	·0003 406	·0002 247
57	·0024 434	·0016 720	·0011 381	·0007 708	·0005 196	·0003 487
58	·0034 410	·0023 947	·0016 578	·0011 419	·0007 829	·0005 344
59	·0047 947	·0033 923	·0023 876	·0016 722	·0011 657	·0008 090
60	·0066 111	·0047 538	·0034 007	·0024 208	·0017 153	·0012 101
61	·0090 218	·0065 912	·0047 907	·0034 652	·0024 950 <sup>-</sup>	·0017 885 <sup>+</sup>
62	·0121 866	·0090 430	·0066 763	·0049 053	·0035 878	·0026 127
63	·0162 961	·0122 784	·0092 049	·0068 679	·0051 012	·0037 727
64	·0215 744	·0165 005 <sup>-</sup>	·0125 573	·0095 115 <sup>+</sup>	·0071 724	·0053 855 <sup>+</sup>
65	·0282 802	·0219 487	·0169 513	·0130 310	·0099 731	·0076 008
66	·0367 062	·0289 006	·0226 449	·0176 619	·0137 154	·0106 066
67	·0471 769	·0376 711	·0299 375 <sup>-</sup>	·0236 839	·0186 560	·0146 353
68	·0600 432	·0486 102	·0391 697	·0314 221	·0251 000	·0199 689
69	·0756 743	·0620 965 <sup>+</sup>	·0507 203	·0412 467	·0334 025 <sup>-</sup>	·0269 424
70	·0944 455 <sup>+</sup>	·0785 283	·0649 987	·0535 685 <sup>+</sup>	·0439 672	·0359 454
71	·1167 226	·0983 093	·0824 344	·0688 312	·0572 414	·0474 199
72	·1428 417	·1218 304	·1034 604	·0874 976	·0737 057	·0618 536
73	·1730 859	·1494 467	·1284 921	·1100 305 <sup>-</sup>	·0938 581	·0797 672
74	·2076 585 <sup>-</sup>	·1814 501	·1579 000	·1368 673	·1181 903	·1016 943
75	·2466 545 <sup>+</sup>	·2180 388	·1919 781	·1683 886	·1471 581	·1281 534
76	·2900 323	·2592 843	·2309 084	·2048 808	·1811 436	·1596 117
77	·3375 867	·3050 993	·2747 236	·2464 951	·2204 120	·1964 408
78	·3889 279	·3552 083	·3232 709	·2932 057	·2650 647	·2388 664
79	·4434 678	·4091 249	·3761 802	·3447 692	·3149 016	·2869 141

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .42$  to  $1.00$  $q = 7$ 

	$p = 33$	$p = 34$	$p = 35$	$p = 36$	$p = 37$
$B(p, q) = .92879350 \times \frac{x}{108}$		$.76625464 \times \frac{x}{108}$	$.63543067 \times \frac{x}{108}$	$.52952556 \times \frac{x}{108}$	$.4433237$
$x$					
.42	.0000 001				
.43	.0000 001	.0000 001			
.44	.0000 002	.0000 001	.0000 001		
.45	.0000 004	.0000 002	.0000 001	.0000 001	
.46	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
.47	.0000 013	.0000 007	.0000 004	.0000 002	.0000 001
.48	.0000 023	.0000 013	.0000 007	.0000 004	.0000 002
.49	.0000 041	.0000 024	.0000 013	.0000 008	.0000 004
.50	.0000 071	.0000 042	.0000 024	.0000 014	.0000 008
.51	.0000 123	.0000 073	.0000 044	.0000 026	.0000 015
.52	.0000 208	.0000 126	.0000 076	.0000 046	.0000 028
.53	.0000 346	.0000 214	.0000 132	.0000 081	.0000 050
.54	.0000 569	.0000 359	.0000 226	.0000 141	.0000 088
.55	.0000 922	.0000 593	.0000 379	.0000 242	.0000 153
.56	.0001 475 <sup>+</sup>	.0000 965 <sup>+</sup>	.0000 629	.0000 408	.0000 263
.57	.0002 330	.0001 550 <sup>+</sup>	.0001 028	.0000 679	.0000 447
.58	.0003 632	.0002 459	.0001 658	.0001 114	.0000 746
.59	.0005 591	.0003 849	.0002 639	.0001 803	.0001 228
.60	.0008 501	.0005 948	.0004 146	.0002 880	.0001 994
.61	.0012 768	.0009 079	.0006 432	.0004 540	.0003 194
.62	.0018 949	.0013 689	.0009 852	.0007 066	.0005 050
.63	.0027 789	.0020 389	.0014 904	.0010 857	.0007 882
.64	.0040 276	.0030 004	.0022 271	.0016 473	.0012 143
.65	.0057 696	.0043 630	.0032 873	.0024 683	.0018 472
.66	.0081 700	.0062 695 <sup>+</sup>	.0047 938	.0036 529	.0027 744
.67	.0114 364	.0089 034	.0069 068	.0053 398	.0041 149
.68	.0158 256	.0124 959	.0098 321	.0077 102	.0060 268
.69	.0216 492	.0173 329	.0138 290	.0109 068	.0087 170
.70	.0292 773	.0237 609	.0192 181	.0154 929	.0124 597
.71	.0391 391	.0321 908	.0263 870	.0215 598	.0175 613
.72	.0517 200	.0430 973	.0357 933	.0296 329	.0244 582
.73	.0675 522	.0570 140	.0479 635 <sup>-</sup>	.0402 240	.0339 326
.74	.0871 984	.0745 211	.0634 844	.0539 171	.0456 574
.75	.1112 273	.0962 246	.0829 869	.0713 567	.0611 803
.76	.1401 791	.1227 251	.1071 192	.0932 255 <sup>+</sup>	.0809 094
.77	.1745 221	.1545 762	.1365 074	.1202 094	.1055 684
.78	.2146 002	.1922 311	.1717 042	.1529 485 <sup>-</sup>	.1358 808
.79	.2605 740	.2359 819	.2131 257	.1919 739	.1724 790
.80	.3123 609	.2858 914	.2609 789	.2376 323	.2158 421
.81	.3695 782	.3417 268	.3151 860	.2900 026	.2662 043
.82	.4314 988	.4029 010	.3753 126	.3488 120	.3244 597
.83	.4970 276	.4684 329	.4405 100	.4133 628	.3870 744
.84	.5647 102	.5369 372	.5094 909	.4824 837	.4560 167
.85	.6327 799	.6066 505 <sup>+</sup>	.5805 331	.5545 216	.5287 266
.86	.6999 632	.6747 664	.6494 464	.6244 404	.5994 454
.87	.7662 546	.7417 816	.7187 464	.6948 404	.6708 454
.88	.8317 546	.8079 816	.7853 464	.7614 404	.7374 454
.89	.8964 546	.8723 816	.8493 464	.8254 404	.8014 454
.90	.9603 546	.9360 816	.9133 464	.8894 404	.8654 454
.91	.1.0234 546	.1.0000 816	.9777 464	.9538 404	.9298 454
.92	.1.0857 546	.1.0623 816	.1.0393 464	.1.0154 404	.9914 454
.93	.1.1472 546	.1.1238 816	.1.1003 464	.1.0764 404	.1.0524 454
.94	.1.2079 546	.1.1845 816	.1.1613 464	.1.1374 404	.1.1134 454
.95	.1.2678 546	.1.2444 816	.1.2213 464	.1.1974 404	.1.1734 454
.96	.1.3269 546	.1.3035 816	.1.2803 464	.1.2564 404	.1.2324 454
.97	.1.3852 546	.1.3618 816	.1.3383 464	.1.3144 404	.1.2904 454
.98	.1.4427 546	.1.4193 816	.1.3953 464	.1.3714 404	.1.3474 454
.99	.1.4994 546	.1.4760 816	.1.4523 464	.1.4284 404	.1.4044 454
1.00	.1.5553 546	.1.5319 816	.1.5083 464	.1.4844 404	.1.4604 454



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\Gamma=00$

$q = 7$

$p = 39$  to 44

$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$
$\cdot 3148\ 0463 \times \frac{1}{10^8}$	$\cdot 2668\ 9957 \times \frac{1}{10^8}$	$\cdot 2271\ 4857 \times \frac{1}{10^8}$	$\cdot 1940\ 2274 \times \frac{1}{10^8}$	$\cdot 1663\ 0521 \times \frac{1}{10^8}$	$\cdot 1430\ 2248 \times \frac{1}{10^8}$
$\cdot 0000\ 001$					
$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$		
$\cdot 0000\ 005^+$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	
$\cdot 0000\ 010$	$\cdot 0000\ 006$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$
$\cdot 0000\ 019$	$\cdot 0000\ 011$	$\cdot 0000\ 007$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 034$	$\cdot 0000\ 021$	$\cdot 0000\ 013$	$\cdot 0000\ 008$	$\cdot 0000\ 005^-$	$\cdot 0000\ 003$
$\cdot 0000\ 061$	$\cdot 0000\ 039$	$\cdot 0000\ 024$	$\cdot 0000\ 015^+$	$\cdot 0000\ 009$	$\cdot 0000\ 006$
$\cdot 0000\ 109$	$\cdot 0000\ 070$	$\cdot 0000\ 045^-$	$\cdot 0000\ 029$	$\cdot 0000\ 018$	$\cdot 0000\ 011$
$\cdot 0000\ 192$	$\cdot 0000\ 125^-$	$\cdot 0000\ 081$	$\cdot 0000\ 053$	$\cdot 0000\ 034$	$\cdot 0000\ 022$
$\cdot 0000\ 331$	$\cdot 0000\ 220$	$\cdot 0000\ 145^+$	$\cdot 0000\ 096$	$\cdot 0000\ 063$	$\cdot 0000\ 041$
$\cdot 0000\ 564$	$\cdot 0000\ 380$	$\cdot 0000\ 256$	$\cdot 0000\ 171$	$\cdot 0000\ 115^-$	$\cdot 0000\ 077$
$\cdot 0000\ 946$	$\cdot 0000\ 649$	$\cdot 0000\ 444$	$\cdot 0000\ 302$	$\cdot 0000\ 206$	$\cdot 0000\ 139$
$\cdot 0001\ 566$	$\cdot 0001\ 091$	$\cdot 0000\ 758$	$\cdot 0000\ 525^+$	$\cdot 0000\ 363$	$\cdot 0000\ 250^+$
$\cdot 0002\ 555^+$	$\cdot 0001\ 810$	$\cdot 0001\ 278$	$\cdot 0000\ 900$	$\cdot 0000\ 632$	$\cdot 0000\ 443$
$\cdot 0004\ 115^-$	$\cdot 0002\ 960$	$\cdot 0002\ 123$	$\cdot 0001\ 519$	$\cdot 0001\ 084$	$\cdot 0000\ 771$
$\cdot 0006\ 537$	$\cdot 0004\ 775^+$	$\cdot 0003\ 478$	$\cdot 0002\ 527$	$\cdot 0001\ 831$	$\cdot 0001\ 323$
$\cdot 0010\ 249$	$\cdot 0007\ 600$	$\cdot 0005\ 621$	$\cdot 0004\ 145^+$	$\cdot 0003\ 050^-$	$\cdot 0002\ 238$
$\cdot 0015\ 856$	$\cdot 0011\ 935^-$	$\cdot 0008\ 958$	$\cdot 0006\ 706$	$\cdot 0005\ 008$	$\cdot 0003\ 730$
$\cdot 0024\ 212$	$\cdot 0018\ 492$	$\cdot 0014\ 084$	$\cdot 0010\ 699$	$\cdot 0008\ 107$	$\cdot 0006\ 128$
$\cdot 0036\ 490$	$\cdot 0028\ 272$	$\cdot 0021\ 845^-$	$\cdot 0016\ 835^-$	$\cdot 0012\ 942$	$\cdot 0009\ 925^-$
$\cdot 0054\ 281$	$\cdot 0042\ 652$	$\cdot 0033\ 425^-$	$\cdot 0026\ 126$	$\cdot 0020\ 370$	$\cdot 0015\ 845^-$
$\cdot 0079\ 699$	$\cdot 0063\ 497$	$\cdot 0050\ 454$	$\cdot 0039\ 989$	$\cdot 0031\ 616$	$\cdot 0024\ 937$
$\cdot 0115\ 496$	$\cdot 0093\ 276$	$\cdot 0075\ 132$	$\cdot 0060\ 366$	$\cdot 0048\ 384$	$\cdot 0038\ 680$
$\cdot 0165\ 184$	$\cdot 0135\ 195^-$	$\cdot 0110\ 363$	$\cdot 0089\ 869$	$\cdot 0073\ 004$	$\cdot 0059\ 168$
$\cdot 0233\ 138$	$\cdot 0193\ 325^-$	$\cdot 0159\ 901$	$\cdot 0131\ 931$	$\cdot 0108\ 596$	$\cdot 0089\ 185^+$
$\cdot 0324\ 675^+$	$\cdot 0272\ 706$	$\cdot 0228\ 481$	$\cdot 0190\ 964$	$\cdot 0159\ 236$	$\cdot 0132\ 481$
$\cdot 0446\ 074$	$\cdot 0379\ 414$	$\cdot 0321\ 920$	$\cdot 0272\ 489$	$\cdot 0230\ 120$	$\cdot 0193\ 909$
$\cdot 0604\ 506$	$\cdot 0520\ 540$	$\cdot 0447\ 154$	$\cdot 0383\ 219$	$\cdot 0327\ 686$	$\cdot 0279\ 592$
$\cdot 0807\ 854$	$\cdot 0704\ 071$	$\cdot 0612\ 171$	$\cdot 0531\ 052$	$\cdot 0459\ 668$	$\cdot 0397\ 032$
$\cdot 1064\ 358$	$\cdot 0938\ 598$	$\cdot 0825\ 791$	$\cdot 0724\ 926$	$\cdot 0635\ 011$	$\cdot 0555\ 092$
$\cdot 1382\ 082$	$\cdot 1232\ 842$	$\cdot 1097\ 257$	$\cdot 0974\ 470$	$\cdot 0863\ 611$	$\cdot 0763\ 812$
$\cdot 1768\ 170$	$\cdot 1594\ 944$	$\cdot 1435\ 576$	$\cdot 1289\ 423$	$\cdot 1155\ 799$	$\cdot 1033\ 982$
$\cdot 2227\ 904$	$\cdot 2031\ 530$	$\cdot 1848\ 607$	$\cdot 1678\ 756$	$\cdot 1521\ 523$	$\cdot 1376\ 393$
$\cdot 2763\ 604$	$\cdot 2546\ 563$	$\cdot 2341\ 893$	$\cdot 2149\ 504$	$\cdot 1969\ 207$	$\cdot 1800\ 735^-$
$\cdot 3373\ 467$	$\cdot 3140\ 077$	$\cdot 2917\ 306$	$\cdot 2705\ 348$	$\cdot 2504\ 290$	$\cdot 2314\ 127$
$\cdot 4050\ 488$	$\cdot 3806\ 917$	$\cdot 3571\ 627$	$\cdot 3345\ 060$	$\cdot 3127\ 556$	$\cdot 2919\ 358$
$\cdot 4781\ 672$	$\cdot 4535\ 708$	$\cdot 4295\ 279$	$\cdot 4061\ 007$	$\cdot 3833\ 428$	$\cdot 3612\ 994$
$\cdot 5547\ 773$	$\cdot 5308\ 314$	$\cdot 5071\ 490$	$\cdot 4838\ 016$	$\cdot 4608\ 544$	$\cdot 4383\ 662$
$\cdot 6323\ 817$	$\cdot 6100\ 075^-$	$\cdot 5876\ 220$	$\cdot 5652\ 967$	$\cdot 5430\ 988$	$\cdot 5210\ 920$
$\cdot 7080\ 613$	$\cdot 6881\ 093$	$\cdot 6679\ 178$	$\cdot 6475\ 490$	$\cdot 6270\ 639$	$\cdot 6065\ 216$
$\cdot 7787\ 315^+$	$\cdot 7618\ 717$	$\cdot 7446\ 156$	$\cdot 7270\ 103$	$\cdot 7091\ 031$	$\cdot 6909\ 419$
$\cdot 8414\ 926$	$\cdot 8281\ 158$	$\cdot 8142\ 708$	$\cdot 7999\ 869$	$\cdot 7852\ 948$	$\cdot 7702\ 268$
$\cdot 8040\ 204$	$\cdot 8841\ 856$	$\cdot 8688\ 888$	$\cdot 8588\ 888$		

$x = .53$  to  $1.00$  $q = 7$ 

	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) = .1233\ 9194 \times \frac{1}{10^8}$		$.1067\ 8149 \times \frac{1}{10^8}$	$.9267\ 8272 \times \frac{1}{10^9}$	$.8066\ 4422 \times \frac{1}{10^9}$	$.7039\ 804$
$x$					
.53	.0000 001	.0000 001			
.54	.0000 002	.0000 001	.0000 001		
.55	.0000 004	.0000 002	.0000 001	.0000 001	.0000 001
.56	.0000 007	.0000 005-	.0000 003	.0000 002	.0000 001
.57	.0000 014	.0000 009	.0000 006	.0000 004	.0000 002
.58	.0000 027	.0000 018	.0000 011	.0000 007	.0000 005
.59	.0000 051	.0000 034	.0000 022	.0000 015-	.0000 010
.60	.0000 094	.0000 064	.0000 043	.0000 029	.0000 019
.61	.0000 172	.0000 118	.0000 081	.0000 055+	.0000 038
.62	.0000 309	.0000 216	.0000 150+	.0000 104	.0000 072
.63	.0000 548	.0000 388	.0000 274	.0000 193	.0000 136
.64	.0000 954	.0000 686	.0000 493	.0000 353	.0000 252
.65	.0001 638	.0001 197	.0000 872	.0000 634	.0000 460
.66	.0002 772	.0002 055+	.0001 521	.0001 123	.0000 827
.67	.0004 621	.0003 477	.0002 611	.0001 956	.0001 463
.68	.0007 593	.0005 797	.0004 416	.0003 357	.0002 547
.69	.0012 296	.0009 522	.0007 357	.0005 673	.0004 366
.70	.0019 625-	.0015 410	.0012 075-	.0009 442	.0007 369
.71	.0030 868	.0024 574	.0019 522	.0015 478	.0012 247
.72	.0047 847	.0038 609	.0031 090	.0024 986	.0020 041
.73	.0073 083	.0059 760	.0048 767	.0039 717	.0032 284
.74	.0109 983	.0091 114	.0075 330	.0062 159	.0051 194
.75	.0163 047	.0136 815-	.0114 574	.0095 764	.0079 894
.76	.0238 057	.0202 281	.0171 545-	.0145 203	.0122 681
.77	.0342 226	.0294 398	.0252 708	.0216 620	.0185 307
.78	.0484 254	.0421 634	.0366 419	.0317 853	.0275 237
.79	.0674 220	.0594 005+	.0522 370	.0458 553	.0401 837
.80	.0923 240	.0822 834	.0732 032	.0650 117	.0576 394
.81	.1242 808	.1120 176	.1007 888	.0905 324	.0811 863
.82	.1643 749	.1497 857	.1362 621	.1237 572	.1122 213
.83	.2134 769	.1966 053	.1807 753	.1659 591	.1521 244
.84	.2720 625-	.2531 430	.2351 778	.2181 605-	.2020 791
.85	.3400 075-	.3194 963	.2997 877	.2808 968	.2628 324
.86	.4163 890	.3949 687	.3741 443	.3539 491	.3344 103
.87	.4993 353	.4778 832	.4567 855-	.4360 873	.4158 289
.88	.5859 793	.5654 917	.5451 111	.5248 866	.5048 643
.89	.6725 743	.6540 475+	.6354 080	.6167 010	.5979 707
.90	.7548 165-	.7390 979	.7231 059	.7068 758	.6904 428
.91	.8283 823	.8160 211	.8033 051	.7902 564	.7768 978
.92	.8896 279	.8807 648	.8715 473	.8619 845+	.8520 871
.93	.9363 151	.9306 537	.9247 019	.9184 601	.9119 297
.94	.9681 529	.9650 390	.9617 300	.9582 226	.9545 134
.95	.9869 141	.9855 150-	.9840 124	.9824 028	.9806 825
.96	.9959 579	.9954 879	.9949 778	.9944 256	.9938 292
.97	.9992 053	.9991 053	.9989 956	.9988 756	.9987 447
.98	.9999 315+	.9999 222	.9999 119	.9999 006	.9998 880
.99	.9999 992	.9999 991	.9999 990	.9999 988	.9999 987
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $p = .04$  to  $.70$  $q = 7.5$  $p = 7$ 

	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$B(p, q) = .4016\ 5879 \times \frac{1}{10^4}$	$.2816\ 6171 \times \frac{1}{10^4}$	$.2008\ 2939 \times \frac{1}{10^4}$	$.1453\ 7379 \times \frac{1}{10^4}$	$.1066\ 9062 \times \frac{1}{10^4}$	$.7929\ 479$	
$x$						
.04	.0000 001					
.05	.0000 004					
.06	.0000 010	.0000 005 <sup>+</sup>	.0000 002	.0000 001		
.07	.0000 048	.0000 017	.0000 006	.0000 002	.0000 001	
.08	.0000 123	.0000 046	.0000 017	.0000 006	.0000 002	.0000 001
.09	.0000 279	.0000 111	.0000 044	.0000 017	.0000 007	.0000 003
.10	.0000 577	.0000 243	.0000 101	.0000 041	.0000 017	.0000 007
.11	.0001 108	.0000 489	.0000 213	.0000 092	.0000 039	.0000 017
.12	.0001 998	.0000 920	.0000 418	.0000 188	.0000 084	.0000 037
.13	.0003 417	.0001 636	.0000 774	.0000 362	.0000 168	.0000 077
.14	.0005 586	.0002 774	.0001 361	.0000 661	.0000 317	.0000 151
.15	.0008 783	.0004 512	.0002 290	.0001 150 <sup>-</sup>	.0000 572	.0000 288
.16	.0013 348	.0007 077	.0003 708	.0001 922	.0000 986	.0000 501
.17	.0019 688	.0010 752	.0005 804	.0003 099	.0001 638	.0000 858
.18	.0028 278	.0015 880	.0008 814	.0004 840	.0002 632	.0001 418
.19	.0039 661	.0022 866	.0013 031	.0007 348	.0004 102	.0002 270
.20	.0054 448	.0032 182	.0018 804	.0010 872	.0006 224	.0003 531
.21	.0073 312	.0044 367	.0026 546	.0015 716	.0009 215 <sup>-</sup>	.0005 354
.22	.0096 984	.0060 027	.0036 734	.0022 246	.0013 342	.0007 931
.23	.0126 246	.0079 827	.0049 911	.0030 885 <sup>-</sup>	.0018 928	.0011 498
.24	.0161 917	.0104 494	.0066 688	.0042 124	.0026 356	.0016 349
.25	.0204 845 <sup>-</sup>	.0134 804	.0087 736	.0056 522	.0036 070	.0022 817
.26	.0255 892	.0171 573	.0113 784	.0074 699	.0048 582	.0031 321
.27	.0315 922	.0215 651	.0145 615 <sup>+</sup>	.0097 343	.0064 469	.0042 320
.28	.0385 780	.0267 902	.0184 054	.0125 197	.0084 378	.0056 381
.29	.0466 280	.0329 198	.0229 958	.0159 059	.0109 016	.0074 084
.30	.0558 188	.0400 396	.0284 203	.0199 770	.0139 152	.0096 111
.31	.0662 204	.0482 326	.0347 674	.0248 203	.0175 607	.0123 208
.32	.0778 947	.0575 776	.0421 243	.0305 256	.0219 246	.0156 169
.33	.0908 939	.0681 467	.0505 761	.0371 829	.0270 969	.0195 851
.34	.1052 591	.0800 047	.0602 030	.0448 816	.0331 694	.0243 157
.35	.1210 189	.0932 066	.0710 797	.0537 084	.0402 349	.0298 999
.36	.1381 888	.1077 903	.0832 726	.0637 457	.0483 847	.0364 344
.37	.1567 695 <sup>-</sup>	.1238 056	.0968 389	.0750 696	.0577 077	.0440 131
.38	.1767 469	.1412 524	.1118 243	.0877 481	.0682 879	.0527 321
.39	.1980 915 <sup>-</sup>	.1601 398	.1282 618	.1018 393	.0802 026	.0626 807
.40	.2207 580	.1804 558	.1461 705 <sup>+</sup>	.1173 896	.0935 206	.0739 441
.41	.2446 858	.2021 718	.1655 541	.1344 321	.1083 000	.0866 002
.42	.2697 991	.2252 432	.1864 000	.1529 851	.1245 866	.1007 181
.43	.2960 075 <sup>-</sup>	.2496 089	.2086 789	.1730 507	.1424 119	.1163 552
.44	.3232 069	.2751 915 <sup>+</sup>	.2323 440	.1946 140	.1617 916	.1335 541
.45	.3512 809	.3018 985 <sup>-</sup>	.2573 312	.2176 421	.1827 241	.1523 431
.46	.3801 019	.3296 221	.2835 594	.2420 838	.2051 896	.1727 341
.47	.4095 323	.3582 413	.3109 306	.2678 693	.2291 492	.1947 181
.48	.4394 271	.3876 228	.3393 313	.2949 106	.2545 443	.2182 681
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TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.97$  $q = 7.5$ 

	$p = 7.5$	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$
$B(p, q) = .4016\ 5879 \times \frac{1}{10^1}$	$.2816\ 6171 \times \frac{1}{10^1}$	$.2008\ 2939 \times \frac{1}{10^1}$	$.1453\ 7379 \times \frac{1}{10^1}$	$.1066\ 9062$	
$.71$	.9533 720	.9423 006	.9297 398	.9156 790	.9001 301
$.72$	.9614 220	.9520 067	.9412 495 <sup>+</sup>	.9291 229	.9156 185 <sup>+</sup>
$.73$	.9684 078	.9604 882	.9513 772	.9410 349	.9294 377
$.74$	.9744 108	.9678 266	.9601 999	.9514 834	.9416 422
$.75$	.9795 155 <sup>+</sup>	.9741 092	.9678 046	.9605 504	.9523 049
$.76$	.9838 083	.9794 279	.9742 855 <sup>-</sup>	.9683 291	.9615 136
$.77$	.9873 754	.9838 765 <sup>+</sup>	.9797 420	.9749 215 <sup>-</sup>	.9693 695 <sup>-</sup>
$.78$	.9903 016	.9875 496	.9842 765 <sup>+</sup>	.9804 357	.9759 832
$.79$	.9926 688	.9905 402	.9879 922	.9849 831	.9814 725 <sup>+</sup>
$.80$	.9945 552	.9929 384	.9909 909	.9886 763	.9859 588
$.81$	.9960 339	.9948 300	.9933 709	.9916 259	.9895 644
$.82$	.9971 722	.9962 953	.9952 258	.9939 389	.9924 093
$.83$	.9980 312	.9974 077	.9966 427	.9957 166	.9946 090
$.84$	.9986 652	.9982 338	.9977 012	.9970 526	.9962 722
$.85$	.9991 217	.9988 321	.9984 725 <sup>-</sup>	.9980 319	.9974 986
$.86$	.9994 414	.9992 536	.9990 190	.9987 298	.9983 778
$.87$	.9996 583	.9995 412	.9993 941	.9992 117	.9989 883
$.88$	.9998 002	.9997 304	.9996 422	.9995 323	.9993 969
$.89$	.9998 892	.9998 498	.9997 997	.9997 368	.9996 590
$.90$	.9999 423	.9999 214	.9998 946	.9998 610	.9998 190
$.91$	.9999 721	.9999 619	.9999 486	.9999 319	.9999 109
$.92$	.9999 877	.9999 832	.9999 772	.9999 696	.9999 601
$.93$	.9999 952	.9999 934	.9999 910	.9999 880	.9999 841
$.94$	.9999 984	.9999 978	.9999 970	.9999 959	.9999 946
$.95$	.9999 996	.9999 994	.9999 992	.9999 989	.9999 985 <sup>+</sup>
$.96$	.9999 999	.9999 999	.9999 998	.9999 998	.9999 997
$.97$	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

09 to 70

$q = 7.5$

$p = 10$

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$\beta(q) = .5962\ 1226 \times \frac{1}{10^3}$		$.4531\ 1311 \times \frac{1}{10^3}$	$.2694\ 1860 \times \frac{1}{10^3}$	$.1657\ 9606 \times \frac{1}{10^3}$	$.1051\ 3897 \times \frac{1}{10^3}$	$.6846\ 2583$
09	.0000 001					
10	.0000 003	.0000 001				
11	.0000 007	.0000 003				
12	.0000 016	.0000 007	.0000 001			
13	.0000 035 <sup>+</sup>	.0000 016	.0000 003	.0000 001		
14	.0000 071	.0000 033	.0000 007	.0000 001		
15	.0000 137	.0000 067	.0000 015 <sup>+</sup>	.0000 003	.0000 001	
16	.0000 253	.0000 126	.0000 031	.0000 007	.0000 002	
17	.0000 446	.0000 230	.0000 060	.0000 015 <sup>+</sup>	.0000 004	.0000 001
18	.0000 757	.0000 401	.0000 110	.0000 030	.0000 008	.0000 002
19	.0001 245 <sup>+</sup>	.0000 678	.0000 196	.0000 055 <sup>+</sup>	.0000 015 <sup>+</sup>	.0000 004
20	.0001 987	.0001 109	.0000 338	.0000 100	.0000 029	.0000 008
21	.0003 085 <sup>+</sup>	.0001 764	.0000 564	.0000 176	.0000 054	.0000 016
22	.0004 675 <sup>+</sup>	.0002 735 <sup>-</sup>	.0000 916	.0000 299	.0000 095 <sup>+</sup>	.0000 030
23	.0006 927	.0004 141	.0001 449	.0000 494	.0000 165 <sup>-</sup>	.0000 054
24	.0010 053	.0006 136	.0002 238	.0000 796	.0000 277	.0000 094
25	.0014 316	.0008 914	.0003 384	.0001 253	.0000 454	.0000 161
26	.0020 030	.0012 712	.0005 015 <sup>-</sup>	.0001 929	.0000 726	.0000 268
27	.0027 569	.0017 821	.0007 294	.0002 912	.0001 137	.0000 435
28	.0037 373	.0024 588	.0010 426	.0004 313	.0001 745 <sup>+</sup>	.0000 692
29	.0049 946	.0033 423	.0014 664	.0006 277	.0002 629	.0001 080
30	.0065 864	.0044 803	.0020 314	.0008 988	.0003 891	.0001 652
31	.0085 770	.0059 273	.0027 741	.0012 671	.0005 664	.0002 483
32	.0110 380	.0077 452	.0037 377	.0017 607	.0008 118	.0003 671
33	.0140 474	.0100 031	.0049 726	.0024 133	.0011 464	.0005 343
34	.0176 891	.0127 772	.0065 363	.0032 650 <sup>+</sup>	.0015 967	.0007 661
35	.0220 527	.0161 503	.0084 944	.0043 634	.0021 946	.0010 831
36	.0272 316	.0202 114	.0109 200	.0057 633	.0029 787	.0015 108
37	.0333 229	.0250 544	.0138 942	.0075 282	.0039 950 <sup>+</sup>	.0020 809
38	.0404 246	.0307 777	.0175 050 <sup>+</sup>	.0097 295 <sup>-</sup>	.0052 974	.0028 313
39	.0486 353	.0374 822	.0218 476	.0124 474	.0069 482	.0038 078
40	.0580 511	.0452 699	.0270 228	.0157 703	.0090 189	.0050 645
41	.0687 645 <sup>+</sup>	.0542 422	.0331 358	.0197 949	.0115 902	.0066 644
42	.0808 618	.0644 976	.0402 955 <sup>+</sup>	.0246 247	.0147 520	.0086 804
43	.0944 207	.0761 297	.0486 118	.0303 697	.0186 036	.0111 952
44	.1095 086	.0892 247	.0581 941	.0371 450 <sup>+</sup>	.0232 526	.0143 020
45	.1261 797	.1038 592	.0691 489	.0450 687	.0288 145 <sup>-</sup>	.0181 041
46	.1444 737	.1200 975 <sup>+</sup>	.0815 772	.0542 599	.0354 109	.0227 148
47	.1644 128	.1379 894	.0955 718	.0648 367	.0431 685 <sup>-</sup>	.0282 561
48	.1860 006	.1575 679	.1112 147	.0769 134	.0522 164	.0348 580
49	.2092 207	.1788 469	.1285 740	.0905 970	.0626 840	.0426 563
50	.2340 348	.2018 199	.1477 014	.1059 850 <sup>+</sup>	.0746 977	.0517 910
51	.2603 825 <sup>+</sup>	.2264 577	.1686 290	.1231 612	.0883 784	.0624 030
52	.2881 810	.2527 080	.1913 676	.1421 930	.1038 368	.0746 313

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.98$  $q = .75$ 

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$B(p, q) = .5962 \ 1226 \times \frac{1}{10^5}$	$.4531 \ 1311 \times \frac{1}{10^5}$	$.2694 \ 1860 \times \frac{1}{10^5}$	$.1657 \ 9606 \times \frac{1}{10^5}$	$.1051 \ 3897 \times \frac{1}{10^5}$	
$\pi$					
.71	.8647 231	.8449 917	.8019 165 <sup>+</sup>	.7547 671	.7045 530
.72	.8845 376	.8670 365 <sup>+</sup>	.8284 227	.7855 613	.7392 709
.73	.9024 637	.8871 192	.8529 070	.8144 041	.7722 434
.74	.9185 135 <sup>+</sup>	.9052 234	.8752 836	.8411 272	.8032 136
.75	.9327 262	.9213 639	.8955 042	.8656 038	.8319 659
.76	.9451 657	.9355 851	.9135 591	.8877 520	.8583 320
.77	.9559 180	.9479 585 <sup>-</sup>	.9294 761	.9075 360	.8821 952
.78	.9650 884	.9585 802	.9433 182	.9249 657	.9034 932
.79	.9727 979	.9675 671	.9551 808	.9400 952	.9222 189
.80	.9791 788	.9750 526	.9651 872	.9530 198	.9384 190
.81	.9843 715 <sup>-</sup>	.9811 822	.9734 840	.9638 709	.9521 910
.82	.9885 195 <sup>-</sup>	.9861 089	.9802 354	.9728 103	.9636 775 <sup>-</sup>
.83	.9917 661	.9899 886	.9856 172	.9800 235 <sup>+</sup>	.9730 594
.84	.9942 506	.9929 754	.9898 104	.9857 118	.9805 475 <sup>-</sup>
.85	.9961 045 <sup>-</sup>	.9952 175 <sup>-</sup>	.9929 957	.9900 843	.9863 723
.86	.9974 492	.9968 533	.9953 473	.9933 505 <sup>-</sup>	.9907 746
.87	.9983 938	.9980 091	.9970 281	.9957 122	.9939 951
.88	.9990 334	.9987 961	.9981 858	.9973 578	.9962 648
.89	.9994 483	.9993 096	.9989 498	.9984 560	.9977 968
.90	.9997 043	.9996 283	.9994 292	.9991 529	.9987 800
.91	.9998 531	.9998 145 <sup>+</sup>	.9997 125 <sup>-</sup>	.9995 694	.9993 740
.92	.9999 336	.9999 157	.9998 682	.9998 007	.9997 076
.93	.9999 733	.9999 660	.9999 463	.9999 181	.9998 788
.94	.9999 908	.9999 883	.9999 813	.9999 712	.9999 570
.95	.9999 975 <sup>-</sup>	.9999 967	.9999 947	.9999 918	.9999 876
.96	.9999 995 <sup>-</sup>	.9999 993	.9999 989	.9999 983	.9999 974
.97	.9999 999	.9999 999	.9999 999	.9999 998	.9999 997
.98	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 80

$q = 7.5$

$p = 1$

$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$= .4564\ 1722 \times \frac{1}{10^6}$	$.3107\ 5215 \times \frac{1}{10^6}$	$.2150\ 2394 \times \frac{1}{10^6}$	$.1542\ 0514 \times \frac{1}{10^6}$	$.1041\ 2824 \times \frac{1}{10^6}$	$.7430\ 3927$
.0000 001					
.0000 002	.0000 001				
.0000 005	.0000 001				
.0000 009	.0000 003	.0000 001			
.0000 017	.0000 005 <sup>+</sup>	.0000 002	.0000 001		
.0000 032	.0000 010	.0000 003	.0000 001		
.0000 056	.0000 019	.0000 006	.0000 002	.0000 001	
.0000 097	.0000 035	.0000 012	.0000 004	.0000 001	
.0000 164	.0000 061	.0000 022	.0000 008	.0000 004	.0000 001
.0000 270	.0000 104	.0000 039	.0000 015	.0000 008	.0000 002
.0000 430	.0000 173	.0000 068	.0000 026	.0000 010	.0000 004
.0000 689	.0000 283	.0000 115	.0000 046	.0000 019	.0000 008
.0001 070	.0000 454	.0000 169	.0000 078	.0000 032	.0000 014
.0001 632	.0000 714	.0000 308	.0000 141	.0000 054 <sup>+</sup>	.0000 027
.0002 447	.0001 104	.0000 491	.0000 213 <sup>+</sup>	.0000 084	.0000 040
.0003 613	.0001 678	.0000 768	.0000 341	.0000 125 <sup>+</sup>	.0000 062
.0005 255	.0002 510	.0001 182	.0000 510	.0000 214	.0000 115
.0007 534	.0003 700	.0001 791	.0000 816	.0000 346	.0000 189
.0010 657	.0005 375	.0002 673	.0001 414	.0000 547	.0000 286
.0014 881	.0007 793	.0003 932	.0002 082	.0000 828 <sup>+</sup>	.0000 457
.0020 523	.0010 895 <sup>+</sup>	.0005 795	.0002 930	.0001 507	.0000 762
.0027 973	.0015 220	.0008 168	.0004 329	.0002 268	.0001 127
.0037 697	.0021 007	.0011 548	.0006 270	.0003 365	.0001 782
.0050 252	.0028 664	.0016 130	.0008 965	.0004 926	.0002 678
.0066 291	.0038 680	.0022 268	.0012 663	.0007 119	.0004 961
.0086 571	.0051 643	.0030 399	.0017 676	.00010 162	.0006 582
.0111 959	.0068 244	.0041 930	.0024 394	.00014 433	.0009 436
.0143 438	.0089 288	.0054 835 <sup>+</sup>	.0034 297	.00019 988	.00014 565
.0182 101	.0115 794	.0072 595	.0044 968	.00027 570	.00016 746
.0229 154	.0148 541	.0095 952	.0060 166	.00037 594	.00023 255
.0285 902	.0188 978	.0123 327	.0079 549	.00050 745 <sup>+</sup>	.00032 050
.0353 745 <sup>+</sup>	.0238 317	.0158 535 <sup>+</sup>	.0104 239	.00067 864	.00044 666
.0434 150 <sup>+</sup>	.0297 971	.0201 964	.0135 420	.00089 794	.00058 878
.0528 635 <sup>+</sup>	.0369 458	.0255 038	.0171 054	.00117 535 <sup>+</sup>	.00078 594
.0638 735 <sup>+</sup>	.0454 375 <sup>+</sup>	.0319 305	.0221 866	.00152 577	.00010 504
.0765 967	.0554 374	.0396 426	.0286 114	.00195 068	.00014 593
.0911 791	.0671 125	.0488 148	.0364 169	.00246 055	.00019 712
.1077 560	.0806 278	.0596 274	.0459 294	.00305 050	.00026 042
.1264 474	.0961 414	.0722 620	.0574 436	.00372 029	.00033 620
.1473 523	.1137 990	.0868 971	.0706 595 <sup>+</sup>	.00449 228	.00042 247
.1705 434	.1337 280	.1037 018	.0859 694	.00534 957	.00053 594
.1960 618	.1560 316	.1228 300	.1035 154	.00628 814	.00066 224
.2230 118	.1807 848	.1444 120	.1236 666		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.99$  $q = 7.5$ 

	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$
$H(p, q) = 4564\ 1722 \times 10^6$	$.3107\ 5215 \times 10^6$	$.2156\ 2394 \times 10^6$	$.1522\ 0514 \times 10^6$	$.1091\ 28$	
$x$					
.81	.0222 542	.0039 340	.8834 207	.8608 049	.8362 10
.82	.0398 229	.0249 607	.9081 139	.8893 111	.8686 18
.83	.0545 264	.0427 738	.9292 894	.9140 558	.8970 80
.84	.0665 474	.0575 133	.9470 231	.9350 292	.9215 07
.85	.0761 232	.0693 949	.9614 892	.9523 426	.9419 08
.86	.0835 324	.0786 068	.9729 480	.9662 189	.9584 51
.87	.0890 707	.0857 424	.9817 287	.9769 759	.9714 26
.88	.0930 799	.0908 815 <sup>†</sup>	.9882 073	.9850 041	.9812 21
.89	.0958 419	.0944 704	.9927 840	.9907 390	.9882 97
.90	.0976 545	.0968 521	.9958 538	.9946 300	.9931 53
.91	.0987 741	.0983 306	.9977 039	.9971 160	.9962 89
.92	.0994 169	.0992 020	.9989 008	.9985 001	.9981 69
.93	.0997 537	.0996 603	.9995 402	.9993 882	.9991 08
.94	.0999 110	.0998 761	.9998 308	.9997 728	.9996 99
.95	.0999 749	.0999 634	.9999 306	.9999 317	.9999 08
.96	.0999 944	.0999 921	.9999 899	.9999 849	.9999 79
.97	.0999 993	.0999 989	.9999 983 <sup>†</sup>	.9999 980	.9999 97
.98	1.0000 000	.9999 999	.9999 999	.9999 999	.9999 99
.99		1.0000 000	1.0000 000	1.0000 000	1.0000 00



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.90

$q = 7.5$

$p = 22$  to

$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$
$\cdot 5848\ 0189 \times \frac{1}{10^7}$	$\cdot 4361\ 2345 \times \frac{1}{10^7}$	$\cdot 3288\ 7998 \times \frac{1}{10^7}$	$\cdot 2505\ 7522 \times \frac{1}{10^7}$	$\cdot 1927\ 5017 \times \frac{1}{10^7}$	$\cdot 1495\ 9715 \times \frac{1}{10^7}$
$\cdot 0000\ 001$					
$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 0000\ 003$	$\cdot 0000\ 001$				
$\cdot 0000\ 005^+$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 009$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 0000\ 017$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	
$\cdot 0000\ 030$	$\cdot 0000\ 013$	$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 052$	$\cdot 0000\ 023$	$\cdot 0000\ 010$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 087$	$\cdot 0000\ 040$	$\cdot 0000\ 018$	$\cdot 0000\ 008$	$\cdot 0000\ 004$	$\cdot 0000\ 002$
$\cdot 0000\ 145^+$	$\cdot 0000\ 068$	$\cdot 0000\ 032$	$\cdot 0000\ 015^-$	$\cdot 0000\ 007$	$\cdot 0000\ 003$
$\cdot 0000\ 238$	$\cdot 0000\ 115^-$	$\cdot 0000\ 055^+$	$\cdot 0000\ 026$	$\cdot 0000\ 012$	$\cdot 0000\ 006$
$\cdot 0000\ 382$	$\cdot 0000\ 189$	$\cdot 0000\ 093$	$\cdot 0000\ 045^+$	$\cdot 0000\ 022$	$\cdot 0000\ 011$
$\cdot 0000\ 603$	$\cdot 0000\ 307$	$\cdot 0000\ 155^-$	$\cdot 0000\ 077$	$\cdot 0000\ 038$	$\cdot 0000\ 019$
$\cdot 0000\ 940$	$\cdot 0000\ 490$	$\cdot 0000\ 253$	$\cdot 0000\ 130$	$\cdot 0000\ 066$	$\cdot 0000\ 033$
$\cdot 0001\ 442$	$\cdot 0000\ 769$	$\cdot 0000\ 407$	$\cdot 0000\ 214$	$\cdot 0000\ 111$	$\cdot 0000\ 058$
$\cdot 0002\ 182$	$\cdot 0001\ 192$	$\cdot 0000\ 645^+$	$\cdot 0000\ 347$	$\cdot 0000\ 185^-$	$\cdot 0000\ 098$
$\cdot 0003\ 258$	$\cdot 0001\ 820$	$\cdot 0001\ 008$	$\cdot 0000\ 554$	$\cdot 0000\ 302$	$\cdot 0000\ 164$
$\cdot 0004\ 802$	$\cdot 0002\ 741$	$\cdot 0001\ 552$	$\cdot 0000\ 872$	$\cdot 0000\ 486$	$\cdot 0000\ 269$
$\cdot 0006\ 988$	$\cdot 0004\ 076$	$\cdot 0002\ 358$	$\cdot 0001\ 353$	$\cdot 0000\ 771$	$\cdot 0000\ 437$
$\cdot 0010\ 047$	$\cdot 0005\ 984$	$\cdot 0003\ 535^+$	$\cdot 0002\ 073$	$\cdot 0001\ 206$	$\cdot 0000\ 697$
$\cdot 0014\ 276$	$\cdot 0008\ 679$	$\cdot 0005\ 234$	$\cdot 0003\ 132$	$\cdot 0001\ 861$	$\cdot 0001\ 098$
$\cdot 0020\ 054$	$\cdot 0012\ 439$	$\cdot 0007\ 653$	$\cdot 0004\ 673$	$\cdot 0002\ 833$	$\cdot 0001\ 706$
$\cdot 0027\ 861$	$\cdot 0017\ 624$	$\cdot 0011\ 058$	$\cdot 0006\ 887$	$\cdot 0004\ 259$	$\cdot 0002\ 616$
$\cdot 0038\ 291$	$\cdot 0024\ 690$	$\cdot 0015\ 793$	$\cdot 0010\ 027$	$\cdot 0006\ 321$	$\cdot 0003\ 959$
$\cdot 0052\ 077$	$\cdot 0034\ 214$	$\cdot 0022\ 300$	$\cdot 0014\ 427$	$\cdot 0009\ 269$	$\cdot 0005\ 916$
$\cdot 0070\ 103$	$\cdot 0046\ 910$	$\cdot 0031\ 143$	$\cdot 0020\ 523$	$\cdot 0013\ 432$	$\cdot 0008\ 733$
$\cdot 0093\ 429$	$\cdot 0063\ 650^-$	$\cdot 0043\ 024$	$\cdot 0028\ 870$	$\cdot 0019\ 239$	$\cdot 0012\ 739$
$\cdot 0123\ 301$	$\cdot 0085\ 488$	$\cdot 0058\ 814$	$\cdot 0040\ 169$	$\cdot 0027\ 249$	$\cdot 0018\ 366$
$\cdot 0161\ 170$	$\cdot 0113\ 680$	$\cdot 0079\ 570$	$\cdot 0055\ 295^-$	$\cdot 0038\ 166$	$\cdot 0026\ 177$
$\cdot 0208\ 692$	$\cdot 0149\ 696$	$\cdot 0106\ 564$	$\cdot 0075\ 320$	$\cdot 0052\ 881$	$\cdot 0036\ 893$
$\cdot 0267\ 737$	$\cdot 0195\ 234$	$\cdot 0141\ 298$	$\cdot 0101\ 543$	$\cdot 0072\ 490$	$\cdot 0051\ 427$
$\cdot 0340\ 371$	$\cdot 0252\ 227$	$\cdot 0185\ 525^-$	$\cdot 0135\ 512$	$\cdot 0098\ 333$	$\cdot 0070\ 913$
$\cdot 0428\ 840$	$\cdot 0322\ 831$	$\cdot 0241\ 249$	$\cdot 0179\ 043$	$\cdot 0132\ 016$	$\cdot 0096\ 745^+$
$\cdot 0535\ 539$	$\cdot 0409\ 412$	$\cdot 0310\ 731$	$\cdot 0234\ 232$	$\cdot 0175\ 436$	$\cdot 0130\ 603$
$\cdot 0662\ 955^-$	$\cdot 0514\ 511$	$\cdot 0396\ 405^+$	$\cdot 0303\ 456$	$\cdot 0230\ 796$	$\cdot 0174\ 485^+$
$\cdot 0813\ 606$	$\cdot 0640\ 794$	$\cdot 0501\ 154$	$\cdot 0389\ 356$	$\cdot 0300\ 610$	$\cdot 0230\ 722$
$\cdot 0989\ 958$	$\cdot 0790\ 984$	$\cdot 0627\ 652$	$\cdot 0494\ 809$	$\cdot 0387\ 684$	$\cdot 0301\ 985^-$
$\cdot 1194\ 326$	$\cdot 0967\ 772$	$\cdot 0778\ 894$	$\cdot 0622\ 873$	$\cdot 0495\ 092$	$\cdot 0391\ 270$
$\cdot 1428\ 755^+$	$\cdot 1173\ 704$	$\cdot 0957\ 798$	$\cdot 0776\ 707$	$\cdot 0626\ 111$	$\cdot 0501\ 870$
$\cdot 1694\ 898$	$\cdot 1411\ 061$	$\cdot 1167\ 146$	$\cdot 0959\ 463$	$\cdot 0784\ 137$	$\cdot 0637\ 302$
$\cdot 1993\ 877$	$\cdot 1681\ 708$	$\cdot 1409\ 443$	$\cdot 1174\ 160$	$\cdot 0972\ 570$	$\cdot 0801\ 218$
$\cdot 2326\ 149$	$\cdot 1986\ 945^-$	$\cdot 1686\ 749$	$\cdot 1423\ 515^-$	$\cdot 1194\ 659$	$\cdot 0997\ 271$
$\cdot 2691\ 373$	$\cdot 2327\ 345^-$	$\cdot 2000\ 511$	$\cdot 1709\ 765^+$	$\cdot 1453\ 327$	$\cdot 1228\ 944$
$\cdot 3088\ 303$	$\cdot 2702\ 624$	$\cdot 2385\ 806$	$\cdot 2096\ 965$	$\cdot 1847\ 581$	$\cdot 1628\ 111$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .91$  to  $.99$  $q = 7.5$ 

	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$B(p, q) = .5848\ 0189 \times \frac{x}{10^7}$	$.4361\ 2345 \times \frac{x}{10^7}$	$.3288\ 7998 \times \frac{x}{10^7}$	$.2505\ 7522 \times \frac{x}{10^7}$	$.1927\ 5017 \times \frac{x}{10^7}$	
$x$					
.91	.9941 042	.9927 036	.9910 690	.9891 785 <sup>+</sup>	.9870 109
.92	.9970 388	.9963 025 <sup>-</sup>	.9954 336	.9944 177	.9932 401
.93	.9986 798	.9983 366	.9979 274	.9974 437	.9968 769
.94	.9994 962	.9993 596	.9991 949	.9989 981	.9987 651
.95	.9998 446	.9998 006	.9997 471	.9996 825 <sup>-</sup>	.9996 051
.96	.9999 647	.9999 544	.9999 416	.9999 260	.9999 071
.97	.9999 951	.9999 936	.9999 917	.9999 894	.9999 866
.98	.9999 997	.9999 996	.9999 995 <sup>+</sup>	.9999 994	.9999 992
.99	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.99

$q = 7.5$

$p = 2$

$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$	$p = 33$
$= .1170\ 7603 \times \frac{1}{10^7}$	$.9234\ 1655 \times \frac{1}{10^6}$	$.7336\ 7342 \times \frac{1}{10^6}$	$.5809\ 3874 \times \frac{1}{10^6}$	$.4726\ 0002 \times \frac{1}{10^6}$	$.3728\ 1617 \times \frac{1}{10^6}$
.0000 001	.0000 001				
.0000 001	.0000 001	.0000 001			
.0000 003	.0000 001	.0000 001			
.0000 005 <sup>+</sup>	.0000 002	.0000 001	.0000 001		
.0000 009	.0000 005 <sup>-</sup>	.0000 002	.0000 001	.0000 001	
.0000 017	.0000 008	.0000 004	.0000 002	.0000 001	
.0000 030	.0000 008 <sup>+</sup>	.0000 008	.0000 004	.0000 002	.0000 001
.0000 052	.0000 027	.0000 014	.0000 007	.0000 004	.0000 002
.0000 088	.0000 047	.0000 025 <sup>+</sup>	.0000 013	.0000 007	.0000 004
.0000 148	.0000 081	.0000 044	.0000 024	.0000 013	.0000 007
.0000 246	.0000 137	.0000 076	.0000 042	.0000 023	.0000 014
.0000 401	.0000 220	.0000 130	.0000 073	.0000 041	.0000 024
.0000 644	.0000 376	.0000 218	.0000 126	.0000 072	.0000 041
.0001 021	.0000 608	.0000 360	.0000 212	.0000 124	.0000 072
.0001 597	.0000 969	.0000 585 <sup>-</sup>	.0000 351	.0000 210	.0000 124
.0002 464	.0001 525 <sup>-</sup>	.0000 938	.0000 574	.0000 450	.0000 212
.0003 753	.0002 367	.0001 484	.0000 926	.0000 575 <sup>+</sup>	.0000 355 <sup>+</sup>
.0005 644	.0003 626	.0002 317	.0001 473	.0000 932	.0000 575 <sup>+</sup>
.0008 383	.0005 486	.0003 570	.0002 311	.0001 480	.0000 935 <sup>+</sup>
.0012 304	.0008 196	.0005 430	.0003 579	.0002 348	.0001 513
.0017 846	.0012 098	.0008 157	.0005 472	.0003 654	.0002 428
.0025 586	.0017 645 <sup>-</sup>	.0012 104	.0008 212	.0005 612	.0003 795 <sup>+</sup>
.0036 269	.0025 437	.0017 745 <sup>+</sup>	.0012 318	.0008 510	.0005 854
.0050 841	.0036 249	.0025 710	.0018 145 <sup>+</sup>	.0012 246	.0008 914
.0070 488	.0051 076	.0036 810	.0026 411	.0018 857	.0014 404
.0096 672	.0071 169	.0052 125 <sup>-</sup>	.0037 691	.0027 562	.0020 905
.0131 168	.0098 076	.0072 061	.0054 015 <sup>+</sup>	.0037 806	.0029 207
.0176 094	.0133 688	.0100 985 <sup>-</sup>	.0075 017	.0056 814	.0042 344
.0233 933	.0180 269	.0138 226	.0105 488	.0080 144	.0060 628
.0307 549	.0240 481	.0187 122	.0144 925 <sup>-</sup>	.0111 245 <sup>+</sup>	.0085 298
.0400 130	.0317 393	.0250 548	.0196 872	.0154 017	.0119 987
.0515 238	.0414 470	.0331 822	.0264 451	.0209 848	.0169 812
.0656 648	.0535 519	.0434 686	.0351 270	.0282 650 <sup>+</sup>	.0226 541
.0828 285 <sup>-</sup>	.0684 616	.0563 266	.0461 493	.0376 361	.0305 272
.1034 063	.0865 978	.0721 947	.0599 280	.0495 409	.0407 940
.1277 690	.1083 784	.0915 252	.0769 669	.0644 642	.0541 824
.1562 432	.1341 952	.1147 620	.0972 496	.0829 141	.0700 696
.1890 835 <sup>-</sup>	.1643 866	.1423 169	.1222 154	.1054 060	.0902 615
.2264 420	.1992 957	.1745 326	.1523 185 <sup>+</sup>	.1324 144	.1141 294
.2683 418	.2387 892	.2116 511	.1868 694	.1644 266	.1444 554
.3146 387	.2831 076	.2537 228	.2266 429	.2017 105 <sup>+</sup>	.1789 084
.3650 051	.3319 625 <sup>+</sup>	.3008 171	.2716 391	.2444 611	.2192 844
.4189 083	.3849 306	.3524 879	.3216 907	.2926 495 <sup>+</sup>	.2654 868
.4756 053	.4413 619	.4082 461	.3764 664	.3455 888	.3154 868

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .43$  to  $1.00$  $q = 7.5$ 

	$p = 34$	$p = 35$	$p = 36$	$p = 37$	$p = 38$
$B(p, q) = .3119\ 6476 \times \frac{x}{108}$	$.2555\ 8559 \times \frac{x}{108}$	$.2104\ 8225 \times \frac{x}{108}$	$.1741\ 9221 \times \frac{x}{108}$	$.1448\ 3397 \times \frac{x}{108}$	
.43	.0000 001				
.44	.0000 002	.0000 001	.0000 001		
.45	.0000 004	.0000 002	.0000 001	.0000 001	
.46	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
.47	.0000 013	.0000 007	.0000 004	.0000 002	.0000 001
.48	.0000 023	.0000 013	.0000 007	.0000 004	.0000 002
.49	.0000 042	.0000 024	.0000 014	.0000 008	.0000 005-
.50	.0000 074	.0000 044	.0000 026	.0000 015-	.0000 009
.51	.0000 128	.0000 077	.0000 046	.0000 027	.0000 016
.52	.0000 219	.0000 134	.0000 082	.0000 050-	.0000 030
.53	.0000 368	.0000 230	.0000 143	.0000 088	.0000 055-
.54	.0000 610	.0000 388	.0000 246	.0000 155+	.0000 097
.55	.0000 997	.0000 645+	.0000 416	.0000 267	.0000 171
.56	.0001 607	.0001 059	.0000 695+	.0000 455-	.0000 296
.57	.0002 555-	.0001 713	.0001 144	.0000 761	.0000 505-
.58	.0004 008	.0002 734	.0001 857	.0001 257	.0000 848
.59	.0006 207	.0004 305-	.0002 974	.0002 047	.0001 404
.60	.0009 488	.0006 688	.0004 697	.0003 287	.0002 292
.61	.0014 319	.0010 258	.0007 321	.0005 206	.0003 689
.62	.0021 341	.0015 532	.0011 262	.0008 136	.0005 857
.63	.0031 415-	.0023 220	.0017 099	.0012 547	.0009 175-
.64	.0045 677	.0034 280	.0025 631	.0019 096	.0014 179
.65	.0065 610	.0049 979	.0037 932	.0028 688	.0021 624
.66	.0093 104	.0071 969	.0055 430	.0042 543	.0032 543
.67	.0130 533	.0102 362	.0079 981	.0062 279	.0048 335-
.68	.0180 817	.0143 805-	.0113 963	.0090 006	.0070 854
.69	.0247 471	.0199 553	.0160 350-	.0128 415+	.0102 509
.70	.0334 634	.0273 517	.0222 791	.0180 871	.0146 372
.71	.0447 051	.0370 282	.0305 654	.0251 483	.0206 265-
.72	.0590 009	.0495 079	.0414 035+	.0345 148	.0286 836
.73	.0769 197	.0653 686	.0553 704	.0467 538	.0393 587
.74	.0990 479	.0852 252	.0730 967	.0625 011	.0532 830
.75	.1259 579	.1097 006	.0952 432	.0824 424	.0711 552
.76	.1581 653	.1393 868	.1224 646	.1072 817	.0937 155+
.77	.1960 777	.1747 934	.1553 609	.1376 962	.1217 048
.78	.2399 352	.2162 868	.1944 154	.1742 755-	.1558 066
.79	.2897 484	.2640 218	.2399 234	.2174 483	.1965 738
.80	.3452 376	.3178 724	.2919 144	.2673 986	.2443 405-
.81	.4057 840	.3773 683	.3500 771	.3239 798	.2991 275-
.82	.4703 985+	.4416 490	.4136 963	.3866 365-	.3605 494
.83	.5377 218	.5094 445-	.4816 156	.4543 472	.4277 383
.84	.6060 620	.5790 963	.5522 385+	.5256 045+	.4993 017
.85	.6734 781	.6486 271	.6235 809	.5984 477	.5733 315-
.86	.7379 095+	.7158 639	.6933 836	.6705 599	.6474 832
.87	.7973 158	.7786 112	.7502 851	.7204 355+	.7010 237

.81	.4057 840	.3773 683	.3500 771	.3239 798	.2991 275-
.82	.4703 985+	.4416 490	.4136 963	.3866 365-	.3605 494
.83	.5377 218	.5094 445-	.4816 156	.4543 472	.4277 383
.84	.6060 620	.5790 963	.5522 385+	.5256 045+	.4993 017
.85	.6734 781	.6486 271	.6235 809	.5984 477	.5733 315-
.86	.7379 095+	.7158 639	.6933 836	.6705 599	.6474 832
.87	.7973 158	.7786 112	.7502 851	.7204 355+	.7010 237

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.100

$q = 7.5$

$p =$

$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$= .1014\ 5052 \times \frac{1}{100}$	$.8543\ 2018 \times \frac{1}{100}$	$.7222\ 0881 \times \frac{1}{100}$	$.6127\ 8324 \times \frac{1}{100}$	$.5217\ 2502 \times \frac{1}{100}$	$.4457\ 9200 \times \frac{1}{100}$
.0000 001	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001
.0000 001	.0000 002	.0000 001	.0000 001	.0000 001	.0000 001
.0000 003	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.0000 006	.0000 003	.0000 002	.0000 001	.0000 001	.0000 001
.0000 011	.0000 007	.0000 003	.0000 002	.0000 001	.0000 001
.0000 021	.0000 013	.0000 008	.0000 005	.0000 003	.0000 002
.0000 038	.0000 024	.0000 015	.0000 009	.0000 006	.0000 004
.0000 069	.0000 041	.0000 028	.0000 015	.0000 011	.0000 007
.0000 125	.0000 080	.0000 052	.0000 033	.0000 021	.0000 013
.0000 220	.0000 144	.0000 094	.0000 062	.0000 039	.0000 024
.0000 382	.0000 255 <sup>1</sup>	.0000 170	.0000 113	.0000 070	.0000 042
.0000 654	.0000 444	.0000 301	.0000 203	.0000 137	.0000 085
.0001 103	.0000 762	.0000 525	.0000 360	.0000 242	.0000 150
.0001 834	.0001 287	.0000 901	.0000 629	.0000 415	.0000 270
.0003 007	.0002 144	.0001 525	.0001 091	.0000 705	.0000 459
.0004 859	.0003 520	.0002 542	.0001 831	.0001 415	.0000 924
.0007 743	.0005 696	.0004 178	.0003 025	.0002 279	.0001 622
.0012 169	.0009 087	.0006 767	.0005 075 <sup>1</sup>	.0004 122	.0002 955
.0018 863	.0014 207	.0010 866	.0008 115	.0006 124	.0004 522
.0028 842	.0022 181	.0017 011	.0013 011	.0009 926	.0007 522
.0043 593	.0033 938	.0026 403	.0020 456	.0015 865	.0012 249
.0064 726	.0051 209	.0040 404	.0031 795 <sup>1</sup>	.0024 957	.0019 541
.0094 996	.0076 202	.0066 961	.0048 641	.0038 213	.0030 247
.0137 526	.0111 821	.0096 678	.0073 413	.0058 175	.0047 622
.0196 371	.0161 803	.0132 968	.0108 095	.0089 124	.0073 522
.0276 528	.0230 837	.0192 195 <sup>1</sup>	.0169 621	.0142 245	.0119 415
.0383 982	.0324 655 <sup>+</sup>	.0273 793	.0240 440	.0209 195	.0176 622
.0525 675	.0450 046	.0384 332	.0342 419	.0298 262	.0255 522
.0709 362	.0614 778	.0531 499	.0458 412	.0404 479	.0348 622
.0943 317	.0827 366	.0723 929	.0631 957	.0559 413	.0488 622
.1235 852	.1096 652	.0970 863	.0857 562	.0755 529	.0664 725
.1594 631	.1431 170	.1281 564	.1145 083	.1009 965 <sup>+</sup>	.0898 122
.2025 769	.1838 266	.1664 481	.1501 918	.1356 669	.1220 136
.2532 765 <sup>+</sup>	.2322 098	.2126 149	.1942 025	.1770 440	.1610 741
.3115 339	.2886 803	.2669 870	.2464 199	.2270 366	.2088 622
.3768 312	.3526 667	.3294 406	.3071 998	.2859 529	.2657 622
.4480 717	.4243 113	.3992 153	.3758 121	.3532 469	.3314 622
.5235 387	.4990 499	.4749 166	.4512 174	.4280 655 <sup>+</sup>	.4053 622
.6009 246	.5776 123	.5543 854	.5311 158	.5088 123	.4864 622
.6774 527	.6562 187	.6348 164	.6133 148	.5917 967	.5702 622
.7591 018	.7317 664	.7129 399	.6938 256	.6745 911	.6553 622
.8159 238	.8008 671	.7853 421	.7693 865	.7539 194	.7384 412
.8724 212	.8609 030	.8488 931	.8364 114	.8244 269	.8119 222

$x = .53$  to 1.00 $q = 7.5$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) = .3821\ 0490 \times \frac{x}{10^6}$		$.3285\ 3880 \times \frac{x}{10^6}$	$.2833\ 2703 \times \frac{x}{10^6}$	$.2450\ 39$
.53	.0000 001	.0000 001		
.54	.0000 002	.0000 001	.0000 001	
.55	.0000 004	.0000 003	.0000 002	.0000 00
.56	.0000 009	.0000 005 <sup>+</sup>	.0000 003	.0000 00
.57	.0000 017	.0000 011	.0000 007	.0000 00
.58	.0000 033	.0000 021	.0000 014	.0000 00
.59	.0000 062	.0000 041	.0000 027	.0000 01
.60	.0000 115 <sup>-</sup>	.0000 078	.0000 053	.0000 03
.61	.0000 210	.0000 145 <sup>+</sup>	.0000 100	.0000 06
.62	.0000 379	.0000 266	.0000 187	.0000 13
.63	.0000 674	.0000 480	.0000 342	.0000 24
.64	.0001 178	.0000 853	.0000 616	.0000 44
.65	.0002 027	.0001 490	.0001 093	.0000 80
.66	.0003 435 <sup>+</sup>	.0002 564	.0001 909	.0001 41
.67	.0005 734	.0004 343	.0003 282	.0002 47
.68	.0009 427	.0007 244	.0005 554	.0004 24
.69	.0015 265 <sup>+</sup>	.0011 897	.0009 252	.0007 18
.70	.0024 347	.0019 243	.0015 175 <sup>-</sup>	.0011 94
.71	.0038 248	.0030 646	.0024 502	.0019 54
.72	.0059 174	.0048 055 <sup>+</sup>	.0038 943	.0031 49
.73	.0090 151	.0074 187	.0060 922	.0049 92
.74	.0135 229	.0112 737	.0093 792	.0077 87
.75	.0199 681	.0168 604	.0142 074	.0119 48
.76	.0290 180	.0248 101	.0211 698	.0180 28
.77	.0414 897	.0359 103	.0310 199	.0267 44
.78	.0583 456	.0511 082	.0446 823	.0389 91
.79	.0806 680	.0714 942	.0632 446	.0558 44
.80	.1096 047	.0982 566	.0879 226	.0785 35
.81	.1462 774	.1325 994	.1199 880	.1083 89
.82	.1916 494	.1756 155 <sup>-</sup>	.1606 493	.1467 15
.83	.2463 555 <sup>+</sup>	.2281 158	.2108 831	.1946 43
.84	.3105 039	.2904 219	.2712 201	.2529 06
.85	.3834 753	.3621 442	.3415 051	.3215 86
.86	.4637 584	.4419 838	.4206 680	.3998 53
.87	.5488 692	.5276 113	.5065 592	.4857 63
.88	.6354 131	.6156 833	.5959 200	.5761 73
.89	.7193 332	.7020 572	.6845 550 <sup>+</sup>	.6668 68
.90	.7963 635 <sup>+</sup>	.7822 312	.7677 530	.7529 58
.91	.8626 520	.8519 914	.8409 486	.8295 38
.92	.9154 495 <sup>+</sup>	.9081 638	.9005 339	.8925 63
.93	.9536 898	.9492 892	.9446 307	.9397 11
.94	.9782 514	.9759 864	.9735 628	.9709 76
.95	.9917 283	.9907 894	.9897 741	.9886 78
.96	.9976 837	.9973 986	.9970 871	.9967 47
.97	.9996 004	.9995 473	.9994 887	.9994 24
.98	.9999 715 <sup>+</sup>	.9999 675 <sup>-</sup>	.9999 629	.9999 57
.99	.9999 998	.9999 997	.9999 997	.9999 99
1.00	1.0000 000	1.0000 000	1.0000 000	1.0000 00

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

70

$q = 8$

$p = 8$  to

$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$
$\cdot 1942\ 5019 \times \frac{1}{10^4}$	$\cdot 1362\ 8793 \times \frac{1}{10^4}$	$\cdot 9712\ 5097 \times \frac{1}{10^5}$	$\cdot 7020\ 8932 \times \frac{1}{10^5}$	$\cdot 5141\ 9169 \times \frac{1}{10^5}$	$\cdot 3811\ 3420 \times \frac{1}{10^5}$
$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 0000\ 007$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 024$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 0000\ 065^-$	$\cdot 0000\ 024$	$\cdot 0000\ 009$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 0000\ 155^-$	$\cdot 0000\ 062$	$\cdot 0000\ 025^-$	$\cdot 0000\ 010$	$\cdot 0000\ 004$	$\cdot 0000\ 001$
$\cdot 0000\ 336$	$\cdot 0000\ 142$	$\cdot 0000\ 059$	$\cdot 0000\ 024$	$\cdot 0000\ 010$	$\cdot 0000\ 004$
$\cdot 0000\ 673$	$\cdot 0000\ 298$	$\cdot 0000\ 130$	$\cdot 0000\ 056$	$\cdot 0000\ 024$	$\cdot 0000\ 010$
$\cdot 0001\ 261$	$\cdot 0000\ 583$	$\cdot 0000\ 266$	$\cdot 0000\ 120$	$\cdot 0000\ 054$	$\cdot 0000\ 024$
$\cdot 0002\ 233$	$\cdot 0001\ 073$	$\cdot 0000\ 510$	$\cdot 0000\ 240$	$\cdot 0000\ 112$	$\cdot 0000\ 051$
$\cdot 0003\ 767$	$\cdot 0001\ 877$	$\cdot 0000\ 925^-$	$\cdot 0000\ 451$	$\cdot 0000\ 218$	$\cdot 0000\ 104$
$\cdot 0006\ 096$	$\cdot 0003\ 143$	$\cdot 0001\ 602$	$\cdot 0000\ 808$	$\cdot 0000\ 404$	$\cdot 0000\ 200$
$\cdot 0009\ 515^-$	$\cdot 0005\ 063$	$\cdot 0002\ 664$	$\cdot 0001\ 387$	$\cdot 0000\ 715^+$	$\cdot 0000\ 366$
$\cdot 0014\ 384$	$\cdot 0007\ 884$	$\cdot 0004\ 274$	$\cdot 0002\ 292$	$\cdot 0001\ 218$	$\cdot 0000\ 641$
$\cdot 0021\ 136$	$\cdot 0011\ 913$	$\cdot 0006\ 640$	$\cdot 0003\ 663$	$\cdot 0002\ 002$	$\cdot 0001\ 084$
$\cdot 0030\ 279$	$\cdot 0017\ 522$	$\cdot 0010\ 028$	$\cdot 0005\ 680$	$\cdot 0003\ 187$	$\cdot 0001\ 773$
$\cdot 0042\ 397$	$\cdot 0025\ 153$	$\cdot 0014\ 759$	$\cdot 0008\ 573$	$\cdot 0004\ 932$	$\cdot 0002\ 813$
$\cdot 0058\ 149$	$\cdot 0035\ 323$	$\cdot 0021\ 224$	$\cdot 0012\ 624$	$\cdot 0007\ 439$	$\cdot 0004\ 345^+$
$\cdot 0078\ 264$	$\cdot 0048\ 622$	$\cdot 0029\ 881$	$\cdot 0018\ 180$	$\cdot 0010\ 959$	$\cdot 0006\ 548$
$\cdot 0103\ 536$	$\cdot 0065\ 715^-$	$\cdot 0041\ 263$	$\cdot 0025\ 652$	$\cdot 0015\ 800$	$\cdot 0009\ 649$
$\cdot 0134\ 815^+$	$\cdot 0087\ 335^-$	$\cdot 0055\ 975^+$	$\cdot 0035\ 523$	$\cdot 0022\ 337$	$\cdot 0013\ 925^+$
$\cdot 0172\ 998$	$\cdot 0114\ 280$	$\cdot 0074\ 697$	$\cdot 0048\ 347$	$\cdot 0031\ 008$	$\cdot 0019\ 718$
$\cdot 0219\ 012$	$\cdot 0147\ 407$	$\cdot 0098\ 178$	$\cdot 0064\ 756$	$\cdot 0042\ 326$	$\cdot 0027\ 432$
$\cdot 0273\ 800$	$\cdot 0187\ 616$	$\cdot 0127\ 231$	$\cdot 0085\ 452$	$\cdot 0056\ 878$	$\cdot 0037\ 542$
$\cdot 0338\ 305^-$	$\cdot 0235\ 840$	$\cdot 0162\ 726$	$\cdot 0111\ 208$	$\cdot 0075\ 326$	$\cdot 0050\ 598$
$\cdot 0413\ 451$	$\cdot 0293\ 030$	$\cdot 0205\ 578$	$\cdot 0142\ 864$	$\cdot 0098\ 407$	$\cdot 0067\ 227$
$\cdot 0500\ 125^+$	$\cdot 0360\ 140$	$\cdot 0256\ 735^+$	$\cdot 0181\ 310$	$\cdot 0126\ 927$	$\cdot 0088\ 131$
$\cdot 0599\ 156$	$\cdot 0438\ 104$	$\cdot 0317\ 165^+$	$\cdot 0227\ 488$	$\cdot 0161\ 757$	$\cdot 0114\ 088$
$\cdot 0711\ 294$	$\cdot 0527\ 821$	$\cdot 0387\ 834$	$\cdot 0282\ 367$	$\cdot 0203\ 822$	$\cdot 0145\ 947$
$\cdot 0837\ 194$	$\cdot 0630\ 134$	$\cdot 0469\ 693$	$\cdot 0346\ 936$	$\cdot 0254\ 092$	$\cdot 0184\ 618$
$\cdot 0977\ 397$	$\cdot 0745\ 808$	$\cdot 0563\ 653$	$\cdot 0422\ 180$	$\cdot 0313\ 568$	$\cdot 0231\ 069$
$\cdot 1132\ 311$	$\cdot 0875\ 511$	$\cdot 0670\ 569$	$\cdot 0509\ 066$	$\cdot 0383\ 263$	$\cdot 0286\ 306$
$\cdot 1302\ 201$	$\cdot 1019\ 798$	$\cdot 0791\ 215^+$	$\cdot 0608\ 521$	$\cdot 0404\ 185^-$	$\cdot 0351\ 364$
$\cdot 1487\ 170$	$\cdot 1179\ 087$	$\cdot 0926\ 268$	$\cdot 0721\ 406$	$\cdot 0557\ 319$	$\cdot 0427\ 286$
$\cdot 1687\ 154$	$\cdot 1353\ 649$	$\cdot 1076\ 281$	$\cdot 0848\ 502$	$\cdot 0663\ 602$	$\cdot 0515\ 105^-$
$\cdot 1901\ 914$	$\cdot 1543\ 589$	$\cdot 1241\ 671$	$\cdot 0990\ 479$	$\cdot 0783\ 902$	$\cdot 0615\ 820$
$\cdot 2131\ 032$	$\cdot 1748\ 841$	$\cdot 1422\ 697$	$\cdot 1147\ 883$	$\cdot 0918\ 993$	$\cdot 0730\ 376$
$\cdot 2373\ 908$	$\cdot 1969\ 153$	$\cdot 1619\ 449$	$\cdot 1321\ 111$	$\cdot 1069\ 532$	$\cdot 0859\ 638$
$\cdot 2629\ 766$	$\cdot 2204\ 087$	$\cdot 1831\ 832$	$\cdot 1510\ 395^-$	$\cdot 1236\ 042$	$\cdot 1004\ 365^-$
$\cdot 2897\ 659$	$\cdot 2453\ 016$	$\cdot 2059\ 560$	$\cdot 1715\ 783$	$\cdot 1418\ 880$	$\cdot 1165\ 187$
$\cdot 3176\ 477$	$\cdot 2715\ 125^-$	$\cdot 2302\ 149$	$\cdot 1937\ 132$	$\cdot 1618\ 231$	$\cdot 1342\ 582$
$\cdot 3464\ 961$	$\cdot 2989\ 419$	$\cdot 2558\ 915^-$	$\cdot 2174\ 092$	$\cdot 1834\ 078$	$\cdot 1536\ 850^+$
$\cdot 3761\ 718$	$\cdot 3274\ 731$	$\cdot 2828\ 977$	$\cdot 2426\ 106$	$\cdot 2066\ 202$	$\cdot 1748\ 100$
$\cdot 4065\ 240$	$\cdot 3569\ 733$	$\cdot 3111\ 262$	$\cdot 2692\ 404$	$\cdot 2314\ 160$	$\cdot 1976\ 227$
$\cdot 4373\ 921$	$\cdot 3872\ 958$	$\cdot 3404\ 517$	$\cdot 2972\ 007$	$\cdot 2577\ 291$	$\cdot 2220\ 899$
$\cdot 4686\ 084$	$\cdot 4182\ 812$	$\cdot 3707\ 310$	$\cdot 3262\ 355^+$	$\cdot 2837\ 311$	

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.97$  $q = 8$ 

	$p = 8$	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$
$B(p, q) = .1942\ 5019 \times \frac{1}{10^4}$	$.1362\ 8793 \times \frac{1}{10^4}$	$.9712\ 5097 \times \frac{1}{10^8}$	$.7020\ 8932 \times \frac{1}{10^8}$	$.5141\ 9169$	
$x$					
.71	.9586 549	.9489 321	.9378 675 <sup>+</sup>	.9254 355 <sup>+</sup>	.9116 293
.72	.9661 695 <sup>+</sup>	.9579 879	.9486 116	.9380 024	.9261 375 <sup>+</sup>
.73	.9726 200	.9658 154	.9579 632	.9490 166	.9389 418
.74	.9780 988	.9725 095 <sup>-</sup>	.9660 154	.9585 657	.9501 190
.75	.9827 002	.9781 696	.9728 700	.9667 496	.9597 632
.76	.9865 185 <sup>-</sup>	.9828 978	.9786 345 <sup>-</sup>	.9736 779	.9679 823
.77	.9896 464	.9867 968	.9834 192	.9794 665 <sup>+</sup>	.9748 947
.78	.9921 736	.9899 674	.9873 354	.9842 353	.9806 264
.79	.9941 851	.9925 071	.9904 926	.9881 046	.9853 067
.80	.9957 603	.9945 086	.9929 964	.9911 925 <sup>+</sup>	.9890 657
.81	.9969 721	.9960 581	.9949 469	.9936 131	.9920 307
.82	.9978 864	.9972 344	.9964 368	.9954 735 <sup>+</sup>	.9943 237
.83	.9985 616	.9981 084	.9975 506	.9968 728	.9960 588
.84	.9990 485 <sup>+</sup>	.9987 424	.9983 634	.9979 001	.9973 404
.85	.9993 904	.9991 903	.9989 410	.9986 345 <sup>-</sup>	.9982 619
.86	.9996 233	.9994 972	.9993 392	.9991 437	.9989 047
.87	.9997 767	.9997 005 <sup>+</sup>	.9996 045 <sup>-</sup>	.9994 849	.9993 380
.88	.9998 739	.9998 300	.9997 744	.9997 048	.9996 187
.89	.9999 327	.9999 088	.9998 784	.9998 401	.9997 924
.90	.9999 664	.9999 542	.9999 387	.9999 190	.9998 944
.91	.9999 845 <sup>+</sup>	.9999 788	.9999 715 <sup>-</sup>	.9999 622	.9999 504
.92	.9999 935 <sup>+</sup>	.9999 911	.9999 880	.9999 840	.9999 789
.93	.9999 976	.9999 967	.9999 956	.9999 940	.9999 921
.94	.9999 993	.9999 990	.9999 986	.9999 981	.9999 975 <sup>+</sup>
.95	.9999 998	.9999 997	.9999 997	.9999 995 <sup>+</sup>	.9999 994
.96	1.0000 000	1.0000 000	.9999 999	.9999 999	.9999 999
.97			1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

9 to 70

$q = 8$

$p = 11$

$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$
$q) = .2856\ 6205 \times \frac{1}{10^8}$	$.1653\ 8329 \times \frac{1}{10^8}$	$.9922\ 9975 \times \frac{1}{10^8}$	$.6142\ 8080 \times \frac{1}{10^8}$	$.3909\ 0596 \times \frac{1}{10^8}$	$.2549\ 3867 \times \frac{1}{10^8}$
.0000 001					
.0000 002					
.0000 004	.0000 001				
.0000 010	.0000 002				
.0000 024	.0000 005-	.0000 001			
.0000 049	.0000 011	.0000 002			
.0000 098	.0000 023	.0000 005+	.0000 001		
.0000 185+	.0000 047	.0000 011	.0000 003	.0000 001	
.0000 335-	.0000 089	.0000 023	.0000 006	.0000 001	
.0000 582	.0000 164	.0000 045+	.0000 012	.0000 003	.0000 001
.0000 978	.0000 291	.0000 084	.0000 024	.0000 007	.0000 002
.0001 591	.0000 498	.0000 152	.0000 045+	.0000 013	.0000 004
.0002 518	.0000 826	.0000 264	.0000 082	.0000 025+	.0000 008
.0003 881	.0001 334	.0000 446	.0000 146	.0000 047	.0000 015-
.0005 845-	.0002 098	.0000 733	.0000 250+	.0000 084	.0000 027
.0008 612	.0003 222	.0001 174	.0000 418	.0000 146	.0000 050-
.0012 440	.0004 844	.0001 837	.0000 681	.0000 247	.0000 088
.0017 639	.0007 136	.0002 812	.0001 083	.0000 408	.0000 151
.0024 586	.0010 319	.0004 219	.0001 686	.0000 660	.0000 253
.0033 724	.0014 663	.0006 212	.0002 572	.0001 043	.0000 415+
.0045 572	.0020 500-	.0008 987	.0003 851	.0001 616	.0000 660
.0060 725+	.0028 226	.0012 789	.0005 664	.0002 458	.0001 047
.0079 857	.0038 311	.0017 919	.0008 194	.0003 671	.0001 615+
.0103 719	.0051 302	.0024 744	.0011 669	.0005 393	.0002 447
.0133 140	.0067 826	.0033 700	.0016 374	.0007 798	.0003 047
.0169 019	.0088 597	.0045 303	.0022 657	.0011 108	.0005 348
.0212 315+	.0114 409	.0060 153	.0030 938	.0015 600	.0007 726
.0264 042	.0146 140	.0078 936	.0041 715-	.0021 616	.0011 003
.0325 250+	.0184 743	.0102 430	.0055 574	.0029 569	.0015 457
.0397 012	.0231 241	.0131 503	.0073 194	.0039 958	.0021 434
.0480 401	.0286 713	.0167 110	.0095 348	.0053 368	.0029 355-
.0576 473	.0352 279	.0210 289	.0122 911	.0070 484	.0039 727
.0686 241	.0429 087	.0262 151	.0156 853	.0092 095-	.0053 153
.0810 647	.0518 285+	.0323 865-	.0198 239	.0119 095-	.0070 342
.0950 544	.0621 005-	.0396 644	.0248 220	.0152 487	.0092 111
.1106 660	.0738 328	.0481 724	.0308 023	.0193 380	.0119 398
.1279 578	.0871 264	.0580 341	.0378 932	.0242 982	.0153 254
.1469 711	.1020 717	.0693 699	.0462 271	.0302 586	.0194 852
.1677 276	.1187 454	.0822 945+	.0559 377	.0373 562	.0245 473
.1902 275+	.1372 081	.0969 133	.0671 571	.0457 327	.0306 499
.2144 478	.1575 005+	.1133 189	.0800 128	.0555 328	.0379 398
.2403 412	.1796 417	.1315 880	.0946 236	.0669 003	.0465 698
.2678 349	.2036 260	.1517 773	.1110 960	.0799 748	.0566 961
.2968 307	.2294 211	.1730 206	.1325 225		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.98$  $q = 8$ 

	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .2856\ 6205 \times \frac{1}{10^8}$	$.1653\ 8329 \times \frac{1}{10^8}$	$.9922\ 9975 \times \frac{1}{10^8}$	$.6142\ 8080 \times \frac{1}{10^8}$	$.3909\ 059$	
$.71$	.8799 597	.8431 654	.8018 024	.7566 214	.7085 031
$.72$	.8986 292	.8662 194	.8292 722	.7883 461	.7441 451
$.73$	.9153 363	.8871 384	.8545 463	.8179 430	.7778 621
$.74$	.9301 214	.9059 060	.8775 337	.8452 330	.8093 791
$.75$	.9430 520	.9225 428	.8981 881	.8700 866	.8384 721
$.76$	.9542 197	.9371 040	.9165 081	.8924 268	.8649 741
$.77$	.9637 363	.9496 766	.9325 355 <sup>-</sup>	.9122 299	.8887 769
$.78$	.9717 302	.9603 755 <sup>-</sup>	.9463 524	.9295 247	.9098 363
$.79$	.9783 420	.9693 386	.9580 769	.9443 895 <sup>-</sup>	.9281 699
$.80$	.9837 199	.9767 217	.9678 573	.9569 474	.9438 554
$.81$	.9880 151	.9826 925 <sup>+</sup>	.9758 664	.9673 599	.9570 246
$.82$	.9913 779	.9874 253	.9822 935 <sup>-</sup>	.9758 194	.9678 564
$.83$	.9939 538	.9910 948	.9873 377	.9825 401	.9765 672
$.84$	.9958 795 <sup>-</sup>	.9938 713	.9912 006	.9877 491	.9834 002
$.85$	.9972 806	.9959 157	.9940 789	.9916 768	.9886 141
$.86$	.9982 696	.9973 757	.9961 586	.9945 483	.9924 709
$.87$	.9989 439	.9983 828	.9976 099	.9965 755 <sup>-</sup>	.9952 255
$.88$	.9993 858	.9990 504	.9985 832	.9979 506	.9971 155
$.89$	.9996 625 <sup>-</sup>	.9994 731	.9992 064	.9988 411	.9983 534
$.90$	.9998 265 <sup>+</sup>	.9997 267	.9995 844	.9993 873	.9991 213
$.91$	.9999 178	.9998 693	.9997 993	.9997 014	.9995 677
$.92$	.9999 648	.9999 434	.9999 124	.9998 684	.9998 077
$.93$	.9999 867	.9999 784	.9999 663	.9999 489	.9999 246
$.94$	.9999 957	.9999 930	.9999 890	.9999 832	.9999 750
$.95$	.9999 989	.9999 982	.9999 971	.9999 956	.9999 934
$.96$	.9999 998	.9999 997	.9999 995 <sup>-</sup>	.9999 998	.9999 987
$.97$	I.0000 000	I.0000 000	.9999 999	.9999 999	.9999 999
$.98$			I.0000 000	I.0000 000	I.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.80

$q = 8$

$p = 17$  to

$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$
$= .1699\ 5911 \times \frac{1}{10^4}$	$.1155\ 7220 \times \frac{1}{10^4}$	$.8001\ 1522 \times \frac{1}{10^4}$	$.5630\ 4404 \times \frac{1}{10^4}$	$.4021\ 7432 \times \frac{1}{10^4}$	$.2912\ 2968 \times \frac{1}{10^4}$
.0000 001					
.0000 002	.0000 001				
.0000 005-	.0000 001				
.0000 009	.0000 003	.0000 001			
.0000 017	.0000 006	.0000 002	.0000 001		
.0000 031	.0000 011	.0000 004	.0000 001		
.0000 055-	.0000 020	.0000 007	.0000 002	.0000 001	
.0000 096	.0000 036	.0000 013	.0000 005-	.0000 002	.0000 001
.0000 163	.0000 063	.0000 024	.0000 009	.0000 003	.0000 001
.0000 270	.0000 108	.0000 042	.0000 016	.0000 006	.0000 002
.0000 439	.0000 181	.0000 074	.0000 030	.0000 012	.0000 005-
.0000 699	.0000 298	.0000 125+	.0000 052	.0000 021	.0000 009
.0001 093	.0000 480	.0000 208	.0000 089	.0000 038	.0000 016
.0001 678	.0000 760	.0000 340	.0000 150+	.0000 065+	.0000 028
.0002 534	.0001 182	.0000 544	.0000 247	.0000 111	.0000 049
.0003 765+	.0001 808	.0000 850	.0000 401	.0000 185+	.0000 085-
.0005 511	.0002 720	.0001 325-	.0000 637	.0000 303	.0000 142
.0007 952	.0004 031	.0002 016	.0000 996	.0000 487	.0000 235+
.0011 316	.0005 887	.0003 023	.0001 533	.0000 769	.0000 381
.0015 893	.0008 480	.0004 466	.0002 323	.0001 195-	.0000 608
.0022 041	.0012 054	.0006 506	.0003 469	.0001 829	.0000 954
.0030 202	.0016 918	.0009 353	.0005 109	.0002 759	.0001 475+
.0040 907	.0023 454	.0013 274	.0007 422	.0004 104	.0002 247
.0054 792	.0032 136	.0018 606	.0010 645-	.0006 023	.0003 373
.0072 603	.0043 536	.0025 773	.0015 077	.0008 724	.0004 997
.0095 212	.0058 338	.0035 291	.0021 100	.0012 478	.0007 305+
.0123 612	.0077 349	.0047 792	.0029 186	.0017 632	.0010 545+
.0158 933	.0101 511	.0064 027	.0039 919	.0024 623	.0015 037
.0202 428	.0131 904	.0084 889	.0054 007	.0033 995+	.0021 188
.0255 478	.0169 754	.0111 414	.0072 296	.0046 419	.0029 513
.0319 573	.0216 426	.0144 796	.0095 786	.0062 705-	.0040 650+
.0396 300	.0273 423	.0186 386	.0125 643	.0083 822	.0055 383
.0487 312	.0342 371	.0237 690	.0163 202	.0110 912	.0074 657
.0594 303	.0424 994	.0300 364	.0209 975-	.0145 301	.0099 598
.0718 963	.0523 095-	.0376 194	.0267 642	.0188 508	.0131 531
.0862 933	.0638 508	.0467 071	.0338 043	.0242 240	.0171 984
.1027 756	.0773 058	.0574 960	.0423 155-	.0308 390	.0222 698
.1214 810	.0928 507	.0701 850-	.0525 057	.0389 015+	.0285 623
.1425 255+	.1106 486	.0849 699	.0645 892	.0486 311	.0362 901
.1659 963	.1308 433	.1020 375-	.0787 805-	.0602 563	.0456 840
.1919 452	.1535 517	.1215 571	.0952 879	.0740 098	.0569 873
.2203 832	.1788 562	.1436 736	.1143 054	.0901 207	.0704 504
.2512 742	.2067 973	.1684 978	.1360 037	.1088 061	.0863 228
.2845 308	.2373 669	.1960 084	.1605 227		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.99$  $q = 8$ 

	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$B(p, q) = .1699\ 5911 \times \frac{1}{10^6}$	$.1155\ 7220 \times \frac{1}{10^6}$	$.8001\ 1522 \times \frac{1}{10^7}$	$.5630\ 4404 \times \frac{1}{10^7}$	$.4021\ 743$	
$x$					
.81	.9304 496	.9141 010	.8957 088	.8753 226	.8530 302
.82	.9469 908	.9339 224	.9190 389	.9023 381	.8838 503
.83	.9606 208	.9504 554	.9387 369	.9254 272	.9105 136
.84	.9715 729	.9639 008	.9549 500 <sup>+</sup>	.9446 613	.9329 940
.85	.9801 307	.9745 324	.9679 232	.9602 358	.9514 144
.86	.9866 115 <sup>-</sup>	.9826 786	.9779 811	.9724 528	.9660 345
.87	.9913 487	.9887 028	.9855 056	.9816 993	.9772 287
.88	.9946 744	.9929 806	.9909 104	.9884 174	.9854 558
.89	.9969 025 <sup>+</sup>	.9958 793	.9946 144	.9930 740	.9912 231
.90	.9983 159	.9977 387	.9970 172	.9961 286	.9950 490
.91	.9991 560	.9988 561	.9984 772	.9980 053	.9974 257
.92	.9996 174	.9994 767	.9992 969	.9990 705 <sup>+</sup>	.9987 894
.93	.9998 473	.9997 892	.9997 141	.9996 186	.9994 087
.94	.9999 483	.9999 280	.9999 015 <sup>+</sup>	.9998 674	.9998 242
.95	.9999 861	.9999 804	.9999 730	.9999 633	.9999 508
.96	.9999 973	.9999 962	.9999 947	.9999 927	.9999 901
.97	.9999 997	.9999 996	.9999 994	.9999 991	.9999 988
.98	1.0000 000	1.0000 000	1.0000 000	1.0000 000	.9999 999
.99					1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q = 8$   $p = 2$

$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$
$\beta(q) = .2135\ 6843 \times \frac{1}{10^5}$	$.1584\ 5400 \times \frac{1}{10^5}$	$.1188\ 4050 \times \frac{1}{10^5}$	$.9003\ 0680 \times \frac{1}{10^5}$	$.6884\ 6990 \times \frac{1}{10^5}$	$.5311\ 0535 \times \frac{1}{10^5}$
.29 .0000 001					
.30 .0000 002	.0000 001				
.31 .0000 003	.0000 001	.0000 001			
.32 .0000 007	.0000 003	.0000 001			
.33 .0000 012	.0000 005 <sup>+</sup>	.0000 002	.0000 001		
.34 .0000 022	.0000 010	.0000 004	.0000 002	.0000 001	
.35 .0000 038	.0000 017	.0000 008	.0000 003	.0000 001	.0000 001
.36 .0000 066	.0000 031	.0000 014	.0000 006	.0000 003	.0000 001
.37 .0000 113	.0000 053	.0000 025 <sup>+</sup>	.0000 012	.0000 005 <sup>+</sup>	.0000 002
.38 .0000 187	.0000 091	.0000 044	.0000 021	.0000 010	.0000 005 <sup>-</sup>
.39 .0000 307	.0000 153	.0000 076	.0000 037	.0000 018	.0000 009
.40 .0000 493	.0000 253	.0000 128	.0000 065 <sup>-</sup>	.0000 032	.0000 016
.41 .0000 781	.0000 410	.0000 213	.0000 110	.0000 056	.0000 029
.42 .0001 218	.0000 654	.0000 349	.0000 184	.0000 097	.0000 050 <sup>+</sup>
.43 .0001 871	.0001 029	.0000 561	.0000 304	.0000 163	.0000 087
.44 .0002 835 <sup>+</sup>	.0001 595 <sup>-</sup>	.0000 889	.0000 492	.0000 271	.0000 148
.45 .0004 237	.0002 436	.0001 389	.0000 786	.0000 442	.0000 246
.46 .0006 248	.0003 670	.0002 138	.0001 236	.0000 710	.0000 405 <sup>-</sup>
.47 .0009 098	.0005 457	.0003 247	.0001 917	.0001 124	.0000 655 <sup>-</sup>
.48 .0013 084	.0008 011	.0004 865 <sup>+</sup>	.0002 933	.0001 755 <sup>+</sup>	.0001 044
.49 .0018 593	.0011 614	.0007 197	.0004 426	.0002 703	.0001 640
.50 .0026 114	.0016 634	.0010 512	.0006 594	.0004 107	.0002 541
.51 .0036 264	.0023 546	.0015 168	.0009 699	.0006 159	.0003 886
.52 .0049 806	.0032 950 <sup>+</sup>	.0021 629	.0014 093	.0009 120	.0005 863
.53 .0067 669	.0045 596	.0030 485 <sup>-</sup>	.0020 234	.0013 338	.0008 735 <sup>+</sup>
.54 .0090 974	.0062 407	.0042 482	.0028 710	.0019 271	.0012 852
.55 .0121 048	.0084 507	.0058 548	.0040 273	.0027 516	.0018 680
.56 .0159 443	.0113 238	.0079 817	.0055 861	.0038 834	.0026 827
.57 .0207 939	.0150 182	.0107 658	.0076 633	.0054 187	.0038 076
.58 .0268 549	.0197 169	.0143 694	.0103 994	.0074 769	.0053 424
.59 .0343 507	.0256 289	.0189 823	.0139 627	.0102 038	.0074 111
.60 .0435 241	.0329 877	.0248 219	.0185 507	.0137 748	.0101 664
.61 .0546 337	.0420 492	.0321 336	.0243 915 <sup>+</sup>	.0183 973	.0137 929
.62 .0679 478	.0530 880	.0411 878	.0317 438	.0243 120	.0185 098
.63 .0837 364	.0663 911	.0522 764	.0408 943	.0317 930	.0245 726
.64 .1022 621	.0822 499	.0657 065 <sup>-</sup>	.0521 541	.0411 454	.0322 733
.65 .1237 678	.1009 493	.0817 913	.0658 518	.0527 017	.0419 382
.66 .1484 637	.1227 552	.1008 387	.0823 236	.0668 137	.0539 232
.67 .1765 128	.1478 996	.1231 372	.1019 010	.0838 420	.0686 056
.68 .2080 152	.1765 638	.1489 390	.1248 943	.1041 420	.0863 718
.69 .2429 931	.2088 614	.1784 415 <sup>+</sup>	.1515 747	.1280 456	.1076 014
.70 .2813 767	.2448 206	.2117 678	.1821 524	.1558 404	.1326 467
.71 .3229 920	.2843 677	.2489 459	.2167 546	.1877 454	.1618 080
.72 .3675 521	.3273 131	.2808 000	.2557 545 <sup>+</sup>		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .91$  to  $.99$  $q = 8$ 

	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$
$B(p, q) = .2135\ 6843 \times \frac{1}{10^7}$		$.1584\ 5400 \times \frac{1}{10^7}$	$.1188\ 4050 \times \frac{1}{10^7}$	$.9003\ 0680 \times \frac{1}{10^8}$	$.6884\ 6900 \times \frac{1}{10^8}$
$x$					
.91	.9958 786	.9948 771	.9937 000	.9923 289	.9907 481
.92	.9980 264	.9975 245 <sup>+</sup>	.9969 281	.9962 259	.9954 031
.93	.9991 677	.9989 466	.9986 810	.9983 648	.9979 911
.94	.9997 027	.9996 203	.9995 203	.9993 999	.9992 501
.95	.9999 154	.9998 909	.9998 609	.9998 244	.9997 801
.96	.9999 826	.9999 774	.9999 709	.9999 630	.9999 531
.97	.9999 979	.9999 972	.9999 964	.9999 954	.9999 941
.98	.9999 999	.9999 999	.9999 998	.9999 998	.9999 999
.99	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .48$  to  $.99$  $q = 8$  $p = .41$  to  $.45$ 

	$p = .41$	$p = .42$	$p = .43$	$p = .44$	$p = .45$
$B(p, q) = .3312\ 5834 \times \frac{1}{10^9}$	$.2771\ 7534 \times \frac{1}{10^9}$	$.2328\ 2729 \times \frac{1}{10^9}$	$.1963\ 0536 \times \frac{1}{10^9}$	$.1661\ 0454 \times \frac{1}{10^9}$	
$x$					
.48	.0000 001				
.49	.0000 002	.0000 001			
.50	.0000 003	.0000 002	.0000 001	.0000 001	
.51	.0000 006	.0000 004	.0000 002	.0000 001	.0000 001
.52	.0000 012	.0000 007	.0000 004	.0000 003	.0000 002
.53	.0000 023	.0000 014	.0000 009	.0000 005 <sup>+</sup>	.0000 003
.54	.0000 042	.0000 026	.0000 017	.0000 010	.0000 006
.55	.0000 077	.0000 049	.0000 031	.0000 020	.0000 013
.56	.0000 140	.0000 091	.0000 059	.0000 038	.0000 024
.57	.0000 249	.0000 164	.0000 108	.0000 071	.0000 047
.58	.0000 435 <sup>-</sup>	.0000 292	.0000 196	.0000 131	.0000 087
.59	.0000 749	.0000 512	.0000 349	.0000 237	.0000 161
.60	.0001 270	.0000 883	.0000 612	.0000 423	.0000 292
.61	.0002 121	.0001 499	.0001 056	.0000 742	.0000 520
.62	.0003 492	.0002 507	.0001 794	.0001 281	.0000 912
.63	.0005 603	.0004 129	.0003 002	.0002 177	.0001 574
.64	.0009 051	.0006 702	.0004 949	.0003 644	.0002 675 <sup>+</sup>
.65	.0014 261	.0010 720	.0008 036	.0006 006	.0004 478
.66	.0022 148	.0016 898	.0012 855 <sup>+</sup>	.0009 753	.0007 379
.67	.0033 911	.0026 251	.0020 264	.0015 599	.0011 976
.68	.0051 188	.0040 195 <sup>-</sup>	.0031 474	.0024 578	.0019 143
.69	.0076 175 <sup>-</sup>	.0060 660	.0048 170	.0038 149	.0030 135 <sup>-</sup>
.70	.0111 754	.0090 225 <sup>-</sup>	.0072 642	.0058 331	.0046 719
.71	.0161 623	.0132 259	.0107 935 <sup>-</sup>	.0087 854	.0071 328
.72	.0230 405 <sup>-</sup>	.0191 055 <sup>-</sup>	.0158 000	.0130 327	.0107 233
.73	.0323 724	.0271 940	.0227 837	.0190 401	.0158 725 <sup>+</sup>
.74	.0448 219	.0381 333	.0323 588	.0273 900	.0231 282
.75	.0611 443	.0526 705 <sup>-</sup>	.0452 558	.0387 896	.0331 684
.76	.0821 634	.0716 414	.0623 119	.0540 673	.0468 045 <sup>+</sup>
.77	.1087 294	.0959 359	.0844 430	.0741 528	.0649 689
.78	.1416 576	.1264 413	.1125 945 <sup>-</sup>	.1000 357	.0886 814
.79	.1816 437	.1639 624	.1476 661	.1326 963	.1189 887
.80	.2291 610	.2091 173	.1904 098	.1730 075 <sup>-</sup>	.1568 708
.81	.2843 446	.2622 155 <sup>-</sup>	.2413 035 <sup>-</sup>	.2216 073	.2031 152
.82	.3468 747	.3231 283	.3004 109	.2787 502	.2581 626
.83	.4158 776	.3911 693	.3672 435 <sup>-</sup>	.3441 523	.3219 374
.84	.4898 662	.4650 086	.4406 481	.4168 542	.3936 875 <sup>-</sup>
.85	.5667 445 <sup>-</sup>	.5426 479	.5187 521	.4951 342	.4718 652
.86	.6438 982	.6214 847	.5989 064	.5765 082	.5540 915 <sup>+</sup>
.87	.7183 850 <sup>+</sup>	.6984 835 <sup>+</sup>	.6782 835 <sup>+</sup>	.6578 486	.6372 419
.88	.7872 205 <sup>-</sup>	.7704 599	.7532 524	.7356 447	.7176 848
.89	.8477 328	.8344 533	.8206 648	.8063 953	.7916 750 <sup>-</sup>
.90	.8979 341	.8881 394	.8778 549	.8670 921	.8558 645 <sup>+</sup>
.91	.9368 317	.9301 990	.9231 574	.9157 063	.9078 472
.92	.9645 048	.9605 484	.9562 073		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .52$  to  $1.00$  $q = 8$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) = .1410 \ 3215 \times \frac{1}{10^9}$		$.1201 \ 3850 \times \frac{1}{10^9}$	$.1026 \ 6381 \times \frac{1}{10^9}$	$.8799 \ 7551 \times \frac{1}{10^{10}}$
.52	.0000 001	.0000 001		
.53	.0000 002	.0000 001	.0000 001	
.54	.0000 004	.0000 002	.0000 002	.0000 001
.55	.0000 008	.0000 005 <sup>+</sup>	.0000 003	.0000 002
.56	.0000 016	.0000 010	.0000 006	.0000 004
.57	.0000 030	.0000 020	.0000 013	.0000 008
.58	.0000 058	.0000 039	.0000 025 <sup>+</sup>	.0000 017
.59	.0000 109	.0000 073	.0000 049	.0000 033
.60	.0000 200	.0000 137	.0000 094	.0000 064
.61	.0000 363	.0000 253	.0000 176	.0000 122
.62	.0000 647	.0000 458	.0000 324	.0000 228
.63	.0001 135 <sup>+</sup>	.0000 817	.0000 586	.0000 419
.64	.0001 959	.0001 431	.0001 043	.0000 758
.65	.0003 329	.0002 469	.0001 827	.0001 349
.66	.0005 569	.0004 193	.0003 149	.0002 359
.67	.0009 172	.0007 007	.0005 340	.0004 060
.68	.0014 872	.0011 526	.0008 912	.0006 875 <sup>+</sup>
.69	.0023 745 <sup>-</sup>	.0018 665 <sup>-</sup>	.0014 637	.0011 453
.70	.0037 326	.0029 751	.0023 658	.0018 772
.71	.0057 769	.0046 677	.0037 630	.0030 269
.72	.0088 018	.0072 078	.0058 892	.0048 014
.73	.0132 005 <sup>-</sup>	.0109 530	.0090 681	.0074 914
.74	.0194 838	.0163 766	.0137 348	.0114 948
.75	.0282 967	.0240 868	.0204 592	.0173 418
.76	.0404 262	.0348 410	.0299 640	.0257 170
.77	.0567 972	.0495 475 <sup>+</sup>	.0431 338	.0374 751
.78	.0784 475 <sup>-</sup>	.0692 502	.0610 079	.0536 414
.79	.1064 752	.0950 852	.0847 469	.0753 886
.80	.1419 533	.1282 033	.1155 650 <sup>-</sup>	.1039 799
.81	.1858 067	.1696 533	.1546 203	.1406 679
.82	.2386 548	.2202 241	.2028 601	.1865 451
.83	.3006 310	.2802 555 <sup>+</sup>	.2608 251	.2423 460
.84	.3712 004	.3494 368	.3284 325 <sup>+</sup>	.3082 160
.85	.4490 099	.4266 265 <sup>+</sup>	.4047 671	.3834 769
.86	.5318 143	.5097 405 <sup>+</sup>	.4879 297	.4664 370
.87	.6165 253	.5957 591	.5750 017	.5543 093
.88	.6994 217	.6809 044	.6621 823	.6433 041
.89	.7765 360	.7610 119	.7451 377	.7289 493
.90	.8441 879	.8320 797	.8195 593	.8066 476
.91	.8995 828	.8909 178	.8818 583	.8724 119
.92	.9413 374	.9357 542	.9298 526	.9236 314
.93	.9697 655 <sup>+</sup>	.9666 111	.9632 405 <sup>+</sup>	.9596 488
.94	.9867 833	.9852 801	.9836 566	.9819 080
.95	.9953 887	.9948 199	.9941 000	.9935 222
.96				



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$0.5$  to  $7.0$

$q = 8.5$

$p = 8$

	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$
$(p, q) =$	$9413\ 8778 \times \frac{1}{10^8}$	$6607\ 8995 \times \frac{1}{10^8}$	$4706\ 9389 \times \frac{1}{10^8}$	$3398\ 3483 \times \frac{1}{10^8}$	$2484\ 2178 \times \frac{1}{10^8}$	$1836\ 945$
$x$						
$0.5$	0000 001					
$0.6$	0000 003	0000 001				
$0.7$	0000 012	0000 004	0000 001	0000 001		
$0.8$	0000 034	0000 013	0000 005	0000 002	0000 001	
$0.9$	0000 080	0000 035	0000 014	0000 005+	0000 002	0000 001
$1.0$	0000 196	0000 083	0000 035	0000 014	0000 006	0000 002
$1.1$	0000 410	0000 182	0000 080	0000 035	0000 015	0000 006
$1.2$	0000 797	0000 370	0000 169	0000 077	0000 035	0000 015
$1.3$	0001 461	0000 704	0000 336	0000 159	0000 074	0000 034
$1.4$	0002 544	0001 272	0000 629	0000 308	0000 149	0000 072
$1.5$	0004 238	0002 192	0001 122	0000 568	0000 285+	0000 142
$1.6$	0006 793	0003 627	0001 915+	0001 001	0000 519	0000 266
$1.7$	0010 524	0005 788	0003 149	0001 696	0000 905+	0000 479
$1.8$	0015 821	0008 947	0005 006	0002 773	0001 522	0000 828
$1.9$	0023 151	0013 441	0007 721	0004 392	0002 476	0001 383
$2.0$	0033 062	0019 680	0011 592	0006 761	0003 908	0002 239
$2.1$	0046 187	0028 151	0016 979	0010 142	0006 003	0003 523
$2.2$	0063 244	0039 423	0024 321	0014 859	0008 997	0005 402
$2.3$	0085 025+	0054 149	0034 131	0021 308	0013 184	0008 089
$2.4$	0112 398	0073 060	0047 006	0029 956	0018 922	0011 853
$2.5$	0146 290	0096 967	0063 625	0041 354	0026 643	0017 024
$2.6$	0187 680	0126 753	0084 747	0056 133	0036 856	0024 002
$2.7$	0237 581	0163 358	0111 210	0075 007	0050 152	0033 261
$2.8$	0297 020	0207 776	0143 918	0098 772	0067 206	0045 360
$2.9$	0367 024	0261 029	0183 839	0128 298	0088 775	0060 937
$3.0$	0446 594	0324 158	0231 985	0164 526	0115 700	0080 720
$3.1$	0542 682	0398 198	0289 400	0208 455	0148 896	0105 519
$3.2$	0650 174	0484 160	0357 143	0261 126	0189 344	0136 228
$3.3$	0771 858	0583 002	0436 262	0323 611	0238 084	0173 813
$3.4$	0908 411	0695 614	0527 778	0396 988	0296 193	0219 306
$3.5$	1060 372	0822 787	0632 657	0482 324	0364 772	0273 791
$3.6$	1228 123	0965 194	0751 786	0580 649	0444 926	0338 386
$3.7$	1411 875+	1123 364	0885 953	0692 932	0537 736	0414 228
$3.8$	1611 655	1297 668	1035 814	0820 055+	0644 242	0502 443
$3.9$	1827 289	1488 295+	1201 879	0962 787	0765 408	0604 128
$4.0$	2058 405+	1695 243	1384 485+	1121 761	0902 101	0720 324

$4.1$	2304 421	1918 302	1583 783	1297 446	1055 062	0851 982
$4.2$	2564 552	2157 051	1799 715	1490 128	1224 878	0999 940
$4.3$	2837 815	2410 854	2032 008	1699 892	1411 962	1164 893
$4.4$	3123 034	2678 858	2280 168	1926 601	1616 522	1347 363
$4.5$	3418 860	2960 005+	2543 469	2169 892	1838 550	1547 677
$4.6$	3723 785	3253 035+	2820 965	2429 160	2077 802	1765 940
$4.7$	4036 162	3556 507	3111 487	2703 564	2333 787	2002 019
$4.8$	4354 232	3868 811	3413 662	2992 025+	2605 761	2255 525
$4.9$	4676 147	4188 199	3725 924	3293 236	2802 728	2525 865

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.97$  $q = 8.5$ 

	$p = 8.5$	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$
$B(p, q) = .9413\ 8778 \times \frac{1}{10^8}$		$.6607\ 8995 \times \frac{1}{10^8}$	$.4706\ 9389 \times \frac{1}{10^8}$	$.3398\ 3483 \times \frac{1}{10^8}$	$.2484\ 21$
$\cdot 71$	$.9632\ 976$	$.9547\ 418$	$.9449\ 790$	$.9339\ 734$	$.9217\ 04$
$\cdot 72$	$.9702\ 980$	$.9631\ 740$	$.9549\ 877$	$.9456\ 944$	$.9352\ 61$
$\cdot 73$	$.9762\ 419$	$.9703\ 835^-$	$.9636\ 048$	$.9558\ 559$	$.9470\ 96$
$\cdot 74$	$.9812\ 320$	$.9764\ 775^+$	$.9709\ 386$	$.9645\ 637$	$.9573\ 08$
$\cdot 75$	$.9853\ 710$	$.9815\ 666$	$.9771\ 044$	$.9719\ 341$	$.9660\ 09$
$\cdot 76$	$.9887\ 602$	$.9857\ 616$	$.9822\ 210$	$.9780\ 913$	$.9733\ 27$
$\cdot 77$	$.9914\ 975^-$	$.9891\ 719$	$.9864\ 080$	$.9831\ 630$	$.9793\ 95$
$\cdot 78$	$.9936\ 756$	$.9919\ 033$	$.9897\ 833$	$.9872\ 780$	$.9843\ 50$
$\cdot 79$	$.9953\ 813$	$.9940\ 559$	$.9924\ 604$	$.9905\ 629$	$.9883\ 31$
$\cdot 80$	$.9966\ 938$	$.9957\ 230$	$.9945\ 468$	$.9931\ 392$	$.9914\ 73$
$\cdot 81$	$.9976\ 849$	$.9969\ 896$	$.9961\ 420$	$.9951\ 213$	$.9939\ 05$
$\cdot 82$	$.9984\ 179$	$.9979\ 321$	$.9973\ 363$	$.9966\ 144$	$.9957\ 49$
$\cdot 83$	$.9989\ 476$	$.9986\ 174$	$.9982\ 100$	$.9977\ 133$	$.9971\ 14$
$\cdot 84$	$.9993\ 207$	$.9991\ 031$	$.9988\ 330$	$.9985\ 016$	$.9980\ 99$
$\cdot 85$	$.9995\ 762$	$.9994\ 376$	$.9992\ 646$	$.9990\ 511$	$.9987\ 90$
$\cdot 86$	$.9997\ 456$	$.9996\ 607$	$.9995\ 541$	$.9994\ 218$	$.9992\ 59$
$\cdot 87$	$.9998\ 539$	$.9998\ 042$	$.9997\ 414$	$.9996\ 630$	$.9995\ 66$
$\cdot 88$	$.9999\ 203$	$.9998\ 926$	$.9998\ 575^-$	$.9998\ 133$	$.9997\ 58$
$\cdot 89$	$.9999\ 590$	$.9999\ 445^+$	$.9999\ 260$	$.9999\ 026$	$.9998\ 73$
$\cdot 90$	$.9999\ 804$	$.9999\ 733$	$.9999\ 642$	$.9999\ 527$	$.9999\ 382$
$\cdot 91$	$.9999\ 914$	$.9999\ 882$	$.9999\ 841$	$.9999\ 789$	$.9999\ 724$
$\cdot 92$	$.9999\ 966$	$.9999\ 953$	$.9999\ 937$	$.9999\ 916$	$.9999\ 889$
$\cdot 93$	$.9999\ 988$	$.9999\ 984$	$.9999\ 978$	$.9999\ 970$	$.9999\ 961$
$\cdot 94$	$.9999\ 997$	$.9999\ 995^+$	$.9999\ 994$	$.9999\ 991$	$.9999\ 989$
$\cdot 95$	$.9999\ 999$	$.9999\ 999$	$.9999\ 999$	$.9999\ 998$	$.9999\ 997$
$\cdot 96$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$
$\cdot 97$					

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 70

$q = 8.5$

$p = 12$  to

$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$
$\gamma = .1036\ 2254 \times \frac{1}{10^3}$	$.6065\ 7097 \times \frac{1}{10^3}$	$.3667\ 6384 \times \frac{1}{10^3}$	$.2282\ 0861 \times \frac{1}{10^3}$	$.1456\ 6507 \times \frac{1}{10^3}$	$.9512\ 8210 \times \frac{1}{10^3}$
.0000 001	.0000 001				
.0000 003	.0000 001				
.0000 007	.0000 001				
.0000 016	.0000 004	.0000 001			
.0000 034	.0000 008	.0000 002			
.0000 079	.0000 017	.0000 004	.0000 001		
.0000 131	.0000 035-	.0000 009	.0000 002	.0000 001	
.0000 240	.0000 067	.0000 018	.0000 005-	.0000 001	
.0000 422	.0000 125+	.0000 036	.0000 010	.0000 003	.0000 001
.0000 718	.0000 224	.0000 068	.0000 020	.0000 006	.0000 002
.0001 186	.0000 388	.0000 124	.0000 039	.0000 012	.0000 004
.0001 903	.0000 652	.0000 218	.0000 071	.0000 023	.0000 007
.0002 976	.0001 065-	.0000 372	.0000 127	.0000 043	.0000 014
.0004 546	.0001 606	.0000 617	.0000 220	.0000 077	.0000 026
.0006 794	.0002 638	.0001 000	.0000 371	.0000 135-	.0000 048
.0009 951	.0004 015-	.0001 581	.0000 609	.0000 230	.0000 085+
.0014 305-	.0005 988	.0002 447	.0000 978	.0000 384	.0000 148
.0020 208	.0008 763	.0003 710	.0001 537	.0000 625-	.0000 249
.0028 083	.0012 601	.0005 521	.0002 368	.0000 996	.0000 412
.0038 435+	.0017 822	.0008 070	.0003 577	.0001 556	.0000 665-
.0051 852	.0024 817	.0011 602	.0005 310	.0002 384	.0001 052
.0069 009	.0034 056	.0016 418	.0007 750+	.0003 590	.0001 634
.0090 674	.0046 092	.0022 891	.0011 134	.0005 314	.0002 493
.0117 704	.0061 568	.0031 471	.0015 756	.0007 742	.0003 739
.0151 040	.0081 226	.0042 694	.0021 983	.0011 109	.0005 520
.0191 707	.0105 900	.0057 188	.0030 257	.0015 714	.0008 025-
.0240 797	.0136 523	.0075 682	.0041 112	.0021 925+	.0011 499
.0299 460	.0174 117	.0099 009	.0055 177	.0030 193	.0016 249
.0368 881	.0219 792	.0128 103	.0073 188	.0041 062	.0022 661
.0450 266	.0274 729	.0164 005-	.0095 989	.0055 179	.0031 204
.0544 814	.0340 163	.0207 849	.0124 538	.0073 302	.0042 448
.0653 689	.0417 372	.0260 858	.0159 906	.0096 307	.0057 074
.0777 991	.0507 044	.0324 327	.0203 273	.0125 193	.0075 881
.0918 727	.0612 252	.0399 606	.0255 917	.0161 083	.0099 797
.1076 774	.0732 426	.0488 071	.0319 205-	.0205 220	.0129 883
.1252 849	.0869 314	.0591 103	.0394 567	.0258 958	.0167 336
.1447 480	.1023 950+	.0710 050-	.0483 481	.0323 750-	.0213 485+
.1660 968	.1197 214	.0846 188	.0587 432	.0401 127	.0269 782
.1893 372	.1389 797	.1000 689	.0707 885-	.0492 674	.0337 789
.2144 475+	.1602 163	.1174 570	.0846 240	.0599 993	.0419 155-
.2413 775-	.1834 515+	.1368 657	.1003 790	.0724 666	.0515 586
.2700 460	.2086 770	.1583 537	.1181 670	.0868 213	.0628 811
.3003 450+	.2358 529	.1819 523	.1380 812	.1032 036	.0760 535+
.3321 312	.2649 061	.2076 614	.1601 891	.1217 366	.0912 390
.3652 355-	.2957 294	.2354 462	.1845 284	.1425 212	.1082 380
.3999 613					

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = \cdot 71$  to  $\cdot 98$  $q = 8\cdot 5$ 

	$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$
$B(p, q) = \cdot 1036\ 2254 \times \frac{1}{10^8}$		$\cdot 6065\ 7097 \times \frac{1}{10^8}$	$\cdot 3667\ 6384 \times \frac{1}{10^8}$	$\cdot 2282\ 0861 \times \frac{1}{10^8}$	$\cdot 1456\ 6500 \times \frac{1}{10^8}$
$x$					
$\cdot 71$	$\cdot 8773\ 556$	$\cdot 8418\ 049$	$\cdot 8020\ 019$	$\cdot 7586\ 023$	$\cdot 7123\ 818$
$\cdot 72$	$\cdot 8970\ 009$	$\cdot 8657\ 978$	$\cdot 8303\ 703$	$\cdot 7911\ 976$	$\cdot 7488\ 910$
$\cdot 73$	$\cdot 9145\ 117$	$\cdot 8874\ 827$	$\cdot 8563\ 681$	$\cdot 8214\ 864$	$\cdot 7832\ 909$
$\cdot 74$	$\cdot 9299\ 338$	$\cdot 9068\ 441$	$\cdot 8799\ 001$	$\cdot 8492\ 803$	$\cdot 8152\ 923$
$\cdot 75$	$\cdot 9433\ 447$	$\cdot 9239\ 095^-$	$\cdot 9009\ 236$	$\cdot 8744\ 488$	$\cdot 8446\ 646$
$\cdot 76$	$\cdot 9548\ 504$	$\cdot 9387\ 468$	$\cdot 9194\ 473$	$\cdot 8969\ 219$	$\cdot 8712\ 430$
$\cdot 77$	$\cdot 9645\ 803$	$\cdot 9514\ 601$	$\cdot 9355\ 292$	$\cdot 9166\ 909$	$\cdot 8949\ 326$
$\cdot 78$	$\cdot 9726\ 823$	$\cdot 9621\ 847$	$\cdot 9492\ 725^+$	$\cdot 9338\ 057$	$\cdot 9157\ 092$
$\cdot 79$	$\cdot 9793\ 177$	$\cdot 9710\ 809$	$\cdot 9608\ 199$	$\cdot 9483\ 711$	$\cdot 9336\ 192$
$\cdot 80$	$\cdot 9846\ 549$	$\cdot 9783\ 279$	$\cdot 9703\ 461$	$\cdot 9605\ 399$	$\cdot 9487\ 725$
$\cdot 81$	$\cdot 9888\ 650^-$	$\cdot 9841\ 161$	$\cdot 9780\ 503$	$\cdot 9705\ 049$	$\cdot 9613\ 373$
$\cdot 82$	$\cdot 9921\ 155^+$	$\cdot 9886\ 406$	$\cdot 9841\ 473$	$\cdot 9784\ 888$	$\cdot 9715\ 290$
$\cdot 83$	$\cdot 9945\ 670$	$\cdot 9920\ 946$	$\cdot 9888\ 587$	$\cdot 9847\ 339$	$\cdot 9795\ 987$
$\cdot 84$	$\cdot 9963\ 683$	$\cdot 9946\ 633$	$\cdot 9924\ 047$	$\cdot 9894\ 912$	$\cdot 9858\ 202$
$\cdot 85$	$\cdot 9976\ 540$	$\cdot 9965\ 185^+$	$\cdot 9949\ 966$	$\cdot 9930\ 099$	$\cdot 9904\ 760$
$\cdot 86$	$\cdot 9985\ 421$	$\cdot 9978\ 153$	$\cdot 9968\ 296$	$\cdot 9955\ 278$	$\cdot 9938\ 482$
$\cdot 87$	$\cdot 9991\ 334$	$\cdot 9986\ 886$	$\cdot 9980\ 785^-$	$\cdot 9972\ 632$	$\cdot 9961\ 992$
$\cdot 88$	$\cdot 9995\ 106$	$\cdot 9992\ 523$	$\cdot 9988\ 938$	$\cdot 9984\ 093$	$\cdot 9977\ 698$
$\cdot 89$	$\cdot 9997\ 398$	$\cdot 9995\ 986$	$\cdot 9994\ 004$	$\cdot 9991\ 295^-$	$\cdot 9987\ 678$
$\cdot 90$	$\cdot 9998\ 712$	$\cdot 9997\ 993$	$\cdot 9996\ 974$	$\cdot 9995\ 565^+$	$\cdot 9993\ 663$
$\cdot 91$	$\cdot 9999\ 415^-$	$\cdot 9999\ 080$	$\cdot 9998\ 599$	$\cdot 9997\ 928$	$\cdot 9997\ 011$
$\cdot 92$	$\cdot 9999\ 761$	$\cdot 9999\ 621$	$\cdot 9999\ 417$	$\cdot 9999\ 130$	$\cdot 9998\ 733$
$\cdot 93$	$\cdot 9999\ 915^-$	$\cdot 9999\ 863$	$\cdot 9999\ 788$	$\cdot 9999\ 681$	$\cdot 9999\ 531$
$\cdot 94$	$\cdot 9999\ 974$	$\cdot 9999\ 959$	$\cdot 9999\ 935^+$	$\cdot 9999\ 902$	$\cdot 9999\ 852$
$\cdot 95$	$\cdot 9999\ 994$	$\cdot 9999\ 990$	$\cdot 9999\ 985^-$	$\cdot 9999\ 976$	$\cdot 9999\ 962$
$\cdot 96$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999\ 997$	$\cdot 9999\ 996$	$\cdot 9999\ 992$
$\cdot 97$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$\cdot 9999\ 999$
$\cdot 98$					$1\cdot 0000\ 000$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$21$  to  $80$   $q = 8.5$   $p =$

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$
$b, q) = .6341\ 8806 \times \frac{1}{10^7}$		$.4307\ 6925 \times \frac{1}{10^7}$	$.2976\ 2239 \times \frac{1}{10^7}$	$.2088\ 5782 \times \frac{1}{10^7}$	$.1486\ 7845 \times \frac{1}{10^7}$	$.1072\ 4344 \times \frac{1}{10^7}$
21	.0000 001					
22	.0000 002	.0000 001				
23	.0000 005 <sup>-</sup>	.0000 001				
24	.0000 009	.0000 003	.0000 001			
25	.0000 017	.0000 006	.0000 002	.0000 001		
26	.0000 031	.0000 011	.0000 004	.0000 001		
27	.0000 056	.0000 021	.0000 008	.0000 003	.0000 001	
28	.0000 098	.0000 038	.0000 015 <sup>-</sup>	.0000 005 <sup>+</sup>	.0000 002	.0000 001
29	.0000 167	.0000 067	.0000 027	.0000 010	.0000 004	.0000 002
30	.0000 280	.0000 116	.0000 047	.0000 019	.0000 008	.0000 003
31	.0000 457	.0000 196	.0000 083	.0000 034	.0000 014	.0000 006
32	.0000 732	.0000 323	.0000 141	.0000 061	.0000 026	.0000 011
33	.0001 151	.0000 524	.0000 236	.0000 105 <sup>-</sup>	.0000 046	.0000 020
34	.0001 778	.0000 834	.0000 386	.0000 176	.0000 080	.0000 036
35	.0002 700	.0001 303	.0000 620	.0000 292	.0000 136	.0000 062
36	.0004 035 <sup>+</sup>	.0002 001	.0000 979	.0000 474	.0000 226	.0000 107
37	.0005 939	.0003 024	.0001 520	.0000 755 <sup>+</sup>	.0000 371	.0000 180
38	.0008 613	.0004 502	.0002 323	.0001 184	.0000 597	.0000 298
39	.0012 318	.0006 603	.0003 495 <sup>-</sup>	.0001 828	.0000 946	.0000 484
40	.0017 382	.0009 550 <sup>+</sup>	.0005 181	.0002 778	.0001 473	.0000 773
41	.0024 217	.0013 628	.0007 573	.0004 159	.0002 260	.0001 215
42	.0033 326	.0019 197	.0010 920	.0006 140	.0003 415 <sup>+</sup>	.0001 881
43	.0045 322	.0026 706	.0015 541	.0008 940	.0005 088	.0002 867
44	.0060 934	.0036 709	.0021 843	.0012 849	.0007 478	.0004 309
45	.0081 025 <sup>-</sup>	.0049 878	.0030 329	.0018 233	.0010 846	.0006 388
46	.0106 597	.0067 015 <sup>-</sup>	.0041 620	.0025 557	.0015 530	.0009 345
47	.0138 797	.0089 068	.0056 468	.0035 401	.0021 963	.0013 494
48	.0178 923	.0117 138	.0075 775 <sup>-</sup>	.0048 475 <sup>+</sup>	.0030 691	.0019 245
49	.0228 413	.0152 488	.0100 599	.0065 639	.0042 391	.0027 115
50	.0288 845 <sup>-</sup>	.0196 542	.0132 173	.0087 919	.0057 890	.0037 757
51	.0361 911	.0250 882	.0171 904	.0116 522	.0078 189	.0051 974
52	.0449 400	.0317 234	.0221 378	.0152 842	.0104 475 <sup>-</sup>	.0070 750
53	.0553 157	.0397 450 <sup>+</sup>	.0282 350 <sup>+</sup>	.0198 471	.0138 138	.0095 261
54	.0675 046	.0493 479	.0356 729	.0255 193	.0180 782	.0126 901
55	.0816 897	.0607 319	.0446 552	.0324 971	.0234 220	.0167 290
56	.0980 442	.0740 977	.0553 942	.0409 926	.0300 473	.0218 284
57	.1167 254	.0896 398	.0681 067	.0512 302	.0381 750 <sup>-</sup>	.0281 966
58	.1378 672	.1075 400	.0830 069	.0634 417	.0480 411	.0360 637
59	.1615 723	.1279 592	.1002 992	.0778 600	.0598 926	.0456 779
60	.1879 050 <sup>+</sup>	.1510 292	.1201 699	.0947 111	.0739 803	.0573 014
61	.2168 838	.1768 437	.1427 780	.1142 053	.0905 513	.0712 033
62	.2484 747	.2054 498	.1682 445 <sup>+</sup>	.1365 270	.1098 390	.0876 516
63	.2825 858	.2368 397	.1966 634	.1618 233	.1320 521	.1069 022
64	.3190 632	.2709 438	.2279 909	.1901 923	.1573 616	.1291 869
65	.3576 890	.3076 247	.2622 374	.2216 726	.1858 886	.1546 996
66	.3981 514	.3466 568	.2982 878	.2566 977	.2166 977	.1816 977
67	.4404 514	.3884 514	.3366 977	.2944 977	.2466 977	.2066 977
68	.4844 514	.4324 514	.3766 977	.3344 977	.2866 977	.2446 977
69	.5294 514	.4724 514	.4166 977	.3744 977	.3266 977	.2846 977
70	.5754 514	.5124 514	.4566 977	.4144 977	.3666 977	.3246 977
71	.6224 514	.5524 514	.5066 977	.4644 977	.4166 977	.3746 977
72	.6704 514	.6024 514	.5566 977	.5144 977	.4666 977	.4246 977
73	.7194 514	.6524 514	.6066 977	.5644 977	.5166 977	.4746 977
74	.7694 514	.7024 514	.6566 977	.6144 977	.5666 977	.5246 977
75	.8194 514	.7524 514	.7066 977	.6644 977	.6166 977	.5746 977
76	.8694 514	.8024 514	.7566 977	.7144 977	.6666 977	.6246 977
77	.9194 514	.8524 514	.8066 977	.7644 977	.7166 977	.6746 977
78	.9694 514	.9024 514	.8566 977	.8144 977	.7666 977	.7246 977
79	1.0194 514	.9524 514	.9066 977	.8644 977	.8166 977	.7746 977
80	1.0694 514	.1.0024 514	.9566 977	.9144 977	.8666 977	.8246 977

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.98$  $q = 8.5$ 

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$
$B(p, q) = .6341\ 8806 \times \frac{x}{10^7}$		$.4307\ 6925 \times \frac{x}{10^7}$	$.2976\ 2239 \times \frac{x}{10^7}$	$.2088\ 5782 \times \frac{x}{10^7}$	$.1486\ 78$
$x$					
.81	.9376 988	.9230 889	.9065 837	.8882 009	.8679 93
.82	.9532 408	.9417 334	.9285 727	.9137 339	.8972 20
.83	.9658 501	.9570 451	.9468 521	.9352 193	.9221 15
.84	.9758 082	.9692 836	.9616 395 <sup>-</sup>	.9528 105 <sup>-</sup>	.9427 45
.85	.9834 408	.9787 761	.9732 459	.9667 825 <sup>+</sup>	.9593 26
.86	.9890 981	.9858 950 <sup>+</sup>	.9820 530	.9775 098	.9722 07
.87	.9931 355 <sup>+</sup>	.9910 348	.9884 857	.9854 363	.9818 35
.88	.9958 949	.9945 880	.9929 839	.9910 431	.9887 25
.89	.9976 888	.9969 243	.9959 754	.9948 141	.9934 11
.90	.9987 888	.9983 730	.9978 511	.9972 052	.9964 16
.91	.9994 179	.9992 107	.9989 478	.9986 188	.9982 12
.92	.9997 485 <sup>+</sup>	.9996 559	.9995 370	.9993 865 <sup>+</sup>	.9991 98
.93	.9999 051	.9998 689	.9998 220	.9997 620	.9996 86
.94	.9999 700	.9999 581	.9999 426	.9999 226	.9998 97
.95	.9999 925 <sup>+</sup>	.9999 895 <sup>-</sup>	.9999 855 <sup>-</sup>	.9999 802	.9999 73
.96	.9999 987	.9999 981	.9999 974	.9999 964	.9999 95
.97	.9999 999	.9999 998	.9999 997	.9999 996	.9999 99
.98	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 00

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .90

$q = 8.5$

$p = 24$

$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$
$= .7830\ 4756 \times \frac{1}{10^8}$	$.5782\ 5051 \times \frac{1}{10^8}$	$.4315\ 3023 \times \frac{1}{10^8}$	$.3252\ 1119 \times \frac{1}{10^8}$	$.2473\ 4372 \times \frac{1}{10^8}$	$.1897\ 4313 \times \frac{1}{10^8}$
.0000 001					
.0000 001					
.0000 002	.0000 001				
.0000 005-	.0000 002	.0000 001			
.0000 009	.0000 004	.0000 002	.0000 001		
.0000 016	.0000 007	.0000 003	.0000 001	.0000 001	
.0000 028	.0000 013	.0000 006	.0000 003	.0000 001	
.0000 050+	.0000 023	.0000 011	.0000 005-	.0000 002	.0000 001
.0000 087	.0000 041	.0000 020	.0000 009	.0000 004	.0000 002
.0000 147	.0000 072	.0000 035+	.0000 017	.0000 008	.0000 004
.0000 246	.0000 124	.0000 062	.0000 030	.0000 015-	.0000 007
.0000 402	.0000 207	.0000 106	.0000 054	.0000 027	.0000 014
.0000 648	.0000 342	.0000 179	.0000 093	.0000 048	.0000 025-
.0001 026	.0000 555-	.0000 298	.0000 158	.0000 084	.0000 044
.0001 601	.0000 886	.0000 486	.0000 265-	.0000 143	.0000 077
.0002 460	.0001 393	.0000 782	.0000 436	.0000 241	.0000 132
.0003 728	.0002 157	.0001 238	.0000 705+	.0000 399	.0000 224
.0005 572	.0003 294	.0001 932	.0001 124	.0000 650-	.0000 373
.0008 216	.0004 960	.0002 971	.0001 766	.0001 042	.0000 611
.0011 959	.0007 369	.0004 505-	.0002 734	.0001 647	.0000 986
.0017 190	.0010 806	.0006 740	.0004 173	.0002 566	.0001 567
.0024 408	.0015 647	.0009 953	.0006 284	.0003 941	.0002 455-
.0034 245+	.0022 378	.0014 510	.0009 340	.0005 971	.0003 792
.0047 494	.0031 622	.0020 893	.0013 704	.0008 927	.0005 778
.0065 125+	.0044 162	.0029 719	.0019 856	.0013 176	.0008 688
.0088 318	.0060 972	.0041 775-	.0028 418	.0019 202	.0012 893
.0118 476	.0083 238	.0058 043	.0040 188	.0027 640	.0018 890
.0157 250-	.0112 390	.0079 731	.0056 167	.0039 306	.0027 334
.0206 543	.0150 117	.0108 306	.0077 598	.0055 233	.0039 070
.0268 515-	.0198 387	.0145 510	.0105 995+	.0076 710	.0055 175+
.0345 565-	.0259 443	.0193 388	.0143 173	.0105 317	.0076 999
.0440 308	.0335 802	.0254 288	.0191 271	.0142 957	.0106 205+
.0555 525+	.0430 221	.0330 855-	.0252 757	.0191 884	.0144 805-
.0694 097	.0545 653	.0426 008	.0330 429	.0254 710	.0195 189
.0858 911	.0685 175+	.0542 886	.0427 385+	.0334 407	.0260 141
.1052 751	.0851 892	.0684 777	.0546 972	.0434 278	.0342 834
.1278 160	.1048 810	.0855 009	.0692 700	.0557 894	.0446 799
.1537 290	.1278 692	.1056 814	.0868 133	.0709 012	.0575 862
.1831 732	.1543 883	.1293 167	.1076 731	.0891 439	.0734 039
.2162 347	.1846 127	.1566 589	.1321 670	.1108 863	.0925 393
.2529 098	.2186 367	.1878 938	.1605 621	.1364 647	.1153 837
.2930 898	.2564 558	.2231 189	.1930 515+	.1661 580	.1422 899
.3365 489	.2979 492	.2623 217	.2297 294	.2001 609	.1735 439
.3829 364	.3428 660	.3053 601	.2707 664		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .91$  to  $.99$  $q = 8.5$ 

	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$
$B(p, q) = .7830\ 4756 \times \frac{1}{10^8}$		$.5782\ 5051 \times \frac{1}{10^8}$	$.4315\ 3023 \times \frac{1}{10^8}$	$.3252\ 1119 \times \frac{1}{10^8}$	$.2473\ 437$
$x$					
.91	.9971 184	.9964 038	.9955 584	.9945 672	.9934 147
.92	.9986 843	.9983 430	.9979 348	.9974 509	.9968 820
.93	.9994 752	.9993 330	.9991 611	.9989 551	.9987 103
.94	.9998 246	.9997 750 <sup>-</sup>	.9997 144	.9996 410	.9995 528
.95	.9999 539	.9999 403	.9999 236	.9999 031	.9998 781
.96	.9999 914	.9999 888	.9999 856	.9999 815 <sup>+</sup>	.9999 766
.97	.9999 991	.9999 988	.9999 984	.9999 980	.9999 974
.98	1.0000 000	1.0000 000	.9999 999	.9999 999	.9999 999
.99			1.0000 000	1.0000 000	1.0000 000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to '99

$q = 8.5$

$p = 30$  to

$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$
$\cdot 1467\ 3468 \times \frac{1}{10^8}$	$\cdot 1143\ 3872 \times \frac{1}{10^8}$	$\cdot 8973\ 4182 \times \frac{1}{10^8}$	$\cdot 7090\ 1082 \times \frac{1}{10^8}$	$\cdot 5637\ 9174 \times \frac{1}{10^8}$	$\cdot 4510\ 3339 \times \frac{1}{10^8}$
$\cdot 0000\ 001$					
$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 0000\ 013$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 023$	$\cdot 0000\ 012$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 041$	$\cdot 0000\ 022$	$\cdot 0000\ 012$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$
$\cdot 0000\ 072$	$\cdot 0000\ 039$	$\cdot 0000\ 021$	$\cdot 0000\ 011$	$\cdot 0000\ 006$	$\cdot 0000\ 003$
$\cdot 0000\ 125^+$	$\cdot 0000\ 069$	$\cdot 0000\ 038$	$\cdot 0000\ 021$	$\cdot 0000\ 011$	$\cdot 0000\ 006$
$\cdot 0000\ 213$	$\cdot 0000\ 121$	$\cdot 0000\ 068$	$\cdot 0000\ 038$	$\cdot 0000\ 021$	$\cdot 0000\ 012$
$\cdot 0000\ 356$	$\cdot 0000\ 206$	$\cdot 0000\ 119$	$\cdot 0000\ 068$	$\cdot 0000\ 039$	$\cdot 0000\ 022$
$\cdot 0000\ 587$	$\cdot 0000\ 347$	$\cdot 0000\ 204$	$\cdot 0000\ 119$	$\cdot 0000\ 069$	$\cdot 0000\ 040$
$\cdot 0000\ 951$	$\cdot 0000\ 574$	$\cdot 0000\ 344$	$\cdot 0000\ 206$	$\cdot 0000\ 122$	$\cdot 0000\ 072$
$\cdot 0001\ 520$	$\cdot 0000\ 936$	$\cdot 0000\ 573$	$\cdot 0000\ 349$	$\cdot 0000\ 211$	$\cdot 0000\ 127$
$\cdot 0002\ 394$	$\cdot 0001\ 502$	$\cdot 0000\ 938$	$\cdot 0000\ 582$	$\cdot 0000\ 360$	$\cdot 0000\ 221$
$\cdot 0003\ 717$	$\cdot 0002\ 377$	$\cdot 0001\ 512$	$\cdot 0000\ 957$	$\cdot 0000\ 603$	$\cdot 0000\ 378$
$\cdot 0005\ 693$	$\cdot 0003\ 710$	$\cdot 0002\ 404$	$\cdot 0001\ 550^+$	$\cdot 0000\ 994$	$\cdot 0000\ 635^-$
$\cdot 0008\ 604$	$\cdot 0005\ 709$	$\cdot 0003\ 768$	$\cdot 0002\ 474$	$\cdot 0001\ 617$	$\cdot 0001\ 051$
$\cdot 0012\ 833$	$\cdot 0008\ 669$	$\cdot 0005\ 825^-$	$\cdot 0003\ 893$	$\cdot 0002\ 590$	$\cdot 0001\ 715^-$
$\cdot 0018\ 896$	$\cdot 0012\ 990$	$\cdot 0008\ 882$	$\cdot 0006\ 042$	$\cdot 0004\ 091$	$\cdot 0002\ 757$
$\cdot 0027\ 475^-$	$\cdot 0019\ 213$	$\cdot 0013\ 364$	$\cdot 0009\ 249$	$\cdot 0006\ 371$	$\cdot 0004\ 368$
$\cdot 0039\ 455^-$	$\cdot 0028\ 057$	$\cdot 0019\ 848$	$\cdot 0013\ 970$	$\cdot 0009\ 786$	$\cdot 0006\ 824$
$\cdot 0055\ 971$	$\cdot 0040\ 462$	$\cdot 0029\ 099$	$\cdot 0020\ 823$	$\cdot 0014\ 830$	$\cdot 0010\ 515^-$
$\cdot 0078\ 451$	$\cdot 0057\ 635^-$	$\cdot 0042\ 124$	$\cdot 0030\ 636$	$\cdot 0022\ 176$	$\cdot 0015\ 981$
$\cdot 0108\ 660$	$\cdot 0081\ 099$	$\cdot 0060\ 219$	$\cdot 0044\ 498$	$\cdot 0032\ 728$	$\cdot 0023\ 965^-$
$\cdot 0148\ 742$	$\cdot 0112\ 746$	$\cdot 0085\ 029$	$\cdot 0063\ 817$	$\cdot 0047\ 676$	$\cdot 0035\ 461$
$\cdot 0201\ 253$	$\cdot 0154\ 879$	$\cdot 0118\ 595^-$	$\cdot 0090\ 378$	$\cdot 0068\ 562$	$\cdot 0051\ 785^+$
$\cdot 0269\ 175^+$	$\cdot 0210\ 248$	$\cdot 0163\ 410$	$\cdot 0126\ 409$	$\cdot 0097\ 345^+$	$\cdot 0074\ 642$
$\cdot 0355\ 912$	$\cdot 0282\ 067$	$\cdot 0222\ 454$	$\cdot 0174\ 624$	$\cdot 0136\ 469$	$\cdot 0106\ 198$
$\cdot 0465\ 255^+$	$\cdot 0374\ 005^-$	$\cdot 0299\ 209$	$\cdot 0238\ 274$	$\cdot 0188\ 916$	$\cdot 0149\ 154$
$\cdot 0601\ 307$	$\cdot 0490\ 145^-$	$\cdot 0397\ 647$	$\cdot 0321\ 148$	$\cdot 0258\ 245^+$	$\cdot 0206\ 804$
$\cdot 0768\ 368$	$\cdot 0634\ 896$	$\cdot 0522\ 178$	$\cdot 0427\ 564$	$\cdot 0348\ 605^+$	$\cdot 0283\ 069$
$\cdot 0970\ 760$	$\cdot 0812\ 856$	$\cdot 0677\ 543$	$\cdot 0562\ 294$	$\cdot 0464\ 699$	$\cdot 0382\ 504$
$\cdot 1212\ 614$	$\cdot 1028\ 614$	$\cdot 0868\ 652$	$\cdot 0730\ 436$	$\cdot 0611\ 695^+$	$\cdot 0510\ 242$
$\cdot 1497\ 585^+$	$\cdot 1286\ 489$	$\cdot 1100\ 351$	$\cdot 0937\ 218$	$\cdot 0795\ 069$	$\cdot 0671\ 881$
$\cdot 1828\ 546$	$\cdot 1590\ 224$	$\cdot 1377\ 118$	$\cdot 1187\ 720$	$\cdot 1020\ 361$	$\cdot 0873\ 282$
$\cdot 2207\ 231$	$\cdot 1942\ 610$	$\cdot 1702\ 701$	$\cdot 1486\ 521$	$\cdot 1292\ 844$	$\cdot 1120\ 271$
$\cdot 2633\ 887$	$\cdot 2345\ 105^-$	$\cdot 2079\ 704$	$\cdot 1837\ 278$	$\cdot 1617\ 110$	$\cdot 1418\ 247$
$\cdot 3106\ 942$	$\cdot 2797\ 432$	$\cdot 2509\ 139$	$\cdot 2242\ 243$	$\cdot 1996\ 577$	$\cdot 1771\ 684$
$\cdot 3622\ 724$	$\cdot 3297\ 225^+$	$\cdot 2989\ 996$	$\cdot 2701\ 777$	$\cdot 2432\ 947$	$\cdot 2183\ 568$
$\cdot 4175\ 294$	$\cdot 3839\ 746$	$\cdot 3518\ 865^-$	$\cdot 3213\ 878$	$\cdot 2925\ 664$	$\cdot 2654\ 787$
$\cdot 4756\ 406$	$\cdot 4417\ 736$	$\cdot 4089\ 663$	$\cdot 3773\ 791$	$\cdot 3471\ 414$	$\cdot 3183\ 534$
$\cdot 5355\ 652$	$\cdot 5021\ 439$	$\cdot 4693\ 534$	$\cdot 4373\ 774$	$\cdot 4063\ 753$	$\cdot 3764\ 811$
$\cdot 5960\ 800$	$\cdot 5638\ 841$	$\cdot 5318\ 960$	$\cdot 5003\ 076$	$\cdot 4692\ 936$	$\cdot 4390\ 094$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .43$  to  $.99$  $q = 8.5$ 

$p = .36$	$p = .37$	$p = .38$	$p = .39$
$I(p, q) = .3629\ 0013 \times \frac{1}{10^4}$	$.2935\ 8237 \times \frac{1}{10^4}$	$.2387\ 3731 \times \frac{1}{10^4}$	$.1950\ 9716 \times \frac{1}{10^4}$
.43	.0000 001		
.44	.0000 002	.0000 001	
.45	.0000 003	.0000 002	
.46	.0000 007	.0000 004	.0000 001
.47	.0000 012	.0000 007	.0000 002
.48	.0000 013	.0000 013	.0000 004
.49	.0000 012	.0000 025	.0000 008
.50	.0000 076	.0000 076	.0000 016
.51	.0000 135 <sup>+</sup>	.0000 082	.0000 050 <sup>+</sup>
.52	.0000 236	.0000 146	.0000 091
.53	.0000 404	.0000 256	.0000 161
.54	.0000 681	.0000 439	.0000 282
.55	.0001 131	.0000 742	.0000 485 <sup>+</sup>
.56	.0001 850	.0001 236	.0000 822
.57	.0002 682	.0002 027	.0001 372
.58	.0004 738	.0003 276	.0002 256
.59	.0007 423	.0005 219	.0003 655 <sup>-</sup>
.60	.0011 468	.0008 195 <sup>+</sup>	.0005 834
.61	.0017 474	.0012 689	.0009 179
.62	.0026 265 <sup>+</sup>	.0019 376	.0014 239
.63	.0038 952	.0029 182	.0021 779
.64	.0056 998	.0043 353	.0032 851
.65	.0082 395	.0063 539	.0048 899
.66	.0117 288	.0091 874	.0071 702
.67	.0164 952	.0131 070	.0103 767
.68	.0228 955 <sup>+</sup>	.0184 491	.0148 127
.69	.0313 646	.0246 218	.0208 568
.70	.0424 006	.0331 666	.0289 657
.71	.0569 655 <sup>+</sup>	.0454 559	.0399 754
.72	.0744 694	.0612 870	.0548 915 <sup>-</sup>
.73	.0967 293	.0812 493	.0743 894
.74	.1249 554	.1059 273	.0997 563
.75	.1609 851	.1381 259	.1213 842
.76	.2051 127	.1742 681	.1548 899
.77	.2591 516	.2165 868	.1947 649
.78	.2910 870	.2653 885 <sup>+</sup>	.2412 805 <sup>-</sup>
.79	.3478 040	.3294 293	.2944 196
.80	.4095 994	.3811 520	.3537 897
.81	.4753 449	.4405 847	.4185 678
.82	.5445 843	.5133 329	.4874 719
.83	.6175 041	.5879 171	.5587 780
.84	.6839 816	.6553 953	.6303 951
.85	.7442 293	.7142 293	.6913 951
.86	.8000 000	.7642 293	.7413 951
.87	.8513 951	.8142 293	.7813 951
.88	.8987 293	.8595 847	.8213 951
.89	.9413 951	.8987 293	.8513 951
.90	.9795 994	.9311 520	.8813 951
.91	.9953 449	.9405 847	.8913 951
.92	.9987 293	.9442 293	.8942 293
.93	.9995 847	.9453 847	.8953 847
.94	.9998 449	.9458 449	.8958 449
.95	.9999 000	.9460 000	.8960 000
.96	.9999 555	.9461 555	.8961 555
.97	.9999 847	.9462 847	.8962 847
.98	.9999 951	.9463 951	.8963 951
.99	.9999 994	.9464 994	.8964 994

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $x = .47$  to  $.99$  $q = 8.5$  $p = 41$  to  $45$ 

	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$B(p, q) = .1321\ 1137 \times \frac{1}{10^9}$	$.1094\ 2558 \times \frac{1}{10^9}$	$.9100\ 7411 \times \frac{1}{10^{10}}$	$.7598\ 6771 \times \frac{1}{10^{10}}$	$.6368\ 4151 \times \frac{1}{10^{10}}$	
.47	.0000 001				
.48	.0000 001	.0000 001			
.49	.0000 003	.0000 002	.0000 001	.0000 001	
.50	.0000 006	.0000 003	.0000 002	.0000 001	.0000 001
.51	.0000 011	.0000 007	.0000 004	.0000 002	.0000 001
.52	.0000 021	.0000 013	.0000 008	.0000 005 <sup>-</sup>	.0000 003
.53	.0000 039	.0000 025 <sup>-</sup>	.0000 015 <sup>+</sup>	.0000 009	.0000 006
.54	.0000 073	.0000 046	.0000 029	.0000 018	.0000 011
.55	.0000 133	.0000 085 <sup>+</sup>	.0000 055 <sup>-</sup>	.0000 035 <sup>+</sup>	.0000 022
.56	.0000 237	.0000 155 <sup>+</sup>	.0000 102	.0000 066	.0000 043
.57	.0000 417	.0000 278	.0000 185 <sup>+</sup>	.0000 123	.0000 081
.58	.0000 721	.0000 490	.0000 331	.0000 224	.0000 150 <sup>+</sup>
.59	.0001 228	.0000 848	.0000 583	.0000 400	.0000 274
.60	.0002 059	.0001 445 <sup>+</sup>	.0001 011	.0000 705 <sup>+</sup>	.0000 491
.61	.0003 400	.0002 425 <sup>+</sup>	.0001 725 <sup>-</sup>	.0001 223	.0000 864
.62	.0005 531	.0004 008	.0002 896	.0002 086	.0001 498
.63	.0008 863	.0006 524	.0004 788	.0003 503	.0002 556
.64	.0013 993	.0010 460	.0007 794	.0005 791	.0004 290
.65	.0021 772	.0016 520	.0012 497	.0009 426	.0007 089
.66	.0033 383	.0025 707	.0019 736	.0015 107	.0011 532
.67	.0050 446	.0039 412	.0030 700	.0023 844	.0018 468
.68	.0075 130	.0059 537	.0047 041	.0037 061	.0029 118
.69	.0110 276	.0088 616	.0071 001	.0056 728	.0045 200
.70	.0159 522	.0129 954	.0105 560	.0085 506	.0069 075 <sup>+</sup>
.71	.0227 405 <sup>-</sup>	.0187 754	.0154 575 <sup>-</sup>	.0126 909	.0103 918
.72	.0319 432	.0267 222	.0222 918	.0185 456	.0153 885 <sup>+</sup>
.73	.0442 081	.0374 612	.0316 564	.0266 799	.0224 277
.74	.0602 702	.0517 186	.0442 603	.0377 786	.0321 645 <sup>-</sup>
.75	.0809 282	.0703 045 <sup>-</sup>	.0609 139	.0526 425 <sup>-</sup>	.0453 815 <sup>-</sup>
.76	.1070 039	.0940 789	.0825 019	.0721 687	.0629 768
.77	.1392 819	.1238 974	.1099 360	.0973 107	.0859 321
.78	.1784 301	.1605 345 <sup>-</sup>	.1440 833	.1290 132	.1152 549
.79	.2249 020	.2045 854	.1856 692	.1681 190	.1518 910
.80	.2788 285 <sup>-</sup>	.2563 524	.2351 607	.2152 503	.1966 070
.81	.3399 106	.3157 244	.2926 352	.2706 710	.2498 474
.82	.4073 290	.3820 674	.3576 539	.3341 432	.3115 782
.83	.4796 910	.4541 468	.4291 590	.4048 017	.3811 392
.84	.5550 365 <sup>-</sup>	.5301 051	.5054 230	.4810 739	.4571 342
.85	.6309 208	.6075 201	.5840 777	.5606 762	.5373 943
.86	.7045 863	.6835 582	.6622 447	.6407 180	.6190 495 <sup>+</sup>
.87	.7773 178	.7552 257	.7337 772	.7119 277	.6907 333
.88	.8342 577	.8196 996	.8046 007	.7889 962	.7729 236
.89	.8857 399	.8746 935 <sup>+</sup>	.8631 097	.8510 020	.8383 893
.90	.9265 569	.9187 906	.9105 526	.9018 454	.8926 730
.91	.9566 528	.9506 511 <sup>+</sup>			

	$p = 46$	$p = 47$	$p = 48$	$p = 49$	
$B(p, q) = .5356 \ 6108 \times \frac{1}{10^{10}}$	$.4521 \ 1761 \times \frac{1}{10^{10}}$	$.3828 \ 7437 \times \frac{1}{10^{10}}$	$.3252 \ 7380 \times \frac{1}{10^{10}}$	$.2$	
.51	.0000 001				
.52	.0000 002	.0000 001	.0000 001		
.53	.0000 004	.0000 002	.0000 001	.0000 001	
.54	.0000 007	.0000 004	.0000 003	.0000 002	
.55	.0000 014	.0000 009	.0000 006	.0000 004	
.56	.0000 028	.0000 018	.0000 012	.0000 007	
.57	.0000 054	.0000 035 <sup>+</sup>	.0000 023	.0000 015 <sup>+</sup>	
.58	.0000 101	.0000 067	.0000 045 <sup>+</sup>	.0000 030	
.59	.0000 187	.0000 127	.0000 086	.0000 058	
.60	.0000 340	.0000 235 <sup>+</sup>	.0000 162	.0000 112	
.61	.0000 609	.0000 428	.0000 300	.0000 210	
.62	.0001 073	.0000 767	.0000 546	.0000 388	
.63	.0001 859	.0001 349	.0000 976	.0000 705 <sup>-</sup>	
.64	.0003 170	.0002 336	.0001 717	.0001 259	
.65	.0005 317	.0003 978	.0002 968	.0002 209	
.66	.0008 779	.0006 666	.0005 049	.0003 814	
.67	.0014 266	.0010 991	.0008 447	.0006 476	
.68	.0022 817	.0017 834	.0013 904	.0010 814	
.69	.0035 920	.0028 473	.0022 515 <sup>-</sup>	.0017 761	
.70	.0055 657	.0044 732	.0035 865 <sup>+</sup>	.0028 688	
.71	.0084 873	.0069 147	.0056 200	.0045 571	
.72	.0127 366	.0105 159	.0086 619	.0071 184	
.73	.0188 063	.0157 317	.0131 290	.0109 322	
.74	.0273 018	.0231 462	.0195 668	.0165 042	
.75	.0390 281	.0334 862	.0286 665 <sup>-</sup>	.0244 869	
.76	.0548 267	.0476 227	.0412 740	.0356 951	
.77	.0757 104	.0665 564	.0583 830	.0511 061	
.78	.1027 348	.0913 770	.0811 042	.0718 393	
.79	.1369 341	.1231 917	.1106 028	.0991 037	
.80	.1792 066	.1630 166	.1479 978	.1341 054	
.81	.2301 691	.2116 308	.1942 187	.1779 111	
.82	.2899 910	.2694 034	.2498 277	.2312 674	
.83	.3582 260	.3361 072	.3148 191	.2943 892	
.84	.4336 734	.4107 531	.3884 278	.3667 444	
.85	.5143 065 <sup>-</sup>	.4914 821	.4689 856	.4468 757	
.86	.5973 087	.5755 633	.5538 779	.5323 146	
.87	.6792 509	.6595 378	.6396 506	.6196 453	
.88	.7564 223	.7395 336	.7223 000	.7047 648	
.89	.8252 931	.8117 372	.7977 472	.7833 505 <sup>+</sup>	
.90	.8830 421	.8729 610	.8624 402	.8514 920	
.91	.9280 372	.9211 931	.9139 711	.9063 722	
.92	.9601 355 <sup>-</sup>	.9559 778	.9515 422	.9468 240	
.93	.9806 801	.9784 827	.9761 131	.9735 650 <sup>-</sup>	
.94	.9921 437	.9911 742	.9901 175 <sup>+</sup>	.9889 691	
.95	.9974 869	.9971 520	.9967 831	.9963 779	
.96	.9994 323	.9993 509	.9992 604	.9991 598	
.97	.9999 260	.9999 146	.9999 018	.9998 875 <sup>+</sup>	
.98	.9999 965 <sup>-</sup>	.9999 959	.9999 952	.9999 945 <sup>-</sup>	
.99	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

6 to 70

$q = 9$

$p = 9$  to

$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$
$\cdot 4570\ 5928 \times \frac{1}{10^8}$	$\cdot 3209\ 5512 \times \frac{1}{10^8}$	$\cdot 2285\ 2964 \times \frac{1}{10^8}$	$\cdot 1648\ 1479 \times \frac{1}{10^8}$	$\cdot 1202\ 7876 \times \frac{1}{10^8}$	$\cdot 6615\ 3317 \times \frac{1}{10^8}$
$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 018$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 0000\ 048$	$\cdot 0000\ 019$	$\cdot 0000\ 008$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 0000\ 115-$	$\cdot 0000\ 049$	$\cdot 0000\ 020$	$\cdot 0000\ 009$	$\cdot 0000\ 004$	$\cdot 0000\ 001$
$\cdot 0000\ 250-$	$\cdot 0000\ 111$	$\cdot 0000\ 049$	$\cdot 0000\ 021$	$\cdot 0000\ 009$	$\cdot 0000\ 002$
$\cdot 0000\ 505-$	$\cdot 0000\ 235-$	$\cdot 0000\ 108$	$\cdot 0000\ 049$	$\cdot 0000\ 022$	$\cdot 0000\ 004$
$\cdot 0000\ 958$	$\cdot 0000\ 463$	$\cdot 0000\ 222$	$\cdot 0000\ 105-$	$\cdot 0000\ 049$	$\cdot 0000\ 011$
$\cdot 0001\ 721$	$\cdot 0000\ 863$	$\cdot 0000\ 428$	$\cdot 0000\ 210$	$\cdot 0000\ 102$	$\cdot 0000\ 024$
$\cdot 0002\ 950+$	$\cdot 0001\ 530$	$\cdot 0000\ 786$	$\cdot 0000\ 399$	$\cdot 0000\ 201$	$\cdot 0000\ 050-$
$\cdot 0004\ 856$	$\cdot 0002\ 600$	$\cdot 0001\ 378$	$\cdot 0000\ 723$	$\cdot 0000\ 376$	$\cdot 0000\ 099$
$\cdot 0007\ 711$	$\cdot 0004\ 253$	$\cdot 0002\ 322$	$\cdot 0001\ 255+$	$\cdot 0000\ 673$	$\cdot 0000\ 188$
$\cdot 0011\ 859$	$\cdot 0006\ 726$	$\cdot 0003\ 776$	$\cdot 0002\ 100$	$\cdot 0001\ 157$	$\cdot 0000\ 343$
$\cdot 0017\ 723$	$\cdot 0010\ 321$	$\cdot 0005\ 949$	$\cdot 0003\ 397$	$\cdot 0001\ 922$	$\cdot 0000\ 601$
$\cdot 0025\ 815-$	$\cdot 0015\ 412$	$\cdot 0009\ 109$	$\cdot 0005\ 333$	$\cdot 0003\ 095-$	$\cdot 0001\ 017$
$\cdot 0036\ 733$	$\cdot 0022\ 456$	$\cdot 0013\ 591$	$\cdot 0008\ 148$	$\cdot 0004\ 843$	$\cdot 0001\ 670$
$\cdot 0051\ 170$	$\cdot 0031\ 993$	$\cdot 0019\ 805-$	$\cdot 0012\ 146$	$\cdot 0007\ 385-$	$\cdot 0002\ 665-$
$\cdot 0069\ 608$	$\cdot 0044\ 657$	$\cdot 0028\ 245+$	$\cdot 0017\ 700$	$\cdot 0010\ 997$	$\cdot 0004\ 144$
$\cdot 0093\ 819$	$\cdot 0061\ 170$	$\cdot 0039\ 492$	$\cdot 0025\ 264$	$\cdot 0016\ 024$	$\cdot 0006\ 295-$
$\cdot 0123\ 848$	$\cdot 0082\ 344$	$\cdot 0054\ 218$	$\cdot 0035\ 375-$	$\cdot 0022\ 884$	$\cdot 0009\ 354$
$\cdot 0161\ 012$	$\cdot 0109\ 078$	$\cdot 0073\ 184$	$\cdot 0048\ 660$	$\cdot 0032\ 081$	$\cdot 0013\ 622$
$\cdot 0206\ 378$	$\cdot 0142\ 344$	$\cdot 0097\ 243$	$\cdot 0065\ 839$	$\cdot 0044\ 203$	$\cdot 0019\ 467$
$\cdot 0261\ 050-$	$\cdot 0183\ 183$	$\cdot 0127\ 329$	$\cdot 0087\ 722$	$\cdot 0059\ 933$	$\cdot 0027\ 339$
$\cdot 0326\ 144$	$\cdot 0232\ 682$	$\cdot 0164\ 451$	$\cdot 0115\ 209$	$\cdot 0080\ 047$	$\cdot 0037\ 768$
$\cdot 0402\ 769$	$\cdot 0291\ 961$	$\cdot 0209\ 680$	$\cdot 0149\ 280$	$\cdot 0105\ 412$	$\cdot 0051\ 382$
$\cdot 0491\ 999$	$\cdot 0362\ 150+$	$\cdot 0264\ 132$	$\cdot 0190\ 987$	$\cdot 0136\ 982$	$\cdot 0068\ 899$
$\cdot 0594\ 848$	$\cdot 0444\ 367$	$\cdot 0328\ 950+$	$\cdot 0241\ 441$	$\cdot 0175\ 793$	$\cdot 0091\ 139$
$\cdot 0712\ 244$	$\cdot 0539\ 687$	$\cdot 0405\ 282$	$\cdot 0301\ 791$	$\cdot 0222\ 947$	$\cdot 0119\ 016$
$\cdot 0844\ 999$	$\cdot 0649\ 124$	$\cdot 0494\ 254$	$\cdot 0373\ 208$	$\cdot 0279\ 599$	$\cdot 0153\ 538$
$\cdot 0993\ 789$	$\cdot 0773\ 594$	$\cdot 0596\ 947$	$\cdot 0456\ 857$	$\cdot 0346\ 936$	$\cdot 0195\ 794$
$\cdot 1159\ 125-$	$\cdot 0913\ 894$	$\cdot 0714\ 363$	$\cdot 0553\ 875-$	$\cdot 0426\ 158$	$\cdot 0240\ 946$
$\cdot 1341\ 335+$	$\cdot 1070\ 674$	$\cdot 0847\ 405+$	$\cdot 0665\ 340$	$\cdot 0518\ 448$	$\cdot 0308\ 214$
$\cdot 1540\ 544$	$\cdot 1244\ 411$	$\cdot 0996\ 840$	$\cdot 0792\ 241$	$\cdot 0624\ 947$	$\cdot 0380\ 849$
$\cdot 1756\ 661$	$\cdot 1435\ 387$	$\cdot 1163\ 278$	$\cdot 0935\ 451$	$\cdot 0746\ 723$	$\cdot 0466\ 117$
$\cdot 1989\ 365-$	$\cdot 1643\ 673$	$\cdot 1347\ 141$	$\cdot 1095\ 695-$	$\cdot 0884\ 741$	$\cdot 0565\ 264$
$\cdot 2238\ 105+$	$\cdot 1869\ 111$	$\cdot 1548\ 647$	$\cdot 1273\ 522$	$\cdot 1039\ 827$	$\cdot 0679\ 490$
$\cdot 2502\ 097$	$\cdot 2111\ 305+$	$\cdot 1767\ 785-$	$\cdot 1469\ 280$	$\cdot 1212\ 645-$	$\cdot 0809\ 916$
$\cdot 2780\ 325-$	$\cdot 2369\ 616$	$\cdot 2004\ 302$	$\cdot 1683\ 092$	$\cdot 1403\ 660$	$\cdot 0957\ 546$
$\cdot 3071\ 557$	$\cdot 2643\ 160$	$\cdot 2257\ 694$	$\cdot 1914\ 836$	$\cdot 1613\ 115-$	$\cdot 1123\ 234$
$\cdot 3374\ 356$	$\cdot 2930\ 817$	$\cdot 2527\ 203$	$\cdot 2164\ 136$	$\cdot 1841\ 009$	$\cdot 1307\ 650-$
$\cdot 3687\ 099$	$\cdot 3231\ 239$	$\cdot 2811\ 814$	$\cdot 2430\ 344$	$\cdot 2087\ 079$	$\cdot 1511\ 243$
$\cdot 4008\ 001$	$\cdot 3542\ 865+$	$\cdot 3110\ 266$	$\cdot 2712\ 547$	$\cdot 2350\ 781$	$\cdot 1734\ 218$
$\cdot 4335\ 145+$	$\cdot 3863\ 949$	$\cdot 3421\ 061$	$\cdot 3009\ 563$	$\cdot 2631\ 292$	$\cdot 1976\ 502$
$\cdot 4666\ 509$	$\cdot 4192\ 577$	$\cdot 3742\ 489$	$\cdot 3319\ 955-$	$\cdot 2927\ 504$	$\cdot 2237\ 730$
$\cdot 5000\ 000^*$	$\cdot 4526\ 706$	$\cdot 4072\ 647$	$\cdot 3649\ 361$		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71 \text{ to } .96$  $q = 9$ 

	$p = 9$	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$
$B(p, q) = .4570\ 5928 \times \frac{1}{10^8}$		$.3209\ 5512 \times \frac{1}{10^8}$	$.2285\ 2964 \times \frac{1}{10^8}$	$.1648\ 1479 \times \frac{1}{10^8}$	$.1202\ 7$
$x$					
.71	.9673 856	.9598 433	.9512 162	.9414 623	.9305 5
.72	.9738 950 <sup>+</sup>	.9676 808	.9605 229	.9523 730	.9431 9
.73	.9793 622	.9743 093	.9684 487	.9617 295 <sup>+</sup>	.9541 0
.74	.9838 988	.9798 474	.9751 161	.9696 546	.9634 1
.75	.9876 152	.9844 149	.9806 522	.9762 795 <sup>+</sup>	.9712 5
.76	.9906 181	.9881 302	.9851 855 <sup>+</sup>	.9817 407	.9777 5
.77	.9930 092	.9911 079	.9888 428	.9861 756	.9830 6
.78	.9948 830	.9934 568	.9917 466	.9897 197	.9873 4
.79	.9963 267	.9952 780	.9940 125 <sup>+</sup>	.9925 030	.9907 2
.80	.9974 185 <sup>+</sup>	.9966 641	.9957 480	.9946 483	.9933 4
.81	.9982 277	.9976 978	.9970 502	.9962 681	.9953 3
.82	.9988 141	.9984 516	.9980 058	.9974 641	.9968 1
.83	.9992 289	.9989 880	.9986 900	.9983 256	.9978 8
.84	.9995 144	.9993 594	.9991 665 <sup>+</sup>	.9989 293	.9986 4
.85	.9997 050 <sup>-</sup>	.9996 088	.9994 885 <sup>+</sup>	.9993 396	.9991 5
.86	.9998 279	.9997 707	.9996 987	.9996 090	.9994 9
.87	.9999 042	.9998 718	.9998 306	.9997 791	.9997 1
.88	.9999 495 <sup>+</sup>	.9999 321	.9999 098	.9998 818	.9998 4
.89	.9999 750 <sup>+</sup>	.9999 662	.9999 549	.9999 407	.9999 2
.90	.9999 885 <sup>+</sup>	.9999 844	.9999 791	.9999 724	.9999 6
.91	.9999 952	.9999 934	.9999 912	.9999 883	.9999 8
.92	.9999 982	.9999 975 <sup>+</sup>	.9999 967	.9999 955 <sup>+</sup>	.9999 9
.93	.9999 994	.9999 992	.9999 989	.9999 985 <sup>+</sup>	.9999 9
.94	.9999 998	.9999 998	.9999 997	.9999 996	.9999 9
.95	1.0000 000	1.0000 000	.9999 999	.9999 999	.9999 9
.96			1.0000 000	1.0000 000	1.0000 0

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.70

$q = 9$

$p = 13$  to

$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$\cdot 3780\ 1895 \times \frac{1}{10^8}$	$\cdot 2233\ 7484 \times \frac{1}{10^8}$	$\cdot 1359\ 6729 \times \frac{1}{10^8}$	$\cdot 8497\ 9557 \times \frac{1}{10^7}$	$\cdot 5438\ 6917 \times \frac{1}{10^7}$	$\cdot 3556\ 0676 \times \frac{1}{10^7}$
$\cdot 0000\ 001$					
$\cdot 0000\ 002$					
$\cdot 0000\ 005^+$	$\cdot 0000\ 001$				
$\cdot 0000\ 012$	$\cdot 0000\ 003$	$\cdot 0000\ 001$			
$\cdot 0000\ 025^+$	$\cdot 0000\ 006$	$\cdot 0000\ 002$			
$\cdot 0000\ 051$	$\cdot 0000\ 014$	$\cdot 0000\ 004$	$\cdot 0000\ 001$		
$\cdot 0000\ 099$	$\cdot 0000\ 028$	$\cdot 0000\ 008$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 182$	$\cdot 0000\ 054$	$\cdot 0000\ 016$	$\cdot 0000\ 004$	$\cdot 0000\ 001$	
$\cdot 0000\ 325^-$	$\cdot 0000\ 101$	$\cdot 0000\ 031$	$\cdot 0000\ 009$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0000\ 559$	$\cdot 0000\ 183$	$\cdot 0000\ 058$	$\cdot 0000\ 018$	$\cdot 0000\ 006$	$\cdot 0000\ 002$
$\cdot 0000\ 934$	$\cdot 0000\ 319$	$\cdot 0000\ 107$	$\cdot 0000\ 035^-$	$\cdot 0000\ 011$	$\cdot 0000\ 004$
$\cdot 0001\ 518$	$\cdot 0000\ 542$	$\cdot 0000\ 189$	$\cdot 0000\ 065^-$	$\cdot 0000\ 022$	$\cdot 0000\ 007$
$\cdot 0002\ 403$	$\cdot 0000\ 894$	$\cdot 0000\ 325^+$	$\cdot 0000\ 116$	$\cdot 0000\ 041$	$\cdot 0000\ 014$
$\cdot 0003\ 716$	$\cdot 0001\ 440$	$\cdot 0000\ 545^+$	$\cdot 0000\ 202$	$\cdot 0000\ 074$	$\cdot 0000\ 026$
$\cdot 0005\ 623$	$\cdot 0002\ 263$	$\cdot 0000\ 891$	$\cdot 0000\ 343$	$\cdot 0000\ 130$	$\cdot 0000\ 048$
$\cdot 0008\ 336$	$\cdot 0003\ 481$	$\cdot 0001\ 422$	$\cdot 0000\ 569$	$\cdot 0000\ 223$	$\cdot 0000\ 086$
$\cdot 0012\ 128$	$\cdot 0005\ 248$	$\cdot 0002\ 220$	$\cdot 0000\ 921$	$\cdot 0000\ 375^-$	$\cdot 0000\ 150^+$
$\cdot 0017\ 334$	$\cdot 0007\ 761$	$\cdot 0003\ 398$	$\cdot 0001\ 458$	$\cdot 0000\ 615^-$	$\cdot 0000\ 255^-$
$\cdot 0024\ 367$	$\cdot 0011\ 275^-$	$\cdot 0005\ 103$	$\cdot 0002\ 264$	$\cdot 0000\ 986$	$\cdot 0000\ 423$
$\cdot 0033\ 723$	$\cdot 0016\ 108$	$\cdot 0007\ 527$	$\cdot 0003\ 448$	$\cdot 0001\ 551$	$\cdot 0000\ 686$
$\cdot 0045\ 990$	$\cdot 0022\ 652$	$\cdot 0010\ 916$	$\cdot 0005\ 157$	$\cdot 0002\ 393$	$\cdot 0001\ 093$
$\cdot 0061\ 854$	$\cdot 0031\ 383$	$\cdot 0015\ 581$	$\cdot 0007\ 585^+$	$\cdot 0003\ 627$	$\cdot 0001\ 706$
$\cdot 0082\ 103$	$\cdot 0042\ 869$	$\cdot 0021\ 906$	$\cdot 0010\ 978$	$\cdot 0005\ 405^-$	$\cdot 0002\ 618$
$\cdot 0107\ 627$	$\cdot 0057\ 779$	$\cdot 0030\ 363$	$\cdot 0015\ 649$	$\cdot 0007\ 925^-$	$\cdot 0003\ 949$
$\cdot 0139\ 420$	$\cdot 0076\ 889$	$\cdot 0041\ 514$	$\cdot 0021\ 987$	$\cdot 0011\ 443$	$\cdot 0005\ 860$
$\cdot 0178\ 570$	$\cdot 0101\ 083$	$\cdot 0056\ 029$	$\cdot 0030\ 469$	$\cdot 0016\ 283$	$\cdot 0008\ 564$
$\cdot 0226\ 254$	$\cdot 0131\ 357$	$\cdot 0074\ 689$	$\cdot 0041\ 671$	$\cdot 0022\ 851$	$\cdot 0012\ 334$
$\cdot 0283\ 723$	$\cdot 0168\ 814$	$\cdot 0098\ 392$	$\cdot 0056\ 279$	$\cdot 0031\ 643$	$\cdot 0017\ 514$
$\cdot 0352\ 279$	$\cdot 0214\ 658$	$\cdot 0128\ 154$	$\cdot 0075\ 098$	$\cdot 0043\ 264$	$\cdot 0024\ 538$
$\cdot 0433\ 260$	$\cdot 0270\ 180$	$\cdot 0165\ 109$	$\cdot 0099\ 055^+$	$\cdot 0058\ 432$	$\cdot 0033\ 939$
$\cdot 0528\ 006$	$\cdot 0336\ 741$	$\cdot 0210\ 506$	$\cdot 0129\ 211$	$\cdot 0077\ 995^-$	$\cdot 0046\ 361$
$\cdot 0637\ 832$	$\cdot 0415\ 753$	$\cdot 0265\ 692$	$\cdot 0166\ 751$	$\cdot 0102\ 934$	$\cdot 0062\ 579$
$\cdot 0763\ 989$	$\cdot 0508\ 648$	$\cdot 0332\ 098$	$\cdot 0212\ 986$	$\cdot 0134\ 372$	$\cdot 0083\ 504$
$\cdot 0907\ 630$	$\cdot 0616\ 846$	$\cdot 0411\ 221$	$\cdot 0269\ 339$	$\cdot 0173\ 569$	$\cdot 0110\ 192$
$\cdot 1069\ 769$	$\cdot 0741\ 720$	$\cdot 0504\ 588$	$\cdot 0337\ 330$	$\cdot 0221\ 923$	$\cdot 0143\ 853$
$\cdot 1251\ 243$	$\cdot 0884\ 554$	$\cdot 0613\ 728$	$\cdot 0418\ 553$	$\cdot 0280\ 954$	$\cdot 0185\ 849$
$\cdot 1452\ 670$	$\cdot 1046\ 499$	$\cdot 0740\ 130$	$\cdot 0514\ 642$	$\cdot 0352\ 291$	$\cdot 0237\ 690$
$\cdot 1674\ 414$	$\cdot 1228\ 528$	$\cdot 0885\ 196$	$\cdot 0627\ 239$	$\cdot 0437\ 641$	$\cdot 0301\ 019$
$\cdot 1916\ 552$	$\cdot 1431\ 394$	$\cdot 1050\ 198$	$\cdot 0757\ 948$	$\cdot 0538\ 761$	$\cdot 0377\ 593$
$\cdot 2178\ 843$	$\cdot 1655\ 580$	$\cdot 1236\ 222$	$\cdot 0908\ 285^-$	$\cdot 0657\ 412$	$\cdot 0469\ 258$
$\cdot 2460\ 713$	$\cdot 1901\ 267$	$\cdot 1444\ 120$	$\cdot 1079\ 622$	$\cdot 0795\ 313$	$\cdot 0577\ 901$
$\cdot 2761\ 230$	$\cdot 2168\ 291$	$\cdot 1674\ 457$	$\cdot 1273\ 135^+$	$\cdot 0954\ 084$	$\cdot 0705\ 412$
$\cdot 3079\ 109$	$\cdot 2456\ 117$	$\cdot 1927\ 464$	$\cdot 1489\ 739$	$\cdot 1135\ 182$	$\cdot 0853\ 622$
$\cdot 3412\ 711$	$\cdot 2763\ 821$	$\cdot 2202\ 995^+$	$\cdot 1730\ 032$	$\cdot 1339\ 838$	$\cdot 1024\ 230$
$\cdot 3760\ 057$	$\cdot 3000\ 078$	$\cdot 2500\ 256$			

$\cdot 2178\ 843$	$\cdot 1655\ 580$	$\cdot 1236\ 222$	$\cdot 0908\ 285^-$	$\cdot 0657\ 412$	$\cdot 0469\ 258$
$\cdot 2460\ 713$	$\cdot 1901\ 267$	$\cdot 1444\ 120$	$\cdot 1079\ 622$	$\cdot 0795\ 313$	$\cdot 0577\ 901$
$\cdot 2761\ 230$	$\cdot 2168\ 291$	$\cdot 1674\ 457$	$\cdot 1273\ 135^+$	$\cdot 0954\ 084$	$\cdot 0705\ 412$
$\cdot 3079\ 109$	$\cdot 2456\ 117$	$\cdot 1927\ 464$	$\cdot 1489\ 739$	$\cdot 1135\ 182$	$\cdot 0853\ 622$
$\cdot 3412\ 711$	$\cdot 2763\ 821$	$\cdot 2202\ 995^+$	$\cdot 1730\ 032$	$\cdot 1339\ 838$	$\cdot 1024\ 230$
$\cdot 3760\ 057$	$\cdot 3000\ 078$	$\cdot 2500\ 256$			

TABLE I. THE  $I_{\infty}(p, q)$  FUNCTION $x = .71$  to  $.97$  $q = 9$ 

	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$
$B(p, q) = .3780\ 1895 \times \frac{1}{10^6}$		$.2233\ 7484 \times \frac{1}{10^6}$	$.1359\ 6729 \times \frac{1}{10^6}$	$.8497\ 9557 \times \frac{1}{10^6}$	$.5438\ 691$
$x$					
.71	.8752 216	.8408 276	.8024 536	.7606 772	.7161 852
.72	.8957 771	.8656 964	.8316 623	.7940 886	.7535 090
.73	.9140 268	.8880 842	.8583 243	.8250 130	.7885 372
.74	.9300 225 <sup>-</sup>	.9079 772	.8823 417	.8532 540	.8209 665
.75	.9438 532	.9254 115 <sup>+</sup>	.9036 767	.8786 817	.8505 623
.76	.9556 403	.9404 690	.9223 502	.9012 358	.8771 654
.77	.9655 321	.9532 726	.9384 386	.9209 246	.9006 959
.78	.9736 974	.9639 794	.9520 679	.9378 218	.9211 538
.79	.9803 188	.9727 737	.9634 069	.9520 606	.9386 152
.80	.9855 860	.9798 583	.9726 577	.9638 250 <sup>+</sup>	.9532 258
.81	.9896 893	.9854 467	.9800 465 <sup>+</sup>	.9733 395 <sup>-</sup>	.9651 904
.82	.9928 137	.9897 548	.9858 131	.9808 570	.9747 611
.83	.9951 337	.9929 928	.9902 005 <sup>-</sup>	.9866 467	.9822 223
.84	.9968 092	.9953 596	.9934 461	.9909 815 <sup>+</sup>	.9878 761
.85	.9979 823	.9970 364	.9957 731	.9941 265 <sup>+</sup>	.9920 271
.86	.9987 754	.9981 836	.9973 838	.9963 292	.9949 687
.87	.9992 909	.9989 379	.9984 553	.9978 115 <sup>+</sup>	.9969 714
.88	.9996 112	.9994 119	.9991 363	.9987 645 <sup>+</sup>	.9982 737
.89	.9997 999	.9996 945 <sup>-</sup>	.9995 470	.9993 456	.9990 769
.90	.9999 046	.9998 528	.9997 797	.9996 787	.9995 425
.91	.9999 585 <sup>-</sup>	.9999 353	.9999 022	.9998 561	.9997 931
.92	.9999 838	.9999 746	.9999 612	.9999 424	.9999 164
.93	.9999 946	.9999 914	.9999 867	.9999 800	.9999 707
.94	.9999 985 <sup>-</sup>	.9999 976	.9999 962	.9999 942	.9999 915
.95	.9999 997	.9999 995 <sup>-</sup>	.9999 992	.9999 987	.9999 981
.96	1.0000 000	.9999 999	.9999 999	.9999 998	.9999 997
.97		1.0000 000	1.0000 000	1.0000 000	1.0000 000



## 30 to .98

$$p = 2$$

	$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$	$p = 30$
$q) = .2880\ 9817 \times \frac{1}{10^8}$	$.2118\ 3689 \times \frac{1}{10^8}$	$.1573\ 6455 \times \frac{1}{10^8}$	$.1180\ 2341 \times \frac{1}{10^8}$	$.8931\ 5014 \times \frac{1}{10^9}$	$.6816\ 1458$	
0	.0000 001					
1	.0000 002	.0000 001				
2	.0000 003	.0000 001	.0000 001			
3	.0000 006	.0000 003	.0000 001			
4	.0000 011	.0000 005+	.0000 002	.0000 001		
5	.0000 021	.0000 010	.0000 004	.0000 002	.0000 001	
6	.0000 038	.0000 018	.0000 008	.0000 004	.0000 002	.0000 001
7	.0000 067	.0000 032	.0000 015+	.0000 007	.0000 003	.0000 002
8	.0000 116	.0000 057	.0000 028	.0000 014	.0000 007	.0000 003
9	.0000 196	.0000 099	.0000 050-	.0000 025-	.0000 012	.0000 006
	.0000 327	.0000 170	.0000 087	.0000 045-	.0000 023	.0000 011
1	.0000 536	.0000 285-	.0000 150+	.0000 078	.0000 041	.0000 021
2	.0000 862	.0000 469	.0000 253	.0000 136	.0000 072	.0000 038
3	.0001 366	.0000 760	.0000 420	.0000 230	.0000 125+	.0000 068
4	.0002 130	.0001 213	.0000 685+	.0000 384	.0000 214	.0000 118
5	.0003 273	.0001 905+	.0001 100	.0000 631	.0000 359	.0000 203
6	.0004 957	.0002 948	.0001 739	.0001 019	.0000 592	.0000 342
7	.0007 402	.0004 495+	.0002 709	.0001 620	.0000 962	.0000 568
8	.0010 905-	.0006 759	.0004 157	.0002 538	.0001 539	.0000 927
9	.0015 854	.0010 026	.0006 291	.0003 919	.0002 425-	.0001 490
	.0022 757	.0014 675+	.0009 391	.0005 966	.0003 764	.0002 360
1	.0032 260	.0021 205+	.0013 833	.0008 959	.0005 763	.0003 683
2	.0045 177	.0030 257	.0020 111	.0013 272	.0008 700	.0005 666
3	.0062 520	.0042 645+	.0028 871	.0019 407	.0012 958	.0008 597
4	.0085 520	.0059 388	.0040 934	.0028 016	.0019 048	.0012 869
5	.0115 658	.0081 735+	.0057 336	.0039 941	.0027 640	.0019 008
6	.0154 681	.0111 200	.0079 359	.0056 245-	.0039 602	.0027 712
7	.0204 612	.0149 581	.0108 562	.0078 253	.0056 041	.0039 888
8	.0267 757	.0198 976	.0146 809	.0107 587	.0078 339	.0056 695+
9	.0346 684	.0261 791	.0196 292	.0146 198	.0108 198	.0079 593
	.0444 197	.0340 721	.0259 532	.0196 385-	.0147 671	.0110 380
1	.0563 276	.0438 725+	.0339 372	.0260 809	.0199 194	.0151 240
2	.0707 003	.0558 968	.0438 946	.0342 484	.0265 590	.0204 764
3	.0878 455+	.0704 735+	.0561 617	.0444 737	.0350 065-	.0273 968
4	.1080 576	.0879 324	.0710 891	.0571 154	.0456 170	.0362 280
5	.1316 021	.1085 895+	.0890 288	.0725 471	.0587 731	.0473 502
6	.1586 983	.1327 309	.1103 191	.0911 445-	.0748 735-	.0611 722
7	.1895 004	.1605 926	.1352 649	.1132 673	.0943 180	.0781 196
8	.2240 784	.1923 395+	.1641 163	.1392 381	.1174 874	.0986 168
9	.2624 004	.2280 444	.1970 448	.1693 173	.1447 191	.1230 642
	.3043 157	.2676 667	.2341 188	.2036 772	.1762 797	.1518 110
1	.3495 437	.3110 349	.2752 809	.2423 740	.2123 347	.1851 233
2	.3976 669	.3578 329	.3203 276	.2853 227	.2529 182	.2231 507
3	.4481 317	.4075 934	.3688 950+	.3322 748	.2979 042	.2658 921
4	.5002 568	.4596 992	.4204 519	.3828 035+	.3469 838	.3131 651
5	.5532 514	.5133 938	.4743 027	.4362 975-	.3996 496	.3645 813
6	.6062 421	.5678 041	.5296 027	.4919 672	.4551 919	.4195 326
7	.6583 080	.6219 719	.5853 843	.5488 616		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .37$  to  $.99$  $q = 9$ 

	$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$
$B(p, q) = .5243\ 1891 \times \frac{1}{10^9}$	$.4063\ 4716 \times \frac{1}{10^9}$	$.3171\ 4900 \times \frac{1}{10^9}$	$.2491\ 8850 \times \frac{1}{10^9}$	$.1970\ 3$	
.37	.0000 001				
.38	.0000 001	.0000 001			
.39	.0000 003	.0000 001	.0000 001		
.40	.0000 006	.0000 003	.0000 001	.0000 001	
.41	.0000 011	.0000 005+	.0000 003	.0000 001	.0000 001
.42	.0000 020	.0000 010	.0000 005+	.0000 003	.0000 001
.43	.0000 036	.0000 019	.0000 010	.0000 005+	.0000 001
.44	.0000 065+	.0000 036	.0000 019	.0000 010	.0000 001
.45	.0000 114	.0000 064	.0000 035+	.0000 020	.0000 001
.46	.0000 196	.0000 112	.0000 064	.0000 036	.0000 001
.47	.0000 333	.0000 194	.0000 113	.0000 065-	.0000 001
.48	.0000 555-	.0000 330	.0000 195+	.0000 115+	.0000 001
.49	.0000 910	.0000 553	.0000 334	.0000 201	.0000 115
.50	.0001 470	.0000 911	.0000 561	.0000 344	.0000 215
.51	.0002 340	.0001 478	.0000 928	.0000 580	.0000 361
.52	.0003 668	.0002 361	.0001 512	.0000 963	.0000 611
.53	.0005 670	.0003 718	.0002 425-	.0001 573	.0001 011
.54	.0008 642	.0005 771	.0003 833	.0002 533	.0001 661
.55	.0012 994	.0008 833	.0005 973	.0004 018	.0002 691
.56	.0019 277	.0013 335+	.0009 176	.0006 282	.0004 281
.57	.0028 225-	.0019 862	.0013 903	.0009 683	.0006 711
.58	.0040 794	.0029 192	.0020 780	.0014 718	.0010 371
.59	.0058 215+	.0042 348	.0030 646	.0022 067	.0015 811
.60	.0082 039	.0060 646	.0044 602	.0032 641	.0023 771
.61	.0114 187	.0085 753	.0064 071	.0047 639	.0035 251
.62	.0156 995-	.0119 735+	.0090 859	.0068 616	.0051 581
.63	.0213 242	.0165 112	.0127 210	.0097 542	.0074 451
.64	.0286 167	.0224 883	.0175 856	.0136 872	.0106 051
.65	.0379 453	.0302 545+	.0240 058	.0189 594	.0149 071
.66	.0497 179	.0402 071	.0323 608	.0259 267	.0206 811
.67	.0643 725+	.0527 851	.0430 806	.0350 024	.0283 161
.68	.0823 629	.0684 579	.0566 388	.0466 533	.0382 651
.69	.1041 378	.0877 085-	.0735 382	.0613 904	.0510 361
.70	.1301 153	.1110 092	.0942 913	.0797 518	.0671 791
.71	.1606 511	.1387 920	.1193 920	.1022 788	.0872 691
.72	.1960 027	.1714 119	.1492 802	.1294 824	.1118 741
.73	.2362 915+	.2091 068	.1843 007	.1618 023	.1415 141
.74	.2814 656	.2519 545-	.2246 566	.1995 592	.1766 172
.75	.3312 665-	.2998 323	.2703 628	.2429 025+	.2174 611
.76	.3852 046	.3523 826	.3212 017	.2917 593	.2641 181
.77	.4425 481	.4089 893	.3766 890	.3457 884	.3163 961
.78	.5023 283	.4687 716	.4360 538	.4043 472	.3737 971
.79	.5633 666	.5305 981	.4982 393	.4664 785+	.4354 831
.80	.6243 234	.5931 271	.5619 309	.5309 237	.5002 811

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

	$q = 9$						$p$
	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p =$	
449 $\times \frac{1}{10^8}$	·1008 5274 $\times \frac{1}{10^8}$	·8154 0514 $\times \frac{1}{10^{10}}$	·6625 1667 $\times \frac{1}{10^{10}}$	·5408 2994 $\times \frac{1}{10^{10}}$	·4434 8055 $\times \frac{1}{10^{16}}$	·3652	
001							
002	·0000 001						
003	·0000 002	·0000 001	·0000 001				
006	·0000 003	·0000 002	·0000 001	·0000 001			
012	·0000 007	·0000 004	·0000 002	·0000 001	·0000 001		
023	·0000 013	·0000 008	·0000 004	·0000 003	·0000 001	·0000 001	
042	·0000 025 <sup>-</sup>	·0000 015 <sup>-</sup>	·0000 009	·0000 005 <sup>-</sup>	·0000 003	·0000 001	
077	·0000 046	·0000 028	·0000 017	·0000 010	·0000 006	·0000 001	
37	·0000 084	·0000 051	·0000 031	·0000 019	·0000 011	·0000 001	
42	·0000 151	·0000 094	·0000 058	·0000 036	·0000 022	·0000 001	
18	·0000 266	·0000 169	·0000 107	·0000 067	·0000 042	·0000 001	
10	·0000 461	·0000 298	·0000 192	·0000 123	·0000 079	·0000 001	
89	·0000 785 <sup>+</sup>	·0000 517	·0000 339	·0000 221	·0000 144	·0000 001	
60	·0001 318	·0000 882	·0000 589	·0000 391	·0000 259	·0000 001	
81	·0002 176	·0001 483	·0001 006	·0000 681	·0000 459	·0000 001	
87	·0003 539	·0002 453	·0001 693	·0001 165 <sup>-</sup>	·0000 798	·0000 001	
16	·0005 671	·0003 996	·0002 805 <sup>+</sup>	·0001 962	·0001 368	·0000 001	
49	·0008 952	·0006 412	·0004 576	·0003 253	·0002 305 <sup>+</sup>	·0001 001	
50	·0013 927	·0010 137	·0007 351	·0005 311	·0003 825 <sup>-</sup>	·0002 700	
70	·0021 356	·0015 791	·0011 633	·0008 539	·0006 247	·0004 500	
44	·0032 281	·0024 241	·0018 136	·0013 521	·0010 046	·0007 400	
03	·0048 108	·0036 677	·0027 860	·0021 089	·0015 910	·0011 900	
12	·0070 690	·0054 700	·0042 175 <sup>-</sup>	·0032 405 <sup>-</sup>	·0024 815 <sup>+</sup>	·0018 900	
05 <sup>+</sup>	·0102 424	·0080 419	·0062 917	·0049 056	·0038 122	·0029 500	
03	·0146 339	·0116 554	·0092 504	·0073 169	·0057 686	·0045 300	
44	·0206 178	·0166 530	·0134 039	·0107 527	·0085 981	·0068 500	
7	·0286 445 <sup>+</sup>	·0234 559	·0191 414	·0155 690	·0126 230	·0102 000	
7	·0392 413	·0325 679	·0269 383	·0222 094	·0182 533	·0149 500	
9	·0530 053	·0445 734	·0373 587	·0312 117	·0259 958	·0215 800	
3	·0705 886	·0601 273	·0510 501	·0432 074	·0364 589	·0306 700	
9	·0926 703	·0799 331	·0687 277	·0589 118	·0503 480	·0429 000	
1	·1199 170	·1047 082	·0911 451	·0791 011	·0684 495 <sup>-</sup>	·0590 600	
6	·1529 294	·1351 337	·1190 490	·1045 729	·0915 973	·0800 100	
3	·1921 761	·1717 895 <sup>-</sup>	·1531 174	·1360 884	·1206 211	·1066 200	
7	·2379 189	·2150 773	·1938 815 <sup>-</sup>	·1742 905 <sup>-</sup>	·1562 736	·1397 500	
8	·2901 353	·2651 360	·2416 366	·2196 413	·1991 373	·1800 900	
4	·3484 467	·3217 577	·2963 484	·2722 604	·2495 179	·2281 200	
4	·4120 645 <sup>-</sup>	·3843 173	·3575 663	·3318 852	·3073 316	·2839 400	
3	·4797 657	·4517 281	·4243 590	·3977 563	·3720 035 <sup>+</sup>	·3471 700	
9	·5499 117	·5224 386	·4952 895 <sup>+</sup>	·4685 748	·4423 945 <sup>-</sup>	·4168 300	
8	·6205 179	·5944 842	·5684 438	·5425 075 <sup>+</sup>	·5167 800	·4913 580	
5	·6893 812	·6656 007	·6415 274	·6172 617	·5929 012	·5685 400	
0	·7542 564	·7333 986	·7120 327	·6902 395 <sup>-</sup>	·6681 007	·6450 980	
4	·8130 702	·7955 870	·7774 671	·7587 675 <sup>-</sup>	·7395 479	·7198 980	
4	·8641 427	·8502 101	·8275 028				

TABLE I. THE  $I_x(p, q)$  FUNCTION

to .99

 $q = 9$ 

$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$\cdot 3020\ 0825 \times \frac{1}{10^{10}}$	$\cdot 2507\ 2383 \times \frac{1}{10^{10}}$	$\cdot 2089\ 3652 \times \frac{1}{10^{10}}$	$\cdot 1747\ 4691 \times \frac{1}{10^{10}}$	$\cdot 1466\ 6259 \times \frac{1}{10^{10}}$	$\cdot 1235\ 0533$
$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$			
$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	
$\cdot 0000\ 008$	$\cdot 0000\ 005^+$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$
$\cdot 0000\ 016$	$\cdot 0000\ 010$	$\cdot 0000\ 006$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 032$	$\cdot 0000\ 020$	$\cdot 0000\ 013$	$\cdot 0000\ 008$	$\cdot 0000\ 005^-$	$\cdot 0000\ 003$
$\cdot 0000\ 060$	$\cdot 0000\ 039$	$\cdot 0000\ 025^-$	$\cdot 0000\ 016$	$\cdot 0000\ 010$	$\cdot 0000\ 007$
$\cdot 0000\ 112$	$\cdot 0000\ 074$	$\cdot 0000\ 048$	$\cdot 0000\ 031$	$\cdot 0000\ 020$	$\cdot 0000\ 013$
$\cdot 0000\ 206$	$\cdot 0000\ 138$	$\cdot 0000\ 092$	$\cdot 0000\ 061$	$\cdot 0000\ 040$	$\cdot 0000\ 026$
$\cdot 0000\ 371$	$\cdot 0000\ 252$	$\cdot 0000\ 171$	$\cdot 0000\ 115^+$	$\cdot 0000\ 077$	$\cdot 0000\ 052$
$\cdot 0000\ 658$	$\cdot 0000\ 454$	$\cdot 0000\ 313$	$\cdot 0000\ 214$	$\cdot 0000\ 147$	$\cdot 0000\ 100$
$\cdot 0001\ 146$	$\cdot 0000\ 804$	$\cdot 0000\ 563$	$\cdot 0000\ 393$	$\cdot 0000\ 273$	$\cdot 0000\ 190$
$\cdot 0001\ 964$	$\cdot 0001\ 401$	$\cdot 0000\ 996$	$\cdot 0000\ 706$	$\cdot 0000\ 499$	$\cdot 0000\ 352$
$\cdot 0003\ 311$	$\cdot 0002\ 399$	$\cdot 0001\ 733$	$\cdot 0001\ 249$	$\cdot 0000\ 897$	$\cdot 0000\ 643$
$\cdot 0005\ 493$	$\cdot 0004\ 043$	$\cdot 0002\ 967$	$\cdot 0002\ 171$	$\cdot 0001\ 585^-$	$\cdot 0001\ 154$
$\cdot 0008\ 909$	$\cdot 0006\ 703$	$\cdot 0004\ 995^+$	$\cdot 0003\ 712$	$\cdot 0002\ 751$	$\cdot 0002\ 034$
$\cdot 0014\ 415^-$	$\cdot 0010\ 937$	$\cdot 0008\ 274$	$\cdot 0006\ 242$	$\cdot 0004\ 697$	$\cdot 0003\ 525^+$
$\cdot 0022\ 807$	$\cdot 0017\ 561$	$\cdot 0013\ 484$	$\cdot 0010\ 325^-$	$\cdot 0007\ 885^+$	$\cdot 0006\ 006$
$\cdot 0035\ 523$	$\cdot 0027\ 753$	$\cdot 0021\ 621$	$\cdot 0016\ 798$	$\cdot 0013\ 017$	$\cdot 0010\ 061$
$\cdot 0054\ 471$	$\cdot 0043\ 166$	$\cdot 0034\ 112$	$\cdot 0026\ 885^-$	$\cdot 0021\ 133$	$\cdot 0016\ 571$
$\cdot 0082\ 228$	$\cdot 0066\ 079$	$\cdot 0052\ 955^+$	$\cdot 0042\ 325^+$	$\cdot 0033\ 742$	$\cdot 0026\ 832$
$\cdot 0122\ 195^+$	$\cdot 0099\ 552$	$\cdot 0080\ 885^-$	$\cdot 0065\ 544$	$\cdot 0052\ 979$	$\cdot 0042\ 716$
$\cdot 0178\ 746$	$\cdot 0147\ 595^-$	$\cdot 0121\ 545^+$	$\cdot 0099\ 834$	$\cdot 0081\ 795^-$	$\cdot 0066\ 852$
$\cdot 0257\ 347$	$\cdot 0215\ 315^+$	$\cdot 0179\ 672$	$\cdot 0149\ 546$	$\cdot 0124\ 163$	$\cdot 0102\ 841$
$\cdot 0304\ 620$	$\cdot 0309\ 028$	$\cdot 0261\ 232$	$\cdot 0220\ 272$	$\cdot 0185\ 283$	$\cdot 0155\ 483$
$\cdot 0508\ 302$	$\cdot 0436\ 276$	$\cdot 0373\ 502$	$\cdot 0318\ 971$	$\cdot 0271\ 749$	$\cdot 0230\ 081$
$\cdot 0697\ 063$	$\cdot 0605\ 718$	$\cdot 0525\ 030$	$\cdot 0453\ 990$	$\cdot 0391\ 641$	$\cdot 0337\ 085^-$
$\cdot 0940\ 126$	$\cdot 0826\ 827$	$\cdot 0725\ 411$	$\cdot 0634\ 931$	$\cdot 0554\ 461$	$\cdot 0483\ 111$
$\cdot 1246\ 646$	$\cdot 1109\ 346$	$\cdot 0984\ 830$	$\cdot 0872\ 278$	$\cdot 0770\ 862$	$\cdot 0679\ 757$
$\cdot 1624\ 842$	$\cdot 1462\ 476$	$\cdot 1313\ 316$	$\cdot 1176\ 737$	$\cdot 1052\ 072$	$\cdot 0938\ 627$
$\cdot 2080\ 881$	$\cdot 1893\ 773$	$\cdot 1719\ 678$	$\cdot 1558\ 225^-$	$\cdot 1408\ 966$	$\cdot 1271\ 399$
$\cdot 2617\ 593$	$\cdot 2407\ 816$	$\cdot 2210\ 160$	$\cdot 2024\ 534$	$\cdot 1850\ 757$	$\cdot 1688\ 566$
$\cdot 3233\ 135^-$	$\cdot 3004\ 758$	$\cdot 2786\ 887$	$\cdot 2579\ 720$	$\cdot 2383\ 352$	$\cdot 2197\ 784$
$\cdot 3919\ 817$	$\cdot 3678\ 943$	$\cdot 3446\ 312$	$\cdot 3222\ 379$	$\cdot 3007\ 499$	$\cdot 2801\ 930$
$\cdot 4663\ 337$	$\cdot 4417\ 863$	$\cdot 4177\ 899$	$\cdot 3944\ 091$	$\cdot 3716\ 998$	$\cdot 3497\ 097$
$\cdot 5442\ 708$	$\cdot 5201\ 775^-$	$\cdot 4963\ 412$	$\cdot 4728\ 365^-$	$\cdot 4497\ 318$	$\cdot 4270\ 893$
$\cdot 6231\ 138$	$\cdot 6004\ 265^+$	$\cdot 5777\ 140$	$\cdot 5550\ 509$	$\cdot 5325\ 083$	$\cdot 5101\ 536$
$\cdot 6997\ 999$	$\cdot 6794\ 007$	$\cdot 6587\ 386$	$\cdot 6378\ 789$	$\cdot 6168\ 860$	$\cdot 5958\ 232$
$\cdot 7711\ 856$	$\cdot 7537\ 730$	$\cdot 7359\ 308$	$\cdot 7177\ 084$	$\cdot 6991\ 566$	$\cdot 6803\ 264$
$\cdot 8344\ 239$	$\cdot 8204\ 152$	$\cdot 8058\ 060$	$\cdot 7908\ 970$	$\cdot 7754\ 512$	$\cdot 7595\ 936$
$\cdot 8873\ 570$	$\cdot 8768\ 320$	$\cdot 8657\ 994$	$\cdot 8542\ 728$	$\cdot 8422\ 679$	$\cdot 8298\ 029$
$\cdot 9288\ 437$	$\cdot 9215\ 458$	$\cdot 9138\ 100$	$\cdot 9056\ 370$	$\cdot 8970\ 292$	$\cdot 8879\ 911$
$\cdot 9589\ 318$	$\cdot 9543\ 341$	$\cdot 9494\ 065^+$	$\cdot 9441\ 425^+$	$\cdot 9385\ 370$	$\cdot 9325\ 857$
$\cdot 9788\ 088$	$\cdot 9766\ 088$				

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

6 to 70

$q = 9.5$

$p = 9.5$

$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$
$q = .2222\ 7212 \times \frac{1}{10^3}$	$.1561\ 4033 \times \frac{1}{10^3}$	$.1111\ 3606 \times \frac{1}{10^3}$	$.8007\ 1963 \times \frac{1}{10^3}$	$.4296\ 5443 \times \frac{1}{10^3}$	$.2398\ 0713 \times \frac{1}{10^3}$
.0000 001					
.0000 003	.0000 001				
.0000 010	.0000 004	.0000 001	.0000 001		
.0000 027	.0000 011	.0000 004	.0000 002		
.0000 067	.0000 029	.0000 012	.0000 005 <sup>+</sup>	.0000 001	
.0000 152	.0000 068	.0000 030	.0000 013	.0000 002	
.0000 320	.0000 149	.0000 069	.0000 031	.0000 006	.0000 001
.0000 628	.0000 305 <sup>-</sup>	.0000 146	.0000 069	.0000 015 <sup>+</sup>	.0000 003
.0001 165 <sup>+</sup>	.0000 586	.0000 292	.0000 144	.0000 034	.0000 008
.0002 056	.0001 070	.0000 551	.0000 281	.0000 071	.0000 018
.0003 476	.0001 866	.0000 992	.0000 522	.0000 141	.0000 037
.0005 657	.0003 128	.0001 713	.0000 929	.0000 267	.0000 074
.0008 899	.0005 061	.0002 850 <sup>+</sup>	.0001 590	.0000 483	.0000 142
.0013 585 <sup>-</sup>	.0007 932	.0004 586	.0002 628	.0000 841	.0000 261
.0020 180	.0012 081	.0007 162	.0004 208	.0001 416	.0000 462
.0029 247	.0017 928	.0010 884	.0006 548	.0002 312	.0000 792
.0041 447	.0025 985 <sup>+</sup>	.0016 136	.0009 930	.0003 669	.0001 316
.0057 542	.0036 859	.0023 386	.0014 706	.0005 674	.0002 125 <sup>-</sup>
.0078 394	.0051 254	.0033 195 <sup>+</sup>	.0021 309	.0008 569	.0003 345 <sup>+</sup>
.0104 958	.0069 979	.0046 222	.0030 263	.0012 661	.0005 144
.0138 273	.0093 935 <sup>+</sup>	.0063 224	.0042 184	.0018 332	.0007 737
.0179 450 <sup>+</sup>	.0124 120	.0085 063	.0057 793	.0026 047	.0011 404
.0229 655 <sup>+</sup>	.0161 609	.0112 692	.0077 910	.0036 304	.0016 492
.0290 086	.0207 546	.0147 155 <sup>+</sup>	.0103 454	.0049 941	.0023 429
.0361 950 <sup>+</sup>	.0263 122	.0189 575 <sup>-</sup>	.0135 440	.0067 537	.0032 736
.0446 435 <sup>+</sup>	.0329 556	.0241 133	.0174 970	.0090 019	.0045 030
.0544 683	.0408 066	.0303 053	.0223 216	.0118 356	.0061 032
.0657 756	.0499 845 <sup>-</sup>	.0376 576	.0281 403	.0153 615 <sup>-</sup>	.0081 574
.0786 609	.0606 027	.0462 934	.0350 791	.0196 953	.0107 599
.0932 057	.0727 658	.0563 322	.0432 648	.0249 602	.0140 157
.1094 744	.0865 665 <sup>-</sup>	.0678 862	.0528 207	.0312 847	.0180 401
.1275 135 <sup>+</sup>	.1020 820	.0810 574	.0638 667	.0388 009	.0229 573
.1473 453	.1193 717	.0959 342	.0765 127	.0476 413	.0288 990
.1689 699	.1384 739	.1125 881	.0908 567	.0579 358	.0360 023
.1923 621	.1594 040	.1310 709	.1069 810	.0698 084	.0444 071
.2174 706	.1821 521	.1514 116	.1249 493	.0833 732	.0542 527
.2442 178	.2066 820	.1736 144	.1448 030	.0987 312	.0656 747
.2725 002	.2329 302	.1976 569	.1665 589	.1159 658	.0788 010
.3021 890	.2608 058	.2234 884	.1902 069	.1351 397	.0937 476
.3331 321	.2901 909	.2510 297	.2157 081	.1562 912	.1106 145 <sup>-</sup>
.3651 559	.3209 418	.2801 730	.2429 940	.1794 311	.1294 813
.3980 682	.3528 911	.3107 827	.2719 658	.2045 404	.1504 033
.4316 610	.3858 496	.3426 968	.3024 951	.2315 680	.1734 078
.4657 143	.4196 096	.3757 291	.3344 248	.2604 301	.1984 004
.5000 000 <sup>*</sup>	.4539 484	.4006 722			

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.96$  $q = 9.5$ 

	$p = 9.5$	$p = 10$	$p = 10.5$	$p = 11$	$p =$
$B(p, q) = .2222\ 7212 \times \frac{1}{10^8}$		$.1561\ 4033 \times \frac{1}{10^8}$	$.1111\ 3606 \times \frac{1}{10^8}$	$.8007\ 1963 \times \frac{1}{10^8}$	$.4296$
$\cdot 71$	$\cdot 9709\ 914$	$\cdot 9643\ 319$	$\cdot 9566\ 983$	$\cdot 9480\ 446$	$\cdot 9275$
$\cdot 72$	$\cdot 9770\ 345^-$	$\cdot 9716\ 053$	$\cdot 9653\ 381$	$\cdot 9581\ 834$	$\cdot 9410$
$\cdot 73$	$\cdot 9820\ 550^-$	$\cdot 9776\ 900$	$\cdot 9726\ 162$	$\cdot 9667\ 839$	$\cdot 9526$
$\cdot 74$	$\cdot 9861\ 727$	$\cdot 9827\ 148$	$\cdot 9786\ 679$	$\cdot 9739\ 841$	$\cdot 9625$
$\cdot 75$	$\cdot 9895\ 042$	$\cdot 9868\ 077$	$\cdot 9836\ 307$	$\cdot 9799\ 287$	$\cdot 9707$
$\cdot 76$	$\cdot 9921\ 606$	$\cdot 9900\ 930$	$\cdot 9876\ 407$	$\cdot 9847\ 642$	$\cdot 9775$
$\cdot 77$	$\cdot 9942\ 458$	$\cdot 9926\ 889$	$\cdot 9908\ 302$	$\cdot 9886\ 356$	$\cdot 9831$
$\cdot 78$	$\cdot 9958\ 553$	$\cdot 9947\ 057$	$\cdot 9933\ 242$	$\cdot 9916\ 826$	$\cdot 9875$
$\cdot 79$	$\cdot 9970\ 753$	$\cdot 9962\ 442$	$\cdot 9952\ 391$	$\cdot 9940\ 370$	$\cdot 9909$
$\cdot 80$	$\cdot 9979\ 820$	$\cdot 9973\ 949$	$\cdot 9966\ 803$	$\cdot 9958\ 203$	$\cdot 9935$
$\cdot 81$	$\cdot 9986\ 415^+$	$\cdot 9982\ 371$	$\cdot 9977\ 417$	$\cdot 9971\ 419$	$\cdot 9955$
$\cdot 82$	$\cdot 9991\ 101$	$\cdot 9988\ 390$	$\cdot 9985\ 051$	$\cdot 9980\ 982$	$\cdot 9970$
$\cdot 83$	$\cdot 9994\ 343$	$\cdot 9992\ 583$	$\cdot 9990\ 400$	$\cdot 9987\ 724$	$\cdot 9980$
$\cdot 84$	$\cdot 9996\ 524$	$\cdot 9995\ 418$	$\cdot 9994\ 040$	$\cdot 9992\ 339$	$\cdot 9987$
$\cdot 85$	$\cdot 9997\ 944$	$\cdot 9997\ 276$	$\cdot 9996\ 438$	$\cdot 9995\ 398$	$\cdot 9992$
$\cdot 86$	$\cdot 9998\ 835^-$	$\cdot 9998\ 448$	$\cdot 9997\ 961$	$\cdot 9997\ 353$	$\cdot 9995$
$\cdot 87$	$\cdot 9999\ 372$	$\cdot 9999\ 159$	$\cdot 9998\ 889$	$\cdot 9998\ 551$	$\cdot 9997$
$\cdot 88$	$\cdot 9999\ 680$	$\cdot 9999\ 569$	$\cdot 9999\ 429$	$\cdot 9999\ 251$	$\cdot 9998$
$\cdot 89$	$\cdot 9999\ 848$	$\cdot 9999\ 794$	$\cdot 9999\ 725^+$	$\cdot 9999\ 638$	$\cdot 9999$
$\cdot 90$	$\cdot 9999\ 933$	$\cdot 9999\ 909$	$\cdot 9999\ 878$	$\cdot 9999\ 838$	$\cdot 9999$
$\cdot 91$	$\cdot 9999\ 973$	$\cdot 9999\ 963$	$\cdot 9999\ 951$	$\cdot 9999\ 934$	$\cdot 9999$
$\cdot 92$	$\cdot 9999\ 990$	$\cdot 9999\ 987$	$\cdot 9999\ 982$	$\cdot 9999\ 976$	$\cdot 9999$
$\cdot 93$	$\cdot 9999\ 997$	$\cdot 9999\ 996$	$\cdot 9999\ 995^-$	$\cdot 9999\ 993$	$\cdot 9999$
$\cdot 94$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999$
$\cdot 95$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$\cdot 9999$
$\cdot 96$					$1.0000$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q = 9.5$

$p = 14$

5 10 70

$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$
$\cdot 1375\ 5523 \times \frac{1}{10^7}$	$\cdot 3254\ 3540 \times \frac{1}{10^7}$	$\cdot 5053\ 6861 \times \frac{1}{10^7}$	$\cdot 3170\ 9403 \times \frac{1}{10^7}$	$\cdot 2034\ 1881 \times \frac{1}{10^7}$	$\cdot 1331\ 4686 \times \frac{1}{10^7}$
$\cdot 0000\ 001$					
$\cdot 0000\ 002$					
$\cdot 0000\ 004$	$\cdot 0000\ 001$				
$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 024$	$\cdot 0000\ 005^+$	$\cdot 0000\ 001$			
$\cdot 0000\ 041$	$\cdot 0000\ 011$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 0000\ 071$	$\cdot 0000\ 023$	$\cdot 0000\ 007$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 147$	$\cdot 0000\ 046$	$\cdot 0000\ 014$	$\cdot 0000\ 004$	$\cdot 0000\ 001$	
$\cdot 0000\ 264$	$\cdot 0000\ 086$	$\cdot 0000\ 027$	$\cdot 0000\ 009$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0000\ 459$	$\cdot 0000\ 157$	$\cdot 0000\ 052$	$\cdot 0000\ 017$	$\cdot 0000\ 005^+$	$\cdot 0000\ 002$
$\cdot 0000\ 775^+$	$\cdot 0000\ 276$	$\cdot 0000\ 096$	$\cdot 0000\ 033$	$\cdot 0000\ 011$	$\cdot 0000\ 004$
$\cdot 0001\ 472$	$\cdot 0000\ 472$	$\cdot 0000\ 172$	$\cdot 0000\ 061$	$\cdot 0000\ 021$	$\cdot 0000\ 007$
$\cdot 0002\ 036$	$\cdot 0000\ 787$	$\cdot 0000\ 298$	$\cdot 0000\ 111$	$\cdot 0000\ 040$	$\cdot 0000\ 014$
$\cdot 0003\ 182$	$\cdot 0001\ 278$	$\cdot 0000\ 503$	$\cdot 0000\ 194$	$\cdot 0000\ 074$	$\cdot 0000\ 027$
$\cdot 0004\ 865^+$	$\cdot 0002\ 028$	$\cdot 0000\ 828$	$\cdot 0000\ 331$	$\cdot 0000\ 130$	$\cdot 0000\ 050^+$
$\cdot 0007\ 249$	$\cdot 0003\ 148$	$\cdot 0001\ 331$	$\cdot 0000\ 552$	$\cdot 0000\ 225^+$	$\cdot 0000\ 090$
$\cdot 0010\ 714$	$\cdot 0004\ 788$	$\cdot 0002\ 096$	$\cdot 0000\ 900$	$\cdot 0000\ 380$	$\cdot 0000\ 158$
$\cdot 0015\ 470$	$\cdot 0007\ 145^+$	$\cdot 0003\ 232$	$\cdot 0001\ 435^+$	$\cdot 0000\ 626$	$\cdot 0000\ 269$
$\cdot 0021\ 964$	$\cdot 0010\ 473$	$\cdot 0004\ 892$	$\cdot 0002\ 242$	$\cdot 0001\ 010$	$\cdot 0000\ 448$
$\cdot 0030\ 694$	$\cdot 0015\ 092$	$\cdot 0007\ 270$	$\cdot 0003\ 438$	$\cdot 0001\ 598$	$\cdot 0000\ 731$
$\cdot 0042\ 257$	$\cdot 0021\ 404$	$\cdot 0010\ 624$	$\cdot 0005\ 176$	$\cdot 0002\ 480$	$\cdot 0001\ 169$
$\cdot 0057\ 355^+$	$\cdot 0029\ 900$	$\cdot 0015\ 276$	$\cdot 0007\ 662$	$\cdot 0003\ 779$	$\cdot 0001\ 835^+$
$\cdot 0076\ 806$	$\cdot 0041\ 171$	$\cdot 0021\ 631$	$\cdot 0011\ 159$	$\cdot 0005\ 661$	$\cdot 0002\ 828$
$\cdot 0101\ 545^+$	$\cdot 0055\ 921$	$\cdot 0030\ 189$	$\cdot 0016\ 004$	$\cdot 0008\ 344$	$\cdot 0004\ 284$
$\cdot 0132\ 622$	$\cdot 0074\ 970$	$\cdot 0041\ 552$	$\cdot 0022\ 618$	$\cdot 0012\ 110$	$\cdot 0006\ 386$
$\cdot 0171\ 201$	$\cdot 0099\ 266$	$\cdot 0056\ 441$	$\cdot 0031\ 522$	$\cdot 0017\ 318$	$\cdot 0009\ 372$
$\cdot 0218\ 552$	$\cdot 0129\ 880$	$\cdot 0075\ 701$	$\cdot 0043\ 346$	$\cdot 0024\ 419$	$\cdot 0013\ 551$
$\cdot 0276\ 031$	$\cdot 0168\ 006$	$\cdot 0100\ 310$	$\cdot 0058\ 846$	$\cdot 0033\ 968$	$\cdot 0019\ 317$
$\cdot 0345\ 067$	$\cdot 0214\ 955^+$	$\cdot 0131\ 380$	$\cdot 0078\ 910$	$\cdot 0046\ 641$	$\cdot 0027\ 163$
$\cdot 0427\ 131$	$\cdot 0272\ 141$	$\cdot 0170\ 157$	$\cdot 0104\ 569$	$\cdot 0063\ 248$	$\cdot 0037\ 698$
$\cdot 0523\ 711$	$\cdot 0341\ 061$	$\cdot 0218\ 017$	$\cdot 0136\ 999$	$\cdot 0084\ 743$	$\cdot 0051\ 662$
$\cdot 0636\ 260$	$\cdot 0423\ 272$	$\cdot 0276\ 446$	$\cdot 0177\ 522$	$\cdot 0112\ 233$	$\cdot 0069\ 940$
$\cdot 0766\ 204$	$\cdot 0520\ 354$	$\cdot 0347\ 030$	$\cdot 0227\ 599$	$\cdot 0146\ 983$	$\cdot 0093\ 575^+$
$\cdot 0914\ 810$	$\cdot 0633\ 879$	$\cdot 0431\ 421$	$\cdot 0288\ 815^+$	$\cdot 0190\ 417$	$\cdot 0123\ 786$
$\cdot 1083\ 222$	$\cdot 0765\ 359$	$\cdot 0531\ 306$	$\cdot 0362\ 861$	$\cdot 0244\ 107$	$\cdot 0161\ 936$
$\cdot 1272\ 375^+$	$\cdot 0916\ 205^+$	$\cdot 0648\ 364$	$\cdot 0451\ 502$	$\cdot 0309\ 761$	$\cdot 0209\ 597$
$\cdot 1482\ 954$	$\cdot 1087\ 669$	$\cdot 0784\ 221$	$\cdot 0556\ 541$	$\cdot 0389\ 196$	$\cdot 0268\ 475^+$
$\cdot 1715\ 349$	$\cdot 1280\ 794$	$\cdot 0940\ 393$	$\cdot 0679\ 774$	$\cdot 0484\ 309$	$\cdot 0340\ 426$
$\cdot 1969\ 614$	$\cdot 1496\ 360$	$\cdot 1118\ 230$	$\cdot 0822\ 934$	$\cdot 0597\ 033$	$\cdot 0427\ 419$
$\cdot 2245\ 436$	$\cdot 1734\ 828$	$\cdot 1318\ 853$	$\cdot 0987\ 633$	$\cdot 0729\ 281$	$\cdot 0531\ 498$
$\cdot 2542\ 108$	$\cdot 1996\ 299$	$\cdot 1543\ 095^+$	$\cdot 1175\ 292$	$\cdot 0882\ 888$	$\cdot 0654\ 733$
$\cdot 2858\ 509$	$\cdot 2280\ 405^+$	$\cdot 1791\ 440$	$\cdot 1387\ 077$	$\cdot 1059\ 543$	$\cdot 0799\ 153$
$\cdot 3193\ 107$	$\cdot 2586\ 587$	$\cdot 2063\ 968$	$\cdot 1623\ 825^+$	$\cdot 1260\ 707$	$\cdot 0966\ 683$
$\cdot 3543\ 901$	$\cdot 2913\ 465^+$	$\cdot 2360\ 311$	$\cdot 1885\ 981$	$\cdot 1487\ 544$	$\cdot 1159\ 054$
$\cdot 3908\ 742$	$\cdot 3259\ 439$	$\cdot 2679\ 611$	$\cdot 2173\ 531$	$\cdot 1740\ 832$	$\cdot 1377\ 725^+$
$\cdot 4284\ 771$	$\cdot 3622\ 388$	$\cdot 3020\ 502$	$\cdot 2482\ 815$		
$\cdot 4669\ 061$					

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.97$  $q = 9.5$ 

	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$B(p, q) = .1385\ 5523 \times \frac{1}{10^4}$	$.8254\ 3540 \times \frac{1}{10^7}$	$.5053\ 6861 \times \frac{1}{10^7}$	$.3170\ 9403 \times \frac{1}{10^7}$	$.2034\ 18$	
$x$					
.71	.8734 835 <sup>-</sup>	.8401 679	.8031 100	.7628 210	.7199 13
.72	.8948 900	.8658 561	.8331 059	.7969 988	.7580 03
.73	.9138 203	.8888 901	.8603 783	.8285 075 <sup>+</sup>	.7936 09
.74	.9303 328	.9092 592	.8848 278	.8571 439	.8264 14
.75	.9445 292	.9270 091	.9064 230	.8827 811	.8561 84
.76	.9565 478	.9422 376	.9251 989	.9053 701	.8827 65
.77	.9665 570	.9550 878	.9412 521	.9249 389	.9060 97
.78	.9747 474	.9657 402	.9547 334	.9415 872	.9262 06
.79	.9813 239	.9744 038	.9658 390	.9554 783	.9432 00
.80	.9864 975 <sup>-</sup>	.9813 060	.9747 993	.9668 286	.9572 63
.81	.9904 780	.9866 831	.9818 674	.9758 944	.9686 37
.82	.9934 670	.9907 709	.9873 074	.9829 585 <sup>-</sup>	.9776 09
.83	.9956 523	.9937 962	.9913 826	.9883 151	.9844 96
.84	.9972 034	.9959 695 <sup>-</sup>	.9943 456	.9922 569	.9896 25
.85	.9982 687	.9974 799	.9964 295 <sup>-</sup>	.9950 622	.9933 19
.86	.9989 738	.9984 913	.9978 413	.9969 853	.9958 81
.87	.9994 212	.9991 406	.9987 583	.9982 489	.9975 84
.88	.9996 918	.9995 379	.9993 257	.9990 398	.9986 62
.89	.9998 465 <sup>+</sup>	.9997 676	.9996 577	.9995 078	.9993 07
.90	.9999 295 <sup>-</sup>	.9998 922	.9998 396	.9997 671	.9996 69
.91	.9999 706	.9999 546	.9999 317	.9999 000	.9998 56
.92	.9999 891	.9999 830	.9999 742	.9999 619	.9999 44
.93	.9999 965 <sup>+</sup>	.9999 945 <sup>+</sup>	.9999 916	.9999 875 <sup>-</sup>	.9999 81
.94	.9999 991	.9999 986	.9999 978	.9999 966	.9999 95
.95	.9999 998	.9999 997	.9999 995 <sup>+</sup>	.9999 993	.9999 99
.96	1.0000 000	1.0000 000	.9999 999	.9999 999	.9999 99
.97			1.0000 000	1.0000 000	1.0000 00



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

2 to 80

$q = 9.5$

$p = 20$

$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$q = .8876\ 4573 \times \frac{1}{10^8}$	$.6017\ 9371 \times \frac{1}{10^8}$	$.4143\ 4977 \times \frac{1}{10^8}$	$.2893\ 8714 \times \frac{1}{10^8}$	$.2047\ 9705 \times \frac{1}{10^8}$	$.1467\ 2028 \times \frac{1}{10^8}$
.0000 001					
.0000 001					
.0000 003	.0000 001				
.0000 005 <sup>+</sup>	.0000 002	.0000 001			
.0000 010	.0000 004	.0000 001			
.0000 019	.0000 007	.0000 003	.0000 001		
.0000 036	.0000 014	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
.0000 065 <sup>-</sup>	.0000 026	.0000 010	.0000 004	.0000 002	.0000 001
.0000 114	.0000 048	.0000 020	.0000 008	.0000 003	.0000 001
.0000 196	.0000 085 <sup>-</sup>	.0000 036	.0000 015 <sup>+</sup>	.0000 006	.0000 003
.0000 330	.0000 147	.0000 065 <sup>-</sup>	.0000 028	.0000 012	.0000 005 <sup>+</sup>
.0000 544	.0000 249	.0000 113	.0000 051	.0000 022	.0000 010
.0000 879	.0000 415 <sup>+</sup>	.0000 194	.0000 089	.0000 041	.0000 018
.0001 393	.0000 677	.0000 325 <sup>+</sup>	.0000 154	.0000 073	.0000 034
.0002 169	.0001 084	.0000 535 <sup>+</sup>	.0000 261	.0000 126	.0000 060
.0003 321	.0001 704	.0000 864	.0000 433	.0000 215 <sup>+</sup>	.0000 106
.0005 002	.0002 635 <sup>-</sup>	.0001 372	.0000 706	.0000 360	.0000 182
.0007 417	.0004 008	.0002 140	.0001 130	.0000 591	.0000 306
.0010 835 <sup>+</sup>	.0006 001	.0003 284	.0001 778	.0000 952	.0000 505 <sup>+</sup>
.0015 605 <sup>-</sup>	.0008 852	.0004 962	.0002 752	.0001 510	.0000 821
.0022 167	.0012 871	.0007 387	.0004 193	.0002 356	.0001 311
.0031 073	.0018 458	.0010 837	.0006 294	.0003 619	.0002 061
.0043 005 <sup>+</sup>	.0026 118	.0015 680	.0009 312	.0005 475 <sup>+</sup>	.0003 189
.0058 790	.0036 483	.0022 383	.0013 586	.0008 164	.0004 860
.0079 413	.0050 331	.0031 538	.0019 554	.0012 004	.0007 300
.0106 038	.0068 601	.0043 883	.0027 777	.0017 410	.0010 811
.0140 009	.0092 412	.0060 317	.0038 960	.0024 919	.0015 793
.0182 858	.0123 077	.0081 927	.0053 973	.0035 214	.0022 766
.0236 299	.0162 109	.0110 000	.0073 878	.0049 143	.0032 395 <sup>6</sup>
.0302 217	.0211 226	.0146 038	.0099 947	.0067 753	.0045 519
.0382 641	.0272 337	.0191 763	.0133 677	.0092 309	.0063 179
.0479 713	.0347 529	.0249 116	.0176 804	.0124 315 <sup>-</sup>	.0086 643
.0595 638	.0439 034	.0320 238	.0231 303	.0165 530	.0117 433
.0732 624	.0549 180	.0407 448	.0299 377	.0217 971	.0157 341
.0892 805 <sup>+</sup>	.0680 338	.0513 198	.0383 436	.0283 914	.0208 443
.1078 162	.0834 842	.0640 013	.0486 053	.0365 864	.0273 093
.1290 425 <sup>+</sup>	.1014 902	.0790 416	.0609 909	.0466 523	.0353 906
.1530 973	.1222 500 <sup>-</sup>	.0966 834	.0757 712	.0588 731	.0453 723
.1800 732	.1459 283	.1171 491	.0932 100	.0735 383	.0575 551
.2100 077	.1726 448	.1406 283	.1135 526	.0909 326	.0722 475 <sup>-</sup>
.2428 739	.2024 621	.1672 653	.1370 121	.1113 232	.0897 552
.2785 730	.2353 756	.1971 459	.1637 557	.1349 458	.1103 673
.3169 287	.2713 036	.2302 845 <sup>-</sup>	.1938 893	.1619 882	.1343 406
.3576 842	.3100 801	.2666 131	.2274 434	.1925 738	.1618 813
.4005 028	.3514 595 <sup>+</sup>	.3050 701	.26		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.98$  $q = 9.5$ 

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$
$B(p, q) = .8876\ 4573 \times \frac{1}{10^8}$		$.6017\ 9371 \times \frac{1}{10^8}$	$.4143\ 4977 \times \frac{1}{10^8}$	$.2893\ 8714 \times \frac{1}{10^8}$	$.2047\ 9711 \times \frac{1}{10^8}$
$\pi$					
.81	.9498 372	.9381 263	.9248 010	.9098 372	.8932 372
.82	.9634 874	.9545 318	.9442 157	.9324 883	.9193 171
.83	.9742 232	.9675 926	.9598 615 <sup>+</sup>	.9509 656	.9408 531
.84	.9824 135 <sup>+</sup>	.9776 770	.9720 880	.9655 793	.9580 911
.85	.9884 538	.9852 031	.9813 217	.9767 477	.9714 231
.86	.9927 429	.9906 102	.9880 338	.9849 620	.9813 441
.87	.9956 606	.9943 315 <sup>-</sup>	.9927 070	.9907 478	.9884 131
.88	.9975 507	.9967 698	.9958 044	.9946 267	.9932 071
.89	.9987 078	.9982 795 <sup>-</sup>	.9977 440	.9970 833	.9962 771
.90	.9993 708	.9991 542	.9988 804	.9985 388	.9981 171
.91	.9997 219	.9996 226	.9994 957	.9993 356	.9991 361
.92	.9998 909	.9998 506	.9997 984	.9997 319	.9996 481
.93	.9999 632	.9999 491	.9999 307	.9999 070	.9998 761
.94	.9999 898	.9999 858	.9999 804	.9999 735 <sup>+</sup>	.9999 641
.95	.9999 978	.9999 970	.9999 958	.9999 942	.9999 921
.96	.9999 997	.9999 996	.9999 994	.9999 991	.9999 981
.97	1.0000 000	1.0000 000	1.0000 000	.9999 999	.9999 999
.98				1.0000 000	1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\cdot 30$  to  $\cdot 98$

$q = 9.5$

$p = 1$

	$p = 26$	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$
$p, q) = \cdot 1063\ 1904 \times \frac{1}{10^8}$	$\cdot 7786\ 7467 \times \frac{1}{10^8}$	$\cdot 5760\ 0592 \times \frac{1}{10^8}$	$\cdot 4300\ 8442 \times \frac{1}{10^8}$	$\cdot 3239\ 5969 \times \frac{1}{10^8}$	$\cdot 2460\ 453$	
$z$						
$\cdot 30$	$\cdot 0000\ 001$					
$\cdot 31$	$\cdot 0000\ 001$					
$\cdot 32$	$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 33$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 34$	$\cdot 0000\ 008$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 35$	$\cdot 0000\ 016$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$	
$\cdot 36$	$\cdot 0000\ 029$	$\cdot 0000\ 013$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$
$\cdot 37$	$\cdot 0000\ 051$	$\cdot 0000\ 025^-$	$\cdot 0000\ 012$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 38$	$\cdot 0000\ 091$	$\cdot 0000\ 045^-$	$\cdot 0000\ 022$	$\cdot 0000\ 011$	$\cdot 0000\ 005^+$	$\cdot 0000\ 003$
$\cdot 39$	$\cdot 0000\ 157$	$\cdot 0000\ 080$	$\cdot 0000\ 040$	$\cdot 0000\ 020$	$\cdot 0000\ 010$	$\cdot 0000\ 005$
$\cdot 40$	$\cdot 0000\ 266$	$\cdot 0000\ 139$	$\cdot 0000\ 072$	$\cdot 0000\ 037$	$\cdot 0000\ 019$	$\cdot 0000\ 010$
$\cdot 41$	$\cdot 0000\ 442$	$\cdot 0000\ 236$	$\cdot 0000\ 125^+$	$\cdot 0000\ 066$	$\cdot 0000\ 034$	$\cdot 0000\ 018$
$\cdot 42$	$\cdot 0000\ 723$	$\cdot 0000\ 396$	$\cdot 0000\ 215^-$	$\cdot 0000\ 116$	$\cdot 0000\ 062$	$\cdot 0000\ 033$
$\cdot 43$	$\cdot 0001\ 163$	$\cdot 0000\ 651$	$\cdot 0000\ 362$	$\cdot 0000\ 199$	$\cdot 0000\ 109$	$\cdot 0000\ 059$
$\cdot 44$	$\cdot 0001\ 841$	$\cdot 0001\ 054$	$\cdot 0000\ 599$	$\cdot 0000\ 338$	$\cdot 0000\ 189$	$\cdot 0000\ 105$
$\cdot 45$	$\cdot 0002\ 868$	$\cdot 0001\ 679$	$\cdot 0000\ 975^-$	$\cdot 0000\ 562$	$\cdot 0000\ 322$	$\cdot 0000\ 183$
$\cdot 46$	$\cdot 0004\ 401$	$\cdot 0002\ 632$	$\cdot 0001\ 562$	$\cdot 0000\ 920$	$\cdot 0000\ 538$	$\cdot 0000\ 313$
$\cdot 47$	$\cdot 0006\ 656$	$\cdot 0004\ 064$	$\cdot 0002\ 463$	$\cdot 0001\ 481$	$\cdot 0000\ 885^-$	$\cdot 0000\ 525$
$\cdot 48$	$\cdot 0009\ 923$	$\cdot 0006\ 185^-$	$\cdot 0003\ 825^+$	$\cdot 0002\ 349$	$\cdot 0001\ 432$	$\cdot 0000\ 868$
$\cdot 49$	$\cdot 0014\ 593$	$\cdot 0009\ 279$	$\cdot 0005\ 855^+$	$\cdot 0003\ 668$	$\cdot 0002\ 283$	$\cdot 0001\ 411$
$\cdot 50$	$\cdot 0021\ 174$	$\cdot 0013\ 729$	$\cdot 0008\ 835^-$	$\cdot 0005\ 645^-$	$\cdot 0003\ 582$	$\cdot 0002\ 259$
$\cdot 51$	$\cdot 0030\ 325^-$	$\cdot 0020\ 042$	$\cdot 0013\ 147$	$\cdot 0008\ 563$	$\cdot 0005\ 540$	$\cdot 0003\ 562$
$\cdot 52$	$\cdot 0042\ 882$	$\cdot 0028\ 876$	$\cdot 0019\ 301$	$\cdot 0012\ 810$	$\cdot 0008\ 445^+$	$\cdot 0005\ 533$
$\cdot 53$	$\cdot 0059\ 889$	$\cdot 0041\ 073$	$\cdot 0027\ 962$	$\cdot 0018\ 903$	$\cdot 0012\ 694$	$\cdot 0008\ 472$
$\cdot 54$	$\cdot 0082\ 630$	$\cdot 0057\ 693$	$\cdot 0039\ 987$	$\cdot 0027\ 524$	$\cdot 0018\ 821$	$\cdot 0012\ 790$
$\cdot 55$	$\cdot 0112\ 657$	$\cdot 0080\ 046$	$\cdot 0056\ 463$	$\cdot 0039\ 555^+$	$\cdot 0027\ 530$	$\cdot 0019\ 043$
$\cdot 56$	$\cdot 0151\ 810$	$\cdot 0109\ 727$	$\cdot 0078\ 742$	$\cdot 0056\ 122$	$\cdot 0039\ 743$	$\cdot 0027\ 971$
$\cdot 57$	$\cdot 0202\ 234$	$\cdot 0148\ 641$	$\cdot 0108\ 475^+$	$\cdot 0078\ 631$	$\cdot 0056\ 634$	$\cdot 0040\ 543$
$\cdot 58$	$\cdot 0266\ 380$	$\cdot 0199\ 018$	$\cdot 0147\ 649$	$\cdot 0108\ 810$	$\cdot 0079\ 681$	$\cdot 0057\ 999$
$\cdot 59$	$\cdot 0346\ 986$	$\cdot 0263\ 423$	$\cdot 0198\ 600$	$\cdot 0148\ 743$	$\cdot 0110\ 707$	$\cdot 0081\ 907$
$\cdot 60$	$\cdot 0447\ 046$	$\cdot 0344\ 736$	$\cdot 0264\ 024$	$\cdot 0200\ 897$	$\cdot 0151\ 918$	$\cdot 0114\ 205$
$\cdot 61$	$\cdot 0569\ 743$	$\cdot 0446\ 118$	$\cdot 0346\ 967$	$\cdot 0268\ 124$	$\cdot 0205\ 933$	$\cdot 0157\ 247$
$\cdot 62$	$\cdot 0718\ 365^+$	$\cdot 0570\ 949$	$\cdot 0450\ 778$	$\cdot 0353\ 654$	$\cdot 0275\ 788$	$\cdot 0213\ 832$
$\cdot 63$	$\cdot 0896\ 182$	$\cdot 0722\ 727$	$\cdot 0579\ 049$	$\cdot 0461\ 053$	$\cdot 0364\ 927$	$\cdot 0287\ 209$
$\cdot 64$	$\cdot 1106\ 299$	$\cdot 0904\ 941$	$\cdot 0735\ 502$	$\cdot 0594\ 142$	$\cdot 0477\ 154$	$\cdot 0381\ 068$
$\cdot 65$	$\cdot 1351\ 482$	$\cdot 1120\ 908$	$\cdot 0923\ 853$	$\cdot 0756\ 884$	$\cdot 0616\ 544$	$\cdot 0499\ 477$
$\cdot 66$	$\cdot 1633\ 957$	$\cdot 1373\ 577$	$\cdot 1147\ 625^-$	$\cdot 0953\ 225^+$	$\cdot 0787\ 315^+$	$\cdot 0646\ 790$
$\cdot 67$	$\cdot 1955\ 206$	$\cdot 1665\ 309$	$\cdot 1409\ 932$	$\cdot 1186\ 887$	$\cdot 0993\ 646$	$\cdot 0827\ 490$
$\cdot 68$	$\cdot 2315\ 752$	$\cdot 1997\ 646$	$\cdot 1713\ 235^+$	$\cdot 1461\ 126$	$\cdot 1239\ 443$	$\cdot 1045\ 988$
$\cdot 69$	$\cdot 2714\ 965^+$	$\cdot 2371\ 069$	$\cdot 2059\ 078$	$\cdot 1778\ 453$	$\cdot 1528\ 069$	$\cdot 1306\ 353$
$\cdot 70$	$\cdot 3150\ 896$	$\cdot 2784\ 782$	$\cdot 2447\ 821$	$\cdot 2140\ 345^-$	$\cdot 1862\ 025^+$	$\cdot 1612\ 002$
$\cdot 71$	$\cdot 3620\ 159$	$\cdot 3236\ 528$	$\cdot 2878\ 401$	$\cdot 2546\ 942$	$\cdot 2242\ 629$	$\cdot 1965\ 348$
$\cdot 72$	$\cdot 4117\ 886$	$\cdot 3722\ 463$	$\cdot 3348\ 129$	$\cdot 2996\ 791$	$\cdot 2669\ 682$	$\cdot 2367\ 434$
$\cdot 73$	$\cdot 4637\ 763$	$\cdot 4237\ 113$	$\cdot 3852\ 562$	$\cdot 3486\ 621$	$\cdot 3141\ 186$	$\cdot 2817\ 570$
$\cdot 74$	$\cdot 5172\ 163$	$\cdot 4773\ 426$	$\cdot 4385\ 469$	$\cdot 4011\ 229$	$\cdot 3653\ 120$	$\cdot 3313\ 036$
$\cdot 75$	$\cdot 5712\ 382$	$\cdot 5322\ 943$	$\cdot 4938\ 914$	$\cdot 4563\ 457$	$\cdot 4199\ 328$	$\cdot 3848\ 854$
$\cdot 76$	$\cdot 6248\ 073$	$\cdot 5876\ 087$	$\cdot 5502\ 177$	$\cdot 5131\ 128$	$\cdot 4533\ 112$	

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = \cdot 37$  to  $\cdot 99$  $q = 9\cdot 5$ 

	$p = 32$	$p = 33$	$p = 34$	$p = 35$	$p = 36$
$B(p, q) = \cdot 1883\ 3100 \times \frac{1}{10^9}$		$\cdot 1452\ 1908 \times \frac{1}{10^9}$	$\cdot 1127\ 5835 \times \frac{1}{10^9}$	$\cdot 8813\ 2961 \times \frac{1}{10^9}$	$\cdot 6931\ 8$
$x$					
$\cdot 37$	$\cdot 0000\ 001$				
$\cdot 38$	$\cdot 0000\ 001$	$\cdot 0000\ 001$			
$\cdot 39$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$		
$\cdot 40$	$\cdot 0000\ 005^-$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 41$	$\cdot 0000\ 009$	$\cdot 0000\ 005^-$		$\cdot 0000\ 001$	
$\cdot 42$	$\cdot 0000\ 017$	$\cdot 0000\ 009$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 001$
$\cdot 43$	$\cdot 0000\ 032$	$\cdot 0000\ 017$	$\cdot 0000\ 005^-$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 44$	$\cdot 0000\ 058$	$\cdot 0000\ 032$	$\cdot 0000\ 009$	$\cdot 0000\ 005^-$	$\cdot 0000\ 001$
$\cdot 45$	$\cdot 0000\ 103$	$\cdot 0000\ 058$	$\cdot 0000\ 017$	$\cdot 0000\ 009$	$\cdot 0000\ 001$
$\cdot 46$	$\cdot 0000\ 181$	$\cdot 0000\ 104$	$\cdot 0000\ 032$	$\cdot 0000\ 018$	$\cdot 0000\ 017$
$\cdot 47$	$\cdot 0000\ 310$	$\cdot 0000\ 182$	$\cdot 0000\ 059$	$\cdot 0000\ 034$	$\cdot 0000\ 017$
$\cdot 48$	$\cdot 0000\ 523$	$\cdot 0000\ 313$	$\cdot 0000\ 106$	$\cdot 0000\ 061$	$\cdot 0000\ 032$
$\cdot 49$	$\cdot 0000\ 867$	$\cdot 0000\ 530$	$\cdot 0000\ 186$	$\cdot 0000\ 110$	$\cdot 0000\ 061$
$\cdot 50$	$\cdot 0001\ 416$	$\cdot 0000\ 882$	$\cdot 0000\ 322$	$\cdot 0000\ 195^-$	$\cdot 0000\ 110$
			$\cdot 0000\ 547$	$\cdot 0000\ 337$	$\cdot 0000\ 200$
$\cdot 51$	$\cdot 0002\ 276$	$\cdot 0001\ 446$	$\cdot 0000\ 914$	$\cdot 0000\ 574$	$\cdot 0000\ 337$
$\cdot 52$	$\cdot 0003\ 603$	$\cdot 0002\ 333$	$\cdot 0001\ 502$	$\cdot 0000\ 963$	$\cdot 0000\ 611$
$\cdot 53$	$\cdot 0005\ 620$	$\cdot 0003\ 707$	$\cdot 0002\ 432$	$\cdot 0001\ 587$	$\cdot 0001\ 032$
$\cdot 54$	$\cdot 0008\ 640$	$\cdot 0005\ 804$	$\cdot 0003\ 878$	$\cdot 0002\ 578$	$\cdot 0001\ 700$
$\cdot 55$	$\cdot 0013\ 095^-$	$\cdot 0008\ 954$	$\cdot 0006\ 090$	$\cdot 0004\ 121$	$\cdot 0002\ 777$
$\cdot 56$	$\cdot 0019\ 572$	$\cdot 0013\ 619$	$\cdot 0009\ 426$	$\cdot 0006\ 492$	$\cdot 0004\ 444$
$\cdot 57$	$\cdot 0028\ 856$	$\cdot 0020\ 425^+$	$\cdot 0014\ 382$	$\cdot 0010\ 076$	$\cdot 0007\ 021$
$\cdot 58$	$\cdot 0041\ 975^+$	$\cdot 0030\ 213$	$\cdot 0021\ 634$	$\cdot 0015\ 414$	$\cdot 0010\ 931$
$\cdot 59$	$\cdot 0060\ 255^+$	$\cdot 0044\ 088$	$\cdot 0032\ 092$	$\cdot 0023\ 245^+$	$\cdot 0016\ 751$
$\cdot 60$	$\cdot 0085\ 372$	$\cdot 0063\ 478$	$\cdot 0046\ 957$	$\cdot 0034\ 567$	$\cdot 0025\ 321$
$\cdot 61$	$\cdot 0119\ 407$	$\cdot 0090\ 192$	$\cdot 0067\ 781$	$\cdot 0050\ 693$	$\cdot 0037\ 731$
$\cdot 62$	$\cdot 0164\ 887$	$\cdot 0126\ 480$	$\cdot 0096\ 534$	$\cdot 0073\ 327$	$\cdot 0055\ 441$
$\cdot 63$	$\cdot 0224\ 822$	$\cdot 0175\ 078$	$\cdot 0135\ 667$	$\cdot 0104\ 631$	$\cdot 0080\ 321$
$\cdot 64$	$\cdot 0302\ 711$	$\cdot 0239\ 242$	$\cdot 0188\ 159$	$\cdot 0147\ 293$	$\cdot 0114\ 781$
$\cdot 65$	$\cdot 0402\ 519$	$\cdot 0322\ 756$	$\cdot 0257\ 556$	$\cdot 0204\ 580$	$\cdot 0161\ 781$
$\cdot 66$	$\cdot 0528\ 612$	$\cdot 0429\ 897$	$\cdot 0347\ 963$	$\cdot 0280\ 368$	$\cdot 0224\ 921$
$\cdot 67$	$\cdot 0685\ 642$	$\cdot 0565\ 359$	$\cdot 0464\ 011$	$\cdot 0379\ 133$	$\cdot 0308\ 452$
$\cdot 68$	$\cdot 0878\ 369$	$\cdot 0734\ 111$	$\cdot 0610\ 749$	$\cdot 0505\ 892$	$\cdot 0417\ 271$
$\cdot 69$	$\cdot 1111\ 422$	$\cdot 0941\ 192$	$\cdot 0793\ 479$	$\cdot 0666\ 077$	$\cdot 0556\ 820$
$\cdot 70$	$\cdot 1388\ 997$	$\cdot 1191\ 429$	$\cdot 1017\ 509$	$\cdot 0865\ 329$	$\cdot 0732\ 932$
$\cdot 71$	$\cdot 1714\ 499$	$\cdot 1489\ 087$	$\cdot 1287\ 822$	$\cdot 1109\ 199$	$\cdot 0951\ 577$
$\cdot 72$	$\cdot 2090\ 145^+$	$\cdot 1837\ 466$	$\cdot 1608\ 676$	$\cdot 1402\ 766$	$\cdot 1218\ 505$
$\cdot 73$	$\cdot 2516\ 556$	$\cdot 2238\ 447$	$\cdot 1983\ 134$	$\cdot 1750\ 161$	$\cdot 1538\ 788$
$\cdot 74$	$\cdot 2992\ 370$	$\cdot 2692\ 046$	$\cdot 2412\ 503$	$\cdot 2154\ 040$	$\cdot 1916\ 274$
$\cdot 75$	$\cdot 3513\ 925^+$	$\cdot 3196\ 005^-$	$\cdot 2896\ 148$	$\cdot 2615\ 032$	$\cdot 2352\ 992$
$\cdot 76$	$\cdot 4075\ 056$	$\cdot 3745\ 469$	$\cdot 3430\ 464$	$\cdot 3131\ 209$	$\cdot 2848\ 541$
$\cdot 77$	$\cdot 4667\ 053$	$\cdot 4332\ 823$	$\cdot 4009\ 176$	$\cdot 3697\ 667$	$\cdot 3399\ 552$
$\cdot 78$	$\cdot 5278\ 831$	$\cdot 4947\ 722$	$\cdot 4622\ 935^-$	$\cdot 4306\ 266$	$\cdot 3999\ 279$
$\cdot 79$	$\cdot 5897\ 333$	$\cdot 5577\ 386$	$\cdot 5259\ 523$	$\cdot 4945\ 633$	$\cdot 4637\ 439$
$\cdot 80$	$\cdot 6508\ 167$	$\cdot 6207\ 149$	$\cdot 5904\ 306$	$\cdot 5601\ 464$	$\cdot 5300\ 351$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

43 to 99

$q = 9.5$

$p = 38$  to

	$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$q = .4364 \ 0154 \times 10^{10}$		$.3491 \ 2123 \times 10^{10}$	$.2807 \ 3666 \times 10^{10}$	$.2268 \ 5791 \times 10^{10}$	$.1841 \ 8167 \times 10^{10}$	$.1502 \ 0641 \times 10^{10}$
43	.0000 001					
44	.0000 001	.0000 001				
45	.0000 003	.0000 002	.0000 001			
46	.0000 006	.0000 003	.0000 002	.0000 001	.0000 001	
47	.0000 012	.0000 007	.0000 004	.0000 002	.0000 001	.0000 001
48	.0000 022	.0000 013	.0000 007	.0000 004	.0000 002	.0000 001
49	.0000 042	.0000 025	.0000 015	.0000 009	.0000 005	.0000 003
50	.0000 077	.0000 046	.0000 028	.0000 017	.0000 010	.0000 006
51	.0000 139	.0000 086	.0000 053	.0000 032	.0000 020	.0000 012
52	.0000 246	.0000 155	.0000 097	.0000 060	.0000 038	.0000 023
53	.0000 429	.0000 275	.0000 175	.0000 112	.0000 071	.0000 045
54	.0000 736	.0000 480	.0000 312	.0000 202	.0000 130	.0000 084
55	.0001 242	.0000 825	.0000 546	.0000 360	.0000 236	.0000 155
56	.0002 062	.0001 394	.0000 939	.0000 630	.0000 421	.0000 281
57	.0003 370	.0002 319	.0001 589	.0001 855	.0000 738	.0000 500
58	.0005 423	.0003 795	.0002 646	.0001 837	.0001 271	.0000 876
59	.0008 594	.0006 116	.0004 335	.0003 001	.0001 254	.0001 510
60	.0013 418	.0009 706	.0006 993	.0005 019	.0003 590	.0002 558
61	.0020 641	.0015 171	.0011 107	.0008 101	.0005 888	.0004 264
62	.0031 288	.0023 360	.0017 374	.0012 873	.0009 505	.0006 993
63	.0046 743	.0035 441	.0026 768	.0020 143	.0015 104	.0011 287
64	.0068 832	.0052 981	.0040 626	.0031 038	.0023 630	.0017 929
65	.0099 913	.0078 051	.0060 744	.0047 104	.0036 400	.0028 034
66	.0142 971	.0113 319	.0089 484	.0070 409	.0055 210	.0043 149
67	.0201 688	.0162 146	.0129 879	.0103 666	.0082 462	.0065 381
68	.0280 493	.0228 661	.0185 734	.0150 341	.0121 283	.0097 526
69	.0384 563	.0317 801	.0261 695	.0214 755	.0175 652	.0143 210
70	.0519 758	.0435 287	.0363 270	.0302 145	.0250 488	.0207 009
71	.0692 460	.0587 524	.0496 780	.0418 661	.0351 608	.0294 532
72	.0909 314	.0781 386	.0669 203	.0571 267	.0486 134	.0412 434
73	.1176 830	.1023 878	.0887 888	.0767 520	.0661 435	.0568 320
74	.1500 868	.1321 649	.1160 123	.1015 194	.0885 712	.0770 503
75	.1886 002	.1680 371	.1492 535	.1321 722	.1167 053	.1027 574
76	.2334 809	.2103 999	.1890 353	.1693 477	.1512 832	.1347 757
77	.2847 127	.2593 968	.2356 551	.2134 893	.1928 832	.1738 053
78	.3419 385	.3148 403	.2890 970	.2647 503	.2418 228	.2203 201
79	.4044 099	.3761 453	.3489 497	.3228 997	.2980 537	.2744 529
80	.4709 648	.4422 877	.4143 460	.3872 425	.3610 645	.3358 837
81	.5400 457	.5118 021	.4839 387	.4565 733	.4298 120	.4037 485
82	.6097 654	.5828 286	.5559 264	.5291 780	.5026 988	.4765 997
83	.6780 241	.6532 177	.6281 411	.6029 046	.5776 158	.5523 781
84	.7426 728	.7206 900	.6981 999	.6752 938	.6520 636	.6286 611
85	.8017 091	.7830 403	.7637 133	.7437 944	.7233 533	.7024 610
86	.8521 822	.8383 620	.8225 280	.8060 362	.7888 162	

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .48$  to  $.99$  $q = 9.5$ 

	$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$
$B(p, q) = .1230\ 2620 \times \frac{x}{1010}$	$.1011\ 8043 \times \frac{x}{1010}$	$.8354\ 3471 \times \frac{x}{1011}$	$.6924\ 3237 \times \frac{x}{1011}$	$.5760\ 0569 \times \frac{x}{1010}$	
$x$					
.48	.0000 001				
.49	.0000 002	.0000 001	.0000 001		
.50	.0000 004	.0000 002	.0000 001	.0000 001	
.51	.0000 007	.0000 004	.0000 003	.0000 002	.0000 001
.52	.0000 014	.0000 009	.0000 005 <sup>+</sup>	.0000 003	.0000 002
.53	.0000 028	.0000 018	.0000 011	.0000 007	.0000 004
.54	.0000 054	.0000 034	.0000 022	.0000 014	.0000 009
.55	.0000 101	.0000 066	.0000 043	.0000 027	.0000 018
.56	.0000 186	.0000 123	.0000 081	.0000 053	.0000 035 <sup>+</sup>
.57	.0000 338	.0000 228	.0000 153	.0000 102	.0000 068
.58	.0000 602	.0000 412	.0000 282	.0000 192	.0000 130
.59	.0001 055 <sup>-</sup>	.0000 735 <sup>-</sup>	.0000 510	.0000 353	.0000 244
.60	.0001 817	.0001 287	.0000 908	.0000 639	.0000 448
.61	.0003 078	.0002 215 <sup>-</sup>	.0001 589	.0001 136	.0000 810
.62	.0005 129	.0003 749	.0002 732	.0001 985 <sup>+</sup>	.0001 438
.63	.0008 407	.0006 242	.0004 620	.0003 410	.0002 510
.64	.0013 559	.0010 222	.0007 683	.0005 758	.0004 303
.65	.0021 521	.0016 470	.0012 566	.0009 560	.0007 253
.66	.0033 615 <sup>-</sup>	.0026 107	.0020 215 <sup>+</sup>	.0015 608	.0012 018
.67	.0051 674	.0040 716	.0031 987	.0025 059	.0019 577
.68	.0078 177	.0062 478	.0049 787	.0039 562	.0031 351
.69	.0116 401	.0094 328	.0076 222	.0061 420	.0049 359
.70	.0170 558	.0140 113	.0114 776	.0093 763	.0076 394
.71	.0245 921	.0204 740	.0169 979	.0140 738	.0116 222
.72	.0348 880	.0294 282	.0247 545 <sup>+</sup>	.0207 677	.0173 781
.73	.0486 909	.0415 999	.0354 456	.0301 228	.0255 345 <sup>+</sup>
.74	.0668 397	.0578 241	.0498 925 <sup>-</sup>	.0429 384	.0368 618
.75	.0902 287	.0790 169	.0690 197	.0601 363	.0522 689
.76	.1197 500 <sup>+</sup>	.1061 244	.0938 130	.0827 273	.0727 785 <sup>+</sup>
.77	.1562 116	.1400 473	.1252 498	.1117 503	.0994 758
.78	.2002 326	.1815 375 <sup>-</sup>	.1642 009	.1481 796	.1334 230
.79	.2521 228	.2310 741	.2113 047	.1920 010	.1755 394
.80	.3117 570	.2887 270	.2668 229	.2460 617	.2264 489
.81	.3784 637	.3540 266	.3304 937	.3079 098	.2863 085 <sup>-</sup>
.82	.4509 502	.4258 632	.4014 062	.3776 457	.3546 385 <sup>-</sup>
.83	.5272 901	.5024 445 <sup>-</sup>	.4779 273	.4538 179	.4301 881
.84	.6049 967	.5813 387	.5577 123	.5341 988	.5108 755 <sup>-</sup>
.85	.6811 941	.6596 241	.6378 265 <sup>-</sup>	.6158 748	.5938 414
.86	.7528 846	.7341 476	.7149 902	.6954 704	.6756 475 <sup>-</sup>
.87	.8172 870	.8018 750 <sup>+</sup>	.7859 339	.7695 024	.7526 217
.88	.8721 958	.8602 831	.8478 198	.8348 255 <sup>-</sup>	.8213 224
.89	.9162 912	.9077 220	.8986 548	.8890 939	.8790 457
.90	.9493 257	.9436 604	.9375 986	.9311 348 <sub>1</sub>	.9242 653
.91	.9721 250 <sup>+</sup>	.9687 391	.9650 750	.9611 111	.9572 464

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

7 to 70

$q = 10$

$p = 10$

$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$q) = .1082\ 5088 \times \frac{1}{10^6}$	$.7606\ 8365 \times \frac{1}{10^6}$	$.5412\ 5441 \times \frac{1}{10^6}$	$.2835\ 1422 \times \frac{1}{10^6}$	$.1546\ 4412 \times \frac{1}{10^6}$	$.8740\ 7545 \times \frac{1}{10^6}$
.0000 001	.0000 001				
.0000 005 <sup>+</sup>	.0000 002	.0000 001			
.0000 015 <sup>-</sup>	.0000 006	.0000 002			
.0000 039	.0000 017	.0000 007	.0000 001		
.0000 093	.0000 042	.0000 018	.0000 004	.0000 001	
.0000 203	.0000 095 <sup>-</sup>	.0000 044	.0000 009	.0000 002	
.0000 413	.0000 201	.0000 097	.0000 022	.0000 005 <sup>-</sup>	.0000 001
.0000 790	.0000 398	.0000 199	.0000 048	.0000 011	.0000 003
.0001 435 <sup>+</sup>	.0000 748	.0000 386	.0000 100	.0000 025 <sup>+</sup>	.0000 006
.0002 491	.0001 340	.0000 714	.0000 198	.0000 053	.0000 014
.0004 154	.0002 303	.0001 265 <sup>-</sup>	.0000 371	.0000 106	.0000 029
.0006 686	.0003 811	.0002 152	.0000 669	.0000 201	.0000 059
.0010 423	.0006 101	.0003 538	.0001 159	.0000 368	.0000 114
.0015 791	.0009 477	.0005 634	.0001 941	.0000 648	.0000 210
.0023 310	.0014 324	.0008 721	.0003 151	.0001 103	.0000 376
.0033 605 <sup>+</sup>	.0021 122	.0013 153	.0004 972	.0001 823	.0000 650 <sup>-</sup>
.0047 410	.0030 445 <sup>-</sup>	.0019 372	.0007 647	.0002 927	.0001 090
.0065 569	.0042 977	.0027 914	.0011 483	.0004 582	.0001 779
.0089 033	.0059 512	.0039 421	.0016 871	.0007 005 <sup>-</sup>	.0002 831
.0118 854	.0080 950 <sup>-</sup>	.0054 642	.0024 287	.0010 475 <sup>+</sup>	.0004 398
.0156 176	.0108 299	.0074 436	.0034 309	.0015 349	.0006 686
.0202 213	.0142 665 <sup>+</sup>	.0099 772	.0047 620	.0022 065 <sup>+</sup>	.0009 957
.0258 233	.0185 236	.0131 721	.0065 015 <sup>-</sup>	.0031 161	.0014 547
.0325 534	.0237 267	.0171 448	.0087 402	.0043 277	.0020 875 <sup>+</sup>
.0405 410	.0300 056	.0220 195 <sup>-</sup>	.0115 801	.0059 167	.0029 456
.0499 125 <sup>+</sup>	.0374 921	.0279 259	.0151 338	.0079 701	.0040 908
.0607 877	.0463 164	.0349 974	.0195 232	.0105 869	.0055 963
.0732 761	.0566 042	.0433 674	.0248 784	.0138 774	.0075 477
.0874 736	.0684 732	.0531 666	.0313 349	.0179 630	.0100 427
.1034 592	.0820 289	.0645 194	.0390 316	.0229 742	.0131 916
.1212 913	.0973 619	.0775 400	.0481 073	.0290 496	.0171 166
.1410 055 <sup>+</sup>	.1145 436	.0923 288	.0586 976	.0363 329	.0219 506
.1626 116	.1336 233	.1089 687	.0709 309	.0449 701	.0278 360
.1860 920	.1546 255 <sup>-</sup>	.1275 212	.0849 243	.0551 065 <sup>-</sup>	.0349 221
.2114 003	.1775 471	.1480 239	.1007 797	.0668 820	.0433 622
.2384 607	.2023 562	.1704 869	.1185 796	.0804 278	.0533 107
.2671 682	.2289 903	.1948 909	.1383 832	.0958 612	.0649 183
.2973 894	.2573 565 <sup>+</sup>	.2211 858	.1602 229	.1132 814	.0783 281
.3289 641	.2873 316	.2492 804	.1841 009	.1327 651	.0936 708
.3617 076	.3187 634	.2790 878	.2099 875 <sup>+</sup>	.1543 618	.1110 593
.3954 137	.3514 726	.3104 359	.2378 184	.1780 905 <sup>+</sup>	.1305 839
.4298 582	.3852 557	.3431 591	.2674 945 <sup>-</sup>	.2039 362	.1523 073
.4648 028	.4198 879	.3776 561	.2988 817	.2318 471	.1762 600
.5000 000 <sup>*</sup>	.4551 280	.4119 015 <sup>-</sup>	.3318 119	.2617 336	.2024 264

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.96$  $q = 10$ 

	$p = 10$	$p = 10.5$	$p = 11$	$p = 12$	$p = 12.5$
$B(p, q) = .1082\ 5088 \times \frac{1}{10^8}$		$.7606\ 8365 \times \frac{1}{10^8}$	$.5412\ 5441 \times \frac{1}{10^8}$	$.2835\ 1422 \times \frac{1}{10^8}$	$.1546\ 44$
$\cdot 71$	$\cdot 9741\ 767$	$\cdot 9682\ 883$	$\cdot 9615\ 255^-$	$\cdot 9451\ 939$	$\cdot 9249\ 02$
$\cdot 72$	$\cdot 9797\ 787$	$\cdot 9750\ 286$	$\cdot 9695\ 346$	$\cdot 9561\ 242$	$\cdot 9392\ 27$
$\cdot 73$	$\cdot 9843\ 824$	$\cdot 9806\ 062$	$\cdot 9762\ 084$	$\cdot 9653\ 593$	$\cdot 9514\ 99$
$\cdot 74$	$\cdot 9881\ 146$	$\cdot 9851\ 590$	$\cdot 9816\ 933$	$\cdot 9730\ 538$	$\cdot 9618\ 65$
$\cdot 75$	$\cdot 9910\ 967$	$\cdot 9888\ 215^-$	$\cdot 9861\ 356$	$\cdot 9793\ 704$	$\cdot 9704\ 91$
$\cdot 76$	$\cdot 9934\ 431$	$\cdot 9917\ 224$	$\cdot 9896\ 777$	$\cdot 9844\ 745^-$	$\cdot 9775\ 54$
$\cdot 77$	$\cdot 9952\ 590$	$\cdot 9939\ 823$	$\cdot 9924\ 551$	$\cdot 9885\ 297$	$\cdot 9832\ 40$
$\cdot 78$	$\cdot 9966\ 395^-$	$\cdot 9957\ 115^-$	$\cdot 9945\ 942$	$\cdot 9916\ 937$	$\cdot 9877\ 34$
$\cdot 79$	$\cdot 9976\ 690$	$\cdot 9970\ 093$	$\cdot 9962\ 101$	$\cdot 9941\ 145^+$	$\cdot 9912\ 17$
$\cdot 80$	$\cdot 9984\ 209$	$\cdot 9979\ 632$	$\cdot 9974\ 052$	$\cdot 9959\ 278$	$\cdot 9938\ 59$
$\cdot 81$	$\cdot 9989\ 577$	$\cdot 9986\ 485^-$	$\cdot 9982\ 691$	$\cdot 9972\ 550^-$	$\cdot 9958\ 17$
$\cdot 82$	$\cdot 9993\ 314$	$\cdot 9991\ 285^+$	$\cdot 9988\ 781$	$\cdot 9982\ 022$	$\cdot 9972\ 32$
$\cdot 83$	$\cdot 9995\ 846$	$\cdot 9994\ 557$	$\cdot 9992\ 956$	$\cdot 9988\ 596$	$\cdot 9982\ 26$
$\cdot 84$	$\cdot 9997\ 509$	$\cdot 9996\ 719$	$\cdot 9995\ 733$	$\cdot 9993\ 019$	$\cdot 9989\ 03$
$\cdot 85$	$\cdot 9998\ 565^-$	$\cdot 9998\ 100$	$\cdot 9997\ 516$	$\cdot 9995\ 895^+$	$\cdot 9993\ 48$
$\cdot 86$	$\cdot 9999\ 210$	$\cdot 9998\ 949$	$\cdot 9998\ 619$	$\cdot 9997\ 694$	$\cdot 9996\ 30$
$\cdot 87$	$\cdot 9999\ 587$	$\cdot 9999\ 448$	$\cdot 9999\ 271$	$\cdot 9998\ 770$	$\cdot 9998\ 00$
$\cdot 88$	$\cdot 9999\ 797$	$\cdot 9999\ 727$	$\cdot 9999\ 638$	$\cdot 9999\ 383$	$\cdot 9998\ 99$
$\cdot 89$	$\cdot 9999\ 907$	$\cdot 9999\ 874$	$\cdot 9999\ 832$	$\cdot 9999\ 711$	$\cdot 9999\ 52$
$\cdot 90$	$\cdot 9999\ 961$	$\cdot 9999\ 947$	$\cdot 9999\ 928$	$\cdot 9999\ 876$	$\cdot 9999\ 79$
$\cdot 91$	$\cdot 9999\ 985^+$	$\cdot 9999\ 980$	$\cdot 9999\ 972$	$\cdot 9999\ 952$	$\cdot 9999\ 91$
$\cdot 92$	$\cdot 9999\ 995^-$	$\cdot 9999\ 993$	$\cdot 9999\ 991$	$\cdot 9999\ 983$	$\cdot 9999\ 97$
$\cdot 93$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999\ 997$	$\cdot 9999\ 995^+$	$\cdot 9999\ 99$
$\cdot 94$	$1.0000\ 000$	$1.0000\ 000$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 99$
$\cdot 95$			$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 00$
$\cdot 96$					



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$= .14$  to  $.70$

$q = 10$

$p =$

	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$
$\beta(p, q) = .5098\ 7734 \times \frac{1}{10^7}$	$.3059\ 2641 \times \frac{1}{10^7}$	$.1882\ 6240 \times \frac{1}{10^7}$	$.1185\ 3559 \times \frac{1}{10^7}$	$.7620\ 1449 \times \frac{1}{10^8}$	$.4992\ 50$	
$x$						
.14	.0000 001					
.15	.0000 001					
.16	.0000 004	.0000 001				
.17	.0000 008	.0000 002	.0000 001			
.18	.0000 017	.0000 005	.0000 001			
.19	.0000 034	.0000 010	.0000 003	.0000 001		
.20	.0000 067	.0000 021	.0000 006	.0000 002	.0000 001	
.21	.0000 125	.0000 041	.0000 013	.0000 004	.0000 001	
.22	.0000 226	.0000 077	.0000 026	.0000 008	.0000 003	.0000 001
.23	.0000 396	.0000 141	.0000 049	.0000 017	.0000 006	.0000 001
.24	.0000 674	.0000 250	.0000 091	.0000 032	.0000 011	.0000 001
.25	.0001 117	.0000 431	.0000 163	.0000 061	.0000 022	.0000 001
.26	.0001 803	.0000 723	.0000 284	.0000 110	.0000 042	.0000 001
.27	.0002 843	.0001 183	.0000 482	.0000 193	.0000 076	.0000 001
.28	.0004 387	.0001 891	.0000 799	.0000 332	.0000 136	.0000 001
.29	.0006 631	.0002 959	.0001 294	.0000 556	.0000 235 <sup>+</sup>	.0000 001
.30	.0009 835	.0004 535 <sup>+</sup>	.0002 051	.0000 911	.0000 398	.0000 1
.31	.0014 325	.0006 820	.0003 184	.0001 461	.0000 659	.0000 2
.32	.0020 513	.0010 071	.0004 850 <sup>+</sup>	.0002 295	.0001 068	.0000 4
.33	.0028 907	.0014 621	.0007 255 <sup>+</sup>	.0003 538	.0001 697	.0000 8
.34	.0040 120	.0020 886	.0010 668	.0005 355 <sup>+</sup>	.0002 645 <sup>+</sup>	.0001 2
.35	.0054 885 <sup>+</sup>	.0029 382	.0015 435	.0007 969	.0004 049	.0002 0
.36	.0074 059	.0040 733	.0021 987	.0011 666	.0006 092	.0003 1
.37	.0098 630	.0055 688	.0030 863	.0016 815	.0009 018	.0004 7
.38	.0129 720	.0075 130	.0042 717	.0023 879	.0013 142	.0007 1
.39	.0168 580	.0100 076	.0058 332	.0033 433	.0018 867	.0010 4
.40	.0216 581	.0131 691	.0078 635	.0046 177	.0026 702	.0015 2
.41	.0275 200	.0171 275	.0104 697	.0062 950	.0037 275	.0021 7
.42	.0345 998	.0220 262	.0137 747	.0084 743	.0051 351	.0030 6
.43	.0430 593	.0280 203	.0179 160	.0112 709	.0069 848	.0042 6
.44	.0530 619	.0352 744	.0230 456	.0148 163	.0093 849	.0058 6
.45	.0647 690	.0439 597	.0293 282	.0192 582	.0124 610	.0079 5
.46	.0783 350 <sup>+</sup>	.0542 500	.0369 388	.0247 599	.0163 565	.0106 5
.47	.0939 020	.0663 173	.0460 597	.0314 981	.0212 321	.0141 2
.48	.1115 943	.0803 266	.0568 759	.0396 603	.0272 651	.0184 9
.49	.1315 124	.0964 303	.0695 705 <sup>+</sup>	.0494 415 <sup>+</sup>	.0346 467	.0239 6
.50	.1537 281	.1147 615	.0843 188	.0610 391	.0435 793	.0307 1
.51	.1782 786	.1354 280	.1012 815	.0746 472	.0542 719	.0389 5
.52	.2051 618	.1585 060	.1205 982	.0904 503	.0669 349	.0489 1
.53	.2343 327	.1840 337	.1423 798	.1086 157	.0817 732	.0608 0
.54	.2657 005 <sup>+</sup>	.2120 063	.1667 018	.1292 856	.0989 784	.0748 6
.55	.2991 267	.2423 712	.1935 968	.1525 690	.1187 211	.0912 8
.56	.3344 253	.2750 248	.2230 493	.1785 339	.1411 409	.1102 8
.57	.3713 630	.3008 107	.2540 800	.2071 001	.1663 380	.1320 2

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.97$  $q = 10$ 

	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$
$B(p, q) = .5098\ 7734 \times \frac{1}{10^7}$	$.3059\ 2641 \times \frac{1}{10^7}$	$.1882\ 6240 \times \frac{1}{10^7}$	$.1185\ 3559 \times \frac{1}{10^7}$	$.7620\ 1449$	
$x$					
.71	.8720 810	.8397 739	.8039 332	.7650 144	.7235 659
.72	.8942 851	.8662 301	.8346 682	.7999 129	.7623 773
.73	.9138 430	.8898 589	.8625 021	.8319 589	.7985 142
.74	.9308 213	.9106 538	.8873 351	.8609 438	.8316 495 <sup>+</sup>
.75	.9453 351	.9286 717	.9091 444	.8867 454	.8615 465 <sup>+</sup>
.76	.9575 410	.9440 277	.9279 808	.9093 285 <sup>+</sup>	.8880 450 <sup>+</sup>
.77	.9676 286	.9568 867	.9439 628	.9287 430	.9111 642
.78	.9758 115 <sup>+</sup>	.9674 538	.9572 678	.9451 165 <sup>-</sup>	.9308 995 <sup>+</sup>
.79	.9823 174	.9759 635 <sup>-</sup>	.9681 203	.9586 440	.9474 145 <sup>+</sup>
.80	.9873 789	.9826 681	.9767 797	.9695 749	.9609 293
.81	.9912 250 <sup>-</sup>	.9878 267	.9835 258	.9781 978	.9717 242
.82	.9940 731	.9916 942	.9886 463	.9848 238	.9801 221
.83	.9961 234	.9945 123	.9924 230	.9897 709	.9864 689
.84	.9975 537	.9965 022	.9951 220	.9933 489	.9911 148
.85	.9985 174	.9978 587	.9969 840	.9958 468	.9943 970
.86	.9991 416	.9987 478	.9982 188	.9975 229	.9966 252
.87	.9995 283	.9993 051	.9990 017	.9985 981	.9980 712
.88	.9997 561	.9996 371	.9994 735 <sup>-</sup>	.9992 533	.9989 626
.89	.9998 825 <sup>-</sup>	.9998 234	.9997 413	.9996 295 <sup>+</sup>	.9994 803
.90	.9999 479	.9999 210	.9998 832	.9998 311	.9997 607
.91	.9999 792	.9999 681	.9999 523	.9999 304	.9999 005 <sup>+</sup>
.92	.9999 926	.9999 886	.9999 829	.9999 747	.9999 635 <sup>+</sup>
.93	.9999 978	.9999 965 <sup>+</sup>	.9999 947	.9999 922	.9999 886
.94	.9999 995 <sup>-</sup>	.9999 991	.9999 987	.9999 980	.9999 971
.95	.9999 999	.9999 998	.9999 998	.9999 996	.9999 994
.96	1.0000 000	1.0000 000	1.0000 000	1.0000 000	.9999 999
.97					1.0000 000

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $\mu = .23$  to  $.80$  $q = 10$  $p =$ 

	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$(p, q) =$	$.3328\ 3392 \times \frac{1}{10^8}$	$.2254\ 6814 \times \frac{1}{10^8}$	$.1550\ 0934 \times \frac{1}{10^8}$	$.1080\ 3682 \times \frac{1}{10^8}$	$.7626\ 1281 \times \frac{1}{10^8}$	$.5447\ 23$
$x$						
.23	.0000 001					
.24	.0000 001					
.25	.0000 003	.0000 001				
.26	.0000 006	.0000 002	.0000 001			
.27	.0000 011	.0000 004	.0000 002	.0000 001		
.28	.0000 022	.0000 008	.0000 003	.0000 001		
.29	.0000 040	.0000 016	.0000 007	.0000 003	.0000 001	
.30	.0000 073	.0000 031	.0000 013	.0000 005 <sup>+</sup>	.0000 002	.0000 00
.31	.0000 129	.0000 056	.0000 024	.0000 010	.0000 004	.0000 00
.32	.0000 222	.0000 099	.0000 044	.0000 019	.0000 008	.0000 00
.33	.0000 374	.0000 172	.0000 078	.0000 035 <sup>+</sup>	.0000 016	.0000 00
.34	.0000 618	.0000 293	.0000 137	.0000 064	.0000 029	.0000 01
.35	.0001 001	.0000 488	.0000 236	.0000 112	.0000 053	.0000 02
.36	.0001 592	.0000 798	.0000 396	.0000 194	.0000 094	.0000 04
.37	.0002 486	.0001 280	.0000 652	.0000 328	.0000 164	.0000 08
.38	.0003 816	.0002 017	.0001 054	.0000 545 <sup>-</sup>	.0000 279	.0000 14
.39	.0005 761	.0003 124	.0001 674	.0000 888	.0000 466	.0000 22
.40	.0008 564	.0004 759	.0002 615 <sup>-</sup>	.0001 421	.0000 765 <sup>-</sup>	.0000 40
.41	.0012 539	.0007 138	.0004 017	.0002 237	.0001 233	.0000 67
.42	.0018 096	.0010 544	.0006 075 <sup>-</sup>	.0003 403	.0001 955 <sup>-</sup>	.0001 09
.43	.0025 754	.0015 351	.0009 048	.0005 277	.0003 048	.0001 74
.44	.0036 162	.0022 039	.0013 282	.0007 922	.0004 679	.0002 73
.45	.0050 121	.0031 214	.0019 224	.0011 718	.0007 073	.0004 23
.46	.0068 601	.0043 632	.0027 448	.0017 089	.0010 538	.0006 43
.47	.0092 758	.0060 222	.0038 674	.0024 583	.0015 477	.0009 61
.48	.0123 946	.0082 100	.0053 797	.0034 895 <sup>+</sup>	.0022 420	.0014 27
.49	.0163 727	.0110 593	.0073 906	.0048 896	.0032 045 <sup>-</sup>	.0020 81
.50	.0213 870	.0147 247	.0100 308	.0067 655 <sup>-</sup>	.0045 206	.0029 94
.51	.0276 339	.0193 833	.0134 541	.0092 469	.0062 966	.0042 50
.52	.0353 275 <sup>+</sup>	.0252 342	.0178 385 <sup>+</sup>	.0124 879	.0086 622	.0059 56
.53	.0446 958	.0324 968	.0233 864	.0166 684	.0117 727	.0082 43
.54	.0559 759	.0414 079	.0303 229	.0219 948	.0158 111	.0112 60
.55	.0694 073	.0522 166	.0388 937	.0286 989	.0209 890	.0152 22
.56	.0852 243	.0651 783	.0493 601	.0370 357	.0275 460	.0203 18
.57	.1036 463	.0805 464	.0619 930	.0472 794	.0357 476	.0268 08
.58	.1248 673	.0985 625 <sup>+</sup>	.0770 644	.0597 163	.0458 812	.0349 68
.59	.1490 444	.1194 449	.0948 370	.0746 371	.0582 500 <sup>-</sup>	.0451 00
.60	.1762 865 <sup>-</sup>	.1433 764	.1155 524	.0923 254	.0731 631	.0575 26
.61	.2066 423	.1704 908	.1394 170	.1130 448	.0909 251	.0725 74
.62	.2400 900	.2008 604	.1665 879	.1370 242	.1118 211	.0905 60
.63	.2765 286	.2344 829	.1971 577	.1644 414	.1361 009	.1118 18
.64	.3157 710	.2712 712	.2311 405 <sup>+</sup>	.1954 068	.1639 611	.1365 90
.65	.3575 409	.3110 451	.2684 591	.2299 466	.1955 260	.1650 98
.66	.4014 733	.3535 259	.3089 348	.2679 886	.2308 299	.1974 70

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.98$  $q = 10$ 

	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$B(p, q) = .3328\ 3392 \times \frac{1}{10^8}$	$.2254\ 6814 \times \frac{1}{10^8}$	$.1550\ 0934 \times \frac{1}{10^8}$	$.1080\ 3682 \times \frac{1}{10^8}$	$.7626\ 128$	
$x$					
.81	.9549 209	.9444 188	.9324 322	.9189 237	.9038 787
.82	.9676 850 <sup>+</sup>	.9597 715 <sup>+</sup>	.9506 278	.9401 960	.9284 342
.83	.9775 693	.9718 059	.9650 652	.9572 813	.9483 979
.84	.9849 807	.9809 385 <sup>-</sup>	.9761 540	.9705 624	.9641 040
.85	.9903 424	.9876 243	.9843 688	.9805 187	.9760 190
.86	.9940 687	.9923 257	.9902 135 <sup>-</sup>	.9876 862	.9846 976
.87	.9965 437	.9954 848	.9941 865 <sup>+</sup>	.9926 151	.9907 353
.88	.9981 047	.9975 001	.9967 503	.9958 323	.9947 216
.89	.9990 320	.9987 109	.9983 082	.9978 095 <sup>+</sup>	.9971 993
.90	.9995 456	.9993 891	.9991 906	.9989 420	.9986 343
.91	.9998 074	.9997 386	.9996 503	.9995 385 <sup>+</sup>	.9993 987
.92	.9999 280	.9999 013	.9998 668	.9998 225 <sup>+</sup>	.9997 665
.93	.9999 770	.9999 682	.9999 567	.9999 418	.9999 227
.94	.9999 941	.9999 917	.9999 886	.9999 845 <sup>-</sup>	.9999 792
.95	.9999 988	.9999 984	.9999 977	.9999 969	.9999 958
.96	.9999 999	.9999 998	.9999 997	.9999 996	.9999 994
.97	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 000
.98		.			

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

4  
= .31 to .98

$q = 10$

$p = 1$

	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$
$B(p, q) =$	$.3934\ 1137 \times \frac{1}{10^9}$	$.2870\ 8397 \times \frac{1}{10^9}$	$.2115\ 3556 \times \frac{1}{10^9}$	$.1572\ 9567 \times \frac{1}{10^9}$	$.1179\ 7175 \times \frac{1}{10^9}$	$.8919\ 815$
$\pi$						
.31	.0000 001					
.32	.0000 001	.0000 001				
.33	.0000 003	.0000 001	.0000 001			
.34	.0000 006	.0000 003	.0000 001	.0000 001		
.35	.0000 011	.0000 005 <sup>+</sup>	.0000 002	.0000 001		
.36	.0000 022	.0000 010	.0000 005 <sup>-</sup>	.0000 002	.0000 001	
.37	.0000 040	.0000 019	.0000 009	.0000 004	.0000 002	.0000 001
.38	.0000 071	.0000 035 <sup>+</sup>	.0000 017	.0000 009	.0000 004	.0000 002
.39	.0000 125 <sup>-</sup>	.0000 064	.0000 032	.0000 016	.0000 008	.0000 004
.40	.0000 215 <sup>+</sup>	.0000 113	.0000 059	.0000 030	.0000 016	.0000 008
.41	.0000 365 <sup>-</sup>	.0000 196	.0000 104	.0000 055 <sup>+</sup>	.0000 029	.0000 015
.42	.0000 606	.0000 333	.0000 182	.0000 098	.0000 053	.0000 028
.43	.0000 989	.0000 557	.0000 311	.0000 172	.0000 095 <sup>-</sup>	.0000 053
.44	.0001 588	.0000 914	.0000 522	.0000 296	.0000 167	.0000 093
.45	.0002 509	.0001 476	.0000 862	.0000 499	.0000 287	.0000 162
.46	.0003 901	.0002 345 <sup>-</sup>	.0001 398	.0000 828	.0000 487	.0000 281
.47	.0005 974	.0003 667	.0002 233	.0001 351	.0000 811	.0000 481
.48	.0009 015 <sup>-</sup>	.0005 647	.0003 511	.0002 167	.0001 329	.0000 810
.49	.0013 408	.0008 569	.0005 435 <sup>+</sup>	.0003 423	.0002 142	.0001 333
.50	.0019 666	.0012 816	.0008 290	.0005 325 <sup>+</sup>	.0003 398	.0002 154
.51	.0028 455 <sup>-</sup>	.0018 902	.0012 463	.0008 161	.0005 308	.0003 433
.52	.0040 628	.0027 497	.0018 474	.0012 327	.0008 171	.0005 381
.53	.0057 262	.0039 470	.0027 009	.0018 356	.0012 393	.0008 310
.54	.0079 687	.0055 918	.0038 958	.0026 957	.0018 532	.0012 662
.55	.0109 522	.0078 211	.0055 454	.0039 053	.0027 327	.0019 003
.56	.0148 702	.0108 021	.0077 917	.0055 827	.0039 745 <sup>-</sup>	.0028 121
.57	.0199 488	.0147 357	.0108 091	.0078 764	.0057 032	.0041 045
.58	.0264 475 <sup>+</sup>	.0198 582	.0148 080	.0109 699	.0080 759	.0059 101
.59	.0346 575 <sup>+</sup>	.0264 420	.0200 369	.0150 851	.0112 870	.0083 950
.60	.0448 973	.0347 938	.0267 831	.0204 851	.0155 726	.0117 691
.61	.0575 060	.0452 502	.0353 712	.0274 748	.0212 129	.0162 841
.62	.0728 333	.0581 710	.0461 584	.0363 992	.0285 333	.0222 401
.63	.0912 257	.0739 275 <sup>-</sup>	.0595 267	.0476 386	.0379 023	.0299 870
.64	.1130 101	.0928 880	.0758 705 <sup>-</sup>	.0615 992	.0497 255 <sup>-</sup>	.0399 201
.65	.1384 739	.1153 995 <sup>+</sup>	.0955 803	.0786 997	.0644 357	.0524 721
.66	.1678 428	.1417 654	.1190 221	.0993 531	.0824 772	.0681 051
.67	.2012 575 <sup>+</sup>	.1722 208	.1465 129	.1239 431	.1042 848	.0872 890
.68	.2387 510	.2069 069	.1782 926	.1527 964	.1302 576	.1104 811
.69	.2802 271	.2458 451	.2144 960	.1861 522	.1607 277	.1380 910
.70	.3254 436	.2889 137	.2551 235 <sup>+</sup>	.2241 297	.1959 254	.1704 501
.71	.3740 015 <sup>+</sup>	.3358 296	.3000 161	.2666 972	.2359 438	.2077 691
.72	.4253 424	.3861 369	.3488 356	.3136 438	.2807 043	.2501 021
.73	.4787 556	.4392 058	.4010 543	.3645 605 <sup>+</sup>	.3299 279	.2973 062
.74	.5333 969	.4942 426	.4559 553	.4188 299	.3831 152	.3490 131
.75	.5883 185 <sup>-</sup>	.5503 132	.5126 473	.4756 308	.4395 397	.4046 121
.76	.6425 091	.6063 791	.5700 942	.5339 594	.4982 582	.4632 481
.77	.6949 439	.6613 456	.6271 594	.5926 666	.5581 393	.5238 341
.78	.7446 302	.7113 302	.6766 521	.6417 521	.6072 521	.5728 521

TABLE I. THE  $L_x(p, q)$  FUNCTION $x = .38$  to  $.98$  $q = 10$ 

$p$	$p = 33$	$p = 34$	$p = 35$	$p = 36$	$p = 37$
$B(p, q)$	$.6796\ 0500 \times 10^0$	$.5215\ 5732 \times 10^0$	$.4030\ 2157 \times 10^0$	$.3134\ 0122 \times 10^0$	$.245$
$x$					
.38	.0000 001				
.39	.0000 002	.0000 001			
.40	.0000 004	.0000 002	.0000 001	.0000 001	
.41	.0000 008	.0000 004	.0000 002	.0000 001	.0000
.42	.0000 015	.0000 008	.0000 004	.0000 002	.0000
.43	.0000 028	.0000 015	.0000 008	.0000 004	.0000
.44	.0000 052	.0000 029	.0000 016	.0000 009	.0000
.45	.0000 093	.0000 053	.0000 030	.0000 017	.0000
.46	.0000 165	.0000 095	.0000 055	.0000 031	.0000
.47	.0000 287	.0000 160	.0000 090	.0000 058	.0000
.48	.0000 490	.0000 295	.0000 157	.0000 105	.0000
.49	.0000 823	.0000 505	.0000 300	.0000 188	.0000
.50	.0001 358	.0000 851	.0000 530	.0000 329	.0000
.51	.0002 205	.0001 409	.0000 865	.0000 566	.0000
.52	.0003 524	.0002 295	.0001 486	.0000 957	.0000
.53	.0005 547	.0003 679	.0002 428	.0001 593	.0000
.54	.0008 601	.0005 810	.0003 903	.0002 600	.0001
.55	.0013 140	.0009 045	.0006 180	.0004 205	.0002
.56	.0020 787	.0013 845	.0009 646	.0006 673	.0004
.57	.0030 376	.0020 908	.0014 803	.0010 429	.0007
.58	.0043 006	.0031 125	.0022 410	.0016 056	.0011
.59	.0060 009	.0045 686	.0034 139	.0024 355	.0017
.60	.0088 457	.0069 031	.0049 189	.0036 410	.0026
.61	.0124 322	.0094 416	.0071 344	.0053 651	.0040
.62	.0172 418	.0132 923	.0102 044	.0077 937	.0059
.63	.0235 989	.0184 764	.0133 949	.0111 623	.0086
.64	.0318 797	.0253 393	.0200 293	.0157 643	.0123
.65	.0425 090	.0342 667	.0274 010	.0219 544	.0174
.66	.0559 522	.0457 435	.0372 224	.0301 521	.0243
.67	.0727 008	.0602 603	.0497 189	.0408 399	.0334
.68	.0941 520	.0783 404	.0655 165	.0545 518	.0452
.69	.1180 801	.1005 664	.0854 206	.0718 629	.0603
.70	.1476 028	.1272 475	.1092 271	.0933 691	.0794
.71	.1821 402	.1589 803	.1381 843	.1196 222	.1031
.72	.2218 722	.1960 041	.1724 469	.1511 253	.1319
.73	.2667 947	.2384 468	.2122 750	.1882 567	.1663
.74	.3166 806	.2862 291	.2577 300	.2312 177	.2066
.75	.3710 503	.3390 144	.3086 254	.2799 748	.2531
.76	.4291 570	.3991 815	.3644 863	.3342 038	.3054
.77	.4899 913	.4508 246	.4245 262	.3932 612	.3631
.78	.5523 110	.5102 599	.4826 474	.4504 506	.4250
.79	.6146 052	.5845 697	.5524 212	.5215 599	.4910
.80	.6756 239	.6466 848	.6145 615	.5845 515	

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.99

$q = 10$

$p = 39$  t

$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$
$= .1528\ 8846 \times \frac{1}{10^6}$	$.1216\ 8674 \times \frac{1}{10^6}$	$.9734\ 9389 \times \frac{1}{10^{11}}$	$.7826\ 1273 \times \frac{1}{10^{11}}$	$.6321\ 1028 \times \frac{1}{10^{11}}$	$.5128\ 4419 \times \frac{1}{10^{11}}$
.0000 001					
.0000 001	.0000 001				
.0000 003	.0000 002	.0000 001			
.0000 005	.0000 003	.0000 002	.0000 001	.0000 001	
.0000 011	.0000 006	.0000 004	.0000 002	.0000 001	.0000 001
.0000 022	.0000 013	.0000 007	.0000 004	.0000 002	.0000 001
.0000 041	.0000 024	.0000 015	.0000 009	.0000 005 <sup>+</sup>	.0000 003
.0000 076	.0000 046	.0000 028	.0000 017	.0000 010	.0000 006
.0000 139	.0000 086	.0000 053	.0000 033	.0000 020	.0000 012
.0000 249	.0000 157	.0000 099	.0000 062	.0000 039	.0000 024
.0000 438	.0000 282	.0000 181	.0000 116	.0000 074	.0000 047
.0000 758	.0000 497	.0000 325 <sup>+</sup>	.0000 212	.0000 137	.0000 089
.0001 288	.0000 861	.0000 573	.0000 380	.0000 251	.0000 165 <sup>+</sup>
.0002 155 <sup>+</sup>	.0001 466	.0000 993	.0000 670	.0000 450 <sup>+</sup>	.0000 302
.0003 547	.0002 454	.0001 692	.0001 161	.0000 794	.0000 541
.0005 744	.0004 043	.0002 834	.0001 979	.0001 377	.0000 955 <sup>+</sup>
.0009 156	.0006 552	.0004 670	.0003 316	.0002 346	.0001 654
.0014 371	.0010 453	.0007 573	.0005 466	.0003 931	.0002 817
.0022 210	.0016 415 <sup>+</sup>	.0012 085 <sup>+</sup>	.0008 863	.0006 477	.0004 717
.0033 809	.0025 382	.0018 982	.0014 142	.0010 499	.0007 767
.0050 694	.0038 647	.0029 351	.0022 208	.0016 744	.0012 581
.0074 882	.0057 954	.0044 683	.0034 325 <sup>+</sup>	.0026 275 <sup>+</sup>	.0020 044
.0108 977	.0085 596	.0066 979	.0052 222	.0040 574	.0031 418
.0156 261	.0124 524	.0098 865 <sup>+</sup>	.0078 212	.0061 661	.0048 450 <sup>+</sup>
.0220 768	.0178 441	.0143 701	.0115 316	.0092 223	.0073 512
.0307 323	.0251 872	.0205 682	.0167 377	.0135 749	.0109 740
.0421 521	.0350 188	.0289 894	.0239 158	.0196 649	.0161 178
.0569 628	.0479 554	.0402 316	.0336 381	.0280 337	.0232 894
.0758 372	.0646 784	.0549 731	.0465 697	.0393 247	.0331 042
.0994 618	.0859 065 <sup>+</sup>	.0739 508	.0634 530	.0542 750 <sup>+</sup>	.0462 837
.1284 899	.1123 541	.0979 247	.0850 790	.0736 922	.0636 400
.1634 817	.1446 742	.1276 252	.1122 395 <sup>+</sup>	.0984 144	.0860 425 <sup>+</sup>
.2048 338	.1833 876	.1636 839	.1456 622	.1292 497	.1143 638
.2527 019	.2288 016	.2065 504	.1859 274	.1668 953	.1494 034
.3069 260	.2809 257	.2564 010	.2333 717	.2118 393	.1917 891
.3669 664	.3393 939	.3130 483	.2879 879	.2642 521	.2418 631
.4318 645 <sup>+</sup>	.4034 061	.3758 655 <sup>+</sup>	.3493 325 <sup>+</sup>	.3238 798	.2995 635 <sup>+</sup>
.5002 387	.4717 042	.4437 404	.4164 587	.3899 564	.3643 170
.5703 274	.5425 934	.5150 744	.4878 909	.4611 540	.4349 643
.6400 845 <sup>+</sup>	.6140 214	.5878 410	.5616 606	.5355 924	.5097 425 <sup>+</sup>
.7073 260	.6837 160	.6597 106	.6354 124	.6109 230	.5863 430
.7699 179	.7493 754	.7282 372	.7065 834	.6844 966	.6620 605 <sup>+</sup>
.8259 842	.8088 915 <sup>+</sup>	.7910 937	.7726 447	.7536 628	.7340 294
.8741 062	.8605 767	.8463 234	.8313 748	.8157 643	.7995 203
.9131 885 <sup>+</sup>	.8999 985	.8869 985	.8741 985	.8614 985	.8489 985

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .48$  to  $.99$  $q = 10$ 

	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) =$	$.4178\ 7305\bar{x}_{101}$	$.3418\ 9613\bar{x}_{101}$	$.2808\ 4325\bar{x}_{101}$	$.2315\ 7250\bar{x}_{101}$	$.1916\ 462\bar{x}_{101}$
$.48$	.0000 001				
$.49$	.0000 002	.0000 001	.0000 001		
$.50$	.0000 004	.0000 002	.0000 001	.0000 001	
$.51$	.0000 007	.0000 005-			
$.52$	.0000 015+	.0000 009	.0000 003	.0000 002	.0000 001
$.53$	.0000 030	.0000 019	.0000 006	.0000 004	.0000 002
$.54$	.0000 057	.0000 037	.0000 012	.0000 007	.0000 005-
$.55$	.0000 108	.0000 071	.0000 023	.0000 015-	.0000 010
$.56$	.0000 201	.0000 134	.0000 046	.0000 030	.0000 019
$.57$	.0000 368	.0000 249	.0000 089	.0000 059	.0000 039
$.58$	.0000 660	.0000 454	.0000 168	.0000 113	.0000 076
$.59$	.0001 162	.0000 814	.0000 312	.0000 213	.0000 146
$.60$	.0002 012	.0001 432	.0000 568	.0000 395+	.0000 274
			.0001 016	.0000 719	.0000 507
$.61$	.0003 423	.0002 477	.0001 786	.0001 284	.0000 921
$.62$	.0005 727	.0004 210	.0003 085-	.0002 253	.0001 641
$.63$	.0009 422	.0007 033	.0005 234	.0003 884	.0002 874
$.64$	.0015 242	.0011 553	.0008 730	.0006 578	.0004 942
$.65$	.0024 250-	.0018 658	.0014 313	.0010 947	.0008 350-
$.66$	.0037 948	.0029 630	.0023 066	.0017 905-	.0013 859
$.67$	.0058 411	.0046 271	.0036 545-	.0028 781	.0022 603
$.68$	.0088 436	.0071 053	.0056 919	.0045 467	.0036 221
$.69$	.0131 698	.0107 288	.0087 149	.0070 593	.0057 027
$.70$	.0192 892	.0159 290	.0131 166	.0107 710	.0088 212
$.71$	.0277 842	.0232 516	.0194 038	.0161 488	.0134 043
$.72$	.0393 531	.0333 651	.0282 101	.0237 878	.0200 068
$.73$	.0548 010	.0470 580	.0402 997	.0344 214	.0293 257
$.74$	.0750 146	.0652 219	.0565 574	.0489 180	.0422 048
$.75$	.1009 158	.0888 126	.0779 592	.0682 604	.0596 224
$.76$	.1333 904	.1187 866	.1055 162	.0934 993	.0826 542
$.77$	.1731 926	.1560 094	.1401 806	.1256 758	.1124 048
$.78$	.2208 277	.2011 386	.1827 763	.1657 111	.1499 044
$.79$	.2764 244	.2544 885+	.2337 687	.2142 655+	.1959 691
$.80$	.3396 099	.3158 911	.2932 036	.2715 780	.2510 337
$.81$	.4094 115+	.3845 742	.3605 198	.3373 047	.3149 747
$.82$	.4842 098	.4590 857	.4344 531	.4103 865+	.3869 517
$.83$	.5617 684	.5372 921	.5130 020	.4889 806	.4653 045+
$.84$	.6393 592	.6164 763	.5934 939	.5704 920	.5475 476
$.85$	.7139 890	.6935 478	.6727 732	.6517 335-	.6304 965+
$.86$	.7827 113	.7653 551	.7475 084	.7292 213	.7105 456
$.87$	.8429 840	.8290 670	.8145 904	.7995 840	.7840 805+
$.88$	.8930 113	.8825 575-	.8715 582	.8600 254	.8479 735+
$.89$	.9319 948	.9247 133	.9169 648	.9087 482	.9000 642
$.90$	.9602 248	.9555 833	.9505 887	.9452 327	.9395 085+



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

07 to 70

$q = 10.5$

$p = 10$

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$p, q) = .5278\ 9627 \times \frac{1}{10^5}$		$.3710\ 6519 \times \frac{1}{10^5}$	$.1898\ 4731 \times \frac{1}{10^5}$	$.1012\ 5190 \times \frac{1}{10^5}$	$.5601\ 1688 \times \frac{1}{10^7}$	$.3200\ 66$
$\frac{x}{\pi}$						
.07	.0000 001					
.08	.0000 003	.0000 001				
.09	.0000 008	.0000 003	.0000 001			
.10	.0000 023	.0000 010	.0000 002			
.11	.0000 057	.0000 026	.0000 005 <sup>+</sup>	.0000 001		
.12	.0000 129	.0000 060	.0000 013	.0000 003	.0000 001	
.13	.0000 272	.0000 132	.0000 031	.0000 007	.0000 001	
.14	.0000 536	.0000 271	.0000 067	.0000 016	.0000 004	.0000 00
.15	.0001 002	.0000 524	.0000 139	.0000 036	.0000 009	.0000 00
.16	.0001 787	.0000 964	.0000 273	.0000 075 <sup>-</sup>	.0000 020	.0000 00
.17	.0003 054	.0001 697	.0000 510	.0000 148	.0000 042	.0000 01
.18	.0005 028	.0002 873	.0000 913	.0000 281	.0000 084	.0000 02
.19	.0008 006	.0004 696	.0001 574	.0000 511	.0000 161	.0000 04
.20	.0012 369	.0007 440	.0002 621	.0000 894	.0000 296	.0000 09
.21	.0018 596	.0011 454	.0004 232	.0001 514	.0000 527	.0000 17
.22	.0027 273	.0017 180	.0006 641	.0002 487	.0000 905 <sup>+</sup>	.0000 32
.23	.0039 098	.0025 164	.0010 157	.0003 972	.0001 510	.0000 56
.24	.0054 891	.0036 061	.0015 167	.0006 183	.0002 451	.0000 94
.25	.0075 590	.0050 643	.0022 157	.0009 397	.0003 876	.0001 55
.26	.0102 249	.0069 803	.0031 714	.0013 971	.0005 987	.0002 50
.27	.0136 032	.0094 552	.0044 543	.0020 351	.0009 046	.0003 92
.28	.0178 191	.0126 015 <sup>-</sup>	.0061 466	.0029 084	.0013 392	.0006 01
.29	.0230 055 <sup>+</sup>	.0165 416	.0083 429	.0040 829	.0019 448	.0009 03
.30	.0293 000	.0214 066	.0111 496	.0056 364	.0027 738	.0013 32
.31	.0368 420	.0273 336	.0146 847	.0076 593	.0038 899	.0019 28
.32	.0457 696	.0344 633	.0190 764	.0102 546	.0053 687	.0027 44
.33	.0562 154	.0429 368	.0244 614	.0135 377	.0072 987	.0038 42
.34	.0683 033	.0528 918	.0309 821	.0176 355 <sup>-</sup>	.0097 815 <sup>-</sup>	.0052 99
.35	.0821 437	.0644 588	.0387 847	.0226 849	.0129 321	.0072 02
.36	.0978 300	.0777 570	.0480 147	.0288 310	.0168 780	.0096 55
.37	.1154 349	.0928 902	.0588 140	.0362 242	.0217 581	.0127 73
.38	.1350 066	.1099 427	.0713 158	.0450 174	.0277 211	.0166 88
.39	.1565 659	.1289 755 <sup>+</sup>	.0856 473	.0553 615 <sup>+</sup>	.0349 227	.0215 42
.40	.1801 039	.1500 231	.1018 943	.0674 019	.0435 226	.0274 89
.41	.2055 800	.1730 900	.1201 571	.0812 735 <sup>-</sup>	.0536 811	.0346 91
.42	.2329 213	.1981 492	.1404 868	.0970 958	.0655 538	.0433 16
.43	.2620 221	.2251 403	.1629 109	.1149 682	.0792 879	.0535 34
.44	.2927 449	.2539 688	.1874 245 <sup>+</sup>	.1349 655 <sup>-</sup>	.0950 158	.0655 10
.45	.3249 218	.2845 066	.2139 882	.1571 326	.1128 509	.0794 02
.46	.3583 573	.3165 932	.2425 257	.1814 817	.1328 813	.0953 54
.47	.3928 310	.3500 375 <sup>-</sup>	.2729 240	.2079 880	.1551 651	.1134 91
.48	.4281 024	.3846 212	.3050 336	.2365 882	.1797 258	.1339 11
.49	.4639 146	.4201 028	.3386 697	.2671 783	.2065 480	.1566 79
.50	.5000 000 <sup>6</sup>	.4562 215 <sup>-</sup>	.3736 154	.2996 141	.2355 745 <sup>-</sup>	.1818 27

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.95$  $q = 10.5$ 

	$p = 10.5$	$p = 11$	$p = 12$	$p = 13$	$p = 14$
$B(p, q) = .5278\ 9627 \times \frac{1}{10^8}$		$.3710\ 6519 \times \frac{1}{10^8}$	$.1898\ 4731 \times \frac{1}{10^8}$	$.1012\ 5190 \times \frac{1}{10^8}$	$.5601\ 16$
$x$					
.71	.9769 945 <sup>-</sup>	.9717 812	.9589 526	.9426 336	.9225 80
.72	.9821 809	.9780 195 <sup>-</sup>	.9676 687	.9543 161	.9376 76
.73	.9863 968	.9831 258	.9749 028	.9641 479	.9505 59
.74	.9897 751	.9872 456	.9808 198	.9723 003	.9613 88
.75	.9924 410	.9905 188	.9855 848	.9789 547	.9703 48
.76	.9945 109	.9930 771	.9893 590	.9842 961	.9776 36
.77	.9960 902	.9950 419	.9922 960	.9885 076	.9834 59
.78	.9972 727	.9965 227	.9945 383	.9917 650 <sup>+</sup>	.9880 21
.79	.9981 404	.9976 161	.9962 154	.9942 328	.9915 21
.80	.9987 631	.9984 059	.9974 422	.9960 608	.9941 48
.81	.9991 994	.9989 628	.9983 180	.9973 823	.9960 70
.82	.9994 972	.9993 451	.9989 269	.9983 123	.9974 40
.83	.9996 946	.9996 002	.9993 379	.9989 479	.9983 87
.84	.9998 213	.9997 648	.9996 065 <sup>+</sup>	.9993 683	.9990 21
.85	.9998 998	.9998 674	.9997 758	.9996 364	.9994 31
.86	.9999 464	.9999 287	.9998 782	.9998 005 <sup>-</sup>	.9996 84
.87	.9999 728	.9999 637	.9999 374	.9998 964	.9998 34
.88	.9999 871	.9999 827	.9999 698	.9999 495 <sup>-</sup>	.9999 18
.89	.9999 943	.9999 923	.9999 865 <sup>-</sup>	.9999 771	.9999 62
.90	.9999 977	.9999 969	.9999 944	.9999 905 <sup>+</sup>	.9999 84
.91	.9999 992	.9999 989	.9999 979	.9999 965 <sup>-</sup>	.9999 94
.92	.9999 997	.9999 996	.9999 993	.9999 988	.9999 98
.93	.9999 999	.9999 999	.9999 998	.9999 997	.9999 99
.94	1.0000 000	1.0000 000	1.0000 000	.9999 999	.9999 99
.95				1.0000 000	1.0000 00

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 70

$q = 10.5$

$p = 16$

$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$= .1882\ 7458 \times \frac{1}{10^7}$	$.1136\ 7522 \times \frac{1}{10^7}$	$.7027\ 1954 \times \frac{1}{10^8}$	$.4438\ 2286 \times \frac{1}{10^8}$	$.2858\ 5201 \times \frac{1}{10^8}$	$.1874\ 4394 \times \frac{1}{10^8}$
.0000 001					
.0000 001					
.0000 003	.0000 001				
.0000 007	.0000 002	.0000 001			
.0000 015 <sup>-</sup>	.0000 004	.0000 001			
.0000 030	.0000 009	.0000 003	.0000 001		
.0000 059	.0000 019	.0000 006	.0000 002	.0000 001	
.0000 111	.0000 038	.0000 013	.0000 004	.0000 001	
.0000 203	.0000 072	.0000 025 <sup>+</sup>	.0000 009	.0000 003	.0000 001
.0000 358	.0000 132	.0000 048	.0000 017	.0000 006	.0000 002
.0000 613	.0000 236	.0000 089	.0000 033	.0000 012	.0000 004
.0001 023	.0000 409	.0000 161	.0000 062	.0000 024	.0000 009
.0001 663	.0000 691	.0000 282	.0000 113	.0000 045 <sup>-</sup>	.0000 017
.0002 643	.0001 138	.0000 481	.0000 200	.0000 082	.0000 033
.0004 109	.0001 831	.0000 801	.0000 344	.0000 146	.0000 061
.0006 260	.0002 882	.0001 303	.0000 579	.0000 253	.0000 109
.0009 354	.0004 447	.0002 076	.0000 953	.0000 431	.0000 192
.0013 726	.0006 730	.0003 240	.0001 534	.0000 715 <sup>+</sup>	.0000 329
.0019 799	.0010 002	.0004 962	.0002 421	.0001 163	.0000 551
.0028 099	.0014 610	.0007 461	.0003 748	.0001 854	.0000 904
.0039 268	.0020 997	.0011 029	.0005 698	.0002 900	.0001 455 <sup>+</sup>
.0054 079	.0029 711	.0016 036	.0008 515 <sup>-</sup>	.0004 454	.0002 297
.0073 440	.0041 422	.0022 956	.0012 517	.0006 723	.0003 561
.0098 410	.0056 939	.0032 374	.0018 113	.0009 984	.0005 428
.0130 194	.0077 217	.0045 010	.0025 820	.0014 594	.0008 136
.0170 144	.0103 365 <sup>+</sup>	.0061 727	.0036 281	.0021 014	.0012 006
.0219 749	.0136 654	.0083 547	.0050 281	.0029 822	.0017 450 <sup>-</sup>
.0280 622	.0178 511	.0111 659	.0068 762	.0041 737	.0024 995 <sup>-</sup>
.0354 467	.0230 510	.0147 423	.0092 839	.0057 633	.0035 303
.0443 055 <sup>-</sup>	.0294 357	.0192 367	.0123 808	.0078 558	.0049 192
.0548 178	.0371 860	.0248 178	.0163 146	.0105 750 <sup>+</sup>	.0067 653
.0671 600	.0464 896	.0316 678	.0212 513	.0140 639	.0091 873
.0815 003	.0575 368	.0399 800	.0273 732	.0184 854	.0123 240
.0979 922	.0705 144	.0499 544	.0348 772	.0240 215 <sup>+</sup>	.0163 357
.1167 683	.0856 001	.0617 930	.0439 708	.0308 715 <sup>-</sup>	.0214 039
.1379 332	.1029 553	.0756 931	.0548 678	.0392 489	.0277 299
.1615 571	.1227 179	.0918 407	.0677 823	.0493 776	.0355 326
.1876 698	.1449 947	.1104 027	.0829 213	.0614 858	.0450 448
.2162 553	.1698 540	.1315 184	.1004 772	.0757 994	.0565 076
.2472 472	.1973 190	.1552 912	.1206 183	.0925 333	.0701 635 <sup>+</sup>
.2805 261	.2273 615 <sup>+</sup>	.1817 807	.1434 801	.1118 821	.0862 483
.3159 183	.2598 974	.2109 945 <sup>-</sup>	.1691 553	.1340 104	.1049 807
.3531 958	.2947 834	.2428 823	.1976 851	.1590 415 <sup>-</sup>	.1265 519
.3920 796	.3318 162	.2773 310	.2290 510	.1870 474	.1511 134
.4322 437	.3707 329	.3141 611	.2631 670	.2180 309	.1787 655 <sup>-</sup>

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = \cdot 71$  to  $\cdot 96$  $q = 10\cdot 5$ 

	$p = 16$	$p = 17$	$p = 18$	$p = 19$	$p = 20$
$B(p, q) = \cdot 1882\ 7458 \times \frac{1}{10^7}$		$\cdot 1136\ 7522 \times \frac{1}{10^7}$	$\cdot 7027\ 1954 \times \frac{1}{10^8}$	$\cdot 4438\ 2286 \times \frac{1}{10^8}$	$\cdot 2858$
$x$					
$\cdot 71$	$\cdot 8709\ 651$	$\cdot 8396\ 033$	$\cdot 8048\ 932$	$\cdot 7672\ 423$	$\cdot 7271$
$\cdot 72$	$\cdot 8939\ 179$	$\cdot 8667\ 806$	$\cdot 8363\ 229$	$\cdot 8028\ 194$	$\cdot 7666$
$\cdot 73$	$\cdot 9140\ 555^+$	$\cdot 8909\ 576$	$\cdot 8646\ 734$	$\cdot 8353\ 593$	$\cdot 8032$
$\cdot 74$	$\cdot 9314\ 532$	$\cdot 9121\ 328$	$\cdot 8898\ 461$	$\cdot 8646\ 498$	$\cdot 8366$
$\cdot 75$	$\cdot 9462\ 409$	$\cdot 9303\ 759$	$\cdot 9118\ 279$	$\cdot 8905\ 750^+$	$\cdot 8666$
$\cdot 76$	$\cdot 9585\ 947$	$\cdot 9458\ 208$	$\cdot 9306\ 875^+$	$\cdot 9131\ 160$	$\cdot 8930$
$\cdot 77$	$\cdot 9687\ 269$	$\cdot 9586\ 556$	$\cdot 9465\ 672$	$\cdot 9323\ 465^+$	$\cdot 9159$
$\cdot 78$	$\cdot 9768\ 744$	$\cdot 9691\ 113$	$\cdot 9596\ 721$	$\cdot 9484\ 238$	$\cdot 9352$
$\cdot 79$	$\cdot 9832\ 886$	$\cdot 9774\ 484$	$\cdot 9702\ 565^-$	$\cdot 9615\ 762$	$\cdot 9512$
$\cdot 80$	$\cdot 9882\ 236$	$\cdot 9839\ 446$	$\cdot 9786\ 084$	$\cdot 9720\ 865^-$	$\cdot 9642$
$\cdot 81$	$\cdot 9919\ 272$	$\cdot 9888\ 810$	$\cdot 9850\ 346$	$\cdot 9802\ 748$	$\cdot 9744$
$\cdot 82$	$\cdot 9946\ 319$	$\cdot 9925\ 307$	$\cdot 9898\ 448$	$\cdot 9864\ 800$	$\cdot 9823$
$\cdot 83$	$\cdot 9965\ 489$	$\cdot 9951\ 492$	$\cdot 9933\ 382$	$\cdot 9910\ 417$	$\cdot 9881$
$\cdot 84$	$\cdot 9978\ 635^-$	$\cdot 9969\ 664$	$\cdot 9957\ 918$	$\cdot 9942\ 844$	$\cdot 9923$
$\cdot 85$	$\cdot 9987\ 322$	$\cdot 9981\ 817$	$\cdot 9974\ 523$	$\cdot 9965\ 051$	$\cdot 9952$
$\cdot 86$	$\cdot 9992\ 830$	$\cdot 9989\ 613$	$\cdot 9985\ 301$	$\cdot 9979\ 635^+$	$\cdot 9972$
$\cdot 87$	$\cdot 9996\ 162$	$\cdot 9994\ 384$	$\cdot 9991\ 974$	$\cdot 9988\ 770$	$\cdot 9984$
$\cdot 88$	$\cdot 9998\ 072$	$\cdot 9997\ 151$	$\cdot 9995\ 888$	$\cdot 9994\ 189$	$\cdot 9991$
$\cdot 89$	$\cdot 9999\ 101$	$\cdot 9998\ 658$	$\cdot 9998\ 045^-$	$\cdot 9997\ 210$	$\cdot 9996$
$\cdot 90$	$\cdot 9999\ 616$	$\cdot 9999\ 422$	$\cdot 9999\ 149$	$\cdot 9998\ 774$	$\cdot 9998$
$\cdot 91$	$\cdot 9999\ 853$	$\cdot 9999\ 776$	$\cdot 9999\ 667$	$\cdot 9999\ 516$	$\cdot 9999$
$\cdot 92$	$\cdot 9999\ 951$	$\cdot 9999\ 924$	$\cdot 9999\ 886$	$\cdot 9999\ 832$	$\cdot 9999$
$\cdot 93$	$\cdot 9999\ 986$	$\cdot 9999\ 978$	$\cdot 9999\ 967$	$\cdot 9999\ 951$	$\cdot 9999$
$\cdot 94$	$\cdot 9999\ 997$	$\cdot 9999\ 995^-$	$\cdot 9999\ 992$	$\cdot 9999\ 988$	$\cdot 9999$
$\cdot 95$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999$
$\cdot 96$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

4 to .80

$q = 10.5$

$p = 22$

$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$
$q) = .1249\ 6263 \times \frac{1}{10^8}$	$.8459\ 0088 \times \frac{1}{10^8}$	$.5807\ 6777 \times \frac{1}{10^8}$	$.4040\ 1236 \times \frac{1}{10^8}$	$.2845\ 1575 \times \frac{1}{10^8}$	$.2026\ 6875$
.0000 001	.0000 001				
.0000 002	.0000 001				
.0000 003	.0000 001				
.0000 007	.0000 003	.0000 001			
.0000 013	.0000 005 <sup>+</sup>	.0000 002	.0000 001		
.0000 025 <sup>+</sup>	.0000 010	.0000 004	.0000 002	.0000 001	
.0000 047	.0000 020	.0000 008	.0000 003	.0000 001	.0000 001
.0000 084	.0000 037	.0000 016	.0000 007	.0000 003	.0000 001
.0000 149	.0000 067	.0000 030	.0000 013	.0000 006	.0000 002
.0000 258	.0000 119	.0000 054	.0000 025	.0000 011	.0000 005 <sup>-</sup>
.0000 435 <sup>+</sup>	.0000 207	.0000 097	.0000 045 <sup>+</sup>	.0000 021	.0000 010
.0000 721	.0000 353	.0000 171	.0000 082	.0000 039	.0000 018
.0001 169	.0000 588	.0000 293	.0000 144	.0000 070	.0000 034
.0001 862	.0000 962	.0000 492	.0000 248	.0000 124	.0000 062
.0002 913	.0001 545 <sup>-</sup>	.0000 810	.0000 420	.0000 216	.0000 110
.0004 478	.0002 436	.0001 310	.0000 697	.0000 368	.0000 192
.0006 773	.0003 776	.0002 082	.0001 136	.0000 614	.0000 329
.0010 082	.0005 757	.0003 251	.0001 817	.0001 006	.0000 552
.0014 782	.0008 640	.0004 996	.0002 859	.0001 620	.0000 910
.0021 357	.0012 771	.0007 554	.0004 423	.0002 565 <sup>+</sup>	.0001 474
.0030 425 <sup>-</sup>	.0018 602	.0011 251	.0006 736	.0003 995 <sup>-</sup>	.0002 348
.0042 755 <sup>-</sup>	.0026 712	.0016 511	.0010 103	.0006 124	.0003 679
.0059 292	.0037 834	.0023 885 <sup>+</sup>	.0014 929	.0009 244	.0005 674
.0081 181	.0052 877	.0034 079	.0021 747	.0013 749	.0008 616
.0109 776	.0072 952	.0047 975 <sup>-</sup>	.0031 240	.0020 156	.0012 891
.0146 662	.0099 391	.0066 660	.0044 274	.0029 137	.0019 009
.0193 649	.0133 764	.0091 453	.0061 924	.0041 549	.0027 639
.0252 775 <sup>+</sup>	.0177 889	.0123 923	.0085 505 <sup>-</sup>	.0058 467	.0039 639
.0326 279	.0233 830	.0165 900	.0116 594	.0081 212	.0056 091
.0416 575 <sup>-</sup>	.0303 882	.0219 486	.0157 050 <sup>-</sup>	.0111 384	.0078 338
.0526 192	.0390 543	.0287 037	.0209 019	.0150 880	.0108 014
.0657 717	.0496 464	.0371 142	.0274 932	.0201 908	.0147 070
.0813 699	.0624 383	.0474 577	.0357 478	.0266 984	.0197 791
.0996 551	.0777 037	.0600 232	.0459 562	.0348 911	.0262 795 <sup>-</sup>
.1208 432	.0957 052	.0751 030	.0584 238	.0450 737	.0345 015 <sup>+</sup>
.1451 118	.1166 822	.0929 805 <sup>-</sup>	.0734 611	.0575 685 <sup>-</sup>	.0447 657
.1725 870	.1408 362	.1139 171	.0913 723	.0727 052	.0574 125 <sup>+</sup>
.2033 300	.1683 168	.1381 370	.1124 401	.0908 084	.0727 917
.2373 254	.1992 064	.1658 108	.1369 096	.1121 819	.0912 488
.2744 705 <sup>+</sup>	.2335 060	.1970 387	.1649 703	.1370 900	.1131 076
.3145 680	.2711 236	.2318 349	.1967 369	.1657 384	.1386 505 <sup>-</sup>
.3573 219	.3118 644	.2701 127	.2322 317	.1982 525 <sup>-</sup>	.1680 959
.4023 385 <sup>-</sup>	.3554 256	.3116 742	.2713 682	.2346 576	.2015 756
.4491 314	.4013 961	.3562 033	.3139 386	.2748 606	.2391 118

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.97$  $q = 10.5$ 

	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$B(p, q) = .1249\ 6263 \times \frac{x}{10^8}$	$.8459\ 0088 \times \frac{x}{10^8}$	$.5807\ 6777 \times \frac{x}{10^9}$	$.4040\ 1236 \times \frac{x}{10^9}$	$.2845\ 157$	
$\cdot 81$	$\cdot 9594\ 538$	$\cdot 9500\ 267$	$\cdot 9392\ 367$	$\cdot 9270\ 373$	$\cdot 9134\ 007$
$\cdot 82$	$\cdot 9713\ 741$	$\cdot 9643\ 746$	$\cdot 9562\ 644$	$\cdot 9469\ 816$	$\cdot 9364\ 771$
$\cdot 83$	$\cdot 9804\ 627$	$\cdot 9754\ 482$	$\cdot 9695\ 670$	$\cdot 9627\ 535^-$	$\cdot 9549\ 492$
$\cdot 84$	$\cdot 9871\ 608$	$\cdot 9837\ 077$	$\cdot 9796\ 091$	$\cdot 9748\ 035^-$	$\cdot 9692\ 328$
$\cdot 85$	$\cdot 9919\ 140$	$\cdot 9896\ 391$	$\cdot 9869\ 067$	$\cdot 9836\ 648$	$\cdot 9708\ 620$
$\cdot 86$	$\cdot 9951\ 474$	$\cdot 9937\ 215^-$	$\cdot 9919\ 886$	$\cdot 9899\ 085^-$	$\cdot 9874\ 398$
$\cdot 87$	$\cdot 9972\ 443$	$\cdot 9963\ 998$	$\cdot 9953\ 615^+$	$\cdot 9941\ 007$	$\cdot 9925\ 869$
$\cdot 88$	$\cdot 9985\ 318$	$\cdot 9980\ 632$	$\cdot 9974\ 805^+$	$\cdot 9967\ 647$	$\cdot 9958\ 955$
$\cdot 89$	$\cdot 9992\ 741$	$\cdot 9990\ 331$	$\cdot 9987\ 300$	$\cdot 9983\ 535^+$	$\cdot 9978\ 911$
$\cdot 90$	$\cdot 9996\ 716$	$\cdot 9995\ 583$	$\cdot 9994\ 142$	$\cdot 9992\ 333$	$\cdot 9990\ 085$
$\cdot 91$	$\cdot 9998\ 665^-$	$\cdot 9998\ 187$	$\cdot 9997\ 573$	$\cdot 9996\ 792$	$\cdot 9995\ 812$
$\cdot 92$	$\cdot 9999\ 524$	$\cdot 9999\ 348$	$\cdot 9999\ 119$	$\cdot 9998\ 824$	$\cdot 9998\ 450$
$\cdot 93$	$\cdot 9999\ 857$	$\cdot 9999\ 802$	$\cdot 9999\ 729$	$\cdot 9999\ 635^+$	$\cdot 9999\ 515$
$\cdot 94$	$\cdot 9999\ 965^+$	$\cdot 9999\ 951$	$\cdot 9999\ 933$	$\cdot 9999\ 909$	$\cdot 9999\ 878$
$\cdot 95$	$\cdot 9999\ 994$	$\cdot 9999\ 991$	$\cdot 9999\ 988$	$\cdot 9999\ 983$	$\cdot 9999\ 977$
$\cdot 96$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999\ 997$
$\cdot 97$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .98

$q = 10.5$

$p = 28$  t

$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$	$p = 33$
$\gamma = .1459\ 2150 \times \frac{1}{10^6}$	$.1061\ 2473 \times \frac{1}{10^6}$	$.7791\ 4357 \times \frac{1}{10^{10}}$	$.5771\ 4338 \times \frac{1}{10^{10}}$	$.4311\ 1915 \times \frac{1}{10^{10}}$	$.3246\ 0736 \times \frac{1}{10^{10}}$
.0000 001	.0000 001				
.0000 002	.0000 002				
.0000 004	.0000 002	.0000 001			
.0000 008	.0000 004	.0000 002	.0000 001		
.0000 016	.0000 008	.0000 004	.0000 002	.0000 001	
.0000 030	.0000 015 <sup>-</sup>	.0000 007	.0000 003	.0000 002	.0000 001
.0000 056	.0000 028	.0000 014	.0000 007	.0000 003	.0000 002
.0000 099	.0000 051	.0000 026	.0000 013	.0000 007	.0000 003
.0000 174	.0000 092	.0000 048	.0000 025 <sup>-</sup>	.0000 013	.0000 007
.0000 300	.0000 162	.0000 087	.0000 046	.0000 024	.0000 013
.0000 507	.0000 280	.0000 153	.0000 084	.0000 045 <sup>+</sup>	.0000 024
.0000 840	.0000 475 <sup>-</sup>	.0000 266	.0000 148	.0000 082	.0000 045 <sup>+</sup>
.0001 368	.0000 791	.0000 454	.0000 259	.0000 146	.0000 082
.0002 192	.0001 295 <sup>+</sup>	.0000 760	.0000 443	.0000 256	.0000 147
.0003 453	.0002 085 <sup>+</sup>	.0001 250 <sup>-</sup>	.0000 744	.0000 440	.0000 258
.0005 355 <sup>-</sup>	.0003 302	.0002 021	.0001 228	.0000 741	.0000 445 <sup>-</sup>
.0008 177	.0005 146	.0003 215 <sup>-</sup>	.0001 995 <sup>-</sup>	.0001 229	.0000 753
.0012 301	.0007 898	.0005 034	.0003 186	.0002 004	.0001 252
.0018 237	.0011 941	.0007 762	.0005 010	.0003 213	.0002 048
.0026 658	.0017 792	.0011 788	.0007 757	.0005 071	.0003 295 <sup>-</sup>
.0038 432	.0026 133	.0017 643	.0011 830	.0007 881	.0005 218
.0054 661	.0037 855 <sup>-</sup>	.0026 029	.0017 777	.0012 063	.0008 136
.0076 722	.0054 091	.0037 866	.0026 331	.0018 193	.0012 494
.0106 297	.0076 263	.0054 334	.0038 453	.0027 042	.0018 902
.0145 409	.0106 123	.0076 916	.0055 380	.0039 625 <sup>-</sup>	.0028 182
.0196 438	.0145 781	.0107 448	.0078 679	.0057 255 <sup>-</sup>	.0041 418
.0262 121	.0197 731	.0148 150 <sup>-</sup>	.0110 286	.0081 596	.0060 015 <sup>+</sup>
.0345 544	.0264 856	.0201 655 <sup>-</sup>	.0152 558	.0114 714	.0085 757
.0450 085 <sup>-</sup>	.0350 410	.0271 013	.0208 290	.0159 123	.0120 865 <sup>-</sup>
.0579 347	.0457 970	.0359 675 <sup>+</sup>	.0280 728	.0217 813	.0168 041
.0737 039	.0591 353	.0471 438	.0373 548	.0294 256	.0230 501
.0926 826	.0754 490	.0610 352	.0490 789	.0392 381	.0311 979
.1152 144	.0951 263	.0780 584	.0636 758	.0516 501	.0416 688
.1415 972	.1185 291	.0986 230	.0815 866	.0671 195 <sup>-</sup>	.0549 241
.1720 591	.1459 687	.1231 081	.1032 422	.0861 126	.0714 508
.2067 327	.1776 782	.1518 347	.1290 364	.1090 806	.0917 408
.2456 295 <sup>+</sup>	.2137 841	.1850 354	.1592 957	.1364 289	.1162 632
.2886 182	.2542 786	.2228 224	.1942 444	.1684 827	.1454 300
.3354 068	.2989 951	.2651 573	.2339 702	.2054 490	.1795 572
.3855 333	.3475 900	.3118 251	.2783 909	.2473 780	.2188 219
.4383 655 <sup>-</sup>	.3995 333	.3624 150 <sup>+</sup>	.3272 270	.2941 275 <sup>-</sup>	.2632 208
.4931 125 <sup>-</sup>	.4541 109	.4163 130	.3799 829	.3453 345 <sup>-</sup>	.3125 322
.5488 491	.5104 403	.4727 069	.4359 421	.4003 988	.3662 883
.6045 525 <sup>-</sup>	.5675 008	.5306 088	.4941 779	.4584 816	.4237 613
.6591 497	.6241 776	.5888 920	.5535 829	.5185 244	.4839 698
.7115 746	.6793 190	.6463 459	.6129 168	.5792 881	.5457 068
.7608 282	.7318 007	.7017 422	.6708 722	.6394 146	.6075 933
.8060 398	.7805 058	.7520 104	.7265 182		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .38$  to  $.98$  $q = 10.5$ 

	$p = 34$	$p = 35$	$p = 36$	$p = 37$	$p = 38$
$B(p, q) = .2462\ 5386 \times \frac{1}{10^{10}}$	$.1881\ 4902 \times \frac{1}{10^{10}}$	$.1447\ 3001 \times \frac{1}{10^{10}}$	$.1120\ 4904 \times \frac{1}{10^{10}}$	$.8728\ 030$	
$\cdot 38$	.0000 001				
$\cdot 39$	.0000 002	.0000 001			
$\cdot 40$	.0000 003	.0000 002	.0000 001		
$\cdot 41$	.0000 007	.0000 003	.0000 002	.0000 001	
$\cdot 42$	.0000 013	.0000 007	.0000 004	.0000 002	.0000 001
$\cdot 43$	.0000 025 <sup>-</sup>	.0000 013	.0000 007	.0000 004	.0000 002
$\cdot 44$	.0000 046	.0000 026	.0000 014	.0000 008	.0000 004
$\cdot 45$	.0000 084	.0000 048	.0000 027	.0000 015 <sup>+</sup>	.0000 008
$\cdot 46$	.0000 151	.0000 088	.0000 051	.0000 029	.0000 017
$\cdot 47$	.0000 265 <sup>+</sup>	.0000 157	.0000 093	.0000 054	.0000 032
$\cdot 48$	.0000 458	.0000 277	.0000 167	.0000 100	.0000 060
$\cdot 49$	.0000 778	.0000 480	.0000 295 <sup>-</sup>	.0000 180	.0000 110
$\cdot 50$	.0001 297	.0000 817	.0000 512	.0000 319	.0000 198
$\cdot 51$	.0002 128	.0001 367	.0000 873	.0000 555 <sup>-</sup>	.0000 351
$\cdot 52$	.0003 434	.0002 248	.0001 463	.0000 948	.0000 611
$\cdot 53$	.0005 455 <sup>+</sup>	.0003 637	.0002 412	.0001 592	.0001 045 <sup>+</sup>
$\cdot 54$	.0008 530	.0005 792	.0003 912	.0002 629	.0001 758
$\cdot 55$	.0013 137	.0009 080	.0006 243	.0004 271	.0002 908
$\cdot 56$	.0019 930	.0014 017	.0009 807	.0006 828	.0004 730
$\cdot 57$	.0029 792	.0021 314	.0015 170	.0010 744	.0007 573
$\cdot 58$	.0043 895 <sup>-</sup>	.0031 933	.0023 112	.0016 645 <sup>+</sup>	.0011 932
$\cdot 59$	.0063 755 <sup>-</sup>	.0047 146	.0034 687	.0025 396	.0018 508
$\cdot 60$	.0091 302	.0068 608	.0051 296	.0038 167	.0028 268
$\cdot 61$	.0128 940	.0098 423	.0074 756	.0056 509	.0042 520
$\cdot 62$	.0179 594	.0139 212	.0107 380	.0082 436	.0063 000
$\cdot 63$	.0246 744	.0194 162	.0152 044	.0118 507	.0091 955 <sup>-</sup>
$\cdot 64$	.0334 418	.0267 053	.0212 237	.0167 897	.0132 234
$\cdot 65$	.0447 151	.0362 251	.0292 088	.0234 446	.0187 360
$\cdot 66$	.0589 883	.0484 648	.0396 341	.0322 678	.0261 577
$\cdot 67$	.0767 787	.0639 531	.0530 276	.0437 757	.0359 854
$\cdot 68$	.0986 029	.0832 385 <sup>+</sup>	.0699 549	.0585 384	.0487 819
$\cdot 69$	.1249 446	.1068 601	.0909 949	.0771 591	.0651 612
$\cdot 70$	.1562 153	.1353 106	.1167 053	.1002 449	.0857 642
$\cdot 71$	.1927 104	.1689 912	.1475 795 <sup>-</sup>	.1283 654	.1112 208
$\cdot 72$	.2345 619	.2081 620	.1839 948	.1620 027	.1421 027
$\cdot 73$	.2816 931	.2528 903	.2261 572	.2014 922	.1788 637
$\cdot 74$	.3337 799	.3030 022	.2740 448	.2469 615 <sup>+</sup>	.2217 741
$\cdot 75$	.3902 247	.3580 443	.3273 580	.2982 700	.2708 525 <sup>+</sup>
$\cdot 76$	.4501 482	.4172 616	.3854 838	.3549 595 <sup>-</sup>	.3258 046
$\cdot 77$	.5124 053	.4795 984	.4474 805 <sup>-</sup>	.4162 235 <sup>-</sup>	.3859 760
$\cdot 78$	.5756 273	.5437 272	.5120 913	.4809 037	.4503 313
$\cdot 79$	.6382 917	.6081 087	.5777 921	.5475 218	.5174 683
$\cdot 80$	.6988 164	.6710 824	.6428 730	.6143 501	.5856 730
$\cdot 81$	.7556 721	.7300 800	.7055 525 <sup>+</sup>	.6795 202	.6520 001



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0.98

$q = 10.5$

$p = 4$

$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$= .5387\ 8753 \times \frac{1}{10^{11}}$	$.4267\ 6240 \times \frac{1}{10^{11}}$	$.3397\ 5259 \times \frac{1}{10^{11}}$	$.2718\ 0207 \times \frac{1}{10^{11}}$	$.2184\ 5774 \times \frac{1}{10^{11}}$	$.1763\ 695$
.0000 001					
.0000 001	.0000 001				
.0000 003	.0000 001	.0000 001			
.0000 005 <sup>+</sup>	.0000 003	.0000 002			
.0000 011	.0000 006	.0000 004	.0000 001	.0000 001	.0000 001
.0000 021	.0000 012	.0000 007	.0000 004	.0000 002	.0000 001
.0000 040	.0000 024	.0000 014	.0000 009	.0000 005 <sup>+</sup>	.0000 003
.0000 075 <sup>+</sup>	.0000 046	.0000 028	.0000 017	.0000 010	.0000 006
.0000 138	.0000 086	.0000 054	.0000 033	.0000 020	.0000 013
.0000 250 <sup>+</sup>	.0000 159	.0000 101	.0000 064	.0000 040	.0000 025 <sup>+</sup>
.0000 445 <sup>-</sup>	.0000 288	.0000 186	.0000 119	.0000 076	.0000 049
.0000 776	.0000 512	.0000 336	.0000 220	.0000 144	.0000 093
.0001 330	.0000 893	.0000 598	.0000 398	.0000 265 <sup>-</sup>	.0000 175 <sup>+</sup>
.0002 240	.0001 532	.0001 043	.0000 708	.0000 478	.0000 322
.0003 712	.0002 582	.0001 789	.0001 235 <sup>+</sup>	.0000 849	.0000 582
.0006 050 <sup>-</sup>	.0004 280	.0003 017	.0002 118	.0001 481	.0001 032
.0009 699	.0006 978	.0005 000 <sup>-</sup>	.0003 569	.0002 538	.0001 799
.0015 303	.0011 189	.0008 150 <sup>-</sup>	.0005 913	.0004 275 <sup>-</sup>	.0003 079
.0023 763	.0017 655 <sup>-</sup>	.0013 066	.0009 633	.0007 077	.0005 180
.0036 323	.0027 412	.0020 607	.0015 434	.0011 518	.0008 566
.0054 662	.0041 890	.0031 979	.0024 323	.0018 434	.0013 923
.0080 995 <sup>+</sup>	.0063 011	.0048 834	.0037 708	.0029 014	.0022 249
.0118 177	.0093 301	.0073 386	.0057 512	.0044 915 <sup>+</sup>	.0034 959
.0169 796	.0136 005 <sup>-</sup>	.0108 535 <sup>-</sup>	.0086 304	.0068 389	.0054 013
.0240 249	.0195 177	.0157 981	.0127 423	.0102 426	.0082 061
.0334 759	.0275 745 <sup>-</sup>	.0226 317	.0185 104	.0150 887	.0122 597
.0459 336	.0383 516	.0319 078	.0264 559	.0218 630	.0180 096
.0620 636	.0525 088	.0442 708	.0371 999	.0311 567	.0260 131
.0825 701	.0707 660	.0604 431	.0514 560	.0436 655 <sup>+</sup>	.0369 400
.1081 565 <sup>-</sup>	.0938 681	.0811 968	.0700 099	.0601 757	.0515 664
.1394 701	.1225 362	.1073 102	.0936 811	.0815 337	.0707 516
.1770 352	.1574 011	.1395 053	.1232 670	.1085 959	.0953 953
.2211 749	.1989 242	.1783 695 <sup>-</sup>	.1594 665 <sup>-</sup>	.1421 570	.1263 720
.2719 302	.2473 104	.2242 639	.2027 867	.1828 579	.1644 419
.3289 850 <sup>+</sup>	.3024 210	.2772 273	.2534 402	.2310 776	.2101 407
.3916 095 <sup>+</sup>	.3637 005 <sup>+</sup>	.3368 874	.3112 426	.2868 204	.2636 582
.4586 339	.4301 297	.4023 937	.3755 262	.3496 115 <sup>-</sup>	.3247 186
.5284 660	.5002 213	.4723 899	.4450 887	.4184 223	.3924 833
.5991 612	.5720 695 <sup>-</sup>	.5450 406	.5181 951	.4916 461	.4654 984
.6685 476	.6434 612	.6181 240	.5926 480	.5671 424	.5417 121
.7344 001	.7120 459	.6891 927	.6659 343	.6423 648	.6185 783
.7946 469	.7755 504	.7557 925 <sup>+</sup>	.7354 419	.7145 707	.6932 536
.8475 823	.8320 141	.8157 150 <sup>-</sup>	.7987 270	.7810 971	.7628 761
.8920 515 <sup>+</sup>	.8800 066	.8672 477	.8537 932	.8396 660	.8248 934
.9275 737	.9187 806	.9092 767			

	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) =$	$\cdot 1430\ 0234 \times \frac{x}{10^{11}}$	$\cdot 1164\ 2668 \times \frac{x}{10^{11}}$	$\cdot 9516\ 6158 \times \frac{x}{10^{12}}$	$\cdot 7808\ 5052 \times \frac{x}{10^{12}}$
$x$				
·48	·0000 001			
·49	·0000 002	·0000 001	·0000 001	
·50	·0000 004	·0000 002	·0000 001	·0000 001
·51	·0000 008	·0000 005 <sup>-</sup>	·0000 003	·0000 002
·52	·0000 016	·0000 010	·0000 006	·0000 004
·53	·0000 031	·0000 020	·0000 012	·0000 008
·54	·0000 060	·0000 039	·0000 025 <sup>+</sup>	·0000 016
·55	·0000 115 <sup>+</sup>	·0000 076	·0000 050 <sup>-</sup>	·0000 032
·56	·0000 216	·0000 145 <sup>-</sup>	·0000 096	·0000 064
·57	·0000 397	·0000 270	·0000 183	·0000 124
·58	·0000 717	·0000 496	·0000 343	·0000 236
·59	·0001 270	·0000 894	·0000 628	·0000 439
·60	·0002 211	·0001 582	·0001 129	·0000 803
·61	·0003 780	·0002 749	·0001 993	·0001 440
·62	·0006 349	·0004 691	·0003 455 <sup>-</sup>	·0002 537
·63	·0010 481	·0007 865 <sup>-</sup>	·0005 883	·0004 388
·64	·0017 006	·0012 957	·0009 841	·0007 453
·65	·0027 122	·0020 975 <sup>+</sup>	·0016 173	·0012 433
·66	·0042 522	·0033 371	·0026 111	·0020 371
·67	·0065 537	·0052 179	·0041 420	·0032 785 <sup>+</sup>
·68	·0099 297	·0080 182	·0064 556	·0051 828
·69	·0147 895 <sup>+</sup>	·0121 088	·0098 852	·0080 473
·70	·0216 524	·0179 696	·0148 705 <sup>+</sup>	·0122 718
·71	·0311 568	·0262 027	·0219 744	·0183 781
·72	·0440 591	·0375 375 <sup>+</sup>	·0318 929	·0270 244
·73	·0612 190	·0528 230	·0454 551	·0390 120
·74	·0835 645 <sup>+</sup>	·0730 017	·0636 054	·0552 759
·75	·1120 339	·0990 595 <sup>+</sup>	·0873 619	·0768 522
·76	·1474 909	·1319 477	·1177 471	·1048 186
·77	·1906 158	·1724 767	·1556 865 <sup>+</sup>	·1401 993
·78	·2417 776	·2211 860	·2018 782	·1838 374
·79	·3009 017	·2782 006	·2566 417	·2362 394
·80	·3673 512	·3430 933	·3197 644	·2974 076
·81	·4398 475 <sup>+</sup>	·4147 793	·3903 699	·3666 851
·82	·5164 571	·4914 710	·4668 412	·4426 475 <sup>-</sup>
·83	·5946 676	·5707 232	·5468 322	·5230 781
·84	·6715 670	·6495 881	·6273 940	·6050 613
·85	·7441 187	·7248 821	·7052 260	·6852 116
·86	·8095 070	·7935 419	·7770 367	·7600 328
·87	·8654 969	·8530 177	·8399 662	·8263 641
·88	·9107 398	·9016 301	·8919 932	·8818 343
·89	·9449 499	·9388 057	·9322 321	·9252 237
·90	·9688 968	·9651 206	·9610 350 <sup>+</sup>	·9566 303
·91	·9841 996	·9821 230	·9798 514	·9773 751
·92	·9929 628	·9919 661	·9908 637	·9896 488
·93	·9973 471	·9969 438	·9964 929	·9959 906
·94	·9991 959	·9990 651	·9989 174	·9987 511
·95	·9998 190	·9997 876	·9997 518	·9997 111
·96	·9999 735 <sup>+</sup>	·9999 686	·9999 630	·9999 565 <sup>+</sup>
·97	·9999 980	·9999 977	·9999 972	·9999 967
·98	1·0000 000	1·0000 000	·9999 999	·9999 999
·99			1·0000 000	1·0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 70

$q = 11$

$p = 11$

$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$
$\cdot 2577\ 4020 \times \frac{1}{10^5}$	$\cdot 1288\ 7010 \times \frac{1}{10^5}$	$\cdot 6723\ 6573 \times \frac{1}{10^5}$	$\cdot 3641\ 9810 \times \frac{1}{10^5}$	$\cdot 2039\ 5094 \times \frac{1}{10^5}$	$\cdot 1176\ 6400 \times \frac{1}{10^5}$
$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 0000\ 005^-$	$\cdot 0000\ 001$				
$\cdot 0000\ 014$	$\cdot 0000\ 002$				
$\cdot 0000\ 035^-$	$\cdot 0000\ 007$	$\cdot 0000\ 001$			
$\cdot 0000\ 032$	$\cdot 0000\ 018$	$\cdot 0000\ 004$	$\cdot 0000\ 001$		
$\cdot 0000\ 179$	$\cdot 0000\ 042$	$\cdot 0000\ 010$	$\cdot 0000\ 002$		
$\cdot 0000\ 364$	$\cdot 0000\ 092$	$\cdot 0000\ 023$	$\cdot 0000\ 005^+$	$\cdot 0000\ 001$	
$\cdot 0000\ 701$	$\cdot 0000\ 190$	$\cdot 0000\ 050^-$	$\cdot 0000\ 013$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0001\ 283$	$\cdot 0000\ 371$	$\cdot 0000\ 104$	$\cdot 0000\ 028$	$\cdot 0000\ 007$	$\cdot 0000\ 002$
$\cdot 0002\ 247$	$\cdot 0000\ 690$	$\cdot 0000\ 205^+$	$\cdot 0000\ 059$	$\cdot 0000\ 017$	$\cdot 0000\ 005^-$
$\cdot 0003\ 784$	$\cdot 0001\ 229$	$\cdot 0000\ 386$	$\cdot 0000\ 118$	$\cdot 0000\ 035^-$	$\cdot 0000\ 010$
$\cdot 0006\ 154$	$\cdot 0002\ 108$	$\cdot 0000\ 699$	$\cdot 0000\ 225^-$	$\cdot 0000\ 070$	$\cdot 0000\ 022$
$\cdot 0009\ 697$	$\cdot 0003\ 492$	$\cdot 0001\ 217$	$\cdot 0000\ 412$	$\cdot 0000\ 136$	$\cdot 0000\ 044$
$\cdot 0014\ 848$	$\cdot 0005\ 607$	$\cdot 0002\ 049$	$\cdot 0000\ 727$	$\cdot 0000\ 251$	$\cdot 0000\ 085^-$
$\cdot 0022\ 152$	$\cdot 0008\ 752$	$\cdot 0003\ 347$	$\cdot 0001\ 243$	$\cdot 0000\ 450^-$	$\cdot 0000\ 159$
$\cdot 0032\ 270$	$\cdot 0013\ 310$	$\cdot 0005\ 315^+$	$\cdot 0002\ 062$	$\cdot 0000\ 779$	$\cdot 0000\ 288$
$\cdot 0045\ 989$	$\cdot 0019\ 765^-$	$\cdot 0008\ 226$	$\cdot 0003\ 327$	$\cdot 0001\ 311$	$\cdot 0000\ 505^-$
$\cdot 0064\ 227$	$\cdot 0028\ 710$	$\cdot 0012\ 431$	$\cdot 0005\ 231$	$\cdot 0002\ 145^+$	$\cdot 0000\ 859$
$\cdot 0088\ 032$	$\cdot 0040\ 861$	$\cdot 0018\ 376$	$\cdot 0008\ 032$	$\cdot 0003\ 422$	$\cdot 0001\ 425^-$
$\cdot 0118\ 575^+$	$\cdot 0057\ 061$	$\cdot 0026\ 611$	$\cdot 0012\ 066$	$\cdot 0005\ 333$	$\cdot 0002\ 303$
$\cdot 0157\ 139$	$\cdot 0078\ 285^+$	$\cdot 0037\ 807$	$\cdot 0017\ 755^-$	$\cdot 0008\ 130$	$\cdot 0003\ 638$
$\cdot 0205\ 098$	$\cdot 0105\ 639$	$\cdot 0052\ 760$	$\cdot 0025\ 629$	$\cdot 0012\ 141$	$\cdot 0005\ 621$
$\cdot 0263\ 899$	$\cdot 0140\ 351$	$\cdot 0072\ 399$	$\cdot 0036\ 333$	$\cdot 0017\ 784$	$\cdot 0008\ 510$
$\cdot 0335\ 028$	$\cdot 0183\ 761$	$\cdot 0097\ 791$	$\cdot 0050\ 640$	$\cdot 0025\ 582$	$\cdot 0012\ 636$
$\cdot 0419\ 973$	$\cdot 0237\ 301$	$\cdot 0130\ 133$	$\cdot 0069\ 460$	$\cdot 0036\ 176$	$\cdot 0018\ 425^-$
$\cdot 0520\ 190$	$\cdot 0302\ 468$	$\cdot 0170\ 747$	$\cdot 0093\ 842$	$\cdot 0050\ 335^+$	$\cdot 0026\ 407$
$\cdot 0637\ 053$	$\cdot 0380\ 795^+$	$\cdot 0221\ 062$	$\cdot 0124\ 976$	$\cdot 0068\ 971$	$\cdot 0037\ 235^+$
$\cdot 0771\ 815^+$	$\cdot 0473\ 812$	$\cdot 0282\ 594$	$\cdot 0164\ 185^+$	$\cdot 0093\ 140$	$\cdot 0051\ 697$
$\cdot 0925\ 560$	$\cdot 0583\ 004$	$\cdot 0356\ 916$	$\cdot 0212\ 916$	$\cdot 0124\ 047$	$\cdot 0070\ 726$
$\cdot 1099\ 160$	$\cdot 0709\ 765^-$	$\cdot 0445\ 625^+$	$\cdot 0272\ 716$	$\cdot 0163\ 042$	$\cdot 0095\ 409$
$\cdot 1293\ 231$	$\cdot 0855\ 353$	$\cdot 0550\ 298$	$\cdot 0345\ 207$	$\cdot 0211\ 605^-$	$\cdot 0126\ 990$
$\cdot 1508\ 102$	$\cdot 1020\ 838$	$\cdot 0672\ 445^-$	$\cdot 0432\ 053$	$\cdot 0271\ 334$	$\cdot 0166\ 867$
$\cdot 1743\ 779$	$\cdot 1207\ 057$	$\cdot 0813\ 462$	$\cdot 0534\ 917$	$\cdot 0343\ 915^+$	$\cdot 0216\ 581$
$\cdot 1999\ 925^+$	$\cdot 1414\ 570$	$\cdot 0974\ 578$	$\cdot 0655\ 414$	$\cdot 0431\ 088$	$\cdot 0277\ 798$
$\cdot 2275\ 849$	$\cdot 1643\ 618$	$\cdot 1156\ 801$	$\cdot 0795\ 058$	$\cdot 0534\ 604$	$\cdot 0352\ 285^+$
$\cdot 2570\ 493$	$\cdot 1894\ 097$	$\cdot 1360\ 870$	$\cdot 0955\ 208$	$\cdot 0656\ 177$	$\cdot 0441\ 872$
$\cdot 2882\ 449$	$\cdot 2165\ 526$	$\cdot 1587\ 207$	$\cdot 1137\ 009$	$\cdot 0797\ 430$	$\cdot 0548\ 406$
$\cdot 3209\ 966$	$\cdot 2457\ 040$	$\cdot 1835\ 876$	$\cdot 1341\ 334$	$\cdot 0959\ 830$	$\cdot 0673\ 702$
$\cdot 3550\ 980$	$\cdot 2767\ 383$	$\cdot 2106\ 550^+$	$\cdot 1568\ 733$	$\cdot 1144\ 627$	$\cdot 0819\ 478$
$\cdot 3903\ 150^+$	$\cdot 3094\ 918$	$\cdot 2398\ 491$	$\cdot 1819\ 386$	$\cdot 1352\ 792$	$\cdot 0987\ 294$
$\cdot 4263\ 902$	$\cdot 3437\ 644$	$\cdot 2710\ 538$	$\cdot 2093\ 056$	$\cdot 1584\ 957$	$\cdot 1178\ 478$
$\cdot 4630\ 479$	$\cdot 3793\ 231$	$\cdot 3041\ 104$	$\cdot 2389\ 067$	$\cdot 1841\ 356$	$\cdot 1394\ 059$
$\cdot 5000\ 000^e$	$\cdot 4159\ 060$	$\cdot 3388\ 197$	$\cdot 2706\ 281$	$\cdot 2121\ 781$	$\cdot 1634\ 698$
$\cdot 5369\ 521$	$\cdot 4532\ 273$	$\cdot 3749\ 447$	$\cdot 3043\ 006$	$\cdot 2425\ 544$	$\cdot 1900\ 625^-$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.95$  $q = 11$ 

	$p = 11$	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .2577\ 4020 \times \frac{1}{10^6}$		$.1288\ 7010 \times \frac{1}{10^6}$	$.6723\ 6573 \times \frac{1}{10^7}$	$.3641\ 9810 \times \frac{1}{10^7}$	$.2035\ 1000 \times \frac{1}{10^7}$
$x$					
.71	.9794 902	.9695 443	.9565 980	.9403 355 <sup>-</sup>	.9209 100
.72	.9842 861	.9764 008	.9659 921	.9527 331	.9367 100
.73	.9881 425 <sup>-</sup>	.9819 910	.9737 583	.9631 254	.9491 100
.74	.9911 968	.9864 796	.9800 800	.9717 014	.9610 100
.75	.9935 773	.9900 256	.9851 419	.9786 618	.9703 100
.76	.9954 011	.9927 787	.9891 249	.9842 118	.9778 100
.77	.9967 730	.9948 771	.9922 006	.9885 544	.9837 100
.78	.9977 848	.9964 448	.9945 285 <sup>+</sup>	.9918 841	.9883 100
.79	.9985 152	.9975 910	.9962 526	.9943 818	.9918 100
.80	.9990 303	.9984 098	.9974 997	.9962 116	.9944 100
.81	.9993 846	.9989 800	.9983 791	.9975 180	.9963 100
.82	.9996 216	.9993 661	.9989 820	.9984 248	.9976 100
.83	.9997 753	.9996 196	.9993 827	.9990 349	.9985 100
.84	.9998 717	.9997 805 <sup>+</sup>	.9996 402	.9994 315 <sup>+</sup>	.9991 100
.85	.9999 299	.9998 789	.9997 993	.9996 796	.9995 100
.86	.9999 636	.9999 364	.9998 935 <sup>+</sup>	.9998 283	.9997 100
.87	.9999 821	.9999 685 <sup>-</sup>	.9999 467	.9999 131	.9998 100
.88	.9999 918	.9999 854	.9999 750 <sup>+</sup>	.9999 589	.9999 100
.89	.9999 965 <sup>+</sup>	.9999 937	.9999 892	.9999 820	.9999 100
.90	.9999 986	.9999 975 <sup>+</sup>	.9999 957	.9999 928	.9999 100
.91	.9999 995 <sup>+</sup>	.9999 991	.9999 985 <sup>-</sup>	.9999 974	.9999 100
.92	.9999 999	.9999 997	.9999 995 <sup>+</sup>	.9999 992	.9999 100
.93	1.0000 000	.9999 999	.9999 999	.9999 998	.9999 100
.94		1.0000 000	1.0000 000	1.0000 000	.9999 100
.95					1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$17 \text{ to } 80$   $q = 11$   $p = 1$

	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$
$g = 6972 \ 6816 \times \frac{1}{10^8}$	$4233 \ 4138 \times \frac{1}{10^8}$	$2627 \ 6362 \times \frac{1}{10^8}$	$1664 \ 1696 \times \frac{1}{10^8}$	$1073 \ 6578 \times \frac{1}{10^8}$	$7045 \ 8793$	
$x$						
17	0000 001					
18	0000 003	0000 001				
19	0000 006	0000 002	0000 001			
20	0000 014	0000 004	0000 001			
21	0000 028	0000 009	0000 003	0000 001		
22	0000 055 <sup>-</sup>	0000 019	0000 006	0000 002	0000 001	
23	0000 104	0000 037	0000 013	0000 004	0000 001	
24	0000 190	0000 070	0000 025 <sup>+</sup>	0000 009	0000 003	0000 001
25	0000 337	0000 130	0000 049	0000 018	0000 007	0000 002
26	0000 581	0000 232	0000 091	0000 035 <sup>+</sup>	0000 013	0000 005 <sup>+</sup>
27	0000 974	0000 404	0000 165 <sup>-</sup>	0000 066	0000 026	0000 010
28	0001 594	0000 685 <sup>+</sup>	0000 289	0000 120	0000 049	0000 020
29	0002 549	0001 134	0000 496	0000 213	0000 090	0000 038
30	0003 989	0001 834	0000 829	0000 369	0000 162	0000 070
31	0006 114	0002 903	0001 355 <sup>-</sup>	0000 622	0000 282	0000 126
32	0009 194	0004 503	0002 167	0001 027	0000 479	0000 221
33	0013 575 <sup>+</sup>	0006 850 <sup>+</sup>	0003 398	0001 659	0000 798	0000 379
34	0019 701	0010 233	0005 225 <sup>+</sup>	0002 626	0001 301	0000 636
35	0028 126	0015 024	0007 890	0004 079	0002 079	0001 045 <sup>+</sup>
36	0039 533	0021 698	0011 710	0006 223	0003 259	0001 684
37	0054 745 <sup>+</sup>	0030 849	0017 096	0009 328	0005 018	0002 663
38	0074 741	0043 207	0024 566	0013 755 <sup>+</sup>	0007 593	0004 136
39	0100 661	0059 652	0034 773	0019 964	0011 300	0006 313
40	0133 813	0081 231	0048 514	0028 539	0016 554	0009 477
41	0175 669	0109 164	0066 750 <sup>-</sup>	0040 207	0023 883	0014 003
42	0227 853	0144 849	0090 620	0055 857	0033 956	0020 377
43	0292 126	0189 858	0121 453	0076 557	0047 599	0029 218
44	0370 354	0245 927	0160 763	0103 569	0065 821	0041 303
45	0464 471	0314 932	0210 255 <sup>-</sup>	0138 358	0089 828	0057 590
46	0576 429	0398 861	0271 801	0182 591	0121 037	0079 238
47	0708 144	0499 768	0347 421	0238 132	0161 084	0107 627
48	0861 424	0619 717	0439 243	0307 021	0211 822	0144 366
49	1037 899	0760 722	0549 452	0391 444	0275 308	0191 303
50	1238 943	0924 667	0680 230	0493 686	0353 778	0250 512
51	1465 598	1113 226	0833 678	0616 071	0449 603	0324 275 <sup>+</sup>
52	1718 495 <sup>+</sup>	1327 779	1011 733	0760 892	0505 236	0415 047
53	1997 789	1569 322	1216 074	0930 316	0703 138	0525 398
54	2303 093	1838 384	1448 028	1126 293	0865 687	0657 947
55	2633 441	2134 953	1708 469	1350 447	1055 079	0815 268
56	2987 256	2458 412	1997 731	1603 970	1273 210	0999 782
57	3362 344	2807 492	2315 524	1887 511	1521 561	1213 639
58	3755 912	3180 248	2660 871	2201 085 <sup>+</sup>	1801 072	1458 584
59	4164 612	3574 061	3032 065 <sup>+</sup>	2543 985 <sup>-</sup>	2112 033	1735 824
60	4584 602	3985 663	3426 654	2914 719	2453 977	2045 807

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.96$  $q = 11$ 

	$p = 17$	$p = 18$	$p = 19$	$p = 20$	$p = 21$
$B(p, q) = .6972\ 6816 \times \frac{1}{10^8}$		$.4233\ 4138 \times \frac{1}{10^8}$	$.2627\ 6362 \times \frac{1}{10^8}$	$.1664\ 1696 \times \frac{1}{10^8}$	$.1073\ 6578$
$\cdot 81$	$.9925\ 835^+$	$.9898\ 502$	$.9864\ 063$	$.9821\ 485^-$	$.9769\ 752$
$\cdot 82$	$.9951\ 445^+$	$.9932\ 868$	$.9909\ 171$	$.9879\ 513$	$.9843\ 034$
$\cdot 83$	$.9969\ 318$	$.9957\ 144$	$.9941\ 427$	$.9921\ 515^+$	$.9896\ 725^+$
$\cdot 84$	$.9981\ 363$	$.9973\ 703$	$.9963\ 694$	$.9950\ 862$	$.9934\ 693$
$\cdot 85$	$.9989\ 172$	$.9984\ 566$	$.9978\ 477$	$.9970\ 576$	$.9960\ 503$
$\cdot 86$	$.9994\ 018$	$.9991\ 387$	$.9987\ 868$	$.9983\ 249$	$.9977\ 291$
$\cdot 87$	$.9996\ 880$	$.9995\ 463$	$.9993\ 546$	$.9990\ 999$	$.9987\ 676$
$\cdot 88$	$.9998\ 478$	$.9997\ 764$	$.9996\ 788$	$.9995\ 476$	$.9993\ 745^-$
$\cdot 89$	$.9999\ 313$	$.9998\ 981$	$.9998\ 522$	$.9997\ 897$	$.9997\ 064$
$\cdot 90$	$.9999\ 717$	$.9999\ 577$	$.9999\ 380$	$.9999\ 109$	$.9998\ 744$
$\cdot 91$	$.9999\ 896$	$.9999\ 843$	$.9999\ 767$	$.9999\ 663$	$.9999\ 520$
$\cdot 92$	$.9999\ 967$	$.9999\ 949$	$.9999\ 924$	$.9999\ 889$	$.9999\ 840$
$\cdot 93$	$.9999\ 991$	$.9999\ 986$	$.9999\ 979$	$.9999\ 969$	$.9999\ 955^+$
$\cdot 94$	$.9999\ 998$	$.9999\ 997$	$.9999\ 995^+$	$.9999\ 993$	$.9999\ 990$
$\cdot 95$	$1.0000\ 000$	$1.0000\ 000$	$.9999\ 999$	$.9999\ 999$	$.9999\ 998$
$\cdot 96$			$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

5 to 80

$q = 11$

$p = 23$

	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$
$q) = .4697\ 2528 \times \frac{1}{10^9}$	$.3177\ 5534 \times \frac{1}{10^9}$	$.2178\ 8938 \times \frac{1}{10^9}$	$.1513\ 1207 \times \frac{1}{10^9}$	$.1063\ 2740 \times \frac{1}{10^9}$	$.7554\ 8414$	
5	.0000 001					
6	.0000 002	.0000 001				
7	.0000 004	.0000 001	.0000 001			
8	.0000 008	.0000 003	.0000 001			
9	.0000 016	.0000 006	.0000 003	.0000 001		
0	.0000 030	.0000 013	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
1	.0000 055 <sup>+</sup>	.0000 024	.0000 010	.0000 004	.0000 002	
2	.0000 100	.0000 045 <sup>+</sup>	.0000 020	.0000 009	.0000 004	
3	.0000 177	.0000 082	.0000 038	.0000 017	.0000 008	
4	.0000 307	.0000 146	.0000 069	.0000 032	.0000 015 <sup>-</sup>	
5	.0000 519	.0000 255 <sup>-</sup>	.0000 124	.0000 059	.0000 028	
6	.0000 860	.0000 434	.0000 216	.0000 107	.0000 052	
7	.0001 396	.0000 723	.0000 371	.0000 188	.0000 095 <sup>-</sup>	
8	.0002 225 <sup>+</sup>	.0001 183	.0000 623	.0000 324	.0000 167	
9	.0003 483	.0001 900	.0001 025 <sup>+</sup>	.0000 548	.0000 290	
0	.0005 360	.0002 997	.0001 658	.0000 908	.0000 492	
1	.0008 111	.0004 645 <sup>+</sup>	.0002 632	.0001 476	.0000 820	
2	.0012 082	.0007 083	.0004 108	.0002 359	.0001 342	
3	.0017 721	.0010 628	.0006 307	.0003 706	.0002 158	
4	.0025 611	.0015 705 <sup>+</sup>	.0009 530	.0005 727	.0003 409	
5	.0036 489	.0022 865 <sup>-</sup>	.0014 180	.0008 708	.0005 298	
6	.0051 271	.0032 813	.0020 785 <sup>-</sup>	.0013 038	.0008 104	
7	.0071 082	.0046 438	.0030 029	.0019 232	.0012 205 <sup>+</sup>	
8	.0097 270	.0064 835 <sup>+</sup>	.0042 779	.0027 958	.0018 107	
9	.0131 431	.0089 338	.0060 118	.0040 074	.0026 474	
0	.0175 410	.0121 533	.0083 369	.0056 655 <sup>-</sup>	.0038 160	
1	.0231 305 <sup>+</sup>	.0163 276	.0114 124	.0079 030	.0054 248	
2	.0301 449	.0216 695 <sup>+</sup>	.0154 260	.0108 807	.0076 081	
3	.0388 377	.0284 181	.0205 948	.0147 898	.0105 299	
4	.0494 777	.0368 355 <sup>+</sup>	.0271 643	.0198 528	.0143 861	
5	.0623 419	.0472 025 <sup>+</sup>	.0354 065 <sup>-</sup>	.0263 235 <sup>-</sup>	.0194 064	
6	.0777 063	.0598 112	.0456 145 <sup>+</sup>	.0344 843	.0258 541	
7	.0958 344	.0749 558	.0580 962	.0446 423	.0340 241	
8	.1169 649	.0929 205 <sup>+</sup>	.0731 640	.0571 218	.0442 386	
9	.1412 968	.1139 663	.0911 226	.0722 537	.0568 393	
0	.1689 745 <sup>-</sup>	.1383 148	.1122 541	.0903 632	.0721 768	
1	.2000 731	.1661 321	.1368 013	.1117 529	.0905 966	
2	.2345 845 <sup>-</sup>	.1975 116	.1649 492	.1366 850 <sup>+</sup>	.1124 214	
3	.2724 052	.2324 586	.1968 063	.1653 609	.1379 309	
4	.3133 282	.2708 765 <sup>-</sup>	.2323 869	.1979 003	.1673 398	
5	.3570 379	.3125 559	.2715 955 <sup>-</sup>	.2343 214	.2007 748	
6	.4031 112	.3571 695 <sup>+</sup>	.3142 140	.2745 232	.2382 519	
7	.4510 236	.4042 714	.3598 958	.3182 714	.2796 568	
8	.5001 619	.4533 040	.4081 642	.3651 911	.3247 303	
9	.5498 425 <sup>+</sup>	.5036 112	.4584 202	.4147 656	.3730 587	
1					.0720 115 <sup>+</sup>	
2					.0918 062	
3					.1142 496	
4					.1405 372	
5					.1708 935 <sup>-</sup>	
6					.2054 464	
7					.2442 029	
8					.2870 268	
9					.3336 225 <sup>+</sup>	

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.97$  $q = 11$ 

	$p = 23$	$p = 24$	$p = 25$	$p = 26$	$p =$
$B(p, q) =$	$.4697\ 2528 \times \frac{x}{10^8}$	$.3177\ 5534 \times \frac{x}{10^8}$	$.2178\ 8938 \times \frac{x}{10^8}$	$.1513\ 1207 \times \frac{x}{10^8}$	$.1063$
$x$					
.81	.9635 016	.9550 318	.9453 127	.9342 913	.9219
.82	.9746 209	.9684 244	.9612 262	.9529 626	.9435
.83	.9829 683	.9786 015 <sup>+</sup>	.9734 669	.9675 005 <sup>-</sup>	.9606
.84	.9890 147	.9860 623	.9825 490	.9784 173	.9736
.85	.9932 238	.9913 180	.9890 232	.9862 923	.9830
.86	.9960 263	.9948 587	.9934 361	.9917 234	.9896
.87	.9978 008	.9971 267	.9962 958	.9952 838	.9940
.88	.9988 616	.9984 981	.9980 449	.9974 866	.9968
.89	.9994 551	.9992 741	.9990 459	.9987 615 <sup>-</sup>	.9984
.90	.9997 623	.9996 803	.9995 757	.9994 439	.9992
.91	.9999 073	.9998 741	.9998 314	.9997 768	.9997
.92	.9999 685 <sup>+</sup>	.9999 569	.9999 416	.9999 220	.9998
.93	.9999 910	.9999 876	.9999 830	.9999 771	.9999
.94	.9999 980	.9999 972	.9999 961	.9999 947	.9999
.95	.9999 997	.9999 995 <sup>+</sup>	.9999 993	.9999 991	.9999
.96	I.0000 000	.9999 999	.9999 999	.9999 999	.9999
.97		I.0000 000	I.0000 000	I.0000 000	I.0000



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .98

$q = 11$

$p = 29$

$p = 29$	$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$
$= .5423\ 9887 \times \frac{1}{10^{10}}$	$.3932\ 3918 \times \frac{1}{10^{10}}$	$.2877\ 3599 \times \frac{1}{10^{10}}$	$.2123\ 7656 \times \frac{1}{10^{10}}$	$.1580\ 4767 \times \frac{1}{10^{10}}$	$.1185\ 3576$
.0000 001	.0000 001				
.0000 002	.0000 001	.0000 001			
.0000 003	.0000 001	.0000 001			
.0000 006	.0000 003	.0000 001	.0000 001		
.0000 012	.0000 006	.0000 003	.0000 001	.0000 001	
.0000 023	.0000 011	.0000 006	.0000 003	.0000 001	.0000 001
.0000 043	.0000 022	.0000 011	.0000 005 <sup>+</sup>	.0000 003	.0000 001
.0000 079	.0000 041	.0000 021	.0000 011	.0000 005 <sup>+</sup>	.0000 003
.0000 141	.0000 075 <sup>-</sup>	.0000 039	.0000 020	.0000 011	.0000 005 <sup>+</sup>
.0000 247	.0000 134	.0000 072	.0000 038	.0000 020	.0000 011
.0000 423	.0000 235 <sup>-</sup>	.0000 129	.0000 071	.0000 038	.0000 021
.0000 713	.0000 404	.0000 228	.0000 128	.0000 071	.0000 039
.0001 178	.0000 684	.0000 394	.0000 226	.0000 128	.0000 072
.0001 912	.0001 135 <sup>+</sup>	.0000 669	.0000 391	.0000 228	.0000 131
.0003 052	.0001 851	.0001 115 <sup>-</sup>	.0000 666	.0000 396	.0000 234
.0004 793	.0002 969	.0001 825 <sup>+</sup>	.0001 114	.0000 676	.0000 407
.0007 407	.0004 682	.0002 938	.0001 831	.0001 134	.0000 698
.0011 270	.0007 268	.0004 654	.0002 959	.0001 870	.0001 174
.0016 889	.0011 107	.0007 252	.0004 703	.0003 031	.0001 941
.0024 940	.0016 719	.0011 128	.0007 356	.0004 832	.0003 155 <sup>-</sup>
.0036 303	.0024 795 <sup>-</sup>	.0016 815 <sup>+</sup>	.0011 327	.0007 582	.0005 044
.0052 105 <sup>-</sup>	.0036 243	.0025 034	.0017 176	.0011 711	.0007 936
.0073 761	.0052 231	.0036 730	.0025 659	.0017 812	.0012 291
.0103 016	.0074 233	.0053 125 <sup>-</sup>	.0037 771	.0026 687	.0018 744
.0141 978	.0104 071	.0075 768	.0054 805 <sup>-</sup>	.0039 397	.0028 154
.0193 141	.0143 959	.0106 580	.0078 402	.0057 321	.0041 663
.0259 387	.0196 518	.0147 899	.0110 604	.0082 213	.0060 756
.0343 972	.0264 792	.0202 504	.0153 900	.0116 261	.0087 325 <sup>+</sup>
.0450 474	.0352 225 <sup>-</sup>	.0273 625 <sup>+</sup>	.0211 253	.0162 135 <sup>-</sup>	.0123 733
.0582 708	.0462 604	.0364 919	.0286 109	.0223 012	.0172 859
.0744 601	.0599 970	.0480 408	.0382 367	.0302 587	.0238 134
.0940 021	.0768 476	.0624 379	.0504 313	.0405 033	.0323 533
.1172 572	.0972 203	.0801 222	.0656 494	.0534 923	.0433 540
.1445 338	.1214 919	.1015 222	.0843 547	.0697 087	.0573 040
.1760 622	.1499 811	.1270 298	.1069 955 <sup>-</sup>	.0896 407	.0747 157
.2119 652	.1829 179	.1569 690	.1339 749	.1137 545 <sup>-</sup>	.0961 014
.2522 319	.2204 125 <sup>-</sup>	.1915 629	.1656 170	.1424 602	.1219 418
.2966 933	.2624 255 <sup>+</sup>	.2308 992	.2021 289	.1760 738	.1526 479
.3450 054	.3087 427	.2748 975 <sup>+</sup>	.2435 634	.2147 752	.1885 169
.3966 409	.3589 572	.3232 833	.2897 844	.2585 676	.2296 873
.4508 918	.4124 624	.3755 701	.3404 392	.3072 494	.2760 940
.5068 864	.4684 584	.4310 552	.3949 429	.3603 429	.3274 310
.5636 192	.5259 740	.4888 307	.4524 782	.4171 708	.3831 260
.6199 952	.5839 041	.5478 130	.5120 130	.4767 723	.4423 326
.6748 850 <sup>-</sup>	.6410 628	.6067 897	.5723 397	.5379 759	.5039 452
.7271 884	.6962 484	.6644 833	.6321 341	.5994 413	.5666 395 <sup>-</sup>
.7759 006	.7483 154	.7196 267	.6900 312	.6597 328	.6289 385 <sup>+</sup>

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .38 \text{ to } .98$  $q = 11$ 

	$p = 35$	$p = 36$	$p = 37$	$p = 38$	$p$
$B(p, q) = .8956\ 0349 \times \frac{1}{10^{11}}$		$.6814\ 3744 \times \frac{1}{10^{11}}$	$.5219\ 5208 \times \frac{1}{10^{11}}$	$.4023\ 3806 \times \frac{1}{10^{11}}$	$.31$
$\cdot 38$	$\cdot 0000\ 001$				
$\cdot 39$	$\cdot 0000\ 001$	$\cdot 0000\ 001$			
$\cdot 40$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	$\cdot 0000\ 001$		
$\cdot 41$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 42$	$\cdot 0000\ 011$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 000$
$\cdot 43$	$\cdot 0000\ 021$	$\cdot 0000\ 012$	$\cdot 0000\ 006$	$\cdot 0000\ 003$	$\cdot 000$
$\cdot 44$	$\cdot 0000\ 041$	$\cdot 0000\ 023$	$\cdot 0000\ 013$	$\cdot 0000\ 007$	$\cdot 000$
$\cdot 45$	$\cdot 0000\ 075^+$	$\cdot 0000\ 043$	$\cdot 0000\ 024$	$\cdot 0000\ 014$	$\cdot 000$
$\cdot 46$	$\cdot 0000\ 137$	$\cdot 0000\ 080$	$\cdot 0000\ 046$	$\cdot 0000\ 027$	$\cdot 000$
$\cdot 47$	$\cdot 0000\ 244$	$\cdot 0000\ 145^+$	$\cdot 0000\ 086$	$\cdot 0000\ 051$	$\cdot 000$
$\cdot 48$	$\cdot 0000\ 427$	$\cdot 0000\ 260$	$\cdot 0000\ 157$	$\cdot 0000\ 095^-$	$\cdot 000$
$\cdot 49$	$\cdot 0000\ 733$	$\cdot 0000\ 455^-$	$\cdot 0000\ 281$	$\cdot 0000\ 172$	$\cdot 000$
$\cdot 50$	$\cdot 0001\ 235^+$	$\cdot 0000\ 782$	$\cdot 0000\ 492$	$\cdot 0000\ 308$	$\cdot 000$
$\cdot 51$	$\cdot 0002\ 047$	$\cdot 0001\ 321$	$\cdot 0000\ 848$	$\cdot 0000\ 542$	$\cdot 000$
$\cdot 52$	$\cdot 0003\ 336$	$\cdot 0002\ 194$	$\cdot 0001\ 436$	$\cdot 0000\ 935^-$	$\cdot 000$
$\cdot 53$	$\cdot 0005\ 347$	$\cdot 0003\ 583$	$\cdot 0002\ 388$	$\cdot 0001\ 584$	$\cdot 000$
$\cdot 54$	$\cdot 0008\ 433$	$\cdot 0005\ 754$	$\cdot 0003\ 905^+$	$\cdot 0002\ 638$	$\cdot 000$
$\cdot 55$	$\cdot 0013\ 090$	$\cdot 0009\ 092$	$\cdot 0006\ 282$	$\cdot 0004\ 319$	$\cdot 000$
$\cdot 56$	$\cdot 0020\ 006$	$\cdot 0014\ 140$	$\cdot 0009\ 942$	$\cdot 0006\ 956$	$\cdot 000$
$\cdot 57$	$\cdot 0030\ 113$	$\cdot 0021\ 649$	$\cdot 0015\ 484$	$\cdot 0011\ 021$	$\cdot 000$
$\cdot 58$	$\cdot 0044\ 651$	$\cdot 0032\ 641$	$\cdot 0023\ 740$	$\cdot 0017\ 183$	$\cdot 001$
$\cdot 59$	$\cdot 0065\ 232$	$\cdot 0048\ 473$	$\cdot 0035\ 838$	$\cdot 0026\ 369$	$\cdot 001$
$\cdot 60$	$\cdot 0093\ 915^+$	$\cdot 0070\ 913$	$\cdot 0053\ 278$	$\cdot 0039\ 837$	$\cdot 002$
$\cdot 61$	$\cdot 0133\ 268$	$\cdot 0102\ 218$	$\cdot 0078\ 015^+$	$\cdot 0059\ 262$	$\cdot 004$
$\cdot 62$	$\cdot 0186\ 420$	$\cdot 0145\ 196$	$\cdot 0112\ 538$	$\cdot 0086\ 817$	$\cdot 006$
$\cdot 63$	$\cdot 0257\ 087$	$\cdot 0203\ 266$	$\cdot 0159\ 940$	$\cdot 0125\ 267$	$\cdot 009$
$\cdot 64$	$\cdot 0349\ 571$	$\cdot 0280\ 477$	$\cdot 0223\ 973$	$\cdot 0178\ 037$	$\cdot 014$
$\cdot 65$	$\cdot 0468\ 694$	$\cdot 0381\ 491$	$\cdot 0309\ 064$	$\cdot 0249\ 263$	$\cdot 020$
$\cdot 66$	$\cdot 0619\ 679$	$\cdot 0511\ 506$	$\cdot 0420\ 279$	$\cdot 0343\ 797$	$\cdot 028$
$\cdot 67$	$\cdot 0807\ 952$	$\cdot 0676\ 099$	$\cdot 0563\ 219$	$\cdot 0467\ 149$	$\cdot 038$
$\cdot 68$	$\cdot 1038\ 857$	$\cdot 0880\ 995^+$	$\cdot 0743\ 830$	$\cdot 0625\ 350^-$	$\cdot 052$
$\cdot 69$	$\cdot 1317\ 299$	$\cdot 1131\ 727$	$\cdot 0968\ 114$	$\cdot 0824\ 709$	$\cdot 069$
$\cdot 70$	$\cdot 1647\ 300$	$\cdot 1433\ 217$	$\cdot 1241\ 732$	$\cdot 1071\ 466$	$\cdot 092$
$\cdot 71$	$\cdot 2031\ 514$	$\cdot 1789\ 279$	$\cdot 1569\ 519$	$\cdot 1371\ 318$	$\cdot 119$
$\cdot 72$	$\cdot 2470\ 728$	$\cdot 2202\ 075^-$	$\cdot 1954\ 914$	$\cdot 1728\ 862$	$\cdot 152$
$\cdot 73$	$\cdot 2963\ 388$	$\cdot 2671\ 567$	$\cdot 2399\ 362$	$\cdot 2146\ 947$	$\cdot 191$
$\cdot 74$	$\cdot 3505\ 230$	$\cdot 3195\ 036$	$\cdot 2901\ 731$	$\cdot 2626\ 023$	$\cdot 236$
$\cdot 75$	$\cdot 4089\ 058$	$\cdot 3766\ 728$	$\cdot 3457\ 829$	$\cdot 3163\ 539$	$\cdot 288$
$\cdot 76$	$\cdot 4704\ 751$	$\cdot 4377\ 700$	$\cdot 4060\ 097$	$\cdot 3753\ 481$	$\cdot 345$
$\cdot 77$	$\cdot 5339\ 534$	$\cdot 5015\ 942$	$\cdot 4697\ 563$	$\cdot 4386\ 155^-$	$\cdot 408$
$\cdot 78$	$\cdot 5978\ 545^-$	$\cdot 5666\ 816$	$\cdot 5356\ 126$	$\cdot 5048\ 292$	$\cdot 474$
$\cdot 79$	$\cdot 6605\ 685^-$	$\cdot 6313\ 823$	$\cdot 6019\ 206$	$\cdot 5723\ 553$	$\cdot 542$
$\cdot 80$	$\cdot 7204\ 707$	$\cdot 6939\ 677$	$\cdot 6668\ 757$	$\cdot 6393\ 443$	$\cdot 611$
$\cdot 81$	$\cdot 7760\ 442$	$\cdot 7527\ 580$	$\cdot 7286\ 587$	$\cdot 7008\ 671$	$\cdot 681$

$x = .43$  to  $.98$  $q = 11$  $p = 41$  to

	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$B(p, q) = .1908\ 8115 \times \frac{1}{10^{11}}$		$.1505\ 0245 \times \frac{1}{10^{11}}$	$.1192\ 6609 \times \frac{1}{10^{11}}$	$.9497\ 1147 \times \frac{1}{10^{11}}$	$.7597\ 6917 \times \frac{1}{10^{12}}$
$x$					
.43	.0000 001				
.44	.0000 001	.0000 001			
.45	.0000 002	.0000 001	.0000 001		
.46	.0000 005 <sup>-</sup>	.0000 003	.0000 002	.0000 001	.0000 001
.47	.0000 010	.0000 006	.0000 003	.0000 002	.0000 001
.48	.0000 020	.0000 012	.0000 007	.0000 004	.0000 002
.49	.0000 039	.0000 023	.0000 014	.0000 008	.0000 005 <sup>+</sup>
.50	.0000 074	.0000 045 <sup>+</sup>	.0000 028	.0000 017	.0000 010
.51	.0000 137	.0000 086	.0000 054	.0000 033	.0000 021
.52	.0000 251	.0000 160	.0000 102	.0000 065 <sup>-</sup>	.0000 041
.53	.0000 449	.0000 292	.0000 190	.0000 123	.0000 079
.54	.0000 790	.0000 524	.0000 346	.0000 228	.0000 149
.55	.0001 365 <sup>-</sup>	.0000 922	.0000 620	.0000 415 <sup>+</sup>	.0000 277
.56	.0002 317	.0001 592	.0001 090	.0000 743	.0000 505 <sup>-</sup>
.57	.0003 865 <sup>+</sup>	.0002 703	.0001 882	.0001 306	.0000 902
.58	.0006 339	.0004 508	.0003 193	.0002 253	.0001 584
.59	.0010 222	.0007 391	.0005 322	.0003 818	.0002 729
.60	.0016 212	.0011 914	.0008 720	.0006 359	.0004 620
.61	.0025 293	.0018 885 <sup>+</sup>	.0014 046	.0010 408	.0007 684
.62	.0038 823	.0029 444	.0022 245 <sup>-</sup>	.0016 744	.0012 557
.63	.0058 636	.0045 157	.0034 644	.0026 481	.0020 169
.64	.0087 152	.0068 134	.0053 065 <sup>-</sup>	.0041 177	.0031 840
.65	.0127 485 <sup>-</sup>	.0101 143	.0079 944	.0062 960	.0049 411
.66	.0183 540	.0147 729	.0118 466	.0094 661	.0075 378
.67	.0260 080	.0212 308	.0172 680	.0139 954	.0113 045 <sup>+</sup>
.68	.0362 730	.0300 217	.0247 587	.0203 476	.0166 663
.69	.0497 909	.0417 697	.0349 172	.0290 894	.0241 543
.70	.0672 649	.0571 769	.0484 339	.0408 906	.0344 102
.71	.0894 270	.0769 984	.0660 730	.0565 121	.0481 810
.72	.1169 915 <sup>-</sup>	.1020 007	.0886 375 <sup>+</sup>	.0767 785 <sup>-</sup>	.0662 994
.73	.1505 920	.1329 035 <sup>+</sup>	.1169 165 <sup>-</sup>	.1025 318	.0896 445 <sup>-</sup>
.74	.1907 065 <sup>+</sup>	.1703 051	.1516 135 <sup>-</sup>	.1345 650 <sup>+</sup>	.1190 819
.75	.2375 727	.2145 951	.1932 587	.1735 350 <sup>+</sup>	.1553 802
.76	.2911 049	.2658 623	.2421 102	.2198 606	.1991 078
.77	.3508 212	.3238 078	.2980 550 <sup>+</sup>	.2736 139	.2505 170
.78	.4157 960	.3876 782	.3605 245 <sup>-</sup>	.3344 190	.3094 289
.79	.4846 509	.4562 340	.4284 397	.4013 757	.3751 359
.80	.5555 956	.5277 682	.5002 059	.4730 281	.4463 444
.81	.6265 265 <sup>+</sup>	.6001 858	.5737 697	.5473 967	.5211 795 <sup>+</sup>
.82	.6951 806	.6711 470	.6467 472	.6220 863	.5972 686
.83	.7593 331	.7382 666	.7166 182	.6944 715 <sup>-</sup>	.6719 119
.84	.8170 177	.7993 482	.7809 719	.7619 461	.7423 322
.85	.8667 343	.8526 209	.8377 682	.8222 074	.8059 746
.86	.9076 126	.8969 393	.8855 748	.8735 284	.8608 140
.87	.9304 024	.9210 056	.9022 222	.8910 682	.8786 101
.88	.9451 000	.9363 000	.9232 000	.9100 000	.8966 000
.89	.9511 000	.9423 000	.9292 000	.9160 000	.9026 000
.90	.9581 000	.9493 000	.9362 000	.9230 000	.9096 000
.91	.9651 000	.9563 000	.9432 000	.9300 000	.9166 000
.92	.9721 000	.9633 000	.9502 000	.9370 000	.9236 000
.93	.9791 000	.9703 000	.9572 000	.9440 000	.9306 000
.94	.9861 000	.9773 000	.9642 000	.9510 000	.9376 000
.95	.9931 000	.9843 000	.9712 000	.9580 000	.9446 000
.96	.9991 000	.9903 000	.9772 000	.9640 000	.9506 000
.97	.9991 000	.9903 000	.9772 000	.9640 000	.9506 000
.98	.9991 000	.9903 000	.9772 000	.9640 000	.9506 000

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .47$  to  $.98$  $q = 11$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$	$p = 50$
$B(p, q) = .6105\ 2880 \times \frac{x}{10^{12}}$		$.4927\ 0745 \times \frac{x}{10^{12}}$	$.3992\ 6294 \times \frac{x}{10^{12}}$	$.3248\ 2408 \times \frac{x}{10^{12}}$	$.2600\ 0000 \times \frac{x}{10^{12}}$
.47	.0000 001				
.48	.0000 001	.0000 001			
.49	.0000 003	.0000 002	.0000 001	.0000 001	
.50	.0000 006	.0000 004	.0000 002	.0000 001	.0000 000
.51	.0000 013	.0000 008	.0000 005-	.0000 003	.0000 001
.52	.0000 026	.0000 016	.0000 010	.0000 006	.0000 003
.53	.0000 051	.0000 032	.0000 021	.0000 013	.0000 007
.54	.0000 097	.0000 063	.0000 041	.0000 027	.0000 013
.55	.0000 184	.0000 122	.0000 081	.0000 053	.0000 027
.56	.0000 342	.0000 230	.0000 155-	.0000 104	.0000 053
.57	.0000 621	.0000 426	.0000 292	.0000 199	.0000 104
.58	.0001 109	.0000 774	.0000 539	.0000 374	.0000 199
.59	.0001 944	.0001 380	.0000 976	.0000 688	.0000 374
.60	.0003 345-	.0002 413	.0001 736	.0001 244	.0000 688
.61	.0005 653	.0004 145+	.0003 029	.0002 207	.0000 976
.62	.0009 385+	.0006 991	.0005 191	.0003 842	.0001 736
.63	.0015 309	.0011 581	.0008 733	.0006 565+	.0002 413
.64	.0024 537	.0018 847	.0014 430	.0011 014	.0003 345-
.65	.0038 648	.0030 130	.0023 416	.0018 143	.0005 653
.66	.0059 824	.0047 326	.0037 323	.0029 345+	.0008 733
.67	.0091 010	.0073 036	.0058 432	.0046 608	.0011 581
.68	.0136 068	.0110 740	.0089 853	.0072 691	.0018 847
.69	.0199 924	.0164 964	.0135 709	.0111 318	.0024 537
.70	.0288 658	.0241 410	.0201 300	.0167 374	.0038 648
.71	.0409 515-	.0347 027	.0293 220	.0247 059	.0059 824
.72	.0570 778	.0489 948	.0419 369	.0357 964	.0091 010
.73	.0781 461	.0679 276	.0588 809	.0509 010	.0136 068
.74	.1050 783	.0924 629	.0811 412	.0710 177	.0199 924
.75	.1387 383	.1235 435+	.1097 228	.0971 977	.0288 658
.76	.1798 307	.1619 952	.1455 562	.1304 597	.0409 515-
.77	.2287 802	.2084 044	.1893 768	.1716 732	.0570 778
.78	.2856 050+	.2629 827	.2415 829	.2214 137	.0781 461
.79	.3498 000	.3254 334	.3020 881	.2798 031	.1050 783
.80	.4202 538	.3948 437	.3701 906	.3463 592	.1387 383
.81	.4952 245-	.4696 306	.4444 887	.4198 810	.1798 307
.82	.5723 956	.5475 660	.5228 737	.4984 077	.2287 802
.83	.6490 266	.6259 024	.6026 257	.5792 811	.2856 050+
.84	.7221 953	.7016 030	.6806 253	.6593 328	.3498 000
.85	.7891 106	.7716 600	.7536 710	.7351 948	.4202 538
.86	.8474 498	.8334 580	.8188 648	.8037 001	.4952 245-
.87	.8956 595-	.8851 205+	.8740 008	.8623 111	.5723 956
.88	.9331 540	.9257 625+	.9178 741	.9094 860	.6490 266
.89	.9603 562	.9555 831	.9504 313	.9448 909	.7221 953
.90	.9785 586	.9757 616	.9727 087	.9693 887	.7891 106

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .70

$q = 12$

$p = 12$

$p = 12$	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$
$= .6163\ 3525 \times \frac{1}{10^7}$	$.3081\ 6763 \times \frac{1}{10^7}$	$.1602\ 4717 \times \frac{1}{10^7}$	$.8628\ 6935 \times \frac{1}{10^8}$	$.4793\ 7186 \times \frac{1}{10^8}$	$.2739\ 2678 \times$
.0000 001					
.0000 005 <sup>-</sup>	.0000 001				
.0000 013	.0000 003	.0000 001			
.0000 033	.0000 007	.0000 002			
.0000 078	.0000 018	.0000 004	.0000 001		
.0000 169	.0000 043	.0000 011	.0000 003	.0000 001	
.0000 344	.0000 094	.0000 025 <sup>-</sup>	.0000 006	.0000 002	
.0000 663	.0000 193	.0000 055 <sup>-</sup>	.0000 015 <sup>+</sup>	.0000 004	.0000 001
.0001 220	.0000 378	.0000 113	.0000 033	.0000 009	.0000 003
.0002 149	.0000 704	.0000 223	.0000 069	.0000 021	.0000 006
.0003 646	.0001 258	.0000 421	.0000 137	.0000 043	.0000 013
.0005 974	.0002 168	.0000 763	.0000 261	.0000 087	.0000 028
.0009 489	.0003 611	.0001 333	.0000 479	.0000 168	.0000 057
.0014 648	.0005 833	.0002 253	.0000 847	.0000 310	.0000 111
.0022 031	.0009 160	.0003 695 <sup>-</sup>	.0001 450 <sup>-</sup>	.0000 555 <sup>+</sup>	.0000 208
.0032 352	.0014 016	.0005 892	.0002 410	.0000 962	.0000 375 <sup>+</sup>
.0046 468	.0020 940	.0009 158	.0003 898	.0001 619	.0000 658
.0065 390	.0030 599	.0013 900	.0006 147	.0002 652	.0001 119
.0090 279	.0043 801	.0020 634	.0009 464	.0004 237	.0001 855 <sup>+</sup>
.0122 443	.0061 505 <sup>+</sup>	.0030 005 <sup>-</sup>	.0014 255 <sup>-</sup>	.0006 611	.0002 999
.0163 325 <sup>+</sup>	.0084 824	.0042 795 <sup>+</sup>	.0021 030	.0010 090	.0004 736
.0214 480	.0115 023	.0059 940	.0030 431	.0015 086	.0007 318
.0277 547	.0153 515 <sup>+</sup>	.0082 531	.0043 237	.0022 122 <sup>+</sup>	.0011 077
.0354 211	.0201 838	.0111 821	.0060 382	.0031 851	.0016 444
.0440 165 <sup>-</sup>	.0261 635 <sup>-</sup>	.0149 213	.0082 965 <sup>+</sup>	.0045 071	.0023 969
.0555 050 <sup>+</sup>	.0334 618	.0196 254	.0112 247	.0062 739	.0034 334
.0682 414	.0422 531	.0254 606	.0149 653	.0085 982	.0048 376
.0829 644	.0527 098	.0326 022	.0196 758	.0116 098	.0067 096
.0997 917	.0649 973	.0412 301	.0255 268	.0154 557	.0091 675 <sup>+</sup>
.1188 140	.0792 678	.0515 246	.0326 988	.0202 990	.0123 475 <sup>+</sup>
.1400 904	.0956 544	.0636 604	.0413 790	.0263 167	.0164 038
.1636 434	.1142 651	.0778 011	.0517 553	.0336 970	.0215 076
.1894 561	.1351 771	.0940 920	.0640 119	.0426 350 <sup>-</sup>	.0278 448
.2174 692	.1584 315 <sup>+</sup>	.1126 546	.0783 218	.0533 278	.0356 131
.2475 798	.1840 289	.1335 793	.0948 412	.0659 684	.0450 176
.2796 419	.2119 260	.1569 199	.1137 009	.0807 391	.0562 650 <sup>+</sup>
.3134 674	.2420 335 <sup>-</sup>	.1826 884	.1350 004	.0978 038	.0695 576
.3488 293	.2742 152	.2108 506	.1588 011	.1173 003	.0850 853
.3854 657	.3082 889	.2413 232	.1851 199	.1393 329	.1030 181
.4230 852	.3440 288	.2739 728	.2139 247	.1639 647	.1234 971
.4613 734	.3811 693	.3086 154	.2451 307	.1912 110	.1466 262
.5000 000 <sup>0</sup>	.4194 099	.3450 190	.2785 985 <sup>+</sup>	.2210 342	.1724 642
.5386 266	.4584 224	.3829 072	.3141 343	.2533 391	.2010 172
.5766 148	.4980 241 <sup>-</sup>	.4232 611			

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.94$  $q = 12$ 

	$p = 12$	$p = 13$	$p = 14$	$p = 15$
$B(p, q) = .6163\ 3525 \times \frac{x}{10^7}$		$.3081\ 6763 \times \frac{x}{10^7}$	$.1602\ 4717 \times \frac{x}{10^7}$	$.8628\ 6935 \times \frac{x}{10^8}$
$x$				
.71	.9836 675 <sup>-</sup>	.9758 173	.9655 276	.9524 817
.72	.9877 557	.9816 619	.9735 618	.9631 475 <sup>-</sup>
.73	.9909 721	.9863 244	.9800 607	.9718 955 <sup>-</sup>
.74	.9934 610	.9899 819	.9852 290	.9789 483
.75	.9953 532	.9928 003	.9892 657	.9845 318
.76	.9967 648	.9949 312	.9923 586	.9888 671
.77	.9977 969	.9965 097	.9946 800	.9921 642
.78	.9985 352	.9976 537	.9963 844	.9946 164
.79	.9990 511	.9984 634	.9976 063	.9963 971
.80	.9994 026	.9990 220	.9984 599	.9976 570
.81	.9996 354	.9993 967	.9990 398	.9985 235 <sup>-</sup>
.82	.9997 851	.9996 405 <sup>+</sup>	.9994 217	.9991 012
.83	.9998 780	.9997 938	.9996 648	.9994 736
.84	.9999 337	.9998 867	.9998 139	.9997 046
.85	.9999 656	.9999 407	.9999 015 <sup>+</sup>	.9998 421
.86	.9999 831	.9999 706	.9999 507	.9999 201
.87	.9999 922	.9999 863	.9999 768	.9999 620
.88	.9999 967	.9999 941	.9999 898	.9999 832
.89	.9999 987	.9999 976	.9999 959	.9999 932
.90	.9999 995 <sup>+</sup>	.9999 992	.9999 985 <sup>+</sup>	.9999 975 <sup>-</sup>
.91	.9999 999	.9999 997	.9999 995 <sup>+</sup>	.9999 992
.92	1.0000 000	.9999 999	.9999 999	.9999 998
.93		1.0000 000	1.0000 000	.9999 999
.94				1.0000 000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$\cdot 17$  to  $\cdot 80$   $q = 12$   $p = 1$

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$
$p, q) = \cdot 1605\ 7777 \times \frac{1}{10^8}$	$\cdot 9634\ 6660 \times \frac{1}{10^8}$	$\cdot 5905\ 1179 \times \frac{1}{10^8}$	$\cdot 3690\ 6987 \times \frac{1}{10^8}$	$\cdot 2348\ 6264 \times \frac{1}{10^8}$	$\cdot 1519\ 699$	
$\cdot 17$	$\cdot 0000\ 001$					
$\cdot 18$	$\cdot 0000\ 002$					
$\cdot 19$	$\cdot 0000\ 004$	$\cdot 0000\ 001$				
$\cdot 20$	$\cdot 0000\ 009$	$\cdot 0000\ 003$	$\cdot 0000\ 001$			
$\cdot 21$	$\cdot 0000\ 019$	$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 22$	$\cdot 0000\ 039$	$\cdot 0000\ 013$	$\cdot 0000\ 005$	$\cdot 0000\ 002$		
$\cdot 23$	$\cdot 0000\ 076$	$\cdot 0000\ 027$	$\cdot 0000\ 010$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 24$	$\cdot 0000\ 143$	$\cdot 0000\ 054$	$\cdot 0000\ 020$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 25$	$\cdot 0000\ 262$	$\cdot 0000\ 102$	$\cdot 0000\ 039$	$\cdot 0000\ 015$	$\cdot 0000\ 006$	$\cdot 0000\ 002$
$\cdot 26$	$\cdot 0000\ 463$	$\cdot 0000\ 188$	$\cdot 0000\ 075$	$\cdot 0000\ 029$	$\cdot 0000\ 011$	$\cdot 0000\ 004$
$\cdot 27$	$\cdot 0000\ 796$	$\cdot 0000\ 335^+$	$\cdot 0000\ 139$	$\cdot 0000\ 056$	$\cdot 0000\ 023$	$\cdot 0000\ 009$
$\cdot 28$	$\cdot 0001\ 333$	$\cdot 0000\ 582$	$\cdot 0000\ 249$	$\cdot 0000\ 105^+$	$\cdot 0000\ 044$	$\cdot 0000\ 018$
$\cdot 29$	$\cdot 0002\ 179$	$\cdot 0000\ 984$	$\cdot 0000\ 437$	$\cdot 0000\ 191$	$\cdot 0000\ 082$	$\cdot 0000\ 035$
$\cdot 30$	$\cdot 0003\ 479$	$\cdot 0001\ 624$	$\cdot 0000\ 745^+$	$\cdot 0000\ 337$	$\cdot 0000\ 150^-$	$\cdot 0000\ 066$
$\cdot 31$	$\cdot 0005\ 437$	$\cdot 0002\ 620$	$\cdot 0001\ 242$	$\cdot 0000\ 579$	$\cdot 0000\ 266$	$\cdot 0000\ 121$
$\cdot 32$	$\cdot 0008\ 324$	$\cdot 0004\ 138$	$\cdot 0002\ 022$	$\cdot 0000\ 973$	$\cdot 0000\ 461$	$\cdot 0000\ 216$
$\cdot 33$	$\cdot 0012\ 499$	$\cdot 0006\ 401$	$\cdot 0003\ 224$	$\cdot 0001\ 599$	$\cdot 0000\ 781$	$\cdot 0000\ 377$
$\cdot 34$	$\cdot 0018\ 427$	$\cdot 0009\ 714$	$\cdot 0005\ 036$	$\cdot 0002\ 571$	$\cdot 0001\ 294$	$\cdot 0000\ 642$
$\cdot 35$	$\cdot 0026\ 697$	$\cdot 0014\ 473$	$\cdot 0007\ 717$	$\cdot 0004\ 052$	$\cdot 0002\ 098$	$\cdot 0001\ 071$
$\cdot 36$	$\cdot 0038\ 041$	$\cdot 0021\ 190$	$\cdot 0011\ 611$	$\cdot 0006\ 266$	$\cdot 0003\ 333$	$\cdot 0001\ 750$
$\cdot 37$	$\cdot 0053\ 355^-$	$\cdot 0030\ 511$	$\cdot 0017\ 166$	$\cdot 0009\ 513$	$\cdot 0005\ 197$	$\cdot 0002\ 803$
$\cdot 38$	$\cdot 0073\ 709$	$\cdot 0043\ 240$	$\cdot 0024\ 960$	$\cdot 0014\ 192$	$\cdot 0007\ 957$	$\cdot 0004\ 403$
$\cdot 39$	$\cdot 0100\ 363$	$\cdot 0060\ 353$	$\cdot 0035\ 716$	$\cdot 0020\ 822$	$\cdot 0011\ 971$	$\cdot 0006\ 794$
$\cdot 40$	$\cdot 0134\ 769$	$\cdot 0083\ 016$	$\cdot 0050\ 330$	$\cdot 0030\ 064$	$\cdot 0017\ 712$	$\cdot 0010\ 301$
$\cdot 41$	$\cdot 0178\ 571$	$\cdot 0112\ 596$	$\cdot 0069\ 886$	$\cdot 0042\ 744$	$\cdot 0025\ 787$	$\cdot 0015\ 358$
$\cdot 42$	$\cdot 0233\ 588$	$\cdot 0150\ 667$	$\cdot 0095\ 677$	$\cdot 0059\ 878$	$\cdot 0036\ 968$	$\cdot 0022\ 534$
$\cdot 43$	$\cdot 0301\ 795^-$	$\cdot 0199\ 000$	$\cdot 0129\ 207$	$\cdot 0082\ 691$	$\cdot 0052\ 211$	$\cdot 0032\ 552$
$\cdot 44$	$\cdot 0385\ 285^+$	$\cdot 0259\ 553$	$\cdot 0172\ 202$	$\cdot 0112\ 629$	$\cdot 0072\ 686$	$\cdot 0046\ 324$
$\cdot 45$	$\cdot 0486\ 222$	$\cdot 0334\ 440$	$\cdot 0226\ 595^+$	$\cdot 0151\ 373$	$\cdot 0099\ 792$	$\cdot 0064\ 976$
$\cdot 46$	$\cdot 0606\ 778$	$\cdot 0425\ 890$	$\cdot 0294\ 509$	$\cdot 0200\ 834$	$\cdot 0135\ 172$	$\cdot 0089\ 866$
$\cdot 47$	$\cdot 0749\ 062$	$\cdot 0536\ 192$	$\cdot 0378\ 220$	$\cdot 0263\ 138$	$\cdot 0180\ 717$	$\cdot 0122\ 610$
$\cdot 48$	$\cdot 0915\ 039$	$\cdot 0667\ 625^-$	$\cdot 0480\ 111$	$\cdot 0340\ 601$	$\cdot 0238\ 559$	$\cdot 0165\ 089$
$\cdot 49$	$\cdot 1106\ 437$	$\cdot 0822\ 374$	$\cdot 0602\ 600$	$\cdot 0435\ 681$	$\cdot 0311\ 049$	$\cdot 0219\ 443$
$\cdot 50$	$\cdot 1324\ 654$	$\cdot 1002\ 442$	$\cdot 0748\ 064$	$\cdot 0550\ 921$	$\cdot 0400\ 717$	$\cdot 0288\ 063$
$\cdot 51$	$\cdot 1570\ 669$	$\cdot 1209\ 543$	$\cdot 0918\ 742$	$\cdot 0688\ 864$	$\cdot 0510\ 215^+$	$\cdot 0373\ 550$
$\cdot 52$	$\cdot 1844\ 945^+$	$\cdot 1445\ 003$	$\cdot 1116\ 630$	$\cdot 0851\ 961$	$\cdot 0642\ 242$	$\cdot 0478\ 601$
$\cdot 53$	$\cdot 2147\ 363$	$\cdot 1709\ 657$	$\cdot 1343\ 367$	$\cdot 1042\ 459$	$\cdot 0799\ 440$	$\cdot 0606\ 241$
$\cdot 54$	$\cdot 2477\ 148$	$\cdot 2003\ 753$	$\cdot 1600\ 121$	$\cdot 1262\ 281$	$\cdot 0984\ 288$	$\cdot 0759\ 113$
$\cdot 55$	$\cdot 2832\ 836$	$\cdot 2326\ 871$	$\cdot 1887\ 480$	$\cdot 1512\ 900$	$\cdot 1198\ 965^+$	$\cdot 0939\ 970$
$\cdot 56$	$\cdot 3212\ 255$	$\cdot 2677\ 864$	$\cdot 2205\ 351$	$\cdot 1795\ 209$	$\cdot 1445\ 221$	$\cdot 1151\ 231$
$\cdot 57$	$\cdot 3612\ 531$	$\cdot 3054\ 818$	$\cdot 2552\ 876$	$\cdot 2109\ 410$	$\cdot 1724\ 229$	$\cdot 1394\ 899$
$\cdot 58$	$\cdot 4030\ 138$	$\cdot 3455\ 046$	$\cdot 2928\ 382$	$\cdot 2454\ 912$	$\cdot 2036\ 454$	$\cdot 1672\ 396$
$\cdot 59$	$\cdot 4460\ 962$	$\cdot 3875\ 114$	$\cdot 3329\ 351$	$\cdot 2830\ 251$	$\cdot 2381\ 536$	$\cdot 1984\ 423$
$\cdot 60$	$\cdot 4900\ 406$	$\cdot 4310\ 905^-$	$\cdot 3752\ 430$	$\cdot 3233\ 049$	$\cdot 2758\ 186$	$\cdot 2330\ 186$

TABLE I. THE  $I_x(p, q)$  FUNCTION

to .95

 $q = 12$  $p$ 

$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$
$\cdot 1605\ 7777 \times 10^8$	$\cdot 0634\ 6660 \times 10^8$	$\cdot 5905\ 1179 \times 10^8$	$\cdot 3690\ 6987 \times 10^8$	$\cdot 2348\ 6264 \times 10^8$	$\cdot 1519\ 5541 \times 10^8$
$\cdot 9954\ 858$	$\cdot 9937\ 607$	$\cdot 9915\ 544$	$\cdot 9887\ 843$	$\cdot 9853\ 653$	$\cdot 9812\ 463$
$\cdot 9971\ 644$	$\cdot 9960\ 399$	$\cdot 9945\ 839$	$\cdot 9927\ 334$	$\cdot 9904\ 212$	$\cdot 9875\ 090$
$\cdot 9982\ 863$	$\cdot 9975\ 819$	$\cdot 9966\ 588$	$\cdot 9954\ 711$	$\cdot 9939\ 690$	$\cdot 9920\ 568$
$\cdot 9990\ 081$	$\cdot 9985\ 859$	$\cdot 9980\ 260$	$\cdot 9972\ 069$	$\cdot 9963\ 637$	$\cdot 9951\ 105$
$\cdot 9994\ 531$	$\cdot 9992\ 123$	$\cdot 9988\ 801$	$\cdot 9984\ 633$	$\cdot 9979\ 117$	$\cdot 9972\ 090$
$\cdot 9997\ 146$	$\cdot 9995\ 847$	$\cdot 9994\ 083$	$\cdot 9991\ 732$	$\cdot 9988\ 651$	$\cdot 9984\ 019$
$\cdot 9998\ 601$	$\cdot 9997\ 944$	$\cdot 9997\ 041$	$\cdot 9995\ 824$	$\cdot 9994\ 210$	$\cdot 9992\ 598$
$\cdot 9999\ 393$	$\cdot 9999\ 054$	$\cdot 9998\ 624$	$\cdot 9998\ 039$	$\cdot 9997\ 254$	$\cdot 9996\ 206$
$\cdot 9999\ 733$	$\cdot 9999\ 600$	$\cdot 9999\ 413$	$\cdot 9999\ 154$	$\cdot 9998\ 804$	$\cdot 9998\ 356$
$\cdot 9999\ 899$	$\cdot 9999\ 847$	$\cdot 9999\ 773$	$\cdot 9999\ 670$	$\cdot 9999\ 529$	$\cdot 9999\ 356$
$\cdot 9999\ 966$	$\cdot 9999\ 948$	$\cdot 9999\ 923$	$\cdot 9999\ 886$	$\cdot 9999\ 836$	$\cdot 9999\ 773$
$\cdot 9999\ 999$	$\cdot 9999\ 985$	$\cdot 9999\ 977$	$\cdot 9999\ 966$	$\cdot 9999\ 951$	$\cdot 9999\ 931$
$\cdot 9999\ 998$	$\cdot 9999\ 996$	$\cdot 9999\ 994$	$\cdot 9999\ 992$	$\cdot 9999\ 988$	$\cdot 9999\ 983$
$1\cdot 0000\ 000$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999\ 998$	$\cdot 9999\ 999$
	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$



TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $= .25$  to  $.80$  $q = 12$  $p =$ 

	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$
$(p, q) = .9986\ 5964 \times \frac{1}{10^{10}}$		$.6657\ 7309 \times \frac{1}{10^{10}}$	$.4498\ 4668 \times \frac{1}{10^{10}}$	$.3077\ 8984 \times \frac{1}{10^{10}}$	$.2130\ 8527 \times \frac{1}{10^{10}}$	$.1491\ 59$
$\frac{1}{2}$						
$.25$	.0000 001					
$.26$	.0000 002	.0000 001				
$.27$	.0000 004	.0000 001	.0000 001			
$.28$	.0000 007	.0000 003	.0000 001			
$.29$	.0000 015 <sup>-</sup>	.0000 006	.0000 002	.0000 001		
$.30$	.0000 029	.0000 012	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
$.31$	.0000 054	.0000 024	.0000 010	.0000 005 <sup>-</sup>	.0000 002	.0000 00
$.32$	.0000 100	.0000 046	.0000 021	.0000 009	.0000 004	.0000 00
$.33$	.0000 179	.0000 084	.0000 039	.0000 018	.0000 008	.0000 00
$.34$	.0000 315 <sup>-</sup>	.0000 153	.0000 073	.0000 035 <sup>-</sup>	.0000 016	.0000 00
$.35$	.0000 540	.0000 269	.0000 133	.0000 065 <sup>-</sup>	.0000 031	.0000 01
$.36$	.0000 908	.0000 465 <sup>+</sup>	.0000 236	.0000 118	.0000 059	.0000 02
$.37$	.0001 493	.0000 786	.0000 409	.0000 211	.0000 108	.0000 05
$.38$	.0002 407	.0001 301	.0000 695 <sup>+</sup>	.0000 368	.0000 193	.0000 10
$.39$	.0003 809	.0002 111	.0001 157	.0000 628	.0000 338	.0000 18
$.40$	.0005 918	.0003 362	.0001 889	.0001 051	.0000 579	.0000 31
$.41$	.0009 038	.0005 258	.0003 027	.0001 725 <sup>+</sup>	.0000 974	.0000 54
$.42$	.0013 572	.0008 083	.0004 763	.0002 779	.0001 606	.0000 92
$.43$	.0020 055 <sup>+</sup>	.0012 219	.0007 367	.0004 398	.0002 601	.0001 52
$.44$	.0029 178	.0018 175 <sup>+</sup>	.0011 204	.0006 839	.0004 136	.0002 47
$.45$	.0041 815 <sup>-</sup>	.0026 615 <sup>+</sup>	.0016 766	.0010 459	.0006 464	.0003 90
$.46$	.0059 057	.0038 390	.0024 700	.0015 738	.0009 936	.0006 21
$.47$	.0082 239	.0054 567	.0035 839	.0023 313	.0015 028	.0009 60
$.48$	.0112 957	.0076 465 <sup>-</sup>	.0051 241	.0034 012	.0022 372	.0014 59
$.49$	.0153 090	.0105 674	.0072 218	.0048 889	.0032 800	.0021 82
$.50$	.0204 798	.0144 084	.0100 369	.0069 265 <sup>-</sup>	.0047 377	.0032 13
$.51$	.0270 515 <sup>+</sup>	.0193 884	.0137 605 <sup>+</sup>	.0096 760	.0067 443	.0046 61
$.52$	.0352 918	.0257 562	.0186 160	.0133 322	.0094 653	.0066 64
$.53$	.0454 873	.0337 878	.0248 588	.0181 242	.0131 007	.0093 92
$.54$	.0579 364	.0437 813	.0327 742	.0243 157	.0178 872	.0130 52
$.55$	.0729 395 <sup>-</sup>	.0560 496	.0426 728	.0322 029	.0240 985 <sup>-</sup>	.0178 89
$.56$	.0907 859	.0709 105 <sup>-</sup>	.0548 830	.0421 103	.0320 435 <sup>-</sup>	.0241 91
$.57$	.1117 402	.0886 733	.0697 400	.0543 822	.0420 618	.0322 80
$.58$	.1360 256	.1096 237	.0875 729	.0693 725 <sup>-</sup>	.0545 156	.0425 13
$.59$	.1638 071	.1340 065 <sup>-</sup>	.1086 878	.0874 299	.0697 780	.0552 71
$.60$	.1951 745 <sup>+</sup>	.1620 064	.1333 491	.1088 802	.0882 175 <sup>+</sup>	.0709 49
$.61$	.2301 263	.1937 296	.1617 587	.1340 055 <sup>-</sup>	.1101 788	.0899 34
$.62$	.2685 557	.2291 859	.1940 356	.1630 222	.1359 601	.1125 90
$.63$	.3102 455 <sup>+</sup>	.2682 730	.2301 955 <sup>+</sup>	.1960 576	.1657 887	.1392 27
$.64$	.3548 537	.3107 657	.2701 341	.2331 282	.1997 954	.1700 81
$.65$	.4019 242	.3563 092	.3136 135 <sup>-</sup>	.2741 199	.2379 907	.2052 80
$.66$	.4508 905 <sup>+</sup>	.4044 205 <sup>+</sup>	.3602 554	.3187 742	.2802 428	.2448 21
$.67$	.5010 909	.4544 965 <sup>+</sup>	.4095 422	.3666 801	.3262 626	.2885 44
$.68$	.5517 908	.5058 303	.4608 257	.4172 752	.3755 957	.3361 19
$.69$	.6022 100	.5576 351	.5133 456	.4608 566	.4276 240	.3870 35

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81 \text{ to } .96$  $q = 12$ 

	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$
$B(p, q) =$	$.9986\ 5964 \times \frac{1}{10^{10}}$	$.6657\ 7309 \times \frac{1}{10^{10}}$	$.4498\ 4668 \times \frac{1}{10^{10}}$	$.3077\ 8984 \times \frac{1}{10^{10}}$	$.2130\ 85$
$x$					
.81	.9762 371	.9703 615 <sup>-</sup>	.9635 081	.9556 084	.9466 02
.82	.9841 297	.9800 070	.9751 390	.9694 585 <sup>-</sup>	.9629 02
.83	.9898 043	.9870 270	.9837 075 <sup>-</sup>	.9797 866	.9752 06
.84	.9937 278	.9919 392	.9897 757	.9871 895 <sup>-</sup>	.9841 32
.85	.9963 250 <sup>-</sup>	.9952 297	.9938 891	.9922 675 <sup>-</sup>	.9903 27
.86	.9979 624	.9973 288	.9965 440	.9955 836	.9944 21
.87	.9989 395 <sup>-</sup>	.9985 958	.9981 652	.9976 322	.9969 79
.88	.9994 869	.9993 138	.9990 946	.9988 200	.9984 79
.89	.9997 721	.9996 922	.9995 898	.9994 601	.9992 97
.90	.9999 085 <sup>+</sup>	.9998 752	.9998 321	.9997 769	.9997 06
.91	.9999 675 <sup>+</sup>	.9999 553	.9999 392	.9999 184	.9998 91
.92	.9999 901	.9999 862	.9999 811	.9999 743	.9999 65
.93	.9999 975 <sup>-</sup>	.9999 965 <sup>-</sup>	.9999 951	.9999 933	.9999 91
.94	.9999 995 <sup>+</sup>	.9999 993	.9999 990	.9999 987	.9999 98
.95	.9999 999	.9999 999	.9999 999	.9999 998	.9999 99
.96	1.0000 000	1.0000 000	1.0000 000	1.0000 000	1.0000 00

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

2 to '97

$q = 12$

$p = 3$

$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$
$q) = .1055\ 0320 \times \frac{1}{100}$	$.7535\ 9425 \times \frac{1}{100}$	$.5432\ 8888 \times \frac{1}{100}$	$.3951\ 1919 \times \frac{1}{100}$	$.2897\ 5407 \times \frac{1}{100}$	$.2141\ 6602$
2 .0000 001					
3 .0000 002	.0000 001				
4 .0000 004	.0000 002	.0000 001			
5 .0000 007	.0000 003	.0000 002	.0000 001		
6 .0000 014	.0000 007	.0000 003	.0000 002	.0000 001	
7 .0000 027	.0000 014	.0000 007	.0000 003	.0000 002	.0000 001
8 .0000 052	.0000 026	.0000 013	.0000 007	.0000 003	.0000 002
9 .0000 095 <sup>+</sup>	.0000 050 <sup>-</sup>	.0000 026	.0000 013	.0000 007	.0000 003
0 .0000 171	.0000 092	.0000 049	.0000 026	.0000 014	.0000 007
1 .0000 302	.0000 166	.0000 091	.0000 049	.0000 027	.0000 014
2 .0000 523	.0000 295 <sup>-</sup>	.0000 165 <sup>-</sup>	.0000 091	.0000 050 <sup>+</sup>	.0000 028
3 .0000 886	.0000 511	.0000 292	.0000 166	.0000 094	.0000 053
4 .0001 474	.0000 869	.0000 509	.0000 296	.0000 171	.0000 098
5 .0002 407	.0001 451	.0000 868	.0000 516	.0000 304	.0000 178
6 .0003 860	.0002 378	.0001 454	.0000 882	.0000 532	.0000 319
7 .0006 087	.0003 828	.0002 390	.0001 482	.0000 912	.0000 558
8 .0009 438	.0006 058	.0003 860	.0002 443	.0001 535 <sup>+</sup>	.0000 959
9 .0014 398	.0009 428	.0006 129	.0003 957	.0002 537	.0001 617
0 .0021 620	.0014 436	.0009 570	.0006 300	.0004 120	.0002 678
1 .0031 967	.0021 755 <sup>+</sup>	.0014 700	.0009 865 <sup>-</sup>	.0006 577	.0004 357
2 .0046 557	.0032 281	.0022 223	.0015 195 <sup>+</sup>	.0010 323	.0006 969
3 .0066 813	.0047 177	.0033 077	.0023 035 <sup>-</sup>	.0015 938	.0010 960
4 .0094 508	.0067 930	.0048 485 <sup>-</sup>	.0034 375 <sup>+</sup>	.0024 217	.0016 956
5 .0131 799	.0096 396	.0070 014	.0050 517	.0036 219	.0025 811
6 .0181 263	.0134 845 <sup>+</sup>	.0099 627	.0073 126	.0053 338	.0038 672
7 .0245 900	.0185 992	.0139 728	.0104 293	.0077 362	.0057 045 <sup>+</sup>
8 .0329 114	.0253 004	.0193 196	.0146 583	.0110 536	.0082 864
9 .0434 669	.0339 481	.0263 393	.0203 069	.0155 614	.0118 557
0 .0566 587	.0449 402	.0354 143	.0277 339	.0215 897	.0167 104
1 .0729 017	.0587 019	.0469 664	.0373 470	.0295 232	.0232 066
2 .0926 047	.0756 704	.0614 456	.0495 946	.0397 977	.0317 585 <sup>+</sup>
3 .1161 470	.0962 746	.0793 126	.0649 531	.0528 912	.0428 337
4 .1438 516	.1209 090	.1010 155 <sup>+</sup>	.0839 072	.0693 080	.0569 417
5 .1759 547	.1499 033	.1269 613	.1069 229	.0895 563	.0746 159
6 .2125 757	.1834 901	.1574 819	.1344 159	.1141 178	.0963 869
7 .2536 877	.2217 705 <sup>+</sup>	.1927 980	.1667 136	.1434 116	.1227 482
8 .2990 931	.2646 835 <sup>-</sup>	.2329 822	.2040 151	.1777 517	.1541 146
9 .3484 064	.3119 792	.2779 256	.2463 515 <sup>+</sup>	.2173 034	.1907 756
0 .4010 478	.3632 027	.3273 109	.2935 502	.2620 402	.2328 471
1 .4562 496	.4176 894	.3805 969	.3452 084	.3117 073	.2802 261
2 .5130 776	.4745 755 <sup>+</sup>	.4370 173	.4006 798	.3657 958	.3325 533
3 .5704 672	.5328 260	.4955 976	.4590 788	.4235 339	.3891 912
4 .6272 739	.5912 786	.5551 905 <sup>+</sup>	.5193 054	.4838 987	.4492 210
5 .6823 343	.6487 040	.6145 312	.5800 915 <sup>-</sup>	.5456 518	.5114 653
6 .7345 349	.7038 761	.6723 074	.6400 628	.6072 282	.5738 282

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .38$  to  $.97$  $q = 12$ 

	$p = 36$	$p = 37$	$p = 38$	$p = 39$
$B(p, q) = .1594\ 8536 \times \frac{1}{10^{11}}$	$.1196\ 1402 \times \frac{1}{10^{11}}$	$.9032\ 0789 \times \frac{1}{10^{11}}$	$.6864\ 3800 \times \frac{1}{10^{11}}$	
$\cdot 38$	.0000 001			
$\cdot 39$	.0000 002	.0000 001		
$\cdot 40$	.0000 004	.0000 002	.0000 001	
$\cdot 41$	.0000 008	.0000 004		
$\cdot 42$	.0000 015 <sup>+</sup>	.0000 008	.0000 002	.0000 001
$\cdot 43$	.0000 029	.0000 016	.0000 004	.0000 002
$\cdot 44$	.0000 056	.0000 032	.0000 009	.0000 005 <sup>-</sup>
$\cdot 45$	.0000 104	.0000 060	.0000 018	.0000 010
$\cdot 46$	.0000 190	.0000 112	.0000 035 <sup>-</sup>	.0000 020
$\cdot 47$	.0000 339	.0000 205 <sup>+</sup>	.0000 066	.0000 039
$\cdot 48$	.0000 595 <sup>+</sup>	.0000 367	.0000 123	.0000 074
$\cdot 49$	.0001 024	.0000 645 <sup>+</sup>	.0000 226	.0000 138
$\cdot 50$	.0001 730	.0001 111	.0000 404	.0000 252
			.0000 710	.0000 451
$\cdot 51$	.0002 870	.0001 879	.0001 224	.0000 793
$\cdot 52$	.0004 677	.0003 121	.0002 072	.0001 368
$\cdot 53$	.0007 493	.0005 094	.0003 444	.0002 317
$\cdot 54$	.0011 803	.0008 170	.0005 625 <sup>+</sup>	.0003 853
$\cdot 55$	.0018 287	.0012 885 <sup>-</sup>	.0009 030	.0006 297
$\cdot 56$	.0027 878	.0019 986	.0014 253	.0010 113
$\cdot 57$	.0041 825 <sup>-</sup>	.0030 498	.0022 123	.0015 967
$\cdot 58$	.0061 770	.0045 798	.0033 779	.0024 791
$\cdot 59$	.0089 823	.0067 689	.0050 748	.0037 859
$\cdot 60$	.0128 628	.0098 488	.0075 028	.0056 877
$\cdot 61$	.0181 425 <sup>+</sup>	.0141 095 <sup>+</sup>	.0109 180	.0084 077
$\cdot 62$	.0252 077	.0199 052	.0156 403	.0122 306
$\cdot 63$	.0345 061	.0276 566	.0220 585 <sup>+</sup>	.0175 109
$\cdot 64$	.0465 399	.0378 486	.0306 324	.0246 772
$\cdot 65$	.0618 525 <sup>+</sup>	.0510 214	.0418 881	.0342 330
$\cdot 66$	.0810 065 <sup>+</sup>	.0677 538	.0564 066	.0467 495 <sup>+</sup>
$\cdot 67$	.1045 525 <sup>+</sup>	.0886 364	.0748 023	.0628 504
$\cdot 68$	.1329 898	.1142 356	.0976 914	.0831 848
$\cdot 69$	.1667 187	.1450 474	.1256 487	.1083 890
$\cdot 70$	.2059 895 <sup>+</sup>	.1814 446	.1591 552	.1390 361
$\cdot 71$	.2508 496	.2236 193	.1985 379	.1755 753
$\cdot 72$	.3010 965 <sup>+</sup>	.2715 271	.2439 076	.2182 652
$\cdot 73$	.3562 419	.3248 394	.2951 003	.2671 065 <sup>-</sup>
$\cdot 74$	.4154 945 <sup>+</sup>	.3829 109	.3516 307	.3217 828
$\cdot 75$	.4777 672	.4447 711	.4126 668	.3816 186
$\cdot 76$	.5417 128	.5091 440	.4770 330	.4455 642
$\cdot 77$	.6057 908	.5745 029	.5432 488	.5122 168
$\cdot 78$	.6683 618	.6391 570	.6096 049	.5798 817
$\cdot 79$	.7278 027	.7013 675 <sup>-</sup>	.6742 749	.6466 761
$\cdot 80$	.7826 324	.7594 810	.7354 537	.7106 676
$\cdot 81$	.8316 225 <sup>+</sup>	.8120 675 <sup>-</sup>	.7885 080	.7666 676

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

7 to .80

$q = 12$

$p = 1$

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$
$q) = .1605\ 7777 \times \frac{1}{10^3}$	$.9634\ 6660 \times \frac{1}{10^3}$	$.5905\ 1179 \times \frac{1}{10^3}$	$.3690\ 6987 \times \frac{1}{10^3}$	$.2348\ 6264 \times \frac{1}{10^3}$	$.1519\ 6994$	
7	.0000 001					
8	.0000 002					
9	.0000 004	.0000 001				
0	.0000 009	.0000 003	.0000 001			
1	.0000 019	.0000 006	.0000 002	.0000 001		
2	.0000 039	.0000 013	.0000 005 <sup>-</sup>	.0000 002		
3	.0000 076	.0000 027	.0000 010	.0000 003	.0000 001	
4	.0000 143	.0000 054	.0000 020	.0000 007	.0000 003	.0000 001
5	.0000 262	.0000 102	.0000 039	.0000 015 <sup>-</sup>	.0000 006	.0000 002
6	.0000 463	.0000 188	.0000 075 <sup>-</sup>	.0000 029	.0000 011	.0000 004
7	.0000 796	.0000 335 <sup>+</sup>	.0000 139	.0000 056	.0000 023	.0000 009
8	.0001 333	.0000 582	.0000 249	.0000 105 <sup>+</sup>	.0000 044	.0000 018
9	.0002 179	.0000 984	.0000 437	.0000 191	.0000 082	.0000 035 <sup>-</sup>
0	.0003 479	.0001 624	.0000 745 <sup>+</sup>	.0000 337	.0000 150 <sup>-</sup>	.0000 066
1	.0005 437	.0002 620	.0001 242	.0000 579	.0000 266	.0000 121
2	.0008 324	.0004 138	.0002 022	.0000 973	.0000 461	.0000 216
3	.0012 499	.0006 401	.0003 224	.0001 599	.0000 781	.0000 377
4	.0018 427	.0009 714	.0005 036	.0002 571	.0001 294	.0000 642
5	.0026 697	.0014 473	.0007 717	.0004 052	.0002 098	.0001 071
6	.0038 041	.0021 190	.0011 611	.0006 266	.0003 333	.0001 750 <sup>-</sup>
7	.0053 355 <sup>-</sup>	.0030 511	.0017 166	.0009 513	.0005 197	.0002 803
8	.0073 709	.0043 240	.0024 960	.0014 192	.0007 957	.0004 403
9	.0100 363	.0060 353	.0035 716	.0020 822	.0011 971	.0006 794
0	.0134 769	.0083 016	.0050 330	.0030 064	.0017 712	.0010 301
1	.0178 571	.0112 596	.0069 886	.0042 744	.0025 787	.0015 358
2	.0233 588	.0150 667	.0095 677	.0059 878	.0030 968	.0022 534
3	.0301 795 <sup>-</sup>	.0199 000	.0129 207	.0082 691	.0052 211	.0032 552
4	.0385 285 <sup>+</sup>	.0259 553	.0172 202	.0112 629	.0072 686	.0046 324
5	.0486 222	.0334 440	.0226 595 <sup>+</sup>	.0151 373	.0099 792	.0064 976
6	.0606 778	.0425 890	.0294 509	.0200 834	.0135 172	.0089 866
7	.0749 062	.0536 192	.0378 220	.0263 138	.0180 717	.0122 610
8	.0915 039	.0667 625 <sup>-</sup>	.0480 111	.0340 601	.0238 559	.0165 089
9	.1106 437	.0822 374	.0602 600	.0435 681	.0311 049	.0219 443
0	.1324 654	.1002 442	.0748 064	.0550 921	.0400 717	.0288 063
1	.1570 669	.1209 543	.0918 742	.0688 864	.0510 215 <sup>+</sup>	.0373 550 <sup>-</sup>
2	.1844 945 <sup>+</sup>	.1445 003	.1116 630	.0851 961	.0642 242	.0478 601
3	.2147 363	.1709 657	.1343 367	.1042 459	.0799 440	.0606 241
4	.2477 148	.2003 753	.1600 121	.1262 281	.0984 288	.0759 113
5	.2832 836	.2326 871	.1887 480	.1512 900	.1198 965 <sup>+</sup>	.0939 970
6	.3212 255	.2677 864	.2205 351	.1795 209	.1445 221	.1151 231
7	.3612 531	.3054 818	.2552 876	.2109 410	.1724 229	.1394 899
8	.4030 138	.3455 046	.2928 382	.2454 912	.2036 454	.1672 396
9	.4460 962	.3875 114	.3329 351	.2830 251	.2381 536	.1984 423
0	.4900 406	.4310 905 <sup>-</sup>	.3752 430	.3233 049	.2758 186	.2330 810

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.95$  $q = 12$ 

	$p = 18$	$p = 19$	$p = 20$	$p = 21$	$p =$
$B(p, q) = .1605\ 7777 \times \frac{x}{10^8}$	$.9634\ 6660 \times \frac{x}{10^8}$	$.5905\ 1179 \times \frac{x}{10^8}$	$.3690\ 6987 \times \frac{x}{10^8}$	$.2348$	
$x$					
.81	.9954 858	.9937 607	.9915 544	.9887 843	.9853
.82	.9971 644	.9960 399	.9945 839	.9927 334	.9904
.83	.9982 863	.9975 819	.9966 588	.9954 711	.9939
.84	.9990 081	.9985 859	.9980 260	.9972 969	.9963
.85	.9994 531	.9992 123	.9988 891	.9984 633	.9979
.86	.9997 146	.9995 847	.9994 083	.9991 732	.9988
.87	.9998 601	.9997 944	.9997 041	.9995 824	.9994
.88	.9999 363	.9999 054	.9998 624	.9998 039	.9997
.89	.9999 733	.9999 600	.9999 413	.9999 154	.9998
.90	.9999 899	.9999 847	.9999 773	.9999 670	.9999
.91	.9999 966	.9999 948	.9999 923	.9999 886	.9999
.92	.9999 990	.9999 985	.9999 977	.9999 966	.9999
.93	.9999 998	.9999 996	.9999 994	.9999 992	.9999
.94	1.0000 000	.9999 999	.9999 999	.9999 998	.9999
.95		1.0000 000	1.0000 000	1.0000 000	1.0000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

5 to .80

$q = 12$

$p = 2$

$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$
$q = .9986\ 5964 \times \frac{1}{100}$	$.6657\ 7309 \times \frac{1}{100}$	$.4498\ 4668 \times \frac{1}{100}$	$.3077\ 8984 \times \frac{1}{100}$	$.2130\ 8527 \times \frac{1}{100}$	$.1491\ 596$
.0000 001					
.0000 002	.0000 001				
.0000 004	.0000 001	.0000 001			
.0000 007	.0000 003	.0000 001			
.0000 015-	.0000 006	.0000 002	.0000 001		
.0000 029	.0000 012	.0000 005+	.0000 002	.0000 001	
.0000 054	.0000 024	.0000 010	.0000 005-	.0000 002	.0000 001
.0000 100	.0000 046	.0000 021	.0000 009	.0000 004	.0000 002
.0000 179	.0000 084	.0000 039	.0000 018	.0000 008	.0000 004
.0000 315-	.0000 153	.0000 073	.0000 035-	.0000 016	.0000 008
.0000 540	.0000 269	.0000 133	.0000 065-	.0000 031	.0000 015+
.0000 908	.0000 465+	.0000 236	.0000 118	.0000 059	.0000 029
.0001 493	.0000 786	.0000 409	.0000 211	.0000 108	.0000 054
.0002 407	.0001 301	.0000 695+	.0000 368	.0000 193	.0000 100
.0003 809	.0002 111	.0001 157	.0000 628	.0000 338	.0000 180
.0005 918	.0003 362	.0001 889	.0001 051	.0000 579	.0000 316
.0009 038	.0005 258	.0003 027	.0001 725+	.0000 974	.0000 545-
.0013 572	.0008 083	.0004 763	.0002 779	.0001 606	.0000 920
.0020 055+	.0012 219	.0007 367	.0004 398	.0002 601	.0001 524
.0029 178	.0018 175+	.0011 204	.0006 839	.0004 136	.0002 479
.0041 815-	.0026 615+	.0016 766	.0010 459	.0006 464	.0003 961
.0059 057	.0038 390	.0024 700	.0015 738	.0009 936	.0006 219
.0082 239	.0054 567	.0035 839	.0023 313	.0015 028	.0009 603
.0112 957	.0076 465-	.0051 241	.0034 012	.0022 372	.0014 590
.0153 090	.0105 674	.0072 218	.0048 889	.0032 800	.0021 820
.0204 798	.0144 084	.0100 369	.0069 265-	.0047 377	.0032 133
.0270 515+	.0193 884	.0137 605+	.0096 760	.0067 443	.0046 617
.0352 918	.0257 562	.0186 160	.0133 322	.0094 653	.0066 645+
.0454 873	.0337 878	.0248 588	.0181 242	.0131 007	.0093 923
.0579 364	.0437 813	.0327 742	.0243 157	.0178 872	.0130 521
.0729 395-	.0560 496	.0426 728	.0322 029	.0240 985-	.0178 899
.0907 859	.0709 105-	.0548 830	.0421 103	.0320 435-	.0241 914
.1117 402	.0886 733	.0697 400	.0543 822	.0420 618	.0322 802
.1360 256	.1096 237	.0875 729	.0693 725-	.0545 156	.0425 133
.1638 071	.1340 065-	.1086 878	.0874 299	.0697 780	.0552 719
.1951 745+	.1620 064	.1333 491	.1088 802	.0882 175+	.0709 495-
.2301 263	.1937 296	.1617 587	.1340 055-	.1101 788	.0899 347
.2685 567	.2291 859	.1940 356	.1630 222	.1359 601	.1125 901
.3102 455+	.2682 730	.2301 955+	.1960 576	.1657 887	.1392 276
.3548 537	.3107 657	.2701 341	.2331 282	.1997 954	.1700 817
.4019 242	.3563 092	.3136 135-	.2741 199	.2379 907	.2052 808
.4508 905+	.4044 205+	.3602 554	.3187 742	.2802 428	.2448 214
.5010 909	.4544 905+	.4095 422	.3666 801	.3262 626	.2885 445-
.5517 908	.5058 303	.4608 257	.4172 752	.3755 957	.3361 193
.6022 100	.5576 351	.5133 456	.4608 566		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.96$  $q = 12$ 

	$p = 24$	$p = 25$	$p = 26$	$p = 27$	$p =$
$B(p, q) = .9986\ 5964 \times \frac{1}{10^{10}}$		$.6657\ 7309 \times \frac{1}{10^{10}}$	$.4498\ 4668 \times \frac{1}{10^{10}}$	$.3077\ 8984 \times \frac{1}{10^{10}}$	$.2130$
$\cdot 81$	$.9762\ 371$	$.9703\ 615^-$	$.9635\ 081$	$.9556\ 084$	$.9466$
$\cdot 82$	$.9841\ 297$	$.9800\ 070$	$.9751\ 390$	$.9694\ 585^-$	$.9629$
$\cdot 83$	$.9898\ 043$	$.9870\ 270$	$.9837\ 075^-$	$.9797\ 866$	$.9752$
$\cdot 84$	$.9937\ 278$	$.9919\ 392$	$.9897\ 757$	$.9871\ 895^-$	$.9841$
$\cdot 85$	$.9963\ 250^-$	$.9952\ 297$	$.9938\ 891$	$.9922\ 675^-$	$.9903$
$\cdot 86$	$.9979\ 624$	$.9973\ 288$	$.9965\ 440$	$.9955\ 836$	$.9944$
$\cdot 87$	$.9989\ 395^-$	$.9985\ 958$	$.9981\ 652$	$.9976\ 322$	$.9969$
$\cdot 88$	$.9994\ 869$	$.9993\ 138$	$.9990\ 946$	$.9988\ 200$	$.9984$
$\cdot 89$	$.9997\ 721$	$.9996\ 922$	$.9995\ 898$	$.9994\ 601$	$.9992$
$\cdot 90$	$.9999\ 085^+$	$.9998\ 752$	$.9998\ 321$	$.9997\ 769$	$.9997$
$\cdot 91$	$.9999\ 675^+$	$.9999\ 553$	$.9999\ 392$	$.9999\ 184$	$.9998$
$\cdot 92$	$.9999\ 901$	$.9999\ 862$	$.9999\ 811$	$.9999\ 743$	$.9999$
$\cdot 93$	$.9999\ 975^-$	$.9999\ 965^-$	$.9999\ 951$	$.9999\ 933$	$.9999$
$\cdot 94$	$.9999\ 995^+$	$.9999\ 993$	$.9999\ 990$	$.9999\ 987$	$.9999$
$\cdot 95$	$.9999\ 999$	$.9999\ 999$	$.9999\ 999$	$.9999\ 998$	$.9999$
$\cdot 96$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$1.0000$



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

2 to '97

$q = 12$

$p = 3$

$p = 30$	$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$
$q) = \cdot 1055\ 0320 \times \frac{1}{10^6}$	$\cdot 7535\ 9425 \times \frac{1}{10^{11}}$	$\cdot 5432\ 8888 \times \frac{1}{10^{11}}$	$\cdot 3951\ 1919 \times \frac{1}{10^{11}}$	$\cdot 2897\ 5407 \times \frac{1}{10^{11}}$	$\cdot 2141\ 6605$
$\cdot 0000\ 001$					
$\cdot 0000\ 002$	$\cdot 0000\ 001$				
$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 002$			
$\cdot 0000\ 014$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 0000\ 027$	$\cdot 0000\ 014$	$\cdot 0000\ 007$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	
$\cdot 0000\ 052$	$\cdot 0000\ 026$	$\cdot 0000\ 013$	$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 095^+$	$\cdot 0000\ 050^-$	$\cdot 0000\ 026$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 002$
$\cdot 0000\ 171$	$\cdot 0000\ 092$	$\cdot 0000\ 049$	$\cdot 0000\ 013$	$\cdot 0000\ 007$	$\cdot 0000\ 003$
			$\cdot 0000\ 026$	$\cdot 0000\ 014$	$\cdot 0000\ 007$
$\cdot 0000\ 302$	$\cdot 0000\ 166$	$\cdot 0000\ 091$			
$\cdot 0000\ 523$	$\cdot 0000\ 295^-$	$\cdot 0000\ 165^-$	$\cdot 0000\ 049$	$\cdot 0000\ 027$	$\cdot 0000\ 014$
$\cdot 0000\ 886$	$\cdot 0000\ 511$	$\cdot 0000\ 292$	$\cdot 0000\ 091$	$\cdot 0000\ 050^+$	$\cdot 0000\ 028$
$\cdot 0001\ 474$	$\cdot 0000\ 869$	$\cdot 0000\ 509$	$\cdot 0000\ 166$	$\cdot 0000\ 094$	$\cdot 0000\ 053$
$\cdot 0002\ 407$	$\cdot 0001\ 451$	$\cdot 0000\ 868$	$\cdot 0000\ 296$	$\cdot 0000\ 171$	$\cdot 0000\ 098$
$\cdot 0003\ 860$	$\cdot 0002\ 378$	$\cdot 0001\ 454$	$\cdot 0000\ 516$	$\cdot 0000\ 304$	$\cdot 0000\ 178$
$\cdot 0006\ 087$	$\cdot 0003\ 828$	$\cdot 0002\ 390$	$\cdot 0000\ 882$	$\cdot 0000\ 532$	$\cdot 0000\ 319$
$\cdot 0009\ 438$	$\cdot 0006\ 038$	$\cdot 0003\ 860$	$\cdot 0001\ 482$	$\cdot 0000\ 912$	$\cdot 0000\ 558$
$\cdot 0014\ 398$	$\cdot 0009\ 428$	$\cdot 0006\ 129$	$\cdot 0002\ 443$	$\cdot 0001\ 535^+$	$\cdot 0000\ 959$
$\cdot 0021\ 620$	$\cdot 0014\ 436$	$\cdot 0009\ 570$	$\cdot 0003\ 957$	$\cdot 0002\ 537$	$\cdot 0001\ 617$
			$\cdot 0006\ 300$	$\cdot 0004\ 120$	$\cdot 0002\ 678$
$\cdot 0031\ 967$	$\cdot 0021\ 755^+$	$\cdot 0014\ 700$	$\cdot 0009\ 865^-$		
$\cdot 0046\ 557$	$\cdot 0032\ 281$	$\cdot 0022\ 223$	$\cdot 0015\ 195^+$	$\cdot 0006\ 577$	$\cdot 0004\ 357$
$\cdot 0066\ 813$	$\cdot 0047\ 177$	$\cdot 0033\ 077$	$\cdot 0023\ 035^-$	$\cdot 0010\ 323$	$\cdot 0006\ 969$
$\cdot 0094\ 508$	$\cdot 0067\ 930$	$\cdot 0048\ 485^-$	$\cdot 0034\ 375^+$	$\cdot 0015\ 938$	$\cdot 0010\ 960$
$\cdot 0131\ 799$	$\cdot 0096\ 396$	$\cdot 0070\ 014$	$\cdot 0050\ 517$	$\cdot 0024\ 217$	$\cdot 0016\ 956$
$\cdot 0181\ 263$	$\cdot 0134\ 845^+$	$\cdot 0099\ 627$	$\cdot 0073\ 126$	$\cdot 0036\ 219$	$\cdot 0025\ 811$
$\cdot 0245\ 900$	$\cdot 0185\ 992$	$\cdot 0139\ 728$	$\cdot 0104\ 293$	$\cdot 0053\ 338$	$\cdot 0038\ 672$
$\cdot 0329\ 114$	$\cdot 0253\ 004$	$\cdot 0193\ 196$	$\cdot 0146\ 583$	$\cdot 0077\ 362$	$\cdot 0057\ 045^+$
$\cdot 0434\ 669$	$\cdot 0339\ 481$	$\cdot 0263\ 393$	$\cdot 0110\ 536$	$\cdot 0110\ 536$	$\cdot 0082\ 864$
$\cdot 0566\ 587$	$\cdot 0449\ 402$	$\cdot 0354\ 143$	$\cdot 0203\ 069$	$\cdot 0155\ 614$	$\cdot 0118\ 557$
			$\cdot 0277\ 339$	$\cdot 0215\ 897$	$\cdot 0167\ 104$
$\cdot 0729\ 017$	$\cdot 0587\ 019$	$\cdot 0469\ 664$			
$\cdot 0926\ 047$	$\cdot 0756\ 704$	$\cdot 0614\ 456$	$\cdot 0373\ 470$	$\cdot 0295\ 232$	$\cdot 0232\ 066$
$\cdot 1161\ 470$	$\cdot 0962\ 746$	$\cdot 0793\ 126$	$\cdot 0495\ 946$	$\cdot 0397\ 977$	$\cdot 0317\ 585^+$
$\cdot 1438\ 516$	$\cdot 1209\ 090$	$\cdot 1010\ 155^+$	$\cdot 0649\ 531$	$\cdot 0528\ 912$	$\cdot 0428\ 337$
$\cdot 1759\ 547$	$\cdot 1499\ 033$	$\cdot 1269\ 613$	$\cdot 0839\ 072$	$\cdot 0693\ 080$	$\cdot 0569\ 417$
$\cdot 2125\ 757$	$\cdot 1834\ 901$	$\cdot 1574\ 819$	$\cdot 1069\ 229$	$\cdot 0895\ 563$	$\cdot 0746\ 159$
$\cdot 2536\ 877$	$\cdot 2217\ 705^+$	$\cdot 1927\ 980$	$\cdot 1344\ 159$	$\cdot 1141\ 178$	$\cdot 0963\ 869$
$\cdot 2990\ 931$	$\cdot 2646\ 835^-$	$\cdot 2329\ 822$	$\cdot 1667\ 136$	$\cdot 1434\ 116$	$\cdot 1227\ 482$
$\cdot 3484\ 064$	$\cdot 3119\ 792$	$\cdot 2779\ 256$	$\cdot 2040\ 151$	$\cdot 1777\ 517$	$\cdot 1541\ 146$
$\cdot 4010\ 478$	$\cdot 3632\ 027$	$\cdot 3273\ 109$	$\cdot 2463\ 515^+$	$\cdot 2173\ 034$	$\cdot 1907\ 756$
			$\cdot 2935\ 502$	$\cdot 2620\ 402$	$\cdot 2328\ 471$
$\cdot 4562\ 496$	$\cdot 4176\ 894$	$\cdot 3805\ 969$			
$\cdot 5130\ 776$	$\cdot 4745\ 755^+$	$\cdot 4370\ 173$	$\cdot 3452\ 084$	$\cdot 3117\ 073$	$\cdot 2802\ 261$
$\cdot 5704\ 672$	$\cdot 5328\ 260$	$\cdot 4955\ 976$	$\cdot 4006\ 798$	$\cdot 3657\ 958$	$\cdot 3325\ 533$
$\cdot 6272\ 739$	$\cdot 5912\ 786$	$\cdot 5551\ 905^+$	$\cdot 4590\ 788$	$\cdot 4235\ 339$	$\cdot 3891\ 912$
$\cdot 6823\ 343$	$\cdot 6487\ 040$	$\cdot 6145\ 312$	$\cdot 5193\ 054$	$\cdot 4838\ 987$	$\cdot 4492\ 210$
$\cdot 7345\ 349$	$\cdot 7038\ 761$	$\cdot 6723\ 074$	$\cdot 5800\ 915^-$	$\cdot 5456\ 518$	$\cdot 5114\ 653$
$\cdot 7838\ 824$			$\cdot 6400\ 678$	$\cdot 6073\ 084$	$\cdot 5745\ 228$

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .38$  to  $.97$  $q = 12$ 

	$p = 36$	$p = 37$	$p = 38$	$p = 39$	
$B(p, q) = .1594\ 8536 \times \frac{1}{10^{11}}$		$.1196\ 1402 \times \frac{1}{10^{11}}$	$.9032\ 0789 \times \frac{1}{10^{12}}$	$.6864\ 3800 \times \frac{1}{10^{12}}$	
$\cdot 38$	.0000 001				
$\cdot 39$	.0000 002	.0000 001			
$\cdot 40$	.0000 004	.0000 002	.0000 001		
$\cdot 41$	.0000 008	.0000 004	.0000 002	.0000 001	
$\cdot 42$	.0000 015 <sup>+</sup>	.0000 008	.0000 004	.0000 002	
$\cdot 43$	.0000 029	.0000 016	.0000 009	.0000 005 <sup>-</sup>	
$\cdot 44$	.0000 056	.0000 032	.0000 018	.0000 010	
$\cdot 45$	.0000 104	.0000 060	.0000 035 <sup>-</sup>	.0000 020	
$\cdot 46$	.0000 190	.0000 112	.0000 066	.0000 039	
$\cdot 47$	.0000 339	.0000 205 <sup>+</sup>	.0000 123	.0000 074	
$\cdot 48$	.0000 595 <sup>+</sup>	.0000 367	.0000 226	.0000 138	
$\cdot 49$	.0001 024	.0000 645 <sup>+</sup>	.0000 404	.0000 252	
$\cdot 50$	.0001 730	.0001 111	.0000 710	.0000 451	
$\cdot 51$	.0002 870	.0001 879	.0001 224	.0000 793	
$\cdot 52$	.0004 677	.0003 121	.0002 072	.0001 368	
$\cdot 53$	.0007 493	.0005 094	.0003 444	.0002 317	
$\cdot 54$	.0011 803	.0008 170	.0005 625 <sup>+</sup>	.0003 853	
$\cdot 55$	.0018 287	.0012 885 <sup>-</sup>	.0009 030	.0006 297	
$\cdot 56$	.0027 878	.0019 986	.0014 253	.0010 113	
$\cdot 57$	.0041 825 <sup>-</sup>	.0030 498	.0022 123	.0015 967	
$\cdot 58$	.0061 770	.0045 708	.0033 779	.0024 791	
$\cdot 59$	.0089 823	.0067 689	.0050 748	.0037 859	
$\cdot 60$	.0128 628	.0098 488	.0075 028	.0056 877	
$\cdot 61$	.0181 425 <sup>+</sup>	.0141 095 <sup>+</sup>	.0109 180	.0084 077	
$\cdot 62$	.0252 077	.0199 032	.0156 403	.0122 306	
$\cdot 63$	.0345 061	.0276 566	.0220 585 <sup>+</sup>	.0175 109	
$\cdot 64$	.0465 399	.0378 486	.0306 324	.0246 772	
$\cdot 65$	.0618 525 <sup>+</sup>	.0510 214	.0418 881	.0342 330	
$\cdot 66$	.0810 065 <sup>+</sup>	.0677 538	.0564 066	.0467 495 <sup>+</sup>	
$\cdot 67$	.1045 525 <sup>+</sup>	.0886 364	.0748 023	.0628 504	
$\cdot 68$	.1329 898	.1142 356	.0976 914	.0831 848	
$\cdot 69$	.1667 187	.1450 474	.1256 487	.1083 890	
$\cdot 70$	.2059 895 <sup>+</sup>	.1814 446	.1591 552	.1390 361	
$\cdot 71$	.2508 496	.2236 193	.1985 379	.1755 753	
$\cdot 72$	.3010 965 <sup>+</sup>	.2715 271	.2439 076	.2182 652	
$\cdot 73$	.3562 419	.3248 394	.2951 003	.2671 065 <sup>-</sup>	
$\cdot 74$	.4154 945 <sup>+</sup>	.3829 109	.3516 307	.3217 828	
$\cdot 75$	.4777 672	.4447 711	.4126 668	.3816 186	
$\cdot 76$	.5417 128	.5091 440	.4770 330	.4455 642	
$\cdot 77$	.6057 908	.5745 029	.5432 488	.5122 168	
$\cdot 78$	.6683 618	.6391 570	.6096 049	.5798 817	
$\cdot 79$	.7278 027	.7013 675 <sup>-</sup>	.6742 749	.6466 761	
$\cdot 80$	.7826 324	.7594 810	.7354 537	.7106 676	
$\cdot 81$	.8316 325 <sup>+</sup>	.8120 675 <sup>-</sup>	.7915 082	.7688 676	

$x = .42$  to  $.98$  $q = 12$  $p = .41$  to  $.49$ 

	$p = 41$	$p = 42$	$p = 43$	$p = 44$	$p = 45$
$B(p, q) = .4037\ 8706 \times \frac{1}{10^{11}}$	$.3123\ 6357 \times \frac{1}{10^{11}}$	$.2429\ 4945 \times \frac{1}{10^{11}}$	$.1899\ 4229 \times \frac{1}{10^{11}}$	$.1492\ 4037 \times \frac{1}{10^{11}}$	
$\cdot 42$	.0000 001				
$\cdot 43$	.0000 001	.0000 001			
$\cdot 44$	.0000 003	.0000 002	.0000 001	.0000 001	
$\cdot 45$	.0000 006	.0000 004	.0000 002	.0000 001	.0000 001
$\cdot 46$	.0000 013	.0000 008	.0000 004	.0000 002	.0000 001
$\cdot 47$	.0000 026	.0000 015 <sup>+</sup>	.0000 009	.0000 005 <sup>+</sup>	.0000 003
$\cdot 48$	.0000 051	.0000 030	.0000 018	.0000 011	.0000 006
$\cdot 49$	.0000 096	.0000 059	.0000 036	.0000 022	.0000 013
$\cdot 50$	.0000 179	.0000 112	.0000 070	.0000 044	.0000 027
$\cdot 51$	.0000 328	.0000 209	.0000 133	.0000 084	.0000 053
$\cdot 52$	.0000 588	.0000 382	.0000 248	.0000 160	.0000 103
$\cdot 53$	.0001 033	.0000 685 <sup>-</sup>	.0000 452	.0000 297	.0000 195
$\cdot 54$	.0001 782	.0001 203	.0000 809	.0000 542	.0000 361
$\cdot 55$	.0003 017	.0002 074	.0001 420	.0000 968	.0000 637
$\cdot 56$	.0005 018	.0003 511	.0002 446	.0001 607	.0001 172
$\cdot 57$	.0008 199	.0005 836	.0004 136	.0002 919	.0002 052
$\cdot 58$	.0013 165 <sup>-</sup>	.0009 529	.0006 868	.0004 929	.0003 524
$\cdot 59$	.0020 776	.0015 288	.0011 202	.0008 175 <sup>-</sup>	.0005 942
$\cdot 60$	.0032 234	.0024 106	.0017 952	.0013 315 <sup>-</sup>	.0009 837
$\cdot 61$	.0049 175 <sup>+</sup>	.0037 362	.0028 269	.0021 303	.0015 091
$\cdot 62$	.0073 779	.0056 931	.0043 751	.0033 488	.0025 534
$\cdot 63$	.0108 873	.0085 298	.0066 556	.0051 728	.0040 051
$\cdot 64$	.0158 036	.0125 671	.0099 533	.0078 525 <sup>-</sup>	.0061 718
$\cdot 65$	.0225 669	.0182 085 <sup>-</sup>	.0146 335 <sup>+</sup>	.0117 154	.0093 444
$\cdot 66$	.0317 020	.0259 461	.0211 522	.0171 789	.0139 010
$\cdot 67$	.0438 139	.0363 615 <sup>-</sup>	.0300 606	.0247 591	.0203 191
$\cdot 68$	.0595 731	.0501 167	.0420 021	.0350 727	.0291 827
$\cdot 69$	.0796 885 <sup>-</sup>	.0679 337	.0576 985 <sup>+</sup>	.0488 297	.0411 803
$\cdot 70$	.1048 658	.0905 591	.0779 216	.0668 123	.0570 916
$\cdot 71$	.1357 519	.1187 134	.1034 477	.0898 364	.0777 593
$\cdot 72$	.1728 661	.1530 238	.1349 957	.1186 948	.1040 241
$\cdot 73$	.2165 220	.1939 450 <sup>+</sup>	.1731 473	.1540 811	.1366 832
$\cdot 74$	.2667 482	.2416 730	.2182 575 <sup>+</sup>	.1964 975 <sup>-</sup>	.1763 694
$\cdot 75$	.3232 166	.2960 612	.2703 606	.2461 542	.2234 607
$\cdot 76$	.3851 909	.3565 521	.3290 860	.3028 719	.2779 685 <sup>+</sup>
$\cdot 77$	.4515 074	.4221 305 <sup>+</sup>	.3935 978	.3660 015 <sup>-</sup>	.3394 401
$\cdot 78$	.5205 987	.4913 562	.4625 732	.4343 793	.4068 902
$\cdot 79$	.5905 683	.5623 581	.5342 352	.5063 347	.4787 830
$\cdot 80$	.6593 159	.6330 064	.6064 463	.5797 626	.5530 790
$\cdot 81$	.7247 058	.7010 470	.6768 643	.6522 654	.6273 591
$\cdot 82$	.7847 603	.7643 099	.7431 486	.7213 574	.6990 214
$\cdot 83$	.8378 538	.8209 240	.8031 920	.7847 095 <sup>+</sup>	.7655 339
$\cdot 84$	.8828 766	.8695 121	.8553 456	.8404 017	.8247 106
$\cdot 85$	.9193 390	.9093 313	.8985 968	.8871 384	.8739 638
$\cdot 86$	.9473 948	.9403 310	.9266 652		
$\cdot 87$					

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .46$  to  $.98$  $q = 12$ 

	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) = .1178\ 2135 \times \frac{1}{10^{13}}$		$.9344\ 4517 \times \frac{1}{10^{13}}$	$.7443\ 8852 \times \frac{1}{10^{13}}$	$.5955\ 1082 \times \frac{1}{10^{13}}$
$x$				
.46	.0000 001			
.47	.0000 002	.0000 001	.0000 001	
.48	.0000 004	.0000 002	.0000 001	.0000 001
.49	.0000 008	.0000 005	.0000 003	.0000 002
.50	.0000 017	.0000 010	.0000 006	.0000 004
.51	.0000 033	.0000 021	.0000 013	.0000 008
.52	.0000 066	.0000 042	.0000 027	.0000 017
.53	.0000 127	.0000 082	.0000 053	.0000 034
.54	.0000 240	.0000 159	.0000 105	.0000 069
.55	.0000 444	.0000 299	.0000 201	.0000 134
.56	.0000 807	.0000 553	.0000 378	.0000 257
.57	.0001 437	.0001 002	.0000 697	.0000 483
.58	.0002 510	.0001 781	.0001 259	.0000 887
.59	.0004 303	.0003 104	.0002 232	.0001 599
.60	.0007 240	.0005 309	.0003 880	.0002 826
.61	.0011 960	.0008 912	.0006 618	.0004 898
.62	.0019 398	.0014 683	.0011 076	.0008 327
.63	.0030 897	.0023 750	.0018 194	.0013 891
.64	.0048 333	.0037 718	.0029 334	.0022 739
.65	.0074 265 <sup>+</sup>	.0058 818	.0046 427	.0036 528
.66	.0112 086	.0090 068	.0072 135	.0057 586
.67	.0166 171	.0135 437	.0110 025 <sup>+</sup>	.0089 098
.68	.0241 984	.0199 986	.0164 743	.0135 286
.69	.0346 121	.0289 962	.0242 144	.0201 587
.70	.0486 239	.0412 790	.0349 343	.0294 752
.71	.0670 829	.0576 926	.0494 652	.0422 850
.72	.0908 791	.0791 515 <sup>+</sup>	.0687 314	.0595 096
.73	.1208 782	.1065 815 <sup>+</sup>	.0937 024	.0821 460
.74	.1578 337	.1408 373	.1253 163	.1111 987
.75	.2022 801	.1825 960	.1643 777	.1475 827
.76	.2544 154	.2322 346	.2114 318	.1919 985
.77	.3139 883	.2897 040	.2666 287	.2447 890
.78	.3802 075	.3544 180	.3295 943	.3057 946
.79	.4516 965 <sup>+</sup>	.4251 813	.3993 317	.3742 307
.80	.5265 139	.5001 799	.4741 820	.4486 175
.81	.6022 535 <sup>+</sup>	.5770 552	.5518 675 <sup>+</sup>	.5267 901
.82	.6762 288	.6530 695	.6296 342	.6060 134
.83	.7457 277	.7253 574	.7044 930	.6832 070
.84	.8083 081	.7912 353	.7735 377	.7552 649
.85	.8620 858	.8485 220	.8342 943	.8194 294
.86	.9059 609	.8958 105	.8850 381	.8736 508
.87	.9397 316	.9326 321	.9250 101	.9168 592
.88	.9640 639	.9594 678	.9544 765 <sup>+</sup>	.9490 777
.89	.9803 162	.9775 956	.9746 075	.9713 387
.90	.0002 551	.0002 551	.0002 551	.0002 551

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION $p = .10$  to  $.70$  $q = 13$  $p =$ 

	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$\beta(p, q) = .1479\ 2046 \times \frac{1}{10^3}$	$.7396\ 0230 \times \frac{1}{10^3}$	$.3834\ 9749 \times \frac{1}{10^3}$	$.2054\ 4508 \times \frac{1}{10^3}$	$.1133\ 4901 \times \frac{1}{10^3}$	$.6423\ 1000 \times \frac{1}{10^3}$	
$x$						
.10	.0000 002					
.11	.0000 005	.0000 001				
.12	.0000 014	.0000 003	.0000 001			
.13	.0000 034	.0000 008	.0000 002			
.14	.0000 078	.0000 020	.0000 005 <sup>+</sup>	.0000 001		
.15	.0000 169	.0000 047	.0000 012	.0000 003	.0000 001	
.16	.0000 344	.0000 101	.0000 029	.0000 008	.0000 002	.0000 000
.17	.0000 664	.0000 207	.0000 063	.0000 018	.0000 005 <sup>+</sup>	.0000 000
.18	.0001 224	.0000 403	.0000 129	.0000 040	.0000 012	.0000 000
.19	.0002 166	.0000 753	.0000 254	.0000 083	.0000 027	.0000 000
.20	.0003 690	.0001 348	.0000 479	.0000 165 <sup>+</sup>	.0000 056	.0000 000
.21	.0006 080	.0002 330	.0000 867	.0000 315	.0000 111	.0000 000
.22	.0009 712	.0003 894	.0001 517	.0000 576	.0000 213	.0000 000
.23	.0015 081	.0006 313	.0002 569	.0001 018	.0000 394	.0000 000
.24	.0022 818	.0009 954	.0004 221	.0001 744	.0000 704	.0000 000
.25	.0033 704	.0015 295	.0006 748	.0002 901	.0001 218	.0000 500
.26	.0048 691	.0022 945 <sup>+</sup>	.0010 514	.0004 696	.0002 049	.0000 800
.27	.0068 899	.0033 666	.0015 999	.0007 413	.0003 356	.0001 400
.28	.0095 630	.0048 380	.0023 810	.0011 426	.0005 359	.0002 400
.29	.0130 355	.0068 187	.0034 706	.0017 229	.0008 359	.0003 900
.30	.0174 697	.0094 367	.0049 612	.0025 444	.0012 756	.0006 200
.31	.0230 415	.0128 375 <sup>+</sup>	.0069 630	.0036 850	.0019 066	.0009 600
.32	.0299 357	.0171 832	.0096 046	.0052 394	.0027 948	.0014 600
.33	.0383 424	.0226 503	.0130 333	.0073 207	.0040 218	.0021 600
.34	.0484 512	.0294 262	.0174 132	.0100 613	.0056 869	.0031 400
.35	.0604 449	.0377 051	.0229 242	.0136 123	.0079 087	.0045 000
.36	.0744 932	.0476 829	.0297 584	.0181 433	.0108 257	.0063 300
.37	.0907 452	.0595 507	.0381 156	.0238 398	.0145 963	.0087 600
.38	.1093 229	.0734 880	.0481 987	.0309 009	.0193 978	.0119 400
.39	.1303 145	.0896 555 <sup>+</sup>	.0602 068	.0395 338	.0254 245 <sup>+</sup>	.0160 300
.40	.1537 678	.1081 877	.0743 283	.0499 495	.0328 843	.0212 300
.41	.1796 859	.1291 855 <sup>+</sup>	.0907 331	.0623 552	.0419 941	.0277 500
.42	.2080 232	.1527 094	.1095 647	.0769 473	.0529 735	.0357 900
.43	.2386 827	.1787 738	.1309 322	.0939 029	.0660 383	.0455 900
.44	.2715 160	.2073 422	.1549 030	.1133 713	.0813 918	.0573 800
.45	.3063 240	.2383 244	.1814 962	.1354 654	.0992 161	.0713 800
.46	.3428 601	.2715 750	.2106 771	.1602 536	.1196 628	.0878 100
.47	.3808 351	.3068 938	.2423 536	.1877 527	.1428 434	.1068 300
.48	.4199 229	.3440 288	.2763 747	.2179 215	.1688 208	.1286 100
.49	.4597 693	.3826 808	.3125 302	.2506 575	.1976 015 <sup>+</sup>	.1532 500
.50	.5000 000	.4225 095	.3505 540	.2857 941	.2291 292	.1807 900
.51	.5402 307	.4631 422	.3901 283	.3231 016	.2632 803	.2112 300
.52	.5800 771	.5041 830	.4308 910	.3622 897	.2998 625	.2444 800
.53	.6101 610	.5452 610				

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.93$  $q = 13$ 

	$p = 13$	$p = 14$	$p = 15$	$p = 16$	$p =$
$B(p, q) = .1479\ 2046 \times \frac{x}{10^7}$		$.7396\ 0230 \times \frac{x}{10^8}$	$.3834\ 9749 \times \frac{x}{10^8}$	$.2054\ 4508 \times \frac{x}{10^8}$	$.1133$
$x$					
.71	.9869 645 <sup>+</sup>	.9807 478	.9725 507	.9620 747	.9490
.72	.9904 370	.9857 119	.9793 939	.9712 057	.9608
.73	.9931 101	.9895 867	.9848 101	.9785 336	.9705
.74	.9951 309	.9925 564	.9890 183	.9843 056	.9782
.75	.9966 296	.9947 886	.9922 244	.9887 627	.9842
.76	.9977 182	.9964 319	.9946 164	.9921 327	.9888
.77	.9984 919	.9976 152	.9963 614	.9946 238	.9922
.78	.9990 288	.9984 471	.9976 043	.9964 211	.9948
.79	.9993 920	.9990 170	.9984 668	.9976 845 <sup>-</sup>	.9966
.80	.9996 310	.9993 968	.9990 488	.9985 477	.9978
.81	.9997 834	.9996 422	.9994 296	.9991 197	.9986
.82	.9998 776	.9997 955 <sup>+</sup>	.9996 706	.9994 861	.9992
.83	.9999 336	.9998 879	.9998 175 <sup>-</sup>	.9997 123	.9995
.84	.9999 656	.9999 414	.9999 035 <sup>-</sup>	.9998 462	.9997
.85	.9999 831	.9999 709	.9999 516	.9999 220	.9998
.86	.9999 922	.9999 864	.9999 771	.9999 627	.9999
.87	.9999 966	.9999 940	.9999 899	.9999 834	.9999
.88	.9999 986	.9999 976	.9999 959	.9999 931	.9999
.89	.9999 995 <sup>+</sup>	.9999 991	.9999 985 <sup>-</sup>	.9999 974	.9999
.90	.9999 998	.9999 997	.9999 995 <sup>-</sup>	.9999 991	.9999
.91	1.0000 000	.9999 999	.9999 998	.9999 997	.9999
.92		1.0000 000	1.0000 000	.9999 999	.9999
.93				1.0000 000	1.0000

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

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$q = 13$

$p = 19$  to

$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$
$\cdot 3729\ 5481 \times \frac{1}{10^6}$	$\cdot 2214\ 4192 \times \frac{1}{10^6}$	$\cdot 1342\ 0722 \times \frac{1}{10^6}$	$\cdot 8289\ 2697 \times \frac{1}{10^6}$	$\cdot 5210\ 3981 \times \frac{1}{10^6}$	$\cdot 3328\ 8655 \times \frac{1}{10^6}$
$\cdot 0000\ 001$	$\cdot 0000\ 001$				
$\cdot 0000\ 003$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 006$					
$\cdot 0000\ 013$	$\cdot 0000\ 004$	$\cdot 0000\ 001$			
$\cdot 0000\ 027$	$\cdot 0000\ 010$	$\cdot 0000\ 003$	$\cdot 0000\ 001$		
$\cdot 0000\ 055^+$	$\cdot 0000\ 020$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 0000\ 108$	$\cdot 0000\ 041$	$\cdot 0000\ 015^+$	$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$
$\cdot 0000\ 202$	$\cdot 0000\ 080$	$\cdot 0000\ 031$	$\cdot 0000\ 012$	$\cdot 0000\ 004$	$\cdot 0000\ 002$
$\cdot 0000\ 367$	$\cdot 0000\ 151$	$\cdot 0000\ 061$	$\cdot 0000\ 024$	$\cdot 0000\ 010$	$\cdot 0000\ 004$
$\cdot 0000\ 646$	$\cdot 0000\ 276$	$\cdot 0000\ 116$	$\cdot 0000\ 048$	$\cdot 0000\ 019$	$\cdot 0000\ 008$
$\cdot 0001\ 108$	$\cdot 0000\ 490$	$\cdot 0000\ 213$	$\cdot 0000\ 091$	$\cdot 0000\ 038$	$\cdot 0000\ 016$
$\cdot 0001\ 850^+$	$\cdot 0000\ 847$	$\cdot 0000\ 381$	$\cdot 0000\ 169$	$\cdot 0000\ 074$	$\cdot 0000\ 032$
$\cdot 0003\ 016$	$\cdot 0001\ 426$	$\cdot 0000\ 664$	$\cdot 0000\ 304$	$\cdot 0000\ 137$	$\cdot 0000\ 061$
$\cdot 0004\ 803$	$\cdot 0002\ 346$	$\cdot 0001\ 127$	$\cdot 0000\ 533$	$\cdot 0000\ 249$	$\cdot 0000\ 114$
$\cdot 0007\ 487$	$\cdot 0003\ 771$	$\cdot 0001\ 868$	$\cdot 0000\ 912$	$\cdot 0000\ 439$	$\cdot 0000\ 208$
$\cdot 0011\ 432$	$\cdot 0005\ 933$	$\cdot 0003\ 029$	$\cdot 0001\ 523$	$\cdot 0000\ 755^-$	$\cdot 0000\ 369$
$\cdot 0017\ 120$	$\cdot 0009\ 145^-$	$\cdot 0004\ 806$	$\cdot 0002\ 488$	$\cdot 0001\ 270$	$\cdot 0000\ 640$
$\cdot 0025\ 169$	$\cdot 0013\ 825^+$	$\cdot 0007\ 473$	$\cdot 0003\ 979$	$\cdot 0002\ 089$	$\cdot 0001\ 082$
$\cdot 0036\ 356$	$\cdot 0020\ 519$	$\cdot 0011\ 397$	$\cdot 0006\ 236$	$\cdot 0003\ 365^+$	$\cdot 0001\ 792$
$\cdot 0051\ 642$	$\cdot 0029\ 922$	$\cdot 0017\ 064$	$\cdot 0009\ 588$	$\cdot 0005\ 313$	$\cdot 0002\ 906$
$\cdot 0072\ 185^+$	$\cdot 0042\ 905^+$	$\cdot 0025\ 103$	$\cdot 0014\ 473$	$\cdot 0008\ 230$	$\cdot 0004\ 620$
$\cdot 0099\ 362$	$\cdot 0060\ 538$	$\cdot 0036\ 311$	$\cdot 0021\ 464$	$\cdot 0012\ 515^+$	$\cdot 0007\ 204$
$\cdot 134\ 769$	$\cdot 0084\ 105^+$	$\cdot 0051\ 681$	$\cdot 0031\ 299$	$\cdot 0018\ 700$	$\cdot 0011\ 031$
$\cdot 180\ 220$	$\cdot 0115\ 123$	$\cdot 0072\ 420$	$\cdot 0044\ 906$	$\cdot 0027\ 473$	$\cdot 0016\ 596$
$\cdot 237\ 734$	$\cdot 0155\ 341$	$\cdot 0099\ 973$	$\cdot 0063\ 430$	$\cdot 0039\ 710$	$\cdot 0024\ 550^-$
$\cdot 309\ 505^-$	$\cdot 0206\ 735^+$	$\cdot 0136\ 030$	$\cdot 0088\ 253$	$\cdot 0056\ 503$	$\cdot 0035\ 728$
$\cdot 397\ 858$	$\cdot 0271\ 491$	$\cdot 0182\ 528$	$\cdot 0121\ 017$	$\cdot 0079\ 189$	$\cdot 0051\ 183$
$\cdot 505\ 194$	$\cdot 0351\ 965^-$	$\cdot 0241\ 639$	$\cdot 0163\ 624$	$\cdot 0109\ 367$	$\cdot 0072\ 213$
$\cdot 633\ 911$	$\cdot 0450\ 634$	$\cdot 0315\ 742$	$\cdot 0218\ 234$	$\cdot 0148\ 915^+$	$\cdot 0100\ 392$
$\cdot 786\ 314$	$\cdot 0570\ 025^-$	$\cdot 0407\ 375^-$	$\cdot 0287\ 246$	$\cdot 0199\ 989$	$\cdot 0137\ 581$
$\cdot 964\ 522$	$\cdot 0712\ 628$	$\cdot 0519\ 174$	$\cdot 0373\ 254$	$\cdot 0265\ 008$	$\cdot 0185\ 941$
$\cdot 170\ 351$	$\cdot 0880\ 798$	$\cdot 0653\ 788$	$\cdot 0478\ 991$	$\cdot 0340\ 622$	$\cdot 0247\ 920$
$\cdot 405\ 208$	$\cdot 1076\ 636$	$\cdot 0813\ 778$	$\cdot 0607\ 247$	$\cdot 0447\ 655^+$	$\cdot 0326\ 227$
$\cdot 669\ 978$	$\cdot 1301\ 872$	$\cdot 1001\ 498$	$\cdot 0760\ 770$	$\cdot 0571\ 032$	$\cdot 0423\ 779$
$\cdot 964\ 928$	$\cdot 1557\ 745^-$	$\cdot 1218\ 969$	$\cdot 0942\ 140$	$\cdot 0719\ 670$	$\cdot 0543\ 629$
$\cdot 289\ 617$	$\cdot 1844\ 879$	$\cdot 1467\ 742$	$\cdot 1153\ 640$	$\cdot 0896\ 362$	$\cdot 0688\ 862$
$\cdot 542\ 836$	$\cdot 2163\ 187$	$\cdot 1748\ 771$	$\cdot 1397\ 108$	$\cdot 1103\ 630$	$\cdot 0862\ 468$
$\cdot 922\ 573$	$\cdot 2511\ 781$	$\cdot 2062\ 285^-$	$\cdot 1673\ 791$	$\cdot 1343\ 571$	$\cdot 1067\ 192$
$\cdot 126\ 011$	$\cdot 2888\ 920$	$\cdot 2407\ 687$	$\cdot 1984\ 202$	$\cdot 1617\ 695^-$	$\cdot 1305\ 367$
$\cdot 449\ 559$	$\cdot 3291\ 986$	$\cdot 2783\ 478$	$\cdot 2328\ 001$	$\cdot 1926\ 767$	$\cdot 1578\ 740$
$\cdot 888\ 929$	$\cdot 3717\ 500^-$	$\cdot 3187\ 213$	$\cdot 2703\ 894$	$\cdot 2270\ 665^+$	$\cdot 1888\ 293$
$\cdot 939\ 239$	$\cdot 4161\ 185^+$	$\cdot 3615\ 502$	$\cdot 3109\ 576$	$\cdot 2648\ 263$	$\cdot 2234\ 084$
$\cdot 95\ 156$	$\cdot 4618\ 066$	$\cdot 4064\ 059$	$\cdot 3541\ 710$	$\cdot 3057\ 350^-$	$\cdot 2615\ 108$
$\cdot 51\ 065^+$	$\cdot 5082\ 609$	$\cdot 4527\ 796$	$\cdot 3995\ 968$	$\cdot 3494\ 599$	$\cdot 3020\ 108$
$\cdot 01\ 258$	$\cdot 5548\ 002$	$\cdot 5000\ 065^+$			

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71 \text{ to } .95$  $q = 13$ 

	$p = 19$	$p = 20$	$p = 21$	$p = 22$	$p =$
$B(p, q) = .3729\ 5481 \times \frac{1}{10^8}$	$.2214\ 4192 \times \frac{1}{10^8}$	$.1342\ 0722 \times \frac{1}{10^8}$	$.8289\ 2697 \times \frac{1}{10^{10}}$	$.5210$	
$\cdot 71$	$\cdot 9146\ 376$	$\cdot 8930\ 267$	$\cdot 8684\ 767$	$\cdot 8410\ 859$	$\cdot 8110$
$\cdot 72$	$\cdot 9330\ 105^-$	$\cdot 9151\ 472$	$\cdot 8945\ 686$	$\cdot 8712\ 855^-$	$\cdot 8453$
$\cdot 73$	$\cdot 9483\ 818$	$\cdot 9339\ 123$	$\cdot 9170\ 119$	$\cdot 8976\ 248$	$\cdot 8757$
$\cdot 74$	$\cdot 9609\ 965^+$	$\cdot 9495\ 244$	$\cdot 9359\ 413$	$\cdot 9201\ 461$	$\cdot 9020$
$\cdot 75$	$\cdot 9711\ 402$	$\cdot 9622\ 487$	$\cdot 9515\ 789$	$\cdot 9390\ 038$	$\cdot 9244$
$\cdot 76$	$\cdot 9791\ 221$	$\cdot 9723\ 955^+$	$\cdot 9642\ 160$	$\cdot 9544\ 473$	$\cdot 9429$
$\cdot 77$	$\cdot 9852\ 595^+$	$\cdot 9803\ 007$	$\cdot 9741\ 915^-$	$\cdot 9667\ 993$	$\cdot 9580$
$\cdot 78$	$\cdot 9898\ 633$	$\cdot 9863\ 080$	$\cdot 9818\ 709$	$\cdot 9764\ 323$	$\cdot 9698$
$\cdot 79$	$\cdot 9932\ 261$	$\cdot 9907\ 523$	$\cdot 9876\ 255^-$	$\cdot 9837\ 438$	$\cdot 9790$
$\cdot 80$	$\cdot 9956\ 125^+$	$\cdot 9939\ 465^-$	$\cdot 9918\ 139$	$\cdot 9891\ 330$	$\cdot 9858$
$\cdot 81$	$\cdot 9972\ 540$	$\cdot 9961\ 711$	$\cdot 9947\ 676$	$\cdot 9929\ 812$	$\cdot 9907$
$\cdot 82$	$\cdot 9983\ 451$	$\cdot 9976\ 681$	$\cdot 9967\ 799$	$\cdot 9956\ 353$	$\cdot 9941$
$\cdot 83$	$\cdot 9990\ 436$	$\cdot 9986\ 382$	$\cdot 9980\ 998$	$\cdot 9973\ 975^+$	$\cdot 9964$
$\cdot 84$	$\cdot 9994\ 725^+$	$\cdot 9992\ 411$	$\cdot 9989\ 300$	$\cdot 9985\ 194$	$\cdot 9979$
$\cdot 85$	$\cdot 9997\ 240$	$\cdot 9995\ 988$	$\cdot 9994\ 284$	$\cdot 9992\ 009$	$\cdot 9989$
$\cdot 86$	$\cdot 9998\ 639$	$\cdot 9998\ 001$	$\cdot 9997\ 124$	$\cdot 9995\ 938$	$\cdot 9994$
$\cdot 87$	$\cdot 9999\ 373$	$\cdot 9999\ 070$	$\cdot 9998\ 648$	$\cdot 9998\ 071$	$\cdot 9997$
$\cdot 88$	$\cdot 9999\ 733$	$\cdot 9999\ 600$	$\cdot 9999\ 413$	$\cdot 9999\ 154$	$\cdot 9998$
$\cdot 89$	$\cdot 9999\ 896$	$\cdot 9999\ 843$	$\cdot 9999\ 767$	$\cdot 9999\ 661$	$\cdot 9999$
$\cdot 90$	$\cdot 9999\ 964$	$\cdot 9999\ 945^-$	$\cdot 9999\ 917$	$\cdot 9999\ 879$	$\cdot 9999$
$\cdot 91$	$\cdot 9999\ 989$	$\cdot 9999\ 983$	$\cdot 9999\ 974$	$\cdot 9999\ 962$	$\cdot 9999$
$\cdot 92$	$\cdot 9999\ 997$	$\cdot 9999\ 996$	$\cdot 9999\ 993$	$\cdot 9999\ 990$	$\cdot 9999$
$\cdot 93$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 999$	$\cdot 9999\ 998$	$\cdot 9999$
$\cdot 94$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$1\cdot 0000\ 000$	$\cdot 9999$
$\cdot 95$					$1\cdot 0000$



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .96

$q = 13$

$p = 25$

$p = 25$	$p = 26$	$p = 27$	$p = 28$	$p = 29$	$p = 30$
$= .2159\ 2641 \times \frac{1}{100}$	$.1420\ 5685 \times \frac{1}{100}$	$.9470\ 4565 \times \frac{1}{101}$	$.6392\ 5581 \times \frac{1}{101}$	$.4365\ 6495 \times \frac{1}{101}$	$.3014\ 3770 \times$
.0000 001					
.0000 001	.0000 001				
.0000 003	.0000 001				
.0000 007	.0000 003	.0000 001			
.0000 014	.0000 006	.0000 002	.0000 001		
.0000 027	.0000 012	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
.0000 052	.0000 023	.0000 010	.0000 005 <sup>-</sup>	.0000 002	.0000 001
.0000 098	.0000 045 <sup>+</sup>	.0000 021	.0000 009	.0000 004	.0000 002
.0000 178	.0000 085 <sup>+</sup>	.0000 040	.0000 019	.0000 009	.0000 004
.0000 318	.0000 156	.0000 076	.0000 037	.0000 017	.0000 008
.0000 554	.0000 280	.0000 140	.0000 069	.0000 034	.0000 017
.0000 943	.0000 490	.0000 252	.0000 128	.0000 065 <sup>-</sup>	.0000 032
.0001 570	.0000 839	.0000 443	.0000 232	.0000 120	.0000 062
.0002 562	.0001 405 <sup>-</sup>	.0000 762	.0000 409	.0000 218	.0000 115 <sup>-</sup>
.0004 097	.0002 304	.0001 282	.0000 706	.0000 385 <sup>+</sup>	.0000 208
.0006 429	.0003 705 <sup>+</sup>	.0002 113	.0001 193	.0000 667	.0000 370
.0009 907	.0005 848	.0003 415 <sup>+</sup>	.0001 975 <sup>-</sup>	.0001 131	.0000 642
.0014 999	.0009 062	.0005 418	.0003 207	.0001 881	.0001 093
.0022 328	.0013 800	.0008 441	.0005 112	.0003 067	.0001 824
.0032 698	.0020 661	.0012 921	.0008 001	.0004 909	.0002 985 <sup>+</sup>
.0047 134	.0030 432	.0019 447	.0012 306	.0007 716	.0004 796
.0066 911	.0044 117	.0028 793	.0018 610	.0011 919	.0007 567
.0093 584	.0062 979	.0041 956	.0027 684	.0018 101	.0011 734
.0129 014	.0088 572	.0060 201	.0040 530	.0027 041	.0017 888
.0175 375 <sup>-</sup>	.0122 765 <sup>-</sup>	.0085 088	.0058 421	.0039 754	.0026 823
.0235 155 <sup>+</sup>	.0167 762	.0118 514	.0082 945 <sup>+</sup>	.0057 539	.0039 579
.0311 130	.0226 103	.0162 725 <sup>+</sup>	.0116 037	.0082 021	.0057 494
.0406 317	.0300 642	.0220 328	.0160 004	.0115 192	.0082 247
.0523 900	.0394 503	.0294 270	.0217 537	.0159 438	.0115 905 <sup>-</sup>
.0667 127	.0511 010	.0387 797	.0291 692	.0217 553	.0160 952
.0839 179	.0653 576	.0504 380	.0385 852	.0292 723	.0220 307
.1043 012	.0825 572	.0647 605 <sup>+</sup>	.0503 650 <sup>+</sup>	.0388 486	.0297 308
.1281 177	.1030 153	.0821 030	.0648 853	.0508 651	.0395 668
.1555 630	.1270 072	.1028 006	.0825 209	.0657 177	.0519 391
.1867 536	.1547 467	.1271 468	.1036 256	.0838 006	.0672 638
.2217 090	.1863 650 <sup>+</sup>	.1553 711	.1285 097	.1054 856	.0859 548
.2603 356	.2218 906	.1876 154	.1574 152	.1310 978	.1084 013
.3024 157	.2612 312	.2239 118	.1904 901	.1608 881	.1349 404
.3476 011	.3041 610	.2641 628	.2277 643	.1950 058	.1658 280
.3954 147	.3503 137	.3081 269	.2691 276	.2334 710	.2012 080
.4452 586	.3991 828	.3554 108	.3143 137	.2761 522	.2410 831
.4964 311	.4501 314	.4054 699	.3628 927	.3227 484	.2852 897
.5481 513	.5024 103	.4576 193	.4142 716	.3727 817	.3334 806
.5995 897	.5551 852	.5110 540	.4677 074	.4255 992	.3851 172
.6499 050 <sup>+</sup>	.6075 716	.5648 800	.5223 306	.4803 891	.4394 745 <sup>-</sup>
.6982 828	.6586 750 <sup>-</sup>	.6181 531	.5771 809	.5362 088	.4956 605 <sup>-</sup>
.7439 748	.7076 347	.6699 248	.6312 512	.5920 251	.5526 502
.7863 350 <sup>-</sup>	.7536 671	.7192 904	.6835 387	.6467 655 <sup>-</sup>	.6093 329
.8248 500 <sup>-</sup>	.7961 055 <sup>-</sup>	.7654 373	.7239 281		

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .32$  to  $.96$  $q = 13$ 

$p = 31$	$p = 32$	$p = 33$	$p = 34$	$p = 35$
$B(p, q) = .2103\ 0537 \times \frac{1}{1011}$	$.1481\ 6969 \times \frac{1}{1011}$	$.1053\ 6512 \times \frac{1}{1011}$	$.7558\ 8018 \times \frac{1}{1011}$	$.5468\ 0$
.32	.0000 001			
.33	.0000 002	.0000 001		
.34	.0000 004	.0000 002	.0000 001	
.35	.0000 008	.0000 004	.0000 002	.0000 001
.36	.0000 016	.0000 008	.0000 004	.0000 002
.37	.0000 031	.0000 016	.0000 008	.0000 004
.38	.0000 060	.0000 031	.0000 016	.0000 008
.39	.0000 112	.0000 059	.0000 031	.0000 016
.40	.0000 203	.0000 111	.0000 060	.0000 032
.41	.0000 361	.0000 202	.0000 112	.0000 062
.42	.0000 630	.0000 360	.0000 204	.0000 115 <sup>+</sup>
.43	.0001 075 <sup>+</sup>	.0000 629	.0000 365 <sup>+</sup>	.0000 211
.44	.0001 800	.0001 077	.0000 640	.0000 377
.45	.0002 956	.0001 808	.0001 097	.0000 661
.46	.0004 765 <sup>-</sup>	.0002 977	.0001 846	.0001 136
.47	.0007 544	.0004 812	.0003 047	.0001 916
.48	.0011 736	.0007 641	.0004 938	.0003 169
.49	.0017 952	.0011 922	.0007 860	.0005 145 <sup>+</sup>
.50	.0027 008	.0018 289	.0012 294	.0008 207
.51	.0039 982	.0027 594	.0018 907	.0012 865 <sup>-</sup>
.52	.0058 263	.0040 964	.0028 595 <sup>-</sup>	.0019 824
.53	.0083 603	.0059 855 <sup>+</sup>	.0042 550 <sup>-</sup>	.0030 042
.54	.0118 162	.0086 110	.0062 312	.0044 788
.55	.0164 547	.0122 007	.0089 837	.0065 709
.56	.0225 824	.0170 298	.0127 542	.0094 892
.57	.0305 507	.0234 222	.0178 353	.0134 927
.58	.0407 508	.0317 497	.0245 713	.0188 939
.59	.0536 044	.0424 257	.0333 570	.0260 608
.60	.0695 490	.0558 951	.0446 306	.0354 143
.61	.0890 185 <sup>+</sup>	.0726 182	.0588 624	.0474 201
.62	.1124 178	.0930 484	.0765 359	.0625 754
.63	.1400 933	.1176 044	.0981 235 <sup>-</sup>	.0813 875 <sup>+</sup>
.64	.1723 004	.1466 378	.1240 548	.1043 460
.65	.2091 702	.1803 971	.1546 811	.1318 874
.66	.2506 778	.2189 912	.1902 356	.1643 556
.67	.2966 163	.2623 563	.2307 942	.2019 579
.68	.3465 784	.3102 276	.2762 396	.2447 235 <sup>-</sup>
.69	.3999 509	.3621 231	.3262 339	.2924 654
.70	.4559 232	.4173 395 <sup>-</sup>	.3802 027	.3447 540
.71	.5135 116	.4749 663	.4373 365 <sup>+</sup>	.4009 041
.72	.5716 008	.5339 174	.4966 109	.4599 827
.73	.6289 994	.5929 809	.5568 273	.5208 381
.74	.6845 063	.6508 842	.6166 737	.5821 522
.75	.7369 836	.7063 705 <sup>+</sup>	.6748 008	.6425 135 <sup>+</sup>
.76	.7854 286	.7582 295 <sup>+</sup>	.7299 087	.7005 062
.77				
.78				
.79				
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.81				
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.95				
.96				

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

$q = 13$

$p =$

$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$9 \times \frac{1}{10^{12}}$	$2 \cdot 167\ 6989 \times \frac{1}{10^{12}}$	$1 \cdot 615\ 1482 \times \frac{1}{10^{12}}$	$1 \cdot 211\ 3612 \times \frac{1}{10^{12}}$	$9 \cdot 142\ 3484 \times \frac{1}{10^{12}}$	$6 \cdot 941\ 4127 \times \frac{1}{10^{12}}$
·0000 001	·0000 001	·0000 001	·0000 001	·0000 001	·0000 001
·0000 001	·0000 001	·0000 001	·0000 001	·0000 001	·0000 001
·0000 002	·0000 001	·0000 001	·0000 001	·0000 001	·0000 001
·0000 005 <sup>+</sup>	·0000 003	·0000 001	·0000 001	·0000 001	·0000 001
·0000 011	·0000 006	·0000 003	·0000 002	·0000 002	·0000 002
·0000 022	·0000 012	·0000 007	·0000 004	·0000 004	·0000 004
·0000 043	·0000 024	·0000 014	·0000 008	·0000 009	·0000 009
·0000 082	·0000 048	·0000 028	·0000 016	·0000 019	·0000 019
·0000 153	·0000 091	·0000 054	·0000 032	·0000 037	·0000 037
·0000 280	·0000 171	·0000 104	·0000 062	·0000 073	·0000 073
·0000 504	·0000 313	·0000 194	·0000 119	·0000 139	·0000 139
·0000 886	·0000 503	·0000 355 <sup>+</sup>	·0000 223	·0000 260	·0000 260
·0001 529	·0000 990	·0000 638	·0000 409	·0000 476	·0000 476
·0002 589	·0001 709	·0001 122	·0000 733	·0000 854	·0000 854
·0004 300	·0002 893	·0001 936	·0001 289	·0001 499	·0001 499
·0007 014	·0004 806	·0003 276	·0002 222	·0002 583	·0002 583
·0011 237	·0007 840	·0005 442	·0003 758	·0004 365 <sup>-</sup>	·0004 365 <sup>-</sup>
·0017 687	·0012 561	·0008 875 <sup>-</sup>	·0006 239	·0007 239	·0007 239
·0027 303	·0019 773	·0014 215 <sup>-</sup>	·0010 168	·0011 786	·0011 786
·0041 617	·0030 587	·0022 366	·0016 274	·0018 843	·0018 843
·0062 243	·0046 513	·0034 583	·0025 587	·0029 588	·0029 588
·0091 502	·0069 544	·0052 556	·0039 526	·0045 645 <sup>-</sup>	·0045 645 <sup>-</sup>
·0132 505 <sup>+</sup>	·0102 254	·0078 518	·0060 004	·0069 188	·0069 188
·0188 675 <sup>+</sup>	·0147 882	·0115 341	·0089 536	·0103 065 <sup>-</sup>	·0103 065 <sup>-</sup>
·0264 377	·0210 390	·0166 619	·0131 340	·0150 895 <sup>-</sup>	·0150 895 <sup>-</sup>
·0364 595 <sup>+</sup>	·0294 485 <sup>+</sup>	·0236 725 <sup>+</sup>	·0189 420	·0217 156	·0217 156
·0494 906	·0405 578	·0330 817	·0268 616	·0307 207	·0307 207
·0661 294	·0549 656	·0454 764	·0374 581	·0427 245 <sup>-</sup>	·0427 245 <sup>-</sup>
·0869 873	·0733 065 <sup>-</sup>	·0614 988	·0513 679	·0584 145 <sup>-</sup>	·0584 145 <sup>-</sup>
·1126 501	·0962 162	·0818 176	·0692 764	·0785 176	·0785 176
·1436 291	·1242 869	·1070 873	·0918 827	·1037 565 <sup>+</sup>	·1037 565 <sup>+</sup>
·1803 046	·1580 107	·1378 948	·1198 509	·1347 906	·1347 906
·2228 658	·1977 169	·1746 959	·1537 469	·1721 434	·1721 434
·2712 530	·2435 064	·2177 448	·1939 669	·2161 224	·2161 224
·3251 088	·2951 926	·2670 253	·2406 607	·2667 370	·2667 370
·3837 475 <sup>-</sup>	·3522 544	·3221 906	·2936 601	·3236 271	·3236 271
·4461 491	·4138 139	·3825 233	·3524 218	·3860 133	·3860 133
·5109 862	·4786 443	·4469 243	·4159 974	·4526 832	·4526 832
·5766 842	·5452 154	·5139 403	·4830 404	·5220 221	·5220 221
·6415 169	·6117 779	·5818 329	·5518 581	·5920 967	·5920 967
·7037 284	·6764 821	·6486 909	·6205 106	·6607 882	·6607 882
·7616 710	·7375 221	·7125 744	·6869 531	·7259 667	·7259 667
·8139 430	·7932 879	·7716 795 <sup>+</sup>	·7492 068	·7856 863	·7856 863
·8595 077	·8425 079	·8245 011	·8055 400	·8383 744	·8383 744
·8977 764	·8843 585 <sup>-</sup>	·8699 703	·8546 325 <sup>-</sup>	·8912 228	·8912 228
·9286 413	·9185 250 <sup>-</sup>	·8975 101		·9424 845	·9424 845
				·5636 015	·5636 015
				·6342 120	·6342 120
				·7020 626	·7020 626

TABLE I. THE  $I_x(p, q)$  FUNCTION

to .97

 $q = 13$ 

$p = 44$	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$\cdot 4070\ 1920 \times \frac{1}{10^{13}}$	$\cdot 3141\ 9026 \times \frac{1}{10^{13}}$	$\cdot 2437\ 6830 \times \frac{1}{10^{13}}$	$\cdot 1900\ 5664 \times \frac{1}{10^{13}}$	$\cdot 1488\ 7770 \times \frac{1}{10^{13}}$	$\cdot 1171\ 496$
•0000 001					
•0000 001	•0000 001				
•0000 003	•0000 002	•0000 001	•0000 001		
•0000 006	•0000 004	•0000 002	•0000 001	•0000 001	
•0000 013	•0000 008	•0000 005	•0000 003	•0000 002	•0000 001
•0000 027	•0000 016	•0000 010	•0000 006	•0000 004	•0000 002
•0000 054	•0000 033	•0000 020	•0000 012	•0000 008	•0000 005
•0000 104	•0000 066	•0000 041	•0000 026	•0000 016	•0000 010
•0000 198	•0000 127	•0000 081	•0000 052	•0000 033	•0000 021
•0000 369	•0000 241	•0000 157	•0000 102	•0000 066	•0000 042
•0000 673	•0000 448	•0000 297	•0000 196	•0000 129	•0000 085
•0001 203	•0000 816	•0000 551	•0000 370	•0000 248	•0000 166
•0002 107	•0001 455	•0001 000	•0000 685	•0000 467	•0000 317
•0003 619	•0002 542	•0001 779	•0001 239	•0000 860	•0000 595
•0006 098	•0004 358	•0003 102	•0002 199	•0001 553	•0001 093
•0010 082	•0007 328	•0005 304	•0003 824	•0002 747	•0001 966
•0016 361	•0012 089	•0008 897	•0006 522	•0004 763	•0003 466
•0026 066	•0019 574	•0014 641	•0010 908	•0008 097	•0005 988
•0040 778	•0031 111	•0023 642	•0017 897	•0013 498	•0010 144
•0062 651	•0048 547	•0037 470	•0028 811	•0022 072	•0016 849
•0094 544	•0074 382	•0058 292	•0045 512	•0035 404	•0027 444
•0140 148	•0111 914	•0089 025	•0070 554	•0055 715	•0043 844
•0204 090	•0165 364	•0133 479	•0107 348	•0086 026	•0068 701
•0291 982	•0239 971	•0196 490	•0160 307	•0130 328	•0105 596
•0410 392	•0342 015	•0283 987	•0234 965	•0193 735	•0159 205
•0566 695	•0478 737	•0402 976	•0338 019	•0282 571	•0235 440
•0768 774	•0658 113	•0561 396	•0477 251	•0404 368	•0341 506
•1024 546	•0888 457	•0767 792	•0661 291	•0567 707	•0485 821
•1341 299	•1177 816	•1030 790	•0899 169	•0781 859	•0677 746
•1724 875	•1533 178	•1358 350	•1199 636	•1056 186	•0927 081
•2178 734	•1959 521	•1756 821	•1570 250	•1399 279	•1243 268
•2703 003	•2458 783	•2229 867	•2016 278	•1817 868	•1634 339
•3293 637	•3028 879	•2777 360	•2539 510	•2315 576	•2105 639
•3941 844	•3662 926	•3394 421	•3137 123	•2891 649	•2658 450
•4633 934	•4348 842	•4070 781	•3800 821	•3539 878	•3288 721
•5351 726	•5069 505	•4790 670	•4516 442	•4247 933	•3986 135
•6073 578	•5803 571	•5533 384	•5264 254	•4997 357	•4733 797
•6776 026	•6526 979	•6274 611	•6020 049	•5764 403	•5508 757
•7435 886	•7215 050	•6988 473	•6757 068	•6521 773	•6283 537
•8032 560	•7844 938	•7650 061	•7448 575	•7241 173	•7028 586
•8550 200	•8398 074	•8238 139	•8070 763	•7866 371	•7715 440
•8979 358	•8862 198	•8737 539	•8605 510	•8466 288	•8320 106
•9317 785	•9232 563	•9140 807	•9042 469	•8937 540	•8826 053
•9570 196	•9512 049	•9448 707	•9380 023	•9305 873	•9226 162

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

0 70

$q = 14$

$p = 14$  t

$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$	$p = 19$
$= .3561\ 0481 \times \frac{1}{10^8}$	$.1780\ 5241 \times \frac{1}{10^8}$	$.9209\ 6072 \times \frac{1}{10^9}$	$.4911\ 7905 \times \frac{1}{10^8}$	$.2693\ 5625 \times \frac{1}{10^8}$	$.1515\ 1289 \times \frac{1}{10^8}$
.0000 001					
.0000 002					
.0000 006	.0000 001				
.0000 015 <sup>-</sup>	.0000 004	.0000 001			
.0000 036	.0000 009	.0000 002	.0000 001		
.0000 083	.0000 023	.0000 006	.0000 002		
.0000 179	.0000 053	.0000 015 <sup>+</sup>	.0000 004	.0000 001	
.0000 362	.0000 114	.0000 035 <sup>-</sup>	.0000 010	.0000 003	.0000 001
.0000 699	.0000 232	.0000 075 <sup>-</sup>	.0000 023	.0000 007	.0000 002
.0001 289	.0000 451	.0000 153	.0000 051	.0000 016	.0000 005 <sup>+</sup>
.0002 285 <sup>+</sup>	.0000 840	.0000 300	.0000 105 <sup>-</sup>	.0000 036	.0000 012
.0003 905 <sup>-</sup>	.0001 505 <sup>+</sup>	.0000 565 <sup>-</sup>	.0000 207	.0000 074	.0000 026
.0006 454	.0002 603	.0001 022	.0000 391	.0000 146	.0000 054
.0010 346	.0004 357	.0001 786	.0000 714	.0000 279	.0000 107
.0016 129	.0007 079	.0003 024	.0001 261	.0000 514	.0000 205 <sup>+</sup>
.0024 500 <sup>-</sup>	.0011 186	.0004 972	.0002 157	.0000 915 <sup>-</sup>	.0000 380
.0036 333	.0017 227	.0007 954	.0003 585 <sup>-</sup>	.0001 580	.0000 682
.0052 692	.0025 906	.0012 406	.0005 799	.0002 651	.0001 188
.0074 840	.0038 098	.0018 895 <sup>-</sup>	.0009 149	.0004 333	.0002 011
.0104 244	.0054 872	.0028 145 <sup>+</sup>	.0014 097	.0006 907	.0003 317
.0142 565 <sup>+</sup>	.0077 498	.0041 060	.0021 247	.0010 758	.0005 338
.0191 640	.0107 453	.0058 737	.0031 364	.0016 390	.0008 395 <sup>+</sup>
.0253 448	.0146 415 <sup>-</sup>	.0082 480	.0045 398	.0024 458	.0012 918
.0330 071	.0196 246	.0113 810	.0064 593	.0035 789	.0019 470
.0423 632	.0258 962	.0154 452	.0090 047	.0051 404	.0028 777
.0536 230	.0336 688	.0206 321	.0123 619	.0072 539	.0041 748
.0669 863	.0431 604	.0271 494	.0167 023	.0100 652	.0059 503
.0826 346	.0545 876	.0352 165 <sup>+</sup>	.0222 258	.0137 436	.0083 385 <sup>+</sup>
.1007 226	.0681 578	.0450 585 <sup>-</sup>	.0291 489	.0184 800	.0114 979
.1213 695 <sup>-</sup>	.0840 603	.0568 991	.0376 996	.0244 858	.0156 106
.1446 518	.1024 577	.0709 528	.0481 117	.0319 886	.0208 816
.1705 958	.1234 768	.0874 151	.0606 167	.0412 272	.0275 361
.1991 729	.1472 002	.1064 535 <sup>-</sup>	.0754 351	.0524 450 <sup>-</sup>	.0358 154
.2302 954	.1736 584	.1281 977	.0927 668	.0658 810	.0459 706
.2638 151	.2028 244	.1527 306	.1127 809	.0817 611	.0582 549
.2995 240	.2346 087	.1800 799	.1356 048	.1002 864	.0729 146
.3371 573	.2688 580	.2102 116	.1613 152	.1216 229	.0901 777
.3703 986	.3053 548	.2430 256	.1899 290	.1458 901	.1102 430
.4168 872	.3438 207	.2783 531	.2213 963	.1731 506	.1332 674
.4582 276	.3839 219	.3159 570	.2555 957	.2034 010	.1593 544
.5000 000 <sup>e</sup>	.4252 770	.3555 356	.2923 324	.2365 648	.1885 428
.5417 724	.4674 668	.3967 279	.3313 385 <sup>+</sup>	.2724 881	.2207 979
.5831 128	.5100 463	.4391 231	.3722 780	.3109 378	.2560 042
.6236 014	.5525 576	.4822 716	.4147 531	.3516 024	.2920 618

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .71$  to  $.93$  $q = 14$ 

	$p = 14$	$p = 15$	$p = 16$	$p = 17$	$p = 18$
$B(p, q) = .3561\ 0481 \times \frac{1}{10^8}$	$.1780\ 5241 \times \frac{1}{10^8}$	$.9209\ 6072 \times \frac{1}{10^8}$	$.4911\ 7905 \times \frac{1}{10^8}$	$.2693\ 562$	
$x$					
.71	.9895 756	.9846 383	.9780 949	.9696 742	.9591 237
.72	.9925 160	.9888 418	.9839 037	.9774 595 <sup>-</sup>	.9692 715
.73	.9947 308	.9920 522	.9884 022	.9835 727	.9773 513
.74	.9963 667	.9944 561	.9918 170	.9882 772	.9836 547
.75	.9975 500 <sup>+</sup>	.9962 186	.9943 546	.9918 208	.9884 672
.76	.9983 871	.9974 822	.9961 983	.9944 297	.9920 578
.77	.9989 654	.9983 665 <sup>-</sup>	.9975 056	.9963 043	.9946 718
.78	.9993 546	.9989 696	.9984 089	.9976 163	.9965 252
.79	.9996 095 <sup>+</sup>	.9993 696	.9990 157	.9985 090	.9978 026
.80	.9997 715 <sup>-</sup>	.9996 269	.9994 111	.9990 981	.9986 563
.81	.9998 711	.9997 872	.9996 604	.9994 742	.9992 080
.82	.9999 301	.9998 834	.9998 119	.9997 056	.9995 518
.83	.9999 638	.9999 389	.9999 004	.9998 424	.9997 575
.84	.9999 821	.9999 696	.9999 498	.9999 198	.9998 752
.85	.9999 917	.9999 857	.9999 761	.9999 614	.9999 393
.86	.9999 964	.9999 937	.9999 893	.9999 826	.9999 723
.87	.9999 985 <sup>+</sup>	.9999 974	.9999 956	.9999 927	.9999 883
.88	.9999 994	.9999 990	.9999 983	.9999 972	.9999 954
.89	.9999 998	.9999 997	.9999 994	.9999 990	.9999 984
.90	.9999 999	.9999 999	.9999 998	.9999 997	.9999 995
.91	1.0000 000	1.0000 000	1.0000 000	.9999 999	.9999 999
.92				1.0000 000	1.0000 000
.93					

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

18 to .80

$q = 14$

$p = 20$

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$
$\gamma, q) = .8723\ 4696 \times 10^{10}$		$.5131\ 4527 \times 10^{10}$	$.3078\ 8716 \times 10^{10}$	$.1881\ 5327 \times 10^{10}$	$.1169\ 6014 \times 10^{10}$	$.7386\ 9561 \times 10^{10}$
18	.0000 001					
19	.0000 002					
20	.0000 004	.0000 001				
21	.0000 009	.0000 003	.0000 001			
22	.0000 019	.0000 007	.0000 002	.0000 001		
23	.0000 040	.0000 015	.0000 005 <sup>+</sup>	.0000 002	.0000 001	
24	.0000 080	.0000 031	.0000 012	.0000 004	.0000 002	.0000 001
25	.0000 155	.0000 062	.0000 024	.0000 009	.0000 004	.0000 001
26	.0000 289	.0000 120	.0000 049	.0000 020	.0000 008	.0000 003
27	.0000 522	.0000 225 <sup>+</sup>	.0000 096	.0000 040	.0000 017	.0000 007
28	.0000 916	.0000 410	.0000 180	.0000 078	.0000 033	.0000 014
29	.0001 563	.0000 724	.0000 330	.0000 148	.0000 065 <sup>+</sup>	.0000 029
30	.0002 600	.0001 244	.0000 586	.0000 272	.0000 124	.0000 056
31	.0004 221	.0002 086	.0001 014	.0000 486	.0000 230	.0000 107
32	.0006 698	.0003 414	.0001 712	.0000 846	.0000 412	.0000 198
33	.0010 400	.0005 461	.0002 823	.0001 437	.0000 722	.0000 358
34	.0015 820	.0008 551	.0004 549	.0002 385	.0001 233	.0000 629
35	.0023 598	.0013 117	.0007 177	.0003 870	.0002 058	.0001 080
36	.0034 553	.0019 733	.0011 095 <sup>+</sup>	.0006 148	.0003 360	.0001 813
37	.0049 703	.0029 141	.0016 823	.0009 572	.0005 372	.0002 977
38	.0070 293	.0042 275 <sup>+</sup>	.0025 038	.0014 617	.0008 418	.0004 787
39	.0097 809	.0060 296	.0036 608	.0021 911	.0012 940	.0007 546
40	.0133 990	.0084 604	.0052 621	.0032 269	.0019 526	.0011 668
41	.0180 821	.0116 864	.0074 409	.0046 717	.0028 946	.0017 713
42	.0240 523	.0159 004	.0103 571	.0066 531	.0042 182	.0026 416
43	.0315 513	.0213 207	.0141 983	.0093 260	.0060 467	.0038 728
44	.0408 357	.0281 893	.0191 802	.0128 739	.0085 307	.0055 846
45	.0521 696	.0367 665	.0255 442	.0175 101	.0118 513	.0079 255
46	.0658 160	.0473 254	.0335 543	.0234 764	.0162 203	.0110 745 <sup>+</sup>
47	.0820 255 <sup>+</sup>	.0601 428	.0434 912	.0310 403	.0218 807	.0152 439
48	.1010 250 <sup>+</sup>	.0754 891	.0556 440	.0404 895 <sup>+</sup>	.0291 039	.0206 786
49	.1230 043	.0936 153	.0703 001	.0521 247	.0381 851	.0276 548
50	.1481 032	.1147 405 <sup>+</sup>	.0877 326	.0662 491	.0494 359	.0364 757
51	.1763 987	.1390 367	.1081 864	.0831 556	.0631 745 <sup>+</sup>	.0474 644
52	.2078 939	.1666 153	.1318 627	.1031 128	.0797 128	.0609 539
53	.2425 091	.1975 137	.1589 033	.1263 478	.0993 409	.0772 739
54	.2800 751	.2316 840	.1893 764	.1530 302	.1223 099	.0967 351
55	.3203 315 <sup>+</sup>	.2689 851	.2232 624	.1832 550 <sup>+</sup>	.1488 139	.1196 107
56	.3629 279	.3091 779	.2604 445	.2170 275	.1789 715 <sup>+</sup>	.1461 166
57	.4074 304	.3519 249	.3007 012	.2542 507	.2128 087	.1763 916
58	.4533 326	.3967 958	.3437 051	.2947 169	.2502 441	.2104 781
59	.5000 698	.4432 767	.3890 259	.3381 041	.2910 789	.2483 055 <sup>+</sup>
60	.5470 384	.4907 854	.4361 396	.3839 778	.3349 910	.2896 782
61	.5936 168	.5386 903	.4844 438	.4318 001	.3815 368	.3342 683
62	.6420 882	.5866 882	.5344 482	.4806 001	.4303 368	.3832 683

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .81$  to  $.94$  $q = 14$ 

	$p = 20$	$p = 21$	$p = 22$	$p = 23$	$p = 24$
$B(p, q) = .8723\ 4696 \times \frac{1}{10^{10}}$	$.5131\ 4527 \times \frac{1}{10^{10}}$	$.3078\ 8716 \times \frac{1}{10^{10}}$	$.1881\ 5327 \times \frac{1}{10^{10}}$	$.1169\ 6000 \times \frac{1}{10^{10}}$	
$\cdot 81$	$.9983\ 303$	$.9976\ 534$	$.9967\ 657$	$.9956\ 217$	$.9941\ 711$
$\cdot 82$	$.9990\ 346$	$.9986\ 288$	$.9980\ 899$	$.9973\ 870$	$.9964\ 841$
$\cdot 83$	$.9994\ 065^-$	$.9992\ 341$	$.9989\ 219$	$.9985\ 096$	$.9979\ 741$
$\cdot 84$	$.9997\ 196$	$.9995\ 933$	$.9994\ 215^-$	$.9991\ 919$	$.9988\ 900$
$\cdot 85$	$.9998\ 608$	$.9997\ 959$	$.9997\ 067$	$.9995\ 860$	$.9994\ 251$
$\cdot 86$	$.9999\ 352$	$.9999\ 040$	$.9998\ 606$	$.9998\ 011$	$.9997\ 211$
$\cdot 87$	$.9999\ 719$	$.9999\ 580$	$.9999\ 384$	$.9999\ 113$	$.9998\ 741$
$\cdot 88$	$.9999\ 888$	$.9999\ 831$	$.9999\ 750^+$	$.9999\ 636$	$.9999\ 471$
$\cdot 89$	$.9999\ 960$	$.9999\ 939$	$.9999\ 908$	$.9999\ 865^+$	$.9999\ 800$
$\cdot 90$	$.9999\ 987$	$.9999\ 980$	$.9999\ 970$	$.9999\ 956$	$.9999\ 931$
$\cdot 91$	$.9999\ 996$	$.9999\ 994$	$.9999\ 992$	$.9999\ 987$	$.9999\ 981$
$\cdot 92$	$.9999\ 999$	$.9999\ 999$	$.9999\ 998$	$.9999\ 997$	$.9999\ 996$
$\cdot 93$	$1.0000\ 000$	$1.0000\ 000$	$1.0000\ 000$	$.9999\ 999$	$.9999\ 999$
$\cdot 94$				$1.00000000$	$1.0000\ 000$



# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

25 to 95

$q = 14$

$p =$

$p = 26$	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$
$(p, q) = .4735\ 2283 \times \frac{1}{10^{11}}$	$.3077\ 8984 \times \frac{1}{10^{11}}$	$.2026\ 9087 \times \frac{1}{10^{11}}$	$.1351\ 2725 \times \frac{1}{10^{11}}$	$.9113\ 2328 \times \frac{1}{10^{11}}$	$.6213\ 506$
$x$					
.25	.0000 00I				
.26	.0000 00I				
.27	.0000 003				
.28	.0000 006	.0000 00I			
.29	.0000 012	.0000 002	.0000 00I		
.30	.0000 025 <sup>+</sup>	.0000 005 <sup>-</sup>	.0000 002		
.31	.0000 049	.0000 010	.0000 004	.0000 00I	.0000 00
.32	.0000 094	.0000 021	.0000 009	.0000 004	.0000 00
.33	.0000 175 <sup>+</sup>	.0000 041	.0000 019	.0000 009	.0000 00
.34	.0000 317	.0000 078	.0000 038	.0000 018	.0000 00
.35	.0000 560	.0000 146	.0000 073	.0000 036	.0000 01
.36	.0000 967	.0000 266	.0000 137	.0000 070	.0000 03
.37	.0001 630	.0000 473	.0000 251	.0000 132	.0000 06
.38	.0002 690	.0000 822	.0000 447	.0000 241	.0000 12
.39	.0004 348	.0001 397	.0000 780	.0000 431	.0000 23
.40	.0006 890	.0002 325 <sup>+</sup>	.0001 330	.0000 754	.0000 42
.41	.0010 712	.0003 792	.0002 222	.0001 290	.0000 74
.42	.0016 331	.0006 064	.0003 638	.0002 162	.0001 27
.43	.0024 519	.0009 516	.0005 840	.0003 551	.0002 14
.44	.0030 143	.0014 662	.0009 200	.0005 720	.0003 52
.45	.0052 402	.0022 193	.0014 231	.0009 042	.0005 69
.46	.0074 766	.0033 022	.0021 626	.0014 035 <sup>-</sup>	.0009 02
.47	.0105 025 <sup>+</sup>	.0048 324	.0032 306	.0021 403	.0014 05
.48	.0145 315 <sup>-</sup>	.0069 582	.0047 461	.0032 083	.0021 50
.49	.0198 119	.0098 626	.0068 602	.0047 294	.0032 33
.50	.0266 260	.0137 666	.0097 602	.0068 591	.0047 79
.51	.0352 859	.0189 302	.0136 734	.0097 906	.0069 52
.52	.0461 268	.0256 523	.0188 684	.0137 594	.0099 51
.53	.0594 968	.0342 672	.0256 552	.0190 448	.0140 23
.54	.0757 434	.0451 375 <sup>-</sup>	.0343 817	.0259 699	.0194 59
.55	.0951 908	.0586 438	.0454 266	.0348 985 <sup>-</sup>	.0265 98
.56	.1181 504	.0751 696	.0591 884	.0462 270	.0358 23
.57	.1448 398	.0950 826	.0760 691	.0603 731	.0475 49
.58	.1754 203	.1187 124	.0964 546	.0777 581	.0622 15
.59	.2099 464	.1463 256	.1206 904	.0987 855 <sup>-</sup>	.0802 61
.60	.2483 529	.1781 000	.1490 542	.1238 145 <sup>-</sup>	.1021 08
.61	.2904 410	.2140 987	.1817 284	.1531 308	.1281 27
.62	.3358 701	.2542 483	.2187 715 <sup>-</sup>	.1869 157	.1586 06
.63	.3841 576	.2983 213	.2600 945 <sup>+</sup>	.2252 159	.1937 20
.64	.4346 872	.3459 262	.3054 424	.2679 180	.2334 95
.65	.4867 268	.3905 078	.3543 834	.3147 283	.2777 82
.66	.5394 543	.4493 573	.4063 103	.3651 633	.3262 38
.67	.5919 923	.5036 346	.4604 531	.4185 522	.3783 13
.68	.6434 477	.5584 019	.5159 050 <sup>+</sup>	.4740 529	.4332 61
.69	.6929 549	.6126 661	.5716 603	.5306 827	.4901 55
.70	.7397 188	.6654 286	.6266 627	.5873 620	.5470 30
.71	.7830 545 <sup>-</sup>	.7502 469	.7157 381	.6429 701	.6054 27
.72	.8224 204	.7935 440	.7627 425 <sup>+</sup>	.6861 668	.6461 67

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .32$  to  $.96$  $q = 14$ 

	$p = 32$	$p = 33$	$p = 34$	$p = 35$	$p = 36$
$B(p, q) = .4280\ 4578 \times \frac{x}{10^{11}}$		$.2977\ 7098 \times \frac{x}{10^{11}}$	$.2090\ 7324 \times \frac{x}{10^{11}}$	$.1480\ 9355 \times \frac{x}{10^{11}}$	$.1057\ 81$
$x$					
.32	.0000 001				
.33	.0000 002	.0000 001			
.34	.0000 004	.0000 002	.0000 001		
.35	.0000 009	.0000 004	.0000 002	.0000 001	
.36	.0000 018	.0000 009	.0000 004	.0000 002	.0000 001
.37	.0000 035 <sup>+</sup>	.0000 018	.0000 009	.0000 005 <sup>-</sup>	.0000 002
.38	.0000 068	.0000 036	.0000 019	.0000 010	.0000 005
.39	.0000 128	.0000 069	.0000 037	.0000 020	.0000 010
.40	.0000 236	.0000 130	.0000 071	.0000 039	.0000 020
.41	.0000 423	.0000 240	.0000 135 <sup>-</sup>	.0000 075 <sup>-</sup>	.0000 040
.42	.0000 744	.0000 431	.0000 248	.0000 141	.0000 080
.43	.0001 279	.0000 758	.0000 446	.0000 260	.0000 150
.44	.0002 154	.0001 306	.0000 786	.0000 469	.0000 270
.45	.0003 557	.0002 204	.0001 355 <sup>+</sup>	.0000 828	.0000 500
.46	.0005 761	.0003 647	.0002 291	.0001 429	.0000 880
.47	.0009 157	.0005 919	.0003 797	.0002 419	.0001 530
.48	.0014 294	.0009 429	.0006 174	.0004 013	.0002 590
.49	.0021 922	.0014 750 <sup>+</sup>	.0009 852	.0006 534	.0004 300
.50	.0033 044	.0022 669	.0015 438	.0010 441	.0007 010
.51	.0048 978	.0034 243	.0023 768	.0016 383	.0011 210
.52	.0071 411	.0050 859	.0035 962	.0025 254	.0017 610
.53	.0102 454	.0074 299	.0053 498	.0038 259	.0027 180
.54	.0144 690	.0106 797	.0078 273	.0056 981	.0041 210
.55	.0201 197	.0151 085 <sup>-</sup>	.0112 666	.0083 456	.0061 420
.56	.0275 542	.0210 422	.0159 589	.0120 238	.0090 010
.57	.0371 748	.0288 588	.0222 514	.0170 451	.0129 750
.58	.0494 196	.0389 833	.0305 457	.0237 808	.0183 990
.59	.0647 486	.0518 781	.0412 930	.0326 599	.0256 740
.60	.0836 230	.0680 261	.0549 813	.0441 619	.0352 590
.61	.1064 786	.0879 083	.0721 179	.0588 029	.0476 630
.62	.1336 944	.1119 742	.0932 026	.0771 143	.0634 340
.63	.1655 576	.1406 070	.1186 958	.0996 137	.0831 260
.64	.2022 268	.1740 849	.1489 789	.1267 674	.1072 720
.65	.2436 979	.2125 420	.1843 129	.1589 482	.1363 370
.66	.2897 742	.2559 311	.2247 954	.1963 887	.1706 760
.67	.3400 475 <sup>+</sup>	.3039 939	.2703 220	.2391 359	.2104 800
.68	.3938 904	.3562 421	.3205 562	.2870 114	.2557 280
.69	.4504 657	.4119 538	.3749 124	.3395 815 <sup>-</sup>	.3061 480
.70	.5087 531	.4701 880	.4325 578	.3961 450 <sup>+</sup>	.3611 880
.71	.5675 935 <sup>-</sup>	.5298 190	.4924 336	.4557 410	.4200 120
.72	.6257 489	.5895 903	.5533 001	.5171 808	.4815 150
.73	.6819 743	.6481 846	.6138 010	.5791 040	.5443 670
.74	.7350 946	.7043 052	.6725 454	.6390 570	.6070 860
.75	.7840 807	.7567 607	.7281 980	.6985 870	.6681 290
.76	.8281 153	.8015 457	.7705 762	.7382 226	.7065 260

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .96

$q = 1.4$

$p = 38$

$p = 38$	$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$
$\cdot 5525\ 5071 \times \frac{1}{10^{13}}$	$\cdot 4037\ 8706 \times \frac{1}{10^{13}}$	$\cdot 2971\ 2632 \times \frac{1}{10^{13}}$	$\cdot 2200\ 9357 \times \frac{1}{10^{13}}$	$\cdot 1640\ 6976 \times \frac{1}{10^{13}}$	$\cdot 1230\ 5232 \times \frac{1}{10^{13}}$
•0000 001					
•0000 001	•0000 001				
•0000 003	•0000 001	•0000 001			
•0000 006	•0000 003	•0000 002	•0000 001		
•0000 012	•0000 007	•0000 004	•0000 002	•0000 001	•0000 001
•0000 025 <sup>+</sup>	•0000 014	•0000 008	•0000 004	•0000 002	•0000 001
•0000 050 <sup>-</sup>	•0000 028	•0000 016	•0000 009	•0000 005 <sup>+</sup>	•0000 003
•0000 096	•0000 056	•0000 032	•0000 019	•0000 011	•0000 006
•0000 181	•0000 108	•0000 064	•0000 037	•0000 022	•0000 013
•0000 333	•0000 203	•0000 122	•0000 074	•0000 044	•0000 026
•0000 601	•0000 373	•0000 230	•0000 141	•0000 086	•0000 052
•0001 060	•0000 672	•0000 423	•0000 265 <sup>+</sup>	•0000 165 <sup>+</sup>	•0000 103
•0001 832	•0001 185 <sup>-</sup>	•0000 762	•0000 487	•0000 310	•0000 196
•0003 105 <sup>+</sup>	•0002 048	•0001 343	•0000 876	•0000 568	•0000 367
•0005 161	•0003 469	•0002 319	•0001 542	•0001 020	•0000 671
•0008 415 <sup>-</sup>	•0005 764	•0003 926	•0002 660	•0001 793	•0001 203
•0013 468	•0009 397	•0006 520	•0004 500 <sup>-</sup>	•0003 090	•0002 111
•0021 165 <sup>+</sup>	•0015 036	•0010 622	•0007 465 <sup>-</sup>	•0005 219	•0003 631
•0032 670	•0023 621	•0016 985 <sup>+</sup>	•0012 149	•0008 646	•0006 123
•0049 549	•0036 448	•0026 665 <sup>+</sup>	•0019 406	•0014 053	•0010 127
•0073 857	•0055 251	•0041 111	•0030 431	•0022 414	•0016 429
•0108 222	•0082 304	•0062 261	•0046 858	•0035 091	•0026 155 <sup>-</sup>
•0155 923	•0120 508	•0092 647	•0070 868	•0053 943	•0040 867
•0220 932	•0173 461	•0135 484	•0105 292	•0081 433	•0062 687
•0307 918	•0245 504	•0194 740	•0153 711	•0120 747	•0094 415 <sup>+</sup>
•0422 185 <sup>-</sup>	•0341 703	•0275 171	•0220 515 <sup>+</sup>	•0175 884	•0139 648
•0569 533	•0467 765 <sup>+</sup>	•0382 280	•0310 922	•0251 712	•0202 864
•0756 019	•0629 860	•0522 205 <sup>-</sup>	•0430 913	•0353 960	•0289 467
•0987 618	•0834 331	•0701 483	•0587 067	•0489 116	•0405 741
•1269 775 <sup>-</sup>	•1087 293	•0926 710	•0786 279	•0664 207	•0558 703
•1606 878	•1394 122	•1204 059	•1035 332	•0886 440	•0755 805 <sup>+</sup>
•2001 679	•1758 860	•1538 704	•1340 343	•1162 690	•1004 494
•2454 712	•2183 585 <sup>-</sup>	•1934 147	•1706 099	•1498 854	•1311 593
•2963 780	•2667 797	•2391 545 <sup>+</sup>	•2135 322	•1899 097	•1682 558
•3523 582	•3207 912	•2909 078	•2627 949	•2365 060	•2120 635 <sup>+</sup>
•4125 562	•3796 944	•3481 471	•3180 509	•2895 109	•2626 018
•4758 041	•4424 457	•4099 768	•3785 713	•3483 761	•3195 108
•5406 676	•5076 856	•4751 434	•4432 358	•4121 375 <sup>+</sup>	•3820 018
•6055 240	•5738 041	•5420 842	•5105 625 <sup>-</sup>	•4794 252	•4488 439
•6686 699	•6390 408	•6090 167	•5787 824	•5485 186	•5183 989
•7284 464	•7016 126	•6740 633	•6459 561	•6174 513	•5887 089
•7833 714	•7598 557	•7353 995 <sup>-</sup>	•7101 239	•6841 579	•6576 355 <sup>+</sup>
•8322 602	•8123 652	•7914 091	•7694 733	•7466 495 <sup>-</sup>	•7230 376
•8743 194	•8581 131	•8408 264	•8225 025 <sup>-</sup>	•8031 953	•7829 688
•9091 996	•8965 282	•8828 420	•8682 520		

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to .80

$q = 15$

$p = 21$  t

$p = 21$	$p = 22$	$p = 23$	$p = 24$	$p = 25$	$p = 26$
$\cdot 2052\ 581 \times \frac{1}{10^{10}}$	$\cdot 1197\ 3390 \times \frac{1}{10^{10}}$	$\cdot 7119\ 3127 \times \frac{1}{10^{11}}$	$\cdot 4309\ 0577 \times \frac{1}{10^{11}}$	$\cdot 2651\ 7278 \times \frac{1}{10^{11}}$	$\cdot 1657\ 3299 \times \frac{1}{10^{10}}$
$\cdot 0000\ 001$					
$\cdot 0000\ 003$	$\cdot 0000\ 001$				
$\cdot 0000\ 006$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 0000\ 013$	$\cdot 0000\ 005^-$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 0000\ 029$	$\cdot 0000\ 011$	$\cdot 0000\ 004$	$\cdot 0000\ 003$	$\cdot 0000\ 001$	
$\cdot 0000\ 060$	$\cdot 0000\ 023$	$\cdot 0000\ 009$	$\cdot 0000\ 007$	$\cdot 0000\ 003$	$\cdot 0000\ 001$
$\cdot 0000\ 118$	$\cdot 0000\ 048$	$\cdot 0000\ 019$	$\cdot 0000\ 016$	$\cdot 0000\ 006$	$\cdot 0000\ 003$
$\cdot 0000\ 227$	$\cdot 0000\ 095^+$	$\cdot 0000\ 039$	$\cdot 0000\ 033$	$\cdot 0000\ 014$	$\cdot 0000\ 006$
$\cdot 0000\ 420$	$\cdot 0000\ 183$	$\cdot 0000\ 079$	$\cdot 0000\ 067$	$\cdot 0000\ 029$	$\cdot 0000\ 012$
$\cdot 0000\ 754$	$\cdot 0000\ 341$	$\cdot 0000\ 152$	$\cdot 0000\ 129$	$\cdot 0000\ 058$	$\cdot 0000\ 025^+$
$\cdot 0001\ 315^-$	$\cdot 0000\ 615^+$	$\cdot 0000\ 284$	$\cdot 0000\ 241$	$\cdot 0000\ 112$	$\cdot 0000\ 051$
$\cdot 0002\ 232$	$\cdot 0001\ 080$	$\cdot 0000\ 514$			
$\cdot 0003\ 693$	$\cdot 0001\ 845^-$	$\cdot 0000\ 907$	$\cdot 0000\ 440$	$\cdot 0000\ 210$	$\cdot 0000\ 099$
$\cdot 0005\ 966$	$\cdot 0003\ 074$	$\cdot 0001\ 559$	$\cdot 0000\ 779$	$\cdot 0000\ 384$	$\cdot 0000\ 187$
$\cdot 0009\ 419$	$\cdot 0004\ 999$	$\cdot 0002\ 613$	$\cdot 0001\ 346$	$\cdot 0000\ 684$	$\cdot 0000\ 343$
$\cdot 0014\ 553$	$\cdot 0007\ 950^+$	$\cdot 0004\ 277$	$\cdot 0002\ 268$	$\cdot 0001\ 186$	$\cdot 0000\ 613$
$\cdot 0022\ 026$	$\cdot 0012\ 374$	$\cdot 0006\ 846$	$\cdot 0003\ 734$	$\cdot 0002\ 009$	$\cdot 0001\ 067$
$\cdot 0032\ 690$	$\cdot 0018\ 869$	$\cdot 0010\ 728$	$\cdot 0006\ 013$	$\cdot 0003\ 325^-$	$\cdot 0001\ 816$
$\cdot 0047\ 618$	$\cdot 0028\ 217$	$\cdot 0016\ 470$	$\cdot 0009\ 479$	$\cdot 0005\ 383$	$\cdot 0003\ 019$
$\cdot 0068\ 132$	$\cdot 0041\ 413$	$\cdot 0024\ 799$	$\cdot 0014\ 643$	$\cdot 0008\ 533$	$\cdot 0004\ 910$
$\cdot 0095\ 826$	$\cdot 0059\ 703$	$\cdot 0036\ 650^+$	$\cdot 0022\ 187$	$\cdot 0013\ 256$	$\cdot 0007\ 822$
$\cdot 0132\ 579$	$\cdot 0084\ 604$	$\cdot 0053\ 203$	$\cdot 0032\ 997$	$\cdot 0020\ 200$	$\cdot 0012\ 214$
$\cdot 0180\ 547$	$\cdot 0117\ 926$	$\cdot 0075\ 912$	$\cdot 0048\ 202$	$\cdot 0030\ 213$	$\cdot 0018\ 708$
$\cdot 0242\ 153$	$\cdot 0161\ 775^+$	$\cdot 0106\ 534$	$\cdot 0069\ 210$	$\cdot 0044\ 389$	$\cdot 0028\ 127$
$\cdot 0320\ 043$	$\cdot 0218\ 549$	$\cdot 0147\ 134$	$\cdot 0097\ 734$	$\cdot 0064\ 100$	$\cdot 0041\ 539$
$\cdot 0417\ 028$	$\cdot 0290\ 902$	$\cdot 0200\ 090$	$\cdot 0135\ 812$	$\cdot 0091\ 031$	$\cdot 0060\ 294$
$\cdot 0535\ 999$	$\cdot 0381\ 693$	$\cdot 0268\ 067$	$\cdot 0185\ 812$	$\cdot 0127\ 206$	$\cdot 0086\ 064$
$\cdot 0679\ 820$	$\cdot 0493\ 911$	$\cdot 0353\ 972$	$\cdot 0250\ 417$	$\cdot 0174\ 994$	$\cdot 0120\ 871$
$\cdot 0851\ 203$	$\cdot 0630\ 569$	$\cdot 0460\ 881$	$\cdot 0332\ 582$	$\cdot 0237\ 106$	$\cdot 0167\ 103$
$\cdot 1052\ 567$	$\cdot 0794\ 581$	$\cdot 0591\ 944$	$\cdot 0435\ 474$	$\cdot 0316\ 556$	$\cdot 0227\ 510$
$\cdot 1285\ 882$	$\cdot 0988\ 613$	$\cdot 0750\ 256$	$\cdot 0562\ 369$	$\cdot 0416\ 600$	$\cdot 0305\ 175^-$
$\cdot 1552\ 523$	$\cdot 1214\ 925^-$	$\cdot 0938\ 708$	$\cdot 0716\ 533$	$\cdot 0540\ 645^+$	$\cdot 0403\ 452$
$\cdot 1853\ 122$	$\cdot 1475\ 206$	$\cdot 1159\ 818$	$\cdot 0901\ 062$	$\cdot 0692\ 117$	$\cdot 0525\ 881$
$\cdot 2187\ 443$	$\cdot 1770\ 411$	$\cdot 1415\ 555^+$	$\cdot 1118\ 710$	$\cdot 0874\ 308$	$\cdot 0676\ 049$
$\cdot 2554\ 292$	$\cdot 2100\ 620$	$\cdot 1707\ 163$	$\cdot 1371\ 699$	$\cdot 1090\ 188$	$\cdot 0857\ 435^-$
$\cdot 2951\ 455^+$	$\cdot 2464\ 917$	$\cdot 2034\ 994$	$\cdot 1661\ 522$	$\cdot 1342\ 204$	$\cdot 1073\ 210$
$\cdot 3375\ 692$	$\cdot 2861\ 312$	$\cdot 2398\ 369$	$\cdot 1988\ 766$	$\cdot 1632\ 069$	$\cdot 1326\ 024$
$\cdot 3822\ 779$	$\cdot 3286\ 712$	$\cdot 2795\ 480$	$\cdot 2352\ 943$	$\cdot 1960\ 561$	$\cdot 1617\ 776$
$\cdot 4287\ 604$	$\cdot 3736\ 950^-$	$\cdot 3223\ 339$	$\cdot 2752\ 381$	$\cdot 2327\ 341$	$\cdot 1949\ 395^+$
$\cdot 4764\ 318$	$\cdot 4206\ 866$	$\cdot 3677\ 794$	$\cdot 3184\ 146$	$\cdot 2730\ 812$	$\cdot 2320\ 636$
$\cdot 5246\ 530$	$\cdot 4690\ 459$	$\cdot 4153\ 597$	$\cdot 3644\ 046$	$\cdot 3168\ 040$	$\cdot 2729\ 924$
$\cdot 5727\ 541$	$\cdot 5181\ 083$	$\cdot 4644\ 561$	$\cdot 4126\ 700$	$\cdot 3634\ 733$	$\cdot 3174\ 252$
$\cdot 6200\ 600$	$\cdot 5671\ 697$	$\cdot 5143\ 756$	$\cdot 4625\ 684$	$\cdot 4125\ 314$	$\cdot 3649\ 162$
$\cdot 6659\ 177$	$\cdot 6155\ 143$	$\cdot 5643\ 778$	$\cdot 5133\ 746$	$\cdot 4633\ 066$	$\cdot 4148\ 807$
$\cdot 7097\ 220$	$\cdot 6624\ 441$	$\cdot 6137\ 050^-$	$\cdot 5632\ 880$		

TABLES OF THE INCOMPLETE  $\beta$ -FUNCTION

= .26 to .94

 $q = 15$  $p =$ 

	$p = 27$	$p = 28$	$p = 29$	$p = 30$	$p = 31$	$p = 32$
$(p, q) =$	$\cdot 1050\ 9897 \times \frac{1}{10^{11}}$	$\cdot 6756\ 3623 \times \frac{1}{10^{11}}$	$\cdot 4399\ 4917 \times \frac{1}{10^{11}}$	$\cdot 2899\ 6650 \times \frac{1}{10^{11}}$	$\cdot 1933\ 1100 \times \frac{1}{10^{11}}$	$\cdot 1302\ 74$
$\pi$						
.26	·0000 001					
.27	·0000 002	·0000 001				
.28	·0000 005 <sup>+</sup>	·0000 002	·0000 001			
.29	·0000 011	·0000 005 <sup>-</sup>	·0000 002	·0000 001		
.30	·0000 023	·0000 010	·0000 005 <sup>-</sup>	·0000 002	·0000 001	
.31	·0000 046	·0000 021	·0000 010	·0000 004	·0000 002	·0000 001
.32	·0000 090	·0000 043	·0000 020	·0000 009	·0000 004	·0000 002
.33	·0000 170	·0000 083	·0000 040	·0000 019	·0000 009	·0000 004
.34	·0000 313	·0000 158	·0000 079	·0000 039	·0000 019	·0000 009
.35	·0000 560	·0000 291	·0000 149	·0000 076	·0000 038	·0000 019
.36	·0000 980	·0000 523	·0000 276	·0000 144	·0000 075 <sup>-</sup>	·0000 038
.37	·0001 673	·0000 917	·0000 497	·0000 267	·0000 142	·0000 075
.38	·0002 792	·0001 570	·0000 874	·0000 481	·0000 263	·0000 142
.39	·0004 562	·0002 631	·0001 502	·0000 849	·0000 475 <sup>-</sup>	·0000 263
.40	·0007 300	·0004 315 <sup>+</sup>	·0002 524	·0001 462	·0000 839	·0000 475
.41	·0011 450 <sup>+</sup>	·0006 932	·0004 153	·0002 464	·0001 448	·0000 848
.42	·0017 619	·0010 917	·0006 695 <sup>+</sup>	·0004 066	·0002 446	·0001 450
.43	·0026 614	·0016 868	·0010 582	·0006 574	·0004 047	·0002 464
.44	·0039 487	·0025 585 <sup>-</sup>	·0016 409	·0010 423	·0006 560	·0004 066
.45	·0057 581	·0038 118	·0024 980	·0016 214	·0010 428	·0006 574
.46	·0082 568	·0055 813	·0037 352	·0024 761	·0016 266	·0010 428
.47	·0116 486	·0080 360	·0054 891	·0037 142	·0024 907	·0016 266
.48	·0161 754	·0113 824	·0079 315 <sup>+</sup>	·0054 754	·0037 463	·0024 907
.49	·0221 177	·0158 676	·0112 738	·0079 362	·0055 375 <sup>+</sup>	·0037 463
.50	·0297 919	·0217 793	·0157 697	·0113 144	·0080 472	·0055 375
.51	·0395 449	·0294 437	·0217 162	·0158 727	·0115 017	·0082 650
.52	·0517 442	·0392 201	·0294 513	·0219 192	·0161 747	·0118 388
.53	·0667 651	·0514 911	·0393 482	·0298 056	·0223 878	·0166 800
.54	·0849 738	·0666 491	·0518 061	·0399 214	·0305 088	·0231 300
.55	·1067 062	·0850 781	·0672 349	·0526 835 <sup>+</sup>	·0409 454	·0315 730
.56	·1322 454	·1071 320	·0860 368	·0685 205 <sup>+</sup>	·0541 338	·0424 388
.57	·1617 966	·1331 096	·1085 822	·0878 523	·0705 221	·0561 822
.58	·1954 633	·1632 279	·1351 831	·1110 646	·0905 478	·0732 730
.59	·2332 249	·1975 962	·1660 648	·1384 803	·1146 105 <sup>-</sup>	·0941 670
.60	·2749 192	·2361 915 <sup>+</sup>	·2013 366	·1703 277	·1430 400	·1192 730
.61	·3202 311	·2788 395 <sup>-</sup>	·2409 661	·2067 104	·1760 629	·1480 250
.62	·3686 899	·3252 021	·2847 585 <sup>-</sup>	·2475 782	·2137 689	·1833 400
.63	·4196 760	·3747 748	·3323 431	·2927 060	·2560 814	·2225 870
.64	·4724 384	·4268 940	·3831 714	·3416 802	·3027 338	·2665 510
.65	·5261 216	·4807 567	·4365 261	·3938 968	·3532 570	·3149 110
.66	·5798 027	·5354 512	·4915 433	·4485 741	·4069 799	·3671 300
.67	·6325 345 <sup>+</sup>	·5899 975 <sup>+</sup>	·5472 479	·5047 783	·4630 449	·4224 510
.68	·6833 927	·6433 957	·6025 987	·5614 640	·5204 391	·4790 430
.69	·7315 237	·6946 779	·6565 424	·6175 259	·5780 412	·5384 910
.70	·7761 885 <sup>-</sup>	·7429 605 <sup>+</sup>	·7080 712	·6718 584	·6346 800	·5969 010
.71	·8167 995 <sup>-</sup>	·7874 917	·7562 789	·7234 193	·6892 016	·6530 330
.72	·8529 469	·8276 897	·8004 119	·7712 905 <sup>-</sup>	·7405 383	·7083 910

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .32$  to  $.95$  $q = 15$ 

	$p = 33$	$p = 34$	$p = 35$	$p = 36$	$p = 37$
$B(p, q) = .8869\ 7739 \times \frac{1}{10^3}$		$.6097\ 9695 \times \frac{1}{10^3}$	$.4231\ 2442 \times \frac{1}{10^3}$	$.2961\ 8709 \times \frac{1}{10^3}$	$.2090\ 7$
$\cdot 32$	$\cdot 0000\ 001$				
$\cdot 33$	$\cdot 0000\ 002$	$\cdot 0000\ 001$			
$\cdot 34$	$\cdot 0000\ 004$	$\cdot 0000\ 002$	$\cdot 0000\ 001$		
$\cdot 35$	$\cdot 0000\ 009$	$\cdot 0000\ 005^-$	$\cdot 0000\ 002$	$\cdot 0000\ 001$	$\cdot 0000\ 00$
$\cdot 36$	$\cdot 0000\ 019$	$\cdot 0000\ 010$	$\cdot 0000\ 005^-$	$\cdot 0000\ 002$	$\cdot 0000\ 00$
$\cdot 37$	$\cdot 0000\ 039$	$\cdot 0000\ 020$	$\cdot 0000\ 010$	$\cdot 0000\ 005^+$	$\cdot 0000\ 00$
$\cdot 38$	$\cdot 0000\ 076$	$\cdot 0000\ 041$	$\cdot 0000\ 021$	$\cdot 0000\ 011$	$\cdot 0000\ 00$
$\cdot 39$	$\cdot 0000\ 145^-$	$\cdot 0000\ 079$	$\cdot 0000\ 043$	$\cdot 0000\ 023$	$\cdot 0000\ 01$
$\cdot 40$	$\cdot 0000\ 269$	$\cdot 0000\ 150^+$	$\cdot 0000\ 083$	$\cdot 0000\ 046$	$\cdot 0000\ 02$
$\cdot 41$	$\cdot 0000\ 487$	$\cdot 0000\ 279$	$\cdot 0000\ 159$	$\cdot 0000\ 090$	$\cdot 0000\ 03$
$\cdot 42$	$\cdot 0000\ 863$	$\cdot 0000\ 506$	$\cdot 0000\ 294$	$\cdot 0000\ 170$	$\cdot 0000\ 06$
$\cdot 43$	$\cdot 0001\ 494$	$\cdot 0000\ 897$	$\cdot 0000\ 534$	$\cdot 0000\ 316$	$\cdot 0000\ 11$
$\cdot 44$	$\cdot 0002\ 532$	$\cdot 0001\ 554$	$\cdot 0000\ 947$	$\cdot 0000\ 572$	$\cdot 0000\ 23$
$\cdot 45$	$\cdot 0004\ 204$	$\cdot 0002\ 637$	$\cdot 0001\ 642$	$\cdot 0001\ 015^-$	$\cdot 0000\ 6$
$\cdot 46$	$\cdot 0006\ 842$	$\cdot 0004\ 385^-$	$\cdot 0002\ 789$	$\cdot 0001\ 761$	$\cdot 0001\ 1$
$\cdot 47$	$\cdot 0010\ 920$	$\cdot 0007\ 145^-$	$\cdot 0004\ 640$	$\cdot 0002\ 992$	$\cdot 0001\ 9$
$\cdot 48$	$\cdot 0017\ 103$	$\cdot 0011\ 419$	$\cdot 0007\ 568$	$\cdot 0004\ 980$	$\cdot 0003\ 2$
$\cdot 49$	$\cdot 0026\ 296$	$\cdot 0017\ 910$	$\cdot 0012\ 108$	$\cdot 0008\ 128$	$\cdot 0005\ 4$
$\cdot 50$	$\cdot 0039\ 714$	$\cdot 0027\ 576$	$\cdot 0019\ 008$	$\cdot 0013\ 011$	$\cdot 0008\ 8$
$\cdot 51$	$\cdot 0058\ 935^-$	$\cdot 0041\ 703$	$\cdot 0029\ 296$	$\cdot 0020\ 437$	$\cdot 0014\ 1$
$\cdot 52$	$\cdot 0085\ 973$	$\cdot 0061\ 968$	$\cdot 0044\ 345^+$	$\cdot 0031\ 516$	$\cdot 0022\ 2$
$\cdot 53$	$\cdot 0123\ 329$	$\cdot 0090\ 509$	$\cdot 0065\ 951$	$\cdot 0047\ 730$	$\cdot 0034\ 3$
$\cdot 54$	$\cdot 0174\ 031$	$\cdot 0129\ 982$	$\cdot 0096\ 401$	$\cdot 0071\ 014$	$\cdot 0051\ 9$
$\cdot 55$	$\cdot 0241\ 644$	$\cdot 0183\ 604$	$\cdot 0138\ 537$	$\cdot 0103\ 836$	$\cdot 0077\ 3$
$\cdot 56$	$\cdot 0330\ 243$	$\cdot 0255\ 155^+$	$\cdot 0195\ 792$	$\cdot 0149\ 251$	$\cdot 0113\ 0$
$\cdot 57$	$\cdot 0444\ 334$	$\cdot 0348\ 951$	$\cdot 0272\ 196$	$\cdot 0210\ 946$	$\cdot 0162\ 4$
$\cdot 58$	$\cdot 0588\ 719$	$\cdot 0469\ 749$	$\cdot 0372\ 334$	$\cdot 0293\ 233$	$\cdot 0229\ 5$
$\cdot 59$	$\cdot 0768\ 286$	$\cdot 0622\ 590$	$\cdot 0501\ 234$	$\cdot 0400\ 993$	$\cdot 0318\ 8$
$\cdot 60$	$\cdot 0987\ 743$	$\cdot 0812\ 571$	$\cdot 0664\ 190$	$\cdot 0539\ 550^+$	$\cdot 0435\ 6$
$\cdot 61$	$\cdot 1251\ 285^-$	$\cdot 1044\ 543$	$\cdot 0866\ 503$	$\cdot 0714\ 456$	$\cdot 0585\ 6$
$\cdot 62$	$\cdot 1562\ 214$	$\cdot 1322\ 743$	$\cdot 1113\ 135^-$	$\cdot 0931\ 195^+$	$\cdot 0774\ 5$
$\cdot 63$	$\cdot 1922\ 548$	$\cdot 1650\ 379$	$\cdot 1408\ 310$	$\cdot 1194\ 804$	$\cdot 1007\ 9$
$\cdot 64$	$\cdot 2332\ 634$	$\cdot 2029\ 210$	$\cdot 1755\ 057$	$\cdot 1509\ 416$	$\cdot 1291\ 0$
$\cdot 65$	$\cdot 2790\ 820$	$\cdot 2459\ 128$	$\cdot 2154\ 752$	$\cdot 1877\ 770$	$\cdot 1627\ 7$
$\cdot 66$	$\cdot 3293\ 224$	$\cdot 2937\ 832$	$\cdot 2606\ 691$	$\cdot 2300\ 716$	$\cdot 2020\ 2$
$\cdot 67$	$\cdot 3833\ 633$	$\cdot 3460\ 597$	$\cdot 3107\ 749$	$\cdot 2776\ 777$	$\cdot 2468\ 7$
$\cdot 68$	$\cdot 4403\ 591$	$\cdot 4020\ 221$	$\cdot 3652\ 187$	$\cdot 3301\ 818$	$\cdot 2970\ 9$
$\cdot 69$	$\cdot 4992\ 657$	$\cdot 4607\ 162$	$\cdot 4231\ 644$	$\cdot 3868\ 894$	$\cdot 3521\ 2$
$\cdot 70$	$\cdot 5588\ 877$	$\cdot 5209\ 888$	$\cdot 4835\ 356$	$\cdot 4468\ 316$	$\cdot 4111\ 4$
$\cdot 71$	$\cdot 6179\ 415^+$	$\cdot 5815\ 442$	$\cdot 5450\ 613$	$\cdot 5087\ 973$	$\cdot 4730\ 3$
$\cdot 72$	$\cdot 6751\ 313$	$\cdot 6410\ 186$	$\cdot 6063\ 440$	$\cdot 5713\ 920$	$\cdot 5364\ 4$
$\cdot 73$	$\cdot 7292\ 315^+$	$\cdot 6980\ 653$	$\cdot 6659\ 458$	$\cdot 6331\ 196$	$\cdot 5998\ 3$
$\cdot 74$	$\cdot 7791\ 675^-$	$\cdot 7514\ 457$	$\cdot 7224\ 847$	$\cdot 6924\ 810$	$\cdot 6616\ 2$
$\cdot 75$	$\cdot 8240\ 849$	$\cdot 8001\ 134$	$\cdot 7747\ 318$	$\cdot 7480\ 811$	$\cdot 7203\ 1$
$\cdot 76$	$\cdot 8634\ 024$	$\cdot 8432\ 841$	$\cdot 8216\ 984$	$\cdot 7987\ 312$	$\cdot 7744\ 8$

# TABLES OF THE INCOMPLETE $\beta$ -FUNCTION

to 96

$q = 15$

$p = 39$  to

$p = 39$	$p = 40$	$p = 41$	$p = 42$	$p = 43$	$p = 44$
$= .1066\ 6073 \times \frac{1}{10^{13}}$	$.7703\ 2751 \times \frac{1}{10^{14}}$	$.5602\ 3819 \times \frac{1}{10^{14}}$	$.4101\ 7439 \times \frac{1}{10^{14}}$	$.3022\ 3376 \times \frac{1}{10^{14}}$	$.2240\ 6986 \times \frac{1}{10^{14}}$
.0000 001	.0000 001	.0000 001	.0000 001	.0000 001	.0000 001
.0000 002	.0000 002	.0000 002	.0000 002	.0000 002	.0000 002
.0000 003	.0000 003	.0000 003	.0000 003	.0000 003	.0000 003
.0000 007	.0000 004	.0000 002	.0000 001	.0000 001	.0000 001
.0000 015 <sup>+</sup>	.0000 008	.0000 005 <sup>-</sup>	.0000 003	.0000 001	.0000 001
.0000 032	.0000 018	.0000 010	.0000 006	.0000 003	.0000 002
.0000 063	.0000 036	.0000 021	.0000 012	.0000 007	.0000 004
.0000 121	.0000 072	.0000 042	.0000 024	.0000 014	.0000 008
.0000 230	.0000 139	.0000 083	.0000 049	.0000 029	.0000 017
.0000 426	.0000 262	.0000 160	.0000 097	.0000 059	.0000 035 <sup>+</sup>
.0000 770	.0000 484	.0000 302	.0000 188	.0000 116	.0000 071
.0001 364	.0000 875 <sup>-</sup>	.0000 558	.0000 354	.0000 223	.0000 140
.0002 363	.0001 547	.0001 006	.0000 651	.0000 419	.0000 268
.0004 012	.0002 677	.0001 776	.0001 172	.0000 769	.0000 502
.0006 673	.0004 540	.0003 071	.0002 066	.0001 382	.0000 920
.0010 884	.0007 544	.0005 200	.0003 565 <sup>-</sup>	.0002 431	.0001 649
.0017 411	.0012 292	.0008 030	.0006 026	.0004 186	.0002 893
.0027 329	.0019 644	.0014 042	.0009 983	.0007 061	.0004 970
.0042 107	.0030 802	.0022 409	.0016 216	.0011 674	.0008 363
.0063 699	.0047 404	.0035 085 <sup>+</sup>	.0025 831	.0018 921	.0013 792
.0094 642	.0071 623	.0053 911	.0040 367	.0030 074	.0022 296
.0138 139	.0106 270	.0081 317	.0061 902	.0046 888	.0035 345 <sup>-</sup>
.0198 118	.0154 875 <sup>+</sup>	.0120 432	.0093 172	.0071 727	.0054 955 <sup>-</sup>
.0279 254	.0221 746	.0175 164	.0137 672	.0107 678	.0083 822
.0386 916	.0311 968	.0250 247	.0199 742	.0158 665 <sup>-</sup>	.0125 449
.0527 041	.0431 331	.0351 221	.0284 593	.0229 514	.0184 246
.0705 901	.0586 162	.0484 323	.0398 257	.0325 961	.0265 586
.0929 758	.0783 039	.0656 274	.0547 441	.0454 570	.0375 781
.1204 409	.1028 385 <sup>+</sup>	.0873 924	.0739 241	.0622 516	.0521 943
.1534 633	.1327 939	.1143 775 <sup>-</sup>	.0980 722	.0837 235 <sup>+</sup>	.0711 701
.1923 580	.1686 138	.1471 372	.1278 345 <sup>-</sup>	.1105 907	.0952 758
.2372 151	.2105 448	.1860 614	.1637 279	.1434 787	.1252 263
.2878 445 <sup>+</sup>	.2585 713	.2313 033	.2060 637	.1828 433	.1616 048
.3437 353	.3123 611	.2827 124	.2548 716	.2288 869	.2047 754
.4040 379	.3712 302	.3397 839	.3098 333	.2814 801	.2547 951
.4675 763	.4341 361	.4016 322	.3702 383	.3401 001	.3113 356
.5328 947	.4997 069	.4670 003	.4349 717	.4037 973	.3736 306
.5983 389	.5663 081	.5343 093	.5025 446	.4712 035 <sup>-</sup>	.4404 600
.6621 668	.6321 461	.6017 502	.5711 690	.5405 877	.5101 842
.7226 795 <sup>-</sup>	.6954 004	.6674 121	.6388 776	.6099 627	.5808 327
.7783 572	.7543 696	.7294 345 <sup>-</sup>	.7036 784	.6772 354	.6502 451
.8279 839	.8076 153	.7861 672	.7637 252	.7403 854	.7162 533
.8707 428	.8540 827	.8363 147	.8174 850 <sup>+</sup>	.7976 511	.7768 808
.9062 689	.8931 804	.8790 448	.8638 749	.8476 937	.8305 341

TABLE I. THE  $I_x(p, q)$  FUNCTION $x = .42$  to  $.96$  $q = 15$ 

	$p = 45$	$p = 46$	$p = 47$	$p = 48$	$p = 49$
$B(p, q) = .1671 \ 0294 \times \frac{1}{10^{14}}$	$.1253 \ 2721 \times \frac{1}{10^{14}}$	$.9450 \ 9042 \times \frac{1}{10^{13}}$	$.7164 \ 3951 \times \frac{1}{10^{13}}$	$.5458 \ 5111 \times \frac{1}{10^{12}}$	
$x$					
.42	.0000 001				
.43	.0000 002	.0000 001	.0000 001		
.44	.0000 005	.0000 003	.0000 002	.0000 001	
.45	.0000 010	.0000 006	.0000 003	.0000 002	.0000 001
.46	.0000 021	.0000 013	.0000 008	.0000 004	.0000 002
.47	.0000 044	.0000 027	.0000 016	.0000 010	.0000 005
.48	.0000 087	.0000 054	.0000 034	.0000 021	.0000 010
.49	.0000 171	.0000 108	.0000 068	.0000 043	.0000 021
.50	.0000 327	.0000 211	.0000 136	.0000 087	.0000 043
.51	.0000 610	.0000 402	.0000 264	.0000 172	.0000 103
.52	.0001 114	.0000 748	.0000 501	.0000 333	.0000 203
.53	.0001 990	.0001 362	.0000 928	.0000 630	.0000 393
.54	.0003 481	.0002 427	.0001 684	.0001 164	.0000 719
.55	.0005 962	.0004 231	.0002 989	.0002 103	.0001 410
.56	.0010 005 <sup>+</sup>	.0007 225 <sup>+</sup>	.0005 195 <sup>-</sup>	.0003 719	.0002 203
.57	.0016 453	.0012 085 <sup>+</sup>	.0008 839	.0006 437	.0004 103
.58	.0026 519	.0019 808	.0014 731	.0010 909	.0008 005
.59	.0041 911	.0031 820	.0024 055 <sup>+</sup>	.0018 109	.0013 410
.60	.0064 955 <sup>-</sup>	.0050 112	.0038 496	.0029 450 <sup>+</sup>	.0022 203
.61	.0098 741	.0077 380	.0060 385 <sup>-</sup>	.0046 929	.0036 410
.62	.0147 250 <sup>+</sup>	.0117 177	.0092 857	.0073 287	.0057 103
.63	.0215 447	.0174 033	.0140 001	.0112 174	.0089 005
.64	.0309 312	.0253 537	.0206 977	.0168 302	.0136 410
.65	.0435 770	.0362 331	.0300 068	.0247 541	.0203 005
.66	.0602 487	.0507 980	.0426 621	.0350 930	.0297 410
.67	.0817 499	.0698 682	.0594 846	.0504 554	.0426 005
.68	.1088 654	.0942 788	.0813 411	.0699 230	.0598 410
.69	.1422 874	.1248 115 <sup>+</sup>	.1090 833	.0949 981	.0824 005
.70	.1825 272	.1621 082	.1434 648	.1205 270	.1012 410
.71	.2298 203	.2065 716	.1850 413	.1652 013	.1470 005
.72	.2840 355 <sup>-</sup>	.2582 642	.2340 616	.2114 450 <sup>+</sup>	.1904 410
.73	.3446 020	.3168 184	.2903 636	.2652 990	.2416 005
.74	.4104 712	.3813 753	.3532 915 <sup>+</sup>	.3263 191	.3005 410
.75	.4801 263	.4505 693	.4216 548	.3935 094	.3662 005
.76	.5516 496	.5225 703	.4937 439	.4653 100	.4373 410
.77	.6228 500 <sup>-</sup>	.5951 931	.5674 159	.5396 565 <sup>+</sup>	.5120 005
.78	.6914 410	.6660 663	.6402 503	.6141 157	.5877 410
.79	.7552 513	.7328 480	.7097 629	.6860 933	.6619 005
.80	.8124 385 <sup>-</sup>	.7934 582	.7736 527	.7530 886	.7318 410
.81	.8616 721	.8462 929	.8300 444	.8129 628	.7950 005
.82	.9022 552	.8890 835 <sup>-</sup>	.8776 859	.8641 724	.8498 410
.83	.9341 606	.9254 699	.9160 614	.9059 262	.8950 005
.84	.9579 739	.9519 736	.9453 994	.9382 321	.9304 410
.85	.9747 554	.9708 737	.9665 700	.9618 223	.9566 005
.86	.9858 479	.9835 137	.9808 952	.9779 725 <sup>+</sup>	.9747 410
.87				.9822 567	.9802 005
.88				.9882 567	.9862 410
.89				.9948 567	.9928 005
.90				.9998 567	.9978 410
.91					.9998 005
.92					.9998 410
.93					.9998 005
.94					.9998 410
.95					.9998 005
.96					.9998 410